Current-Density Approximation for Efficient Computation of the Electrostatic Field in Wire-Plate Precipitators

François Beux, Angelo Iollo, Maria-Vittoria Salvetti, and Alfredo Soldati

***Abstract*—In this paper, numerical computation of ionic space charge and electric field produced by corona discharge in an electrostatic precipitator is addressed. The problem is defined by a reduced set of the Maxwell equations. The efficiency of numerical iterative computations is significantly improved by deriving an initial field as close as possible to the final solution from an approximation of the current-density field J. Different techniques to approximate J are proposed. A first analytic approximation $\tilde{J}$ is derived, which verifies by construction the boundary conditions of the problem and, in particular, gives the correct average value at the plate. A second approximation is also considered, which contains a free parameter that can be computed by an optimization procedure based on the known value of the potential at the wire. Finally, Karhunen–Loève (KL) decomposition is used and the current-density field is expressed as the sum of $\tilde{J}$ and of a linear combination of few KL basis functions. The coefficients can be determined again by an optimization algorithm. Starting from these approximated J fields, a procedure is proposed to obtain, at negligible computational cost, an estimate of the complete electrostatic field. It is shown that this estimate is in all cases much closer to the exact solution than guesses typically employed in the literature. Hence, when it is used as initialization for standard numerical solvers, this significantly improves the efficiency of the numerical algorithm. In particular, the initialization based on the second approximation gives a significant efficiency gain without any noticeable additional cost.**

***Index Terms*—Efficient computation, initial field from current-density approximation, reduced Maxwell equations, wire-plate precipitators.**

I. Introduction

ELECTROSTATIC precipitators (ESPs) are used to collect airborne particles from process or waste dust-laden gases. The most widespread configuration for industrial use is the wire plate. In this configuration, the gas flows through grounded parallel plates in the middle of which wire electrodes are kept at a voltage high enough to ensure corona discharge. The ions discharged at the wires charge the dust particles, which are thus driven toward the collecting plates. A further effect of the discharged ions is to release momentum to the fluid and generate flows of electrohydrodynamic (EHD) origin [1], [2]. In turn, EHD flows influence particle collection and the overall pressure drop through the duct. Numerical computation is certainly a useful tool in ESP design. As an example, in our previous work, we considered one specific geometry and few operating conditions for the ESP, by solving the reduced Maxwell equations for the electrostatic field coupled with both particle dynamics and Navier–Stokes equations, and we found that the distribution of the electrostatic field might have an optimum for particle collection [3] and for drag reduction [2]. However, the computational effort required by those calculations is important, and, seriously limiting the number of configurations which can be analyzed in practice. Thus, efficient numerical strategies should be devised to reduce the cost of each part of the simulations (electrostatics, fluid dynamics and particle dynamics). Specifically, a strategy to reduce the cost of numerical computation of the electrostatic problem is proposed here.

In the past, several numerical methods have been applied to solve accurately both the elliptic Poisson equation for potential and the charge continuity equation, namely, finite elements, finite volumes, finite differences, the method of characteristics, the charge simulation method, or some hybrid or derivative of these (see, e.g., [4] for a review). All these solvers use an iterative procedure to obtain the self-consistent solution: starting from an initial guess of the charge density and electric potential fields, ρ and V, the Poisson equation for potential and the charge continuity equation are numerically solved, adjusting the value of V (as, for instance, in [5]–[7]) or ρ at the wire ([8]–[12]). This leads to quite time-consuming calculations. Different numerical techniques, such as multigrid [11] or implicit schemes [13], have been used in the literature to accelerate the convergence of the iterative procedure.

In this paper, we restrict to the case of uniform discharge along the wires, which leads to a two-dimensional electrostatic problem. Our aim is to improve the efficiency of numerical iterative computations of coupled space charge and electric field by deriving an initial field as close as possible to the final solution. This initialization is done working on the current density field J. More precisely, an original procedure is proposed based on three different approximations of the current-density field and the reconstruction of the entire electrostatic fields from a given J.

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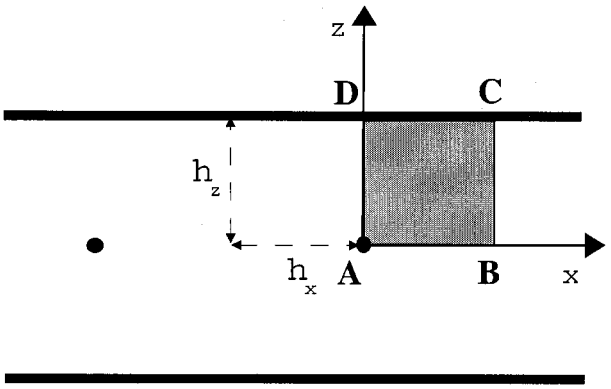


Fig. 1. Computational domain.

II. Problem Description and Numerical Solver

We consider a wire-duct ESP configuration, i.e., a series of equidistant wires at high voltage placed in the middle plane between two parallel grounded plates. The electric phenomenon is characterized by ion emission from the corona around the wire to the plates, which forms a nonuniform electric field. Under certain conditions [14] and specifically under conditions of positive corona, the ion discharge may be assumed to be uniform along the wire and the electrostatic problem can be solved in two dimensions. Neglecting the magnetic effects and considering the steady state, the governing electrostatic equations can be expressed as follows:

$$\Delta V = -\frac{\rho}{\epsilon_0} \tag{1}$$

$$\rho^2 = \epsilon_0 \nabla\rho\nabla V \tag{2}$$

in which ρ is the space-charge density, V is the electrical potential, and ϵ_0 the permittivity of the gas. The electric field and the current density are obtained from V and ρ as follows: $E = -\nabla V$ and $J = \rho\beta E$ where β is the ionic mobility.

Due to symmetry considerations, the computational domain can be reduced to the rectangle ABCD of dimensions $h_x \times h_z$ shown in Fig. 1, with the Neumann boundary conditions $E_z = 0$ along AB and $E_x = 0$ along BC and DA. To close the previous system of equations, Dirichlet conditions are also applied by setting to zero the potential at the grounded plate, i.e., along CD, and by imposing both potential and charge density at the wire (point A). A desired average current density J_p is specified at the plate from V_0, the wire potential value, and a, the wire radius, using a current–voltage formula defined in [15]. More precisely, the following equation is numerically solved by means of a Newton algorithm:

$$J_p\left(V_0 - V_0^S + h_z E_1\right) - \frac{\beta\epsilon_0}{3}\left[\left((E_1)^2 + \frac{2h_z}{\beta\epsilon_0}J_p\right)^{3/2} - (E_1)^3\right] = 0 \tag{3}$$

in which V_0^S is the corona starting voltage, i.e., V_0 corresponding to $J_p = 0$, and E_1, defined by $E_1 = \pi a/(2h_x)E_c$, is an average electric field. E_c is the strength of the idealized corona onset field as given by the semi-empirical Peek formula, modified for wire-plate precipitators (see, e.g., [16]).

The wire charge density ρ_0 is not known *a priori* and, thus, an iterative algorithm is needed to numerically solve the problem. An algorithm close to the one proposed in [8] is adopted here. An initial estimate of ρ_0 is obtained by using the wire condition $\rho_0 = J_p/(\beta E_1)$ [5]. Hence, starting from an initial guess of ρ and V fields, (2) can be solved, and then, using the updated ρ field, (1) is discretized to obtain a new potential field. The value of ρ_0 is adjusted and the computation of ρ and V fields is repeated until the calculated average current density at the plate is close enough to the desired one J_p.

As for the numerical discretization of (1) and (2), the present approach is based on a finite-difference (FD) method close to the one defined in [5], but here a constant value of β and a nonuniform grid system are used. This method is certainly not the most efficient among those proposed in the literature; however, tests on grid independence and comparisons with experimental and numerical data have shown that accurate solutions can be obtained [17]. On the basis of those tests, all the computations presented here are carried out on a 23×34 nonuniform grid, with points clustered near the wire (point A) in both directions.

III. Procedure for the Derivation of an Initial Electrostatic Field

As discussed previously, the computation of the electrostatic variables in a wire-duct ESP requires initialization of charge density and electric potential fields, and thus, a first *a priori* estimation of these two fields. Usually, a uniform distribution of the space charge is assumed and a Laplace equation is solved in turn for the potential. The analytic solution of this problem is known as Cooperman's formula (it can be found, for instance, in [18]). However, this initialization furnishes V, ρ, E, and J fields rather far from the exact solution. We propose here to improve the initialization in order to increase the computational efficiency of the reduced Maxwell solver. This is done working on the current-density field J. Three different techniques to approximate J are considered. First, an analytic approximation $\tilde{J}$ is obtained, which verifies by construction the boundary conditions of the problem, and, in particular, gives the correct average value at the plate. A second approximation is also derived, which contains a free parameter that can be computed by an optimization procedure based on the known value of the potential at the wire. Finally, a parameterization of the homogeneous $J - \tilde{J}$ field is defined through a Karhunen–Loève (KL) decomposition [19]. Following this technique and using the good properties of $\tilde{J}$ at the boundaries, J can be expressed as the sum of $\tilde{J}$ and of a linear combination of few KL functions, which can be obtained from a set of snapshots, i.e., from some J fields calculated for different configurations (for instance, different values of the potential at the wire). The unknown coefficients of the KL modes are computed by an optimization procedure again based on the assigned value of the potential at the wire.

Then, starting from the approximated J field, a procedure is proposed, which permits us to obtain an estimate of the complete electrostatic field at a very low computational cost. A flowchart of the entire procedure is shown in Fig. 2.

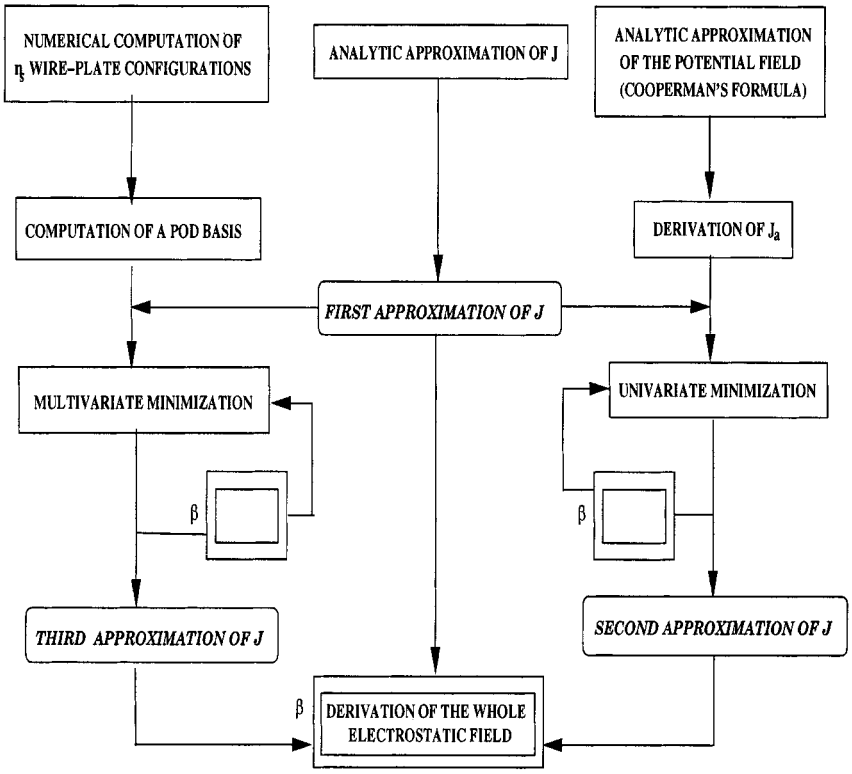


Fig. 2. Flowchart of the different proposed procedures for the initialization of the electrostatic field. The box β in the figure corresponds to the procedure described in Section III-A.

Although the derivation of the electrostatic field from a given J is the last step of the proposed initialization process, it will be described first (Section III-A), because parts of it are used in two of the techniques for approximation of J (Sections III-C and D).

A. Electrostatic Field Construction From a Given Current-Density Field

We search here to define potential, electric, and charge-density fields using the knowledge of a current-density field J_{ap} and of the parameters J_p, a, and V_0. As a first step, the charge density can be obtained by solving the following equation, which can easily be derived from (2):

$$\frac{\beta}{\epsilon_0}\rho^3 + \nabla\rho J = 0. \tag{4}$$

Equation (4) is discretized similarly to (2), i.e., by using an FD approach. The following expression for the charge-density field is, thus, obtained:

$$\rho_{i,k} = \left(-\frac{\mathcal{B}_{i,k}}{2} + \sqrt{\mathcal{C}_{i,k}}\right)^{1/3} - \left(\frac{\mathcal{B}_{i,k}}{2} + \sqrt{\mathcal{C}_{i,k}}\right)^{1/3} \tag{5}$$

in which

$$\begin{cases} \mathcal{A}_{i,k} = \frac{\epsilon_0}{\beta}\left(\frac{J_x^{i,k}}{\Delta_x^i} + \frac{J_z^{i,k}}{\Delta_z^k}\right) \\ \Delta_z^k = z_k - z_{k-1}, \quad \Delta_x^i = x_i - x_{i-1} \\ \mathcal{B}_{i,k} = -\frac{\epsilon_0}{\beta}\left(\frac{J_x^{i,k}}{\Delta_x^i}\rho_{i-1,k} + \frac{J_z^{i,k}}{\Delta_z^k}\rho_{i,k-1}\right) \\ \mathcal{C}_{i,k} = \left(\frac{\mathcal{A}_{i,k}}{3}\right)^3 + \left(\frac{\mathcal{B}_{i,k}}{2}\right)^2. \end{cases}$$

For a given J_{ap}, the charge-density field is totally determined by (5) and by the charge-density wire condition, i.e., $\rho_{1,1} = J_p/(\beta E_1)$.

From the knowledge of ρ and J, the electric field is immediately derived from $E = J/(\beta\rho)$ and, thus, the potential V can also be obtained from $E = -\nabla V$.

A first way to reconstruct V is to use an upwind first-order FD scheme: $V_{i,k-1} = V_{i,k} + \Delta_z^k E_z^{i,k}$. This appears a natural way in our formulation because it is exactly the inverse procedure of that used in the reduced Maxwell solver to obtain E_z from V. Remembering that the potential is equal to zero at the plate, $V_{i,k}$ can simply be expressed as follows:

$$V_{i,k} = \sum_{s=k+1}^{n_k} \Delta_z^s E_z^{i,s}. \tag{6}$$

One possible drawback of this construction is that information from only the second component of the electric field has been used. Moreover, in this way we have an accumulation of approximation errors moving away from the plate, i.e., when k decreases (see also [17]).

As an alternative approach, the E_x field can be used to relate the potential evaluated in two points in the x direction. This is done by integrating with respect to x between x_j and x_i and, using a trapezium formula, the following reconstruction is found:

$$V_{i,k} = V_{j,k} - \frac{1}{2}\sum_{q=j}^{i-1}\left(E_x^{q,k} + E_x^{q+1,k}\right)\Delta_x^{q+1}. \tag{7}$$

However, in this case a Dirichlet boundary condition is not available for V in the x direction to initialize the procedure. Nevertheless, a mixed approach using both E_z and E_x can be envisaged, using (6) in the left part of the domain and (7) in the right part. More precisely, if i_0 is an i index larger than one, for i smaller or equal to i_0, (6) is used, whereas for $i > i_0$, (7) is employed with $j = i_0$.

B. Analytic Approximation of the Current-Density Field

A suitable approximation of the current-density field should satisfy the following governing equation for J, which represents the continuity of steady currents:

$$\nabla \cdot J = 0. \tag{8}$$

The boundary conditions for the considered problem are (see Fig. 1): $J_z = 0$ on AB, $J_x = 0$ on BC, CD, and DA, and $J_r = 2J_p h_x/(\pi a)$ at the wire, where J_r is the radial component of the current density vector. However, it is clear that (8) and these boundary conditions are not sufficient to uniquely determine J.

Nevertheless, we wish to obtain here an approximation, $\tilde{J}$, of the actual current-density field, which satisfies (8) and the boundary conditions above. In order to be able to obtain an analytic expression of $\tilde{J}$, the following assumption is made:

$$\nabla \times \tilde{J} = 0. \tag{9}$$

This implies that a function Φ exists such that $\tilde{J} = \nabla\Phi$; Φ can be determined from a Laplace equation with Neumann boundary conditions derived from those for J. This problem now has a unique solution that can be obtained by classical tools used for hydrodynamic problems, such as singularity distributions and elliptic functions. In particular, using the reflection technique, it can be shown that the Laplace equation for Φ and the corresponding boundary conditions are satisfied by an infinite sequence of sources of intensity $4J_p h_x$ in the x direction located at $(\pm 2kh_x, 0)$, $k = 1, \ldots, \infty$, and by an infinite row of alternating sources and sinks of intensity $4J_p h_x$ along z at $(0, \pm 2jh_z)$, $j = 0, \ldots, \infty$ (even js correspond to sources, while odd js to sinks). Finally, $\tilde{J}_x$ and $\tilde{J}_z$ can be obtained as the real and the imaginary part of the following complex function:

$$\Psi(x,z) = 4J_p h_x \left(\zeta(x + \iota z) - \zeta(x + \iota z - 2h_z)\right) \tag{10}$$

in which ζ denotes a Weierstrass ζ function (see, for instance, [20]) and $\iota = \sqrt{-1}$.

Since assumption (9) is not, in general, satisfied by the exact solution of (1) and (2), the $\tilde{J}$ field obtained in this way can be considered only as an approximation of the exact current-density field. It can be easily verified, however, that $\tilde{J}$ respects the boundary condition of the original nonlinear problem. In particular, we have the correct average value J_p at the plates.

However, $\tilde{J}$ only depends on J_p and, thus, the wire potential V_0 has not been used explicitly in the construction of the initial estimated electrostatic fields. Thus, the obtained initial field may be characterized by a value of V_0 significantly different from the exact one. To solve this problem, a second approximate J field is considered in Section III-C.

C. Semi-Analytic Approximation of the Current-Density Field

Remembering that V_a, the potential field defined by the Cooperman formula [18], is obtained by solving a Laplace equation, the corresponding electric field $E_a = -\nabla V_a$ is divergence free. Thus, the current-density field $J_a = \beta\rho_0 E_a$ satisfies the governing (8). Moreover, the boundary conditions $J_z = 0$ on AB and $J_x = 0$ on BC, CD and DA, are satisfied by J_a. The field J_a is used here to perturb $\tilde{J}$ in order to find a corresponding potential field which has a wire value close to V_0. More precisely, we consider $J^* = \tilde{J} + \alpha J_a$, in which the parameter α is chosen in order to minimize the function $I(\alpha) = (V_{1,1}(\alpha) - V_0)^2/(2V_0^2)$, $V_{1,1}(\alpha)$ being the potential at the wire obtained from J^*. To estimate $V_{1,1}$ from J^*, we apply the procedure previously described in Section III-A, [(5) and (6)], with $J_{ap} = J^*$. More precisely, from (6), $V_{1,1}$ can be expressed as $V_{1,1}(\alpha) = \sum_{k=2}^{n_k} \Delta_z^k \left(\tilde{J}_z^{1,k} + \alpha (J_a)_z^{1,k}\right)/(\beta\rho_{1,k}(\alpha))$, in which $\rho_{1,k}(\alpha)$ is defined by (5). A first estimation of α is obtained by considering the charge density field independent of α, i.e., by taking

$$\bar{I}(\alpha) = \frac{\left(\sum_{k=2}^{n_k} \frac{\Delta_z^k}{\beta\rho_{1,k}} (J^*)_z^{1,k}(\alpha) - V_0\right)^2}{2V_0^2}. \tag{11}$$

$\bar{I}$ is simply a second-order polynomial function in α and the minimum value is obtained for the following value of α:

$$\alpha_0 = -\left(\sum_{k=2}^{n_k} \Delta_z^k \frac{(J_a)_z^{1,k}}{\beta\rho_{1,k}}\right)^{-1} \left(\sum_{k=2}^{n_k} \Delta_z^k \frac{\tilde{J}_z^{1,k}}{\beta\rho_{1,k}} - V_0\right). \tag{12}$$

Then, starting from α_0, an optimum α value is obtained applying a classical univariate minimization on the function $I(\alpha)$.

Note that, for $\alpha \neq 0$, J^* does not give the correct average value at the plate, because the average value of J_a at the plate is, in general, nonequal to zero. However, it will be shown in the following section that it is preferable to have an accurate guess of the potential at the wire, since this permits us to obtain a larger increase in the computation efficiency.

D. KL Decomposition

In this section, we try to obtain an approximation of the J field involving different unknown parameters; they are then computed by an optimization procedure, which is a generalization of that devised to obtain J^*. To this aim, we consider the KL decomposition, classically used in probability theory [19], which has been also utilized in fluid dynamics as a reduced-order model. In that context, it is usually called proper orthogonal decomposition (see e.g., [21] and [22]) and is employed to describe the dynamical behavior of the flow by considering only a finite number of modes. This technique is employed here for the parameterization of the homogeneous part of the current-density field, i.e., $J - \tilde{J}$.

For a fixed configuration of wire-plate precipitator, i.e., for given h_x and h_z, the current density depends on the particular space position $X = (x, z)$ and on the particular electric configuration $T = (V_0, a)$ (J_p can be obtained from T using a current–voltage formula [15]). Thus, starting from

a set of n_s discrete J fields obtained for different T configurations, a KL decomposition can be defined which gives the current-density field as a function of T. More precisely, from a set of n_s snapshots $(J_T)^l$, the KL technique generates n_s modes $\varphi_j = \sum_l \alpha_{lj}\left(J_T^l - \tilde{J}_T^l\right)$ which give an optimal representation of $\left(J_T^l - \tilde{J}_T^l\right)$ in the L^2 norm, as explained, for example, in [22]. The coefficients α_{lj} are obtained as the components of the jth eigenvector of the correlation matrix $\mathcal{K}\left(\mathcal{K}_{lj} = \langle J_T^l - \tilde{J}_T^l, J_T^j - \tilde{J}_T^j\rangle/n_s\right)$, in which $\langle\cdot\rangle$ denotes the inner product in L^2. Then, using n_m of n_s KL modes, an approximation J^R of J field is derived as follows:

$$J^R(x,z) = \tilde{J}(x,z) + \sum_{l=1}^{n_m} c_l \varphi_l(x,z). \tag{13}$$

For the problem under consideration, it has been shown by *a priori* tests [17] that only three or four modes are typically needed to obtain an accurate representation of J.

The coefficients c_l are determined here by an optimization algorithm aimed to obtain a wire potential value close to V_0, i.e., as previously, by minimizing the function $I = (V_{1,1} - V_0)^2/\left(2V_0^2\right)$. Thus, starting with all the coefficients equal to zero, i.e., J^R equal to $\tilde{J}$, the following iterative procedure is carried out:

- computation of I: charge-density and potential fields are obtained using (5) and (6);
- updating of the mode coefficients using a gradient descent method

$$l = 1, \ldots, n_m \quad c_l^{p+1} = c_l^p - \omega \frac{\partial I^p}{\partial c_l} \tag{14}$$

in which the derivatives of I can be explicitly obtained by using the expression of $V_{1,1}$ from (6) and introducing the J field and its parameterization, i.e., $J_z^{1,k} = \tilde{J}_z^{1,k} + \sum_{l=1}^{n_m} c_l \left(\varphi_z\right)_l^{1,k}$ [17].

IV. Results and Discussion

We present now some numerical examples to evaluate the accuracy of the initial fields reconstructed as described previously starting from the different approximations of J. Three configurations have been considered for a classical wire-plate precipitator geometry ($h_z = 0.1143$ m and $h_x = 0.0762$ m). The first one, referred to as configuration $\mathcal{A}$, is characterized by $a = 0.152$ mm, $J_p = 3.77\times10^{-4}$ A/m^2, $\beta = 1.9\times10^{-4}$ m^2/Vs, and $V_0 = 25.415$ kV. This configuration has been studied numerically (e.g., [6]) as well as experimentally [23]. The following parameter values characterize configuration $\mathcal{B}$: $a = 0.85$ mm, $J_p \simeq 1.33 \times 10^{-3}$ A/m^2, $\beta = 1.6 \times 10^{-4}$ m^2/Vs and $V_0 = 55$ kV. Configuration $\mathcal{C}$ is defined by $a = 0.3$ mm, $J_p \simeq 5.94 \times 10^{-4}$ A/m^2, $\beta = 1.6 \times 10^{-4}$ m^2/Vs, and $V_0 = 35$ kV. The data base of snapshots for the KL decomposition has been constructed by considering ten different configurations, which are defined by varying the wire radius between 0.1–1.016 mm, the plate current density between 5×10^{-5}–1.557×10^{-3} A/m^2 and the ionic mobility between 1.6×10^{-4}–1.9×10^{-4} m^2/V·s. In all cases, the snapshots are obtained numerically using the algorithm presented in Section II and six KL modes have been used to construct J^R from (13).

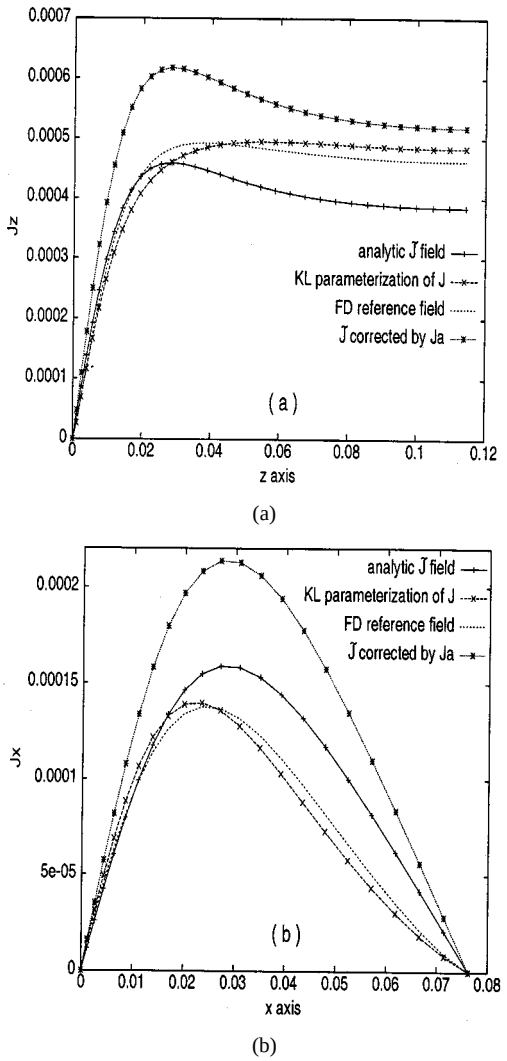
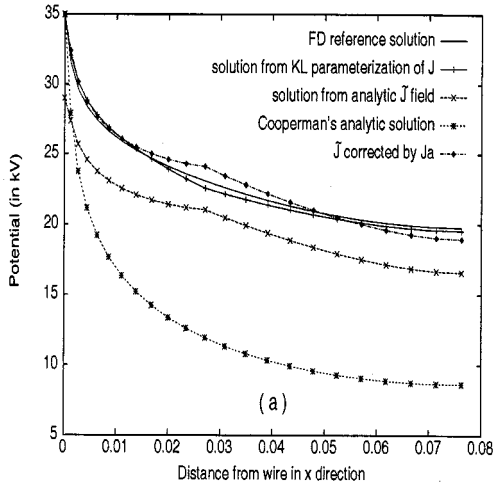


Fig. 3. Comparison of current density fields (configuration $\mathcal{A}$). (a) J_z at $x = 0.0234$ m. (b) J_x at $z = 0.0387$ m.

In Fig. 3(a) and (b), for configuration $\mathcal{A}$, the J_z and J_x components of the current-density field, obtained with the three different approximations described previously, are compared with the reference ones. The reference fields are, here, the fully converged solutions of the reduced Maxwell problem computed by the FD solver. It can be seen that the estimate obtained by KL decomposition is rather close to the FD solution, while more significant differences are observed for the approximations $\tilde{J}$ and J^*. However, all those approximations lead to estimates of the potential field much closer to the FD ones than guesses based on the Cooperman analytic formula, which are classically used as initialization for numerical computations. This is shown, for instance, in Fig. 4(a) and (b), in which the potential along lines AB and AD is plotted for configuration $\mathcal{C}$. The best agreement is again obtained by the KL-based approximation; however, the potential field given by J^* is significantly more accurate than that derived from $\tilde{J}$. In particular, note that, as described previously, $\tilde{J}$ does not give the correct potential value at the wire. Similar considerations can be done for the charge-density field.



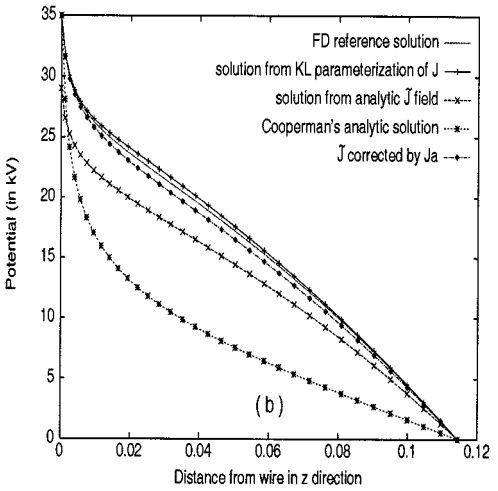
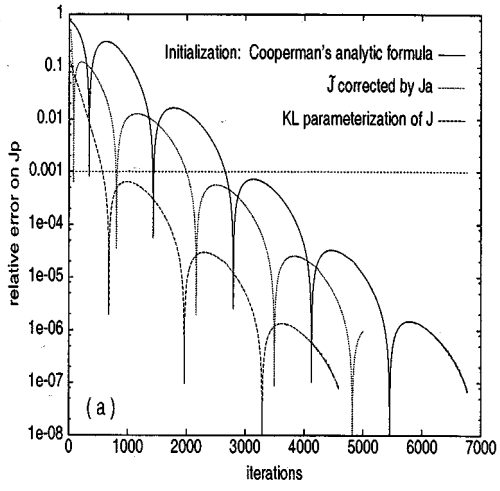
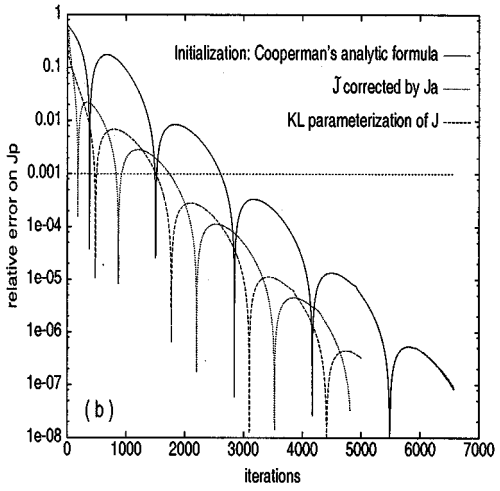


Fig. 4. Comparisons of electric potential fields (configuration $\mathcal{C}$). (a) Potential along AB. (b) Potential along AD.

These estimates of the V and ρ fields can be used as initialization for standard algorithms of solution of the reduced Maxwell problem, such as the solver described in Section II. In Fig. 5(a)–(c), the convergence histories obtained with our approach are compared to those given by the classical initialization [18] for all three considered configurations. As expected on the basis of previous considerations, the initialization based on J^R gives the largest reduction in the number of iterations required to reach the convergence of the numerical solution. Indeed, if we consider the number of iterations needed to obtain a residual of 0.001, i.e., an agreement between computed and specified values of J_p within 0.1%, this is reduced by 80%, 40%, and 85% for configurations $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ respectively. Note that the gain in convergence rate is obtained mostly in the initial iterations, and this is due to the fact that the initialization fields are closer to the exact ones, as shown in Fig. 4. Fig. 6, in which the evolution of ρ_0 is plotted for configuration $\mathcal{C}$, also illustrates the convergence improvement in the early iterations. Indeed, even if the initial estimate of ρ_0 is close to the converged one, a large variation in the first 1000/2000 iterations is observed for the Cooperman initialization. This is reduced using $\tilde{J}$ and almost





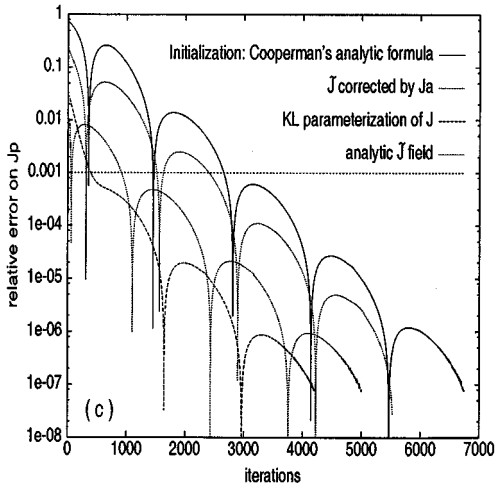


Fig. 5. Convergence histories for different initializations. Plot of $|J_p - J_p^d|/J_p$ (J_p^d: desired J_p). Configurations (a) $\mathcal{A}$, (b) $\mathcal{B}$, and (c) $\mathcal{C}$.

disappears with J^R and J^*. Indeed, J^* also gives a significant reduction in the number of iterations needed to reach convergence, up to 65% for configuration $\mathcal{C}$. Conversely, if only $\tilde{J}$ is used, a more limited gain is obtained: the best case is shown in Fig. 5 for configuration $\mathcal{C}$ with an improvement of only 10%.

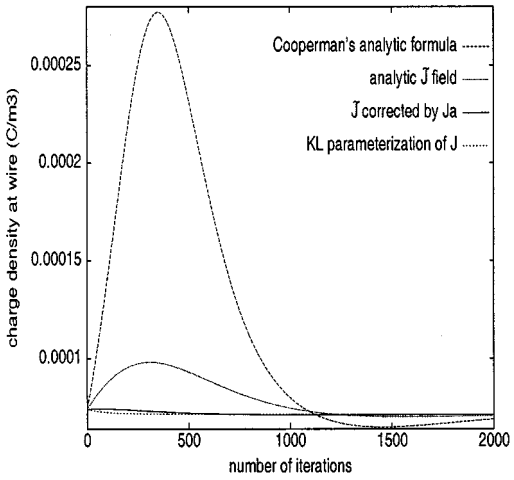


Fig. 6. Evolution of charge density at the wire (configuration $\mathcal{C}$).

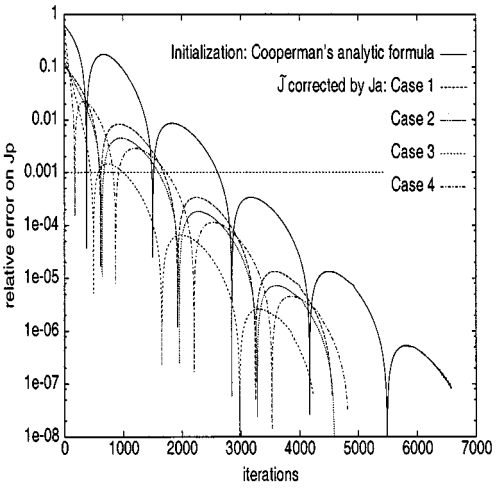


Fig. 7. Convergence histories for different values of x_{i_0} (configuration $\mathcal{B}$). Case 1: $x_{i_0} \simeq 0.35h_x$; case 2: $x_{i_0} \simeq 0.46h_x$; case 3: $x_{i_0} \simeq 0.57h_x$; case 4: $x_{i_0} \simeq 0.69h_x$.

Note that the whole initialization procedure, i.e., both J approximation and derivation of V and ρ fields (see Fig. 2), requires approximately 0.1–0.2 s CPU on a PC, while the numerical solution of the reduced Maxwell equations by the FD solver takes typically about 1 h CPU. Thus, the gain in efficiency obtained with the procedures based both on $\tilde{J}$ and J^* is not decreased by the cost of the computation of these electrostatic field initializations. When the KL decomposition is used to approximate J, since preliminary computations must be carried out in order to construct the snapshot basis, the proposed procedure remains interesting, only when several different configurations have to be studied.

The performance of the present algorithm depends also on the choice of i_0 in the construction of the initial potential field (see Section III-A). For this precipitator geometry and this computational grid, the best results are obtained taking x_{i_0} between approximatively $h_x/3$ and $2h_x/3$, i.e., for i_0 between $n_i/2$ and $3n_i/4$, respectively. In particular, the results, previously shown, have been obtained with $i_0 = 3n_i/4$ for configurations $\mathcal{A}$ and $\mathcal{B}$ and $i_0 = n_i/2$ for configuration $\mathcal{C}$. A systematic sensitivity analysis to the i_0 parameter has been carried out for the configuration $\mathcal{B}$, for the initialization based on J^*. For a sampling of 11 different values of x_{i_0} varying between 0.11 h_x and 0.81 h_x, efficiency gains of at least 27% have been found. As shown in Fig. 7, the gain is between 35%–45% for x_{i_0} between $h_x/3$ and $2h_x/3$, except for the "optimum" value of $x_{i_0} \simeq 0.57h_x$, for which a gain of 62% is obtained.

V. Conclusions

A procedure has been proposed to derive estimates of the global electrostatic field (charge density and potential) in wire-duct precipitators, starting from an approximation of the current-density field J.

Three different approximations have been devised: the first approximation supplies $\tilde{J}$ and is obtained analytically from the supplementary assumption of irrotationality of J; $\tilde{J}$ satisfies by construction the boundary conditions of the problem. A second approximation, J^*, is considered which contains a free parameter. This is determined by an optimization procedure aimed to obtain a potential value at the wire as close as possible to the prescribed one. Finally, the current density field J is parameterized by KL decomposition and, in particular, J is approximated as the sum of $\tilde{J}$ and of a linear combination of few KL modes. The unknown coefficients are determined by an optimization algorithm, again based on the prescribed value of the wire potential.

It has been shown that, for all the considered approximations of J, the estimates of the global electrostatic field obtained by the proposed procedure are much closer to the exact solutions than those typically used in the literature to initialize numerical computations. In particular, when J is derived from its KL decomposition, an accurate estimate is obtained, which in many cases could already be considered as a satisfactory approximation of the desired solution. In all cases, those estimates can be used to initialize standard numerical algorithms for the discretization of the reduced Maxwell equations. It has been shown that, for all the considered approximations of J, the efficiency of the numerical algorithm is improved compared to the case of initialization by the Cooperman analytic formula, typically used in this type of calculations. This is not noticeably decreased by the cost of the computation of the initial V and ρ fields, which is negligible compared to the cost of one iteration in the solution of the reduced Maxwell problem. The initialization based on J^* appears to be particularly well suited. Indeed, it improves significantly the efficiency of the numerical solver, with a reduction in the number of iterations needed to reach a residual of 0.001 up to 65%, and, at the same time, it can be obtained at a negligible cost. The gain in efficiency is further increased when the KL-based approximation of J is considered. However, when the KL decomposition is used to approximate J, the construction of a snapshot basis is needed and, thus, preliminary computations must be carried out, which imply the solution of the reduced Maxwell problem. Nevertheless, as shown in [17], a limited number of snapshots and, thus, of preliminary simulations, is sufficient to obtain an accurate approximation. Thus, the proposed procedure remains interesting when many different configurations must be computed, as in optimization or control applications.

The procedure proposed to estimate the ρ and V fields, starting from a given J field, is strictly linked to the numerical method used to discretize the problem. We have adopted here

an FD approach similar to that employed in the numerical solution of the reduced Maxwell equations. Nevertheless, the adaptation of the whole procedure, i.e., approximations of J and global field reconstruction, to other numerical methods is straightforward. The use of a more suitable discretization method, in which the corona boundary conditions are well imposed, might probably improve substantially the optimization algorithm. However, in that case, the explicit extraction of an exact gradient could be more critical. Nevertheless, thanks to the low cost of the algorithm, an approximate gradient computation by divided FDs can be envisaged without particular drawbacks.

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