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Identification of two phase flow regimes via diffusional analysis of experimental time series

A. Soldati, A. Paglianti, M. Giona

Abstract The problem of identifying different two phase flow regimes from experimental time series by employing the method of diffusional analysis is addressed. This technique, recently applied to the multiphase flow field, is described and compared with other techniques used to characterize multiphase flow regimes. Diffusional analysis is applied to experimental time-series obtained from both a γ-densitometer and capacitance probes. The choice of the appropriate experimental signal to be processed is also discussed. The experimental time series were obtained from a rig with air and light oil. The results obtained confirm the advantages of the method proposed in identifying the features of different flow regimes. The advantages are particularly evident when comparing diffusional analysis with the widely applied Rescaled Range technique.

1 Introduction

The positive assessment of the actual flow regime occurring when gas and liquid flow together in a pipeline is a challenging problem in multiphase fluid dynamics. A number of different flow patterns, with very different fluid dynamics features, may arise depending on the inclination of the pipe and on the specific mass flow rate of each phase. Considering a gas and a liquid flowing concurrently into a horizontal pipe, the two phases may flow separated (stratified, and annular flow), intermittent (elongated bubbles and slug flow) or dispersed (mist and dispersed bubbles flow). Since different flow patterns correspond to different flow features, the correct determination of all transport characteristics is bound to depend on the unambiguous identification of the flow regime.

Customary, the identification of flow regimes is performed by visual observation (Mandhane et al. 1974) and results are reported in the form of flow regime maps (Mandhane et al. 1974; Weisman et al. 1979). In these maps, the coordinates are the phase superficial velocities, j_l and j_g. Transition lines are drawn by means of visual observation or by examination of pressure drop signals (Weisman et al. 1979). Physically, the features of a transition region between two regimes are not unique and do not belong to either of the two. The use of standard statistical methods to identify transition lines leads to lack of objectivity in the determination of the boundaries.

Several researchers attempted to obtain a more reliable tool to identify two phase flow regimes. For instance, wall pressure fluctuations were used by Tutu (1982) and Matsui (1984), while Vince and Lahey (1982) used void fraction fluctuations. Lin and Hanratty (1986) presented an analysis of pressure drop signals suitable to distinguish slug flow from large amplitude waves in stratified flow. However, it appears that conventional analysis of experimental data is not capable to discriminate between different flow regimes. Subsequently, Franca et al. (1991) attempted to develop a new analytical technique for identification and classification of flow regimes. They processed pressure drop time series relative to different flow regimes using the method proposed by Grassberger and Procaccia (1983a, b) to estimate the correlation dimension, and evaluated the Hurst exponent by means of the Rescaled Range (R/S) analysis. The estimation of the Hurst exponent (Hurst 1951; Mandelbrot and Van Ness 1968; Mandelbrot and Wallis 1969) is a means of obtaining information about *anomalous characteristics* presented by the experimental time series, such as, for instance, long term correlations. Rescaled Range analysis has been used in the field of multiphase flow also by Fan et al. (1990, 1993), to examine the features of particle behavior in three phase fluidized beds, by Sæther et al. (1990), who used it to draw information on the slug flow regime, and by Bernicot et al. (1993), who extended the analysis performed by Sæther et al. (1990) including statistics relative to the flow of large bubbles as well.

In a previous paper (Giona et al. 1994b), it was shown that diffusional analysis can provide a thorough characterization of the structure of time series, also coming from two phase flow measurements. In this paper, diffusional analysis is described

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and compared to other analytical methods such as R/S analysis, and then applied to experimental time series of light oil and air flowing in a horizontal rig. Since different flow variables were recorded during the experiments (void fraction, holdup, and pressure drop), a discussion on the choice of the appropriate time series is presented. The diffusional analysis is evaluated by comparison Rescaled Range analysis and Fourier decomposition and it is used to obtain parameters which can univocally characterize the different flow regimes.

2 Experiments

The experiments were performed in the facility[1] depicted in Fig. 1. The loop consists of acrylic pipes with internal diameter of 31.7 mm placed on an inclinable bench 13 m long. Data were obtained in a horizontal set-up at atmospheric conditions, using air and light oil. The density of the oil was 800 kg/m^3, its viscosity was 1.6×10^{-3} Pa s, and its surface tension was 0.027 N/m. The superficial velocities of gas and liquid were varied in order to span all the different flow patterns: stratified, intermittent, dispersed bubbles and annular flow. In Fig. 2, the investigated area is plotted in color over the map obtained by Mandhane et al. (1974) for oil and air[2]. The liquid superficial velocity was varied in the range [0.06–4.12 m/s], while the gas superficial velocity was varied in the range [0.47–16 m/s]. An overall number of 70 experimental runs allowed to investigate over the area which overlapped the dispersed bubbles flow (14 experiments), the intermittent flow (slug and elongated bubble flow, 35 experiments), the annular flow (14 experiments), and the stratified flow (stratified and wave flow, 7 experiments).

For each experiment, three different signals could be recorded from four measuring stations, and for all types of signal a time history of 8000 points was recorded at a frequency of 1000 Hz. A differential pressure transducer (DP cell) recorded the pressure drop, two capacitance probes recorded the liquid holdup, and a γ-densitometer recorded the void fraction. The test section, containing the measuring devices, was located 8 m downstream from the inlet to reduce entrance disturbances. This distance is equivalent to 250 pipe diameters, which is sufficient to obtain a fully developed regime (Nydal et al. 1992). The differential pressure transducer provided the pressure drop measurement in the range [0–0.1] bar. The γ-densitometer was centered between the two pressure taps of the pressure transducer (see Fig. 1). The pressure taps were 2.06 m far apart, while the distance between the two capacitance probes was 1.04 m. The distance between the pressure taps was chosen in order to both minimize the volume over which measurements had to be performed and to guarantee adequate accuracy while recording pressure drop data. The distance between capacitance probes was set as small as possible to minimize the pressure drop between the two sampling points. As a matter of fact, the gas expansion caused by the pressure drop would change the superficial gas velocity, which, in turn, could induce changes in the flow pattern.

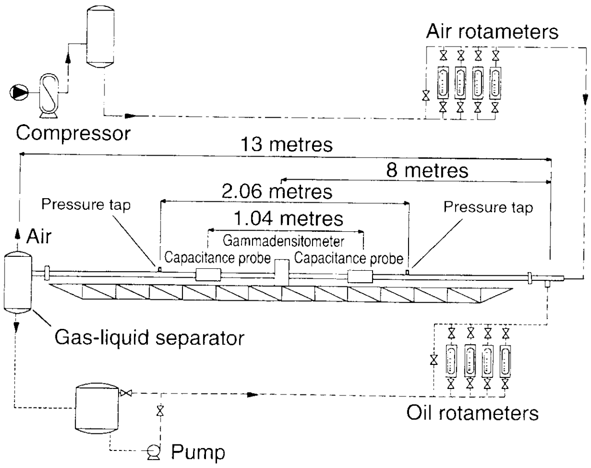

Fig. 1. The experimental setup

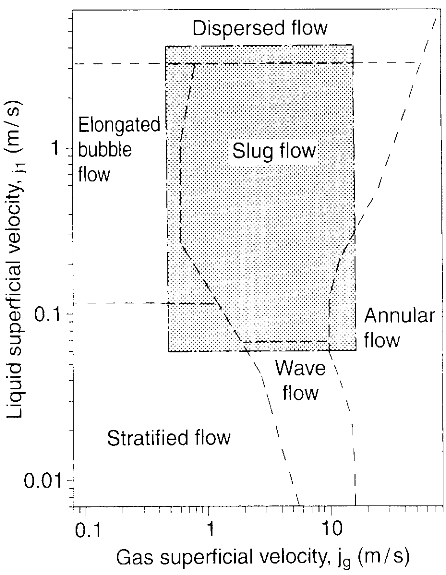

Fig. 2. The flow regime map by Mandhane et al. (1974) for oil and air and the investigated area (gray region)

The γ-densitometer, with a source of Am^{243}, is a one-shot collimator ray densitometer developed at IFE. The collimator is such that γ-rays are distributed over the entire cross section. Since air does not attenuate γ-rays, their attenuation can be directly related to the oil presence and the void fraction may be obtained. More details of the collimator design may be found in Gardner et al. (1970).

Capacitance probes have been developed at the University of Pisa and their detailed description can be found in the work by Andreussi et al. (1988). These probes measure the capacitance of the flowing mixture between two conducting rings 0.1

[1] The experimental facility is assembled at the Institutt for Energyteknikk (IFE), Kjeller, Norway.

[2] In their work, Mandhane et al. (1974) presented the air and water map. However, they also proposed correlations to plot maps for different working fluids (i.e. air and oil, as in the present case) and different pipe geometry.

diameters apart. The experimental device measures the potential difference between two metal rings mounted flush with the pipe wall. Since the current is controlled, the total impedance is measured, and, the flowing fluids being not conductive, the impedance is just the capacitance of the flowing mixture. The volume fraction of the oil (liquid holdup) may be obtained considering that the capacitance of air is negligible compared to the capacitance of the oil. The signal refers to the volume contained between the two measuring rings, which are 6.3 mm apart (corresponding to 0.2 diameters). The control volume is considered to be sufficiently small to assume that the measurements are cross section averaged rather than volume averaged; this is because the flow velocity was sufficiently high to ensure that no modification of the flow regime occurred over the small distance between the rings.

The calibration of both γ-densitometer and capacitance probes was performed for dispersed bubbles flow and for stratified flow employing quick closing valves, following the method described by Andreussi et al. (1988) and by Nydal (1991). Andreussi et al. (1988) proposed correlations to calculate the liquid holdup for stratified, annular and dispersed bubbles flow. The liquid holdup in intermittent flow may be calculated with the same correlations if the slug unit is considered as the superposition of a stratified flow region (in the long bubble zone), and of a dispersed bubbles flow region (in the slug body). In this work, the same correlations were used and the calibration results obtained were in good agreement with the ones presented by Andreussi et al. (1988) and by Nydal (1991).

3 Identification methods

3.1 Time-series analysis of multiphase flows

Experimental analyses of multiphase flows are based on the examination of time series of some measurable variable (which in the following will be called observable). The variable may be pressure, pressure drop, density or any other variable characteristic of the phenomenon. The fundamental underlying concept is that the fluctuations of characteristic physical quantities, which are measured instantaneously along the duct, may contain information about the flow conditions, i.e., *fluctuations bring the signature of the flow regime.*

To perform multiphase flow characterization, two main approaches may be followed: chaotic and stochastic. The analysis of chaotic time series is based on the assumption that fluctuations have a deterministic low-dimensional explanation. This implies that multiphase flow dynamics can be described by a deterministic chaotic system of differential equations. Classical methods of chaotic data analysis are then used to characterize the conditions of flow (see, e.g., Hao 1989; Daw et al. 1990; Van den Bleek and Schouten 1993). The attractiveness of chaotic methods lies in the chance of a low-dimensional reconstruction of the fluid dynamics. However, conclusive results are yet to be obtained with chaotic methods. On the other hand, stochastic methods, as for instance diffusional analysis or R/S techniques, allow the quantitative characterization of the correlation properties of time series. Since no hypothesis on the character of the phenomenon is required, these methods can be applied to both deterministic and stochastic processes[3].

All stochastic methods applied to analyze time series can supply global information about the signal. In particular, a wide class of stochastic processes can be identified by the Hurst exponent (see Feder 1988) that can be determined by R/S analysis (Fan et al. 1990; Sæther et al. 1990) and by diffusional analysis (Giona et al. 1994b). The Hurst exponent gives information on the long range properties of the signal. If a process is random, and the process variable is totally uncorrelated, the value of the Hurst exponent is $H=\frac{1}{2}$. If the signal is persistent (positive correlation) $H>\frac{1}{2}$, while for antipersistent signals (negative correlation) $H<\frac{1}{2}$.

3.2 Diffusional analysis

The fundamental assumption on which the analysis of fluid dynamics time series (turbulent flow, dispersion in random porous media, multiphase flow) is based, is that the fluctuations associated with a characteristic physical observable (such as local density, pressure drop across a given length, liquid holdup) are characteristic of the flow regime. Therefore, from the statistical analysis of these fluctuations, it is possible to achieve a detailed characterization of the macroscopic properties of the flow. Such result cannot be obtained from purely averaged quantities (such as average liquid holdup, average density, etc.)

More precisely, the statistical analysis (and, in this case, diffusional analysis) is aimed at obtaining the correlation properties (second order quantities). Indeed, these are the fundamental quantities describing interaction between fluid elements and transport coefficients[4].

Diffusional analysis of time-series as a statistical method has been introduced by Suzuki (1982, 1984) to characterize the fine structure of turbulence. It is therefore natural to extend a similar analysis to the study of multiphase flow systems (Giona et al. 1994a, b), and there is no physical reason why should it not provide significant results in such flows. There is a strong analogy between the description of fluctuations and diffusion processes (Kac and Logan 1979).

In this subsection, a brief description of the statistical formulation of diffusional analysis is presented, while physical implications relative to two phase flow are analyzed in the subsequent section.

Let $\{x_i\}$, for $i=1, \ldots, N$, be a time series of a generic flow observable sampled at a constant time interval, Δt. Diffusional analysis studies the diffusion process generated by $\{x_i\}$. For this reason, it is convenient to consider the normalized time

[3] Chaotic time series may possess very complex statistical properties depending on the structure of the invariant measure (if any) associated with the dynamical system generating the time series. See, for instance, Lasota and Mackey (1994).

[4] The correlation function of velocity fluctuations is associated, for instance, with the definition of Reynolds stresses, with the expression of the convective contribution to dispersion in random packings and porous media (Bhattacharya and Gupta 1990), with the representation of transport coefficients by means of correlation integrals, Green–Kubo formalism, see Boon and Yip (1980).

series $\{\xi_i\}$ derived from $\{x_i\}$ upon normalization, i.e., $\xi_i = (x_i - \langle x \rangle)/\sigma_x$, so that $\{\xi_i\}$ has zero mean and unit variance ($\langle x \rangle$ is the average of $\{x_i\}$, and σ_x^2 its variance).

Starting from $\{\xi_i\}$, it is possible to generate a diffusion process (random walk) on the real axis, which is linearly driven by $\{\xi_i\}$. Let z_i be the position at time i of the walk, then the diffusion process is described by the dynamic equation

$$z_{i+1} = z_i + \xi_{i+1} \tag{1}$$

with the initial condition $z_0 = 0$ (the random walk starts from the origin)[5]. As in all diffusion phenomena, the correlation properties of the random walk (and ultimately the correlation properties of the driving signal $\{\xi_i\}$) can be obtained from the analysis of the mean square displacement, $R_z^2(n)$, defined as

$$R_z^2(n) = \langle (z_{k+n} - z_k)^2 \rangle = \frac{1}{N_{av}} \sum_{k=1}^{N_{av}} (z_{k+n} - z_k)^2, \tag{2}$$

where N_{av} is the number of averaging points.

The scaling behavior of $R_z^2(n)$ with n allows information about the short-term and long-term properties of $\{\xi_i\}$, and on the crossover (if any) between different scaling regimes to be obtained.

As an example, if the signal $\{\xi_i\}$ is a realization of a fractional Brownian motion (fBm) process characterized by the Hurst exponent H, then (Feder 1988)

$$R_z^2(n) \sim n^{2H} \tag{3}$$

and, in particular, if the driving signal is a regular Brownian motion process, $H = \frac{1}{2}$, $R_z^2(n) \sim n$. The exponent H in Eq. (3) completely describes the scaling properties of trials of fBm. For $H < \frac{1}{2}$, two subsequent displacements of fBm show, on average, a negative correlation $[\langle \xi_i(\xi_{i+j} - \xi_i) \rangle < 0]$ (antipersistent character), while, for $H > \frac{1}{2}$, two subsequent displacements ξ_i and $\xi_{i+j} - \xi_i$ tend to have, on average, the same sign (persistent character). The separation value between persistent and antipersistent behavior, $H = \frac{1}{2}$ (regular Brownian motion), is characterized by the absence of correlation between two arbitrary increments of the process.

The opposite case is represented by deterministic oscillating signals. Here, the mean square displacement exhibits a crossover behavior: at short time scales, $n \leqslant n_c$, the signal is strongly correlated in a deterministic way, and therefore $R_z^2(n) \sim n^2$. This behavior is a typical universal feature of deterministic signals, corresponding, in the theory of transport processes, to the effect of a biasing velocity field[6]. At long time scales, the behavior of $R_z^2(n)$ is oscillating with an average slope equal to zero. This happens because the motion described by Eq. (1) in the presence of a deterministic signal $\{\xi_i\}$ with mean equal to zero is always bounded.

A similar situation arises in the analysis of a signal $\{\xi_i\}$ which can be regarded as a superposition of deterministic oscillating modes and random fluctuations. In multiphase flow, this situation occurs for Intermittent flow. In this case, the mean square displacement $R_z^2(n)$ is characterized by a crossover behavior

$$R_z^2(n) \sim \begin{cases} n^{\beta_1}, & n \leqslant n_c \\ n^{\beta_2}, & n \gg n_c \end{cases} \tag{4}$$

The crossover instant, n_c, is related to the fundamental period of oscillations, T_c, by the relation $T_c = 4n_c\Delta t$, where Δt is the sampling time. For practical purposes, the meaning of the relation $n \gg n_c$ should be interpreted as n larger than $1.5n_c$.

For the physical situations occurring in two phase flow analysis, the mean square displacement can be characterized by the two exponents, β_1 and β_2, and by the crossover instant, n_c, in the case $\beta_1 \neq \beta_2$. The exponents β_1 and β_2 are related to short-term and long-term correlation properties of $\{\xi_i\}$ respectively, and the crossover instant, n_c, is linked to the time scale which discriminates the qualitative notion of short-term and long-term behavior in a quantitative way.

3.3 Identification of two phase flow regimes

In the application of diffusional analysis to fluctuations of two phase flow variables, the exponents β_1 and β_2 appearing in Eq. (4) depend on the superficial gas and liquid velocities j_g, j_l. This dependence enables one to attempt a regime identification which is based on the statistical properties of $R_z^2(n)$. It should be noted that, to identify different flow regimes successfully, diffusional analysis may be applied to an experimental time series only under the hypothesis that the series itself is stationary: this implies that time series have to refer to fully established flow regimes.

Diffusional analysis has been applied to data relative to different two phase flow regimes, which are characterized by different gas and liquid superficial velocities. The flow patterns have been identified examining the correlation exponents $\beta_i (i = 1, 2)$ and the crossover time, n_c, (if any). In Table 1, the results obtained from the analysis of all the experimental time series from the γ-densitometer are presented. These results have been obtained applying diffusional analysis to raw data, without any form of filtering. Each signal is affected by a noise which is characteristic of the particular device employed. However, the noise is filtered out since diffusional analysis itself provide some form of filtering (see also note 5).

Table 1. Behavior of β_1 and β_2 in two phase flow regimes

Flow-regime	β_1	β_2
Dispersed bubbles	$\simeq 1.0$	$\simeq 1.0$
Stratified	$\simeq 2.0$	$\gg 1.0$, [1.5–2.0]
Intermittent	$\gg 1.0$, [1.5–2.0]	$\ll 1.0$ ($\simeq 0$)
Annular	> 1.0	< 1.0

[5] The definition of random walk represented by Eq. (1) can be regarded as a filtering of the signal with a linear filter having a pole at $\lambda = 1$.

[6] In convection/diffusion, phenomena such that the flux $\mathbf{J}$ of the transported entity, the concentration of which be c, is made by the contribution of a diffusive and a convective part $\mathbf{J} = \mathbf{v}c - D\nabla_c$, where $\mathbf{v}$ is the biasing velocity field, and D the corresponding diffusion coefficient, the mean square displacement of a tracer particle $R_z^2(t)$ scales with time t as $R^2(t) \simeq v^2t^2 + 2Dt$. An exponent 2 in the scaling of the mean square displacement is always a signature of the presence of a deterministic contribution in the random motion of transported particles.

The results reported in Table 1 can be physically interpreted in terms of the fluid dynamic properties of the regime. Let us examine each case separately, bearing in mind the qualitative behavior of the void fraction in the various regimes. Also, it should be remembered that the exponents β_i span the interval [0.0–2.0].

Dispersed bubbles flow is characterized by a random distribution of gas bubbles flowing in a liquid stream. It is natural to hypothesize that the motion of a generic gas bubble is not correlated to the motion of other bubbles. This implies that bubble-bubble interactions are negligible and therefore bubbles evolve as a system of noninteracting Brownian particles. Therefore, the density signal exhibits a linear scaling with time, $\beta_1 \simeq \beta_2 \simeq 1$, as in the case of purely Brownian motion ($\beta_1 = \beta_2 = 1$).

In Stratified flow, there is a clear cut separation between the two phases. Fluctuations of stochastic nature at the interface separating the two phases may occur. However, the void fraction signal has a strongly persistent character with the following values $\beta_1 \simeq 2$, $\beta_2 \gg 1$ ($\beta_2 \simeq 1.5 \div 2.0$). The fluctuations at the gas-liquid interface depend on the superficial velocities and in general increase with the total superficial velocity $j_T = j_g + j_l$. For low superficial velocities, the deterministic contribution to interface oscillations is dominant and, therefore, the mean square displacement $R_z^2(n)$ behaves as in the case of a deterministic signal (discussed in the previous subsection). The highly correlated nature of interface fluctuations in stratified flow makes the behavior of the diffusion process associated with $\{\xi_i\}$ highly persistent. This observation explains why the long-term exponent β_2 is greater than 1.5 (this value is, however, empirical and comes from the present analyses, since theoretical considerations only indicate β_2 to be larger than unity). It should be pointed out also that the experimental values found for β_1 vary between 1.85 and 2.0, and decrease with increasing j_T.

Intermittent flow can be interpreted as a superposition of an almost periodic propagation of aerated liquid slugs followed by long gas bubbles. Owing to the *mixed* nature of this flow, the resulting signal exhibits a high value of β_1 in the range [1.5–2.0] which is associated with the deterministic and coherent component of the fluctuations. The long-time exponent, β_2, accounts for the persistent, almost periodic, occurrence of liquid slugs[7], and therefore it should be close to 0.0 (in practice $\beta_2 \simeq 0.0 \div 0.2$). The stochastic component of the oscillations of the slugs may be retrieved when observing the value of the short-term exponent β_1 which is less than 2.0. The more coherent the slug dynamics, the closer is β_1 to 2.0. The value of the crossover point, n_c, is related to the main frequency of slug oscillations (see Sect. 5 below).

A preliminary analysis of Annular flow indicates that $R_z^2(n)$ presents a crossover behavior between the values $\beta_1 > 1$ and $0 < \beta_2 < 1$ (in most of the analyzed cases, it was found $\beta_1 > 1.5$). A possible physical interpretation of this behavior is related to the biasing effect of liquid oscillations at the liquid layer near the walls ($\beta_1 > 1$) and of complex (non-Brownian) fluctuations of the inner gas stream ($\beta_2 < 1$). However, the identification of the Annular regime can be further improved by analyzing the temporal behavior of the relative mean square displacement[8].

The results obtained and gathered in Table 1 can be used as a predictive tool to identify the regimes directly from the behavior of the mean square displacement of the diffusion process generated by the analyzed time-series. The exponents β_1 and β_2 depend continuously on the superficial velocity of gas and liquid. It is just this dependence which allows the identification of the various regimes and the localization of the boundary associated with regime transitions in the j_g-j_l plane.

Finally, it is important to stress that the prediction of the scaling behavior of $R_z^2(n)$ (summarized in Table 1) is directly related to the physical interpretation of the nature of the density signal fluctuations. This phenomenological connection with the physics of multiphase flow evolution makes diffusional analysis a simple but powerful tool to understand the dynamics of the flow from a macroscopic point of view.

3.4
Comparison with R/S analysis

The most popular method applied to fluid dynamic problems is certainly rescaled-range analysis (Feder 1988; Sæther et al. 1990; Fan et al. 1990, 1993). The key quantity in this analysis is the ratio between the maximal cumulative variation (cumulative range) in the time interval $1 < i \leqslant n$, $R(n)$ and the square root of the variance up to time n, $S(n)$. Given the time series $\{\xi_i\}$, $R(n)$ and $S(n)$ can be evaluated by the set of equations

$$\langle \xi \rangle_n = \frac{1}{n} \sum_{i=1}^{n} \xi_i, \quad S^2(n) = \frac{1}{n} \sum_{i=1}^{n} (\xi_i - \langle \xi \rangle_n)^2$$

$$c(n, m) = \sum_{i=1}^{m} (\xi_i - \langle \xi \rangle_n), \quad m \leqslant n \tag{5}$$

$$R(n) = \max_{1 \leqslant m \leqslant n} c(n, m) - \min_{1 \leqslant m \leqslant n} c(n, m)$$

The quantity $R(n)/S(n)$ scales with time. For a wide class of stochastic processes R/S follows a power law with time:

$$R(n)/S(n) \sim n^H \tag{6}$$

which allows for the evaluation of the Hurst exponent, H. The R/S approach has been used by Fan et al. (1990, 1993) in fluidized bed dynamics, by Drahos et al. (1992) in bubble columns (considering pressure drop signals) and in some modified form by Sæther et al. (1990) who described slug-length statistics in terms of fractional Brownian motion.

An examination of the results presented by these authors leads to the conclusion that R/S analyses of pressure drop fluctuations in fluidized beds and bubble columns are characterized by a persistent character (i.e., by a Hurst exponent larger than $\frac{1}{2}$). It is not clear, though, how this might be used to obtain a clear regime identification. More specifically, Fan et al. (1990) show experimentally that the Hurst exponent for the

[7] At this point, it should be clarified that persistent periodic oscillations lead β_2 toward 0 (intermittent flow), while a persistent random drift in the motion implies $\beta_2 \gg 1$.

[8] The scaling of the relative mean square displacement $\langle [x_1(t) - x_2(t)]^2 \rangle$ was considered by Suzuki (1984) in connection with chaotic maps. It has been applied to multiphase flow problems by Giona et al. (1994b).

considered (narrow) range of liquid and velocity fluctuations spans in the range [0.7–1.0], and that this exponent increases with the gas velocity. However, no clear indication is gathered about the way to pursue the identification of flow regimes. On the contrary, by applying diffusional analysis to two phase flow, it was shown (see also Giona et al. 1994a, b) that regime identification can be achieved. This is because there is a quantitative, and physically grounded, difference for the values attained by the exponents β_i for different flow regimes.

There are several reasons why results obtained by applying diffusional analysis of density fluctuations are clearer than those of the R/S approach and pressure drop fluctuations. First, most of the authors consider fluidized bed dynamics (in which three phase, gas, liquid and solid, are involved) which exhibit a more complex physical phenomenology than two phase flow in pipes. Moreover, attention of most authors is focused almost exclusively on pressure drop fluctuations. By the nature of the measurement (which is taken along a given length of the apparatus), pressure drops are volume averaged quantities (averaged across the section of the column or pipe and over a length corresponding to the distance between the probes) while density signals are local quantities averaged across the section of the duct. This can be readily observed in the experimental signal shown in Fig. 3, where the normalized output (with zero mean value and unit variance) obtained from γ-densitometer (case *i*), from the capacitance probe (case *ii*) and from pressure drop measurements (case *iii*) in stratified and intermittent flows are presented. The highly fluctuating

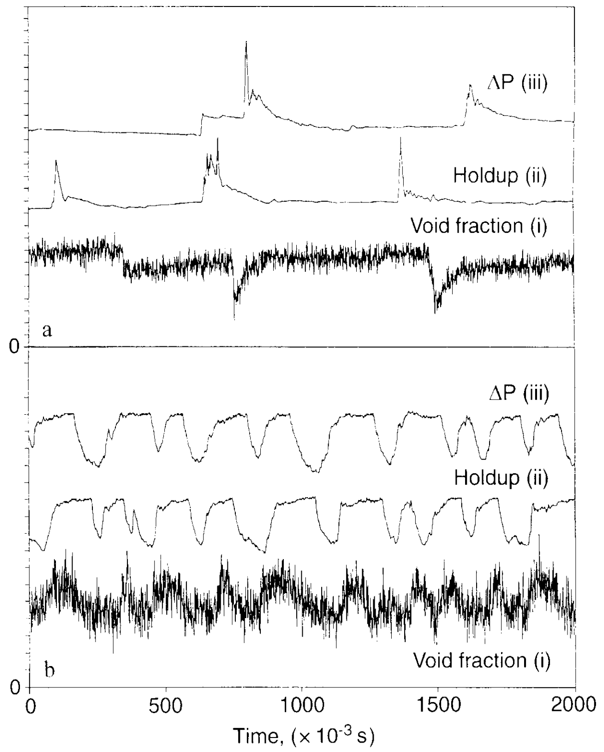

Fig. 3a,b Normalized experimental time series **a**. stratified flow ($j_g = 2.525$ m/s, $j_l = 0.10$ m/s); **b** intermittent flow ($j_g = 2.07$ m/s, $j_l = 2.07$ m/s). Comparison of the different features of experimental data: i) γ-densitometric signal (void fraction); ii) capacitance probe signal (holdup); iii) pressure drop signal (ΔP)

character of the density (void fraction) signals is an intrinsic feature of the measuring technique employed (γ-densitometer), which is very sensitive to the small variations of the void fraction of the flowing mixture. The γ-densitometer output is also affected by noise which is typical of this technique. Such level of noise, though higher than that shown by other measuring techniques, does not influence the interpretation of the results. As a matter of fact, noise is filtered out by diffusional analysis (see also note 5). This is confirmed by the fact that diffusional analysis applied to different experimental signals gives consistent results.

There are also statistical reasons why diffusional techniques should be preferred in many cases to R/S analysis. From its definition in Eq. (5), the quantity $R(n)/S(n)$ is cumulative since the maximum and minimum values of $c(n, m)$ are taken in the range $[1 \leqslant m \leqslant n]$. As with any cumulative quantity, the sensitivity of $R(n)/S(n)$ with respect to the local fluctuating nature of the signal is less than that of $R_z^2(n)$. In many cases, such as in fBm, diffusional and R/S analyses supply the same information about the statistics of the signal. This is usually true for all the stochastic fluctuations which are homogeneous in the sense that they are characterized by a unique time-independent (i.e., with no crossover) scaling behavior. In the case of mixed signals, i.e. signals which come from the superposition of stochastic and deterministic components, R/S analysis does not supply a clear statistical description as the one obtained by diffusional methods. This point has been developed in Giona et al. (1994b), considering simple time series generated by the superposition of two periodic signals, and will not be repeated here.

A comparison between the results from diffusional and R/S analyses is indeed very interesting and can be made considering that $\beta_i = 2H_i$, with $i = 1, 2$. Both kind of analysis were applied to experimental data relative to the four different regimes, stratified, intermittent, dispersed bubbles and annular. The results of such comparison are presented in Table 2. For the Hurst exponents H_i (as in the case of the β_i), the subscript refers to short and long term behavior. In some cases, the absence of any power law scaling did not allow the determination of the Hurst exponent (this situation is indicated in the table with the dash sign). While in the case of dispersed bubbles flow, both diffusional and R/S analysis give the same results, in intermittent flow it is impossible to obtain a clear scaling behavior from R/S analysis as discussed above. In Fig. 4, results from the application of R/S analysis of void fraction fluctuations in stratified flow, a and intermittent flow,

Table 2. Values of the scaling exponents β and H for dispersed bubbles flow ($j_g = 0.55$ m/s, $j_l = 2.89$ m/s), Stratified flow ($j_g = 2.525$ m/s, $j_l = 0.07$ m/s), Intermittent flow ($j_g = 2.07$ m/s, $j_l = 2.07$ m/s), and annular flow ($j_g = 13.0$ m/s, $j_l = 0.19$ m/s). The dash sign indicates the absence of any power-law scaling

Flow regime	β_1	β_2	H_1	H_2
Dispersed bubbles	0.97 ± 0.03	0.90 ± 0.1	0.48 ± 0.04	0.48 ± 0.04
Stratified	1.87 ± 0.02	1.54 ± 0.02	0.7 ± 0.02	—
Intermittent	1.80 ± 0.03	0 ± 0.01	—	—
Annular	1.52 ± 0.03	0.74 ± 0.03	0.77 ± 0.03	0.48 ± 0.03

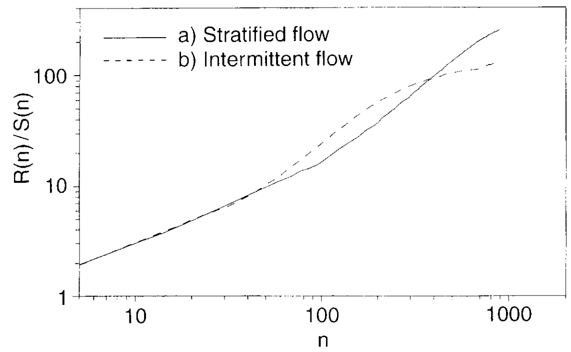

Fig. 4a,b. R/S versus n. **a** Stratified flow ($j_l = 2.525$ m/s, $j_l = 0.07$ m/s); **b** intermittent flow ($j_g = 2.07$ m/s, $j_l = 2.07$ m/s)

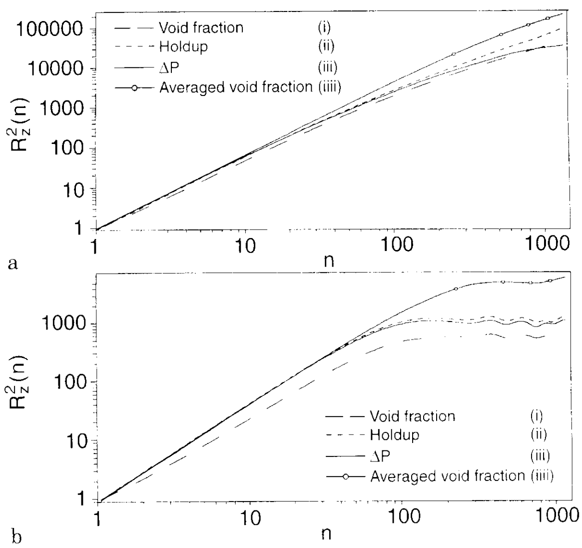

Fig. 5a,b. Mean square displacement, $R_z^2(n)$. **a** Stratified flow ($j_g = 2.525$ m/s, $j_l = 0.10$ m/s); **b** intermittent flow ($j_g = 2.07$ m/s, $j_l = 2.07$ *m/s*). Comparison of the different features of experimental data: i) γ-densitometric signal (void fraction); ii) capacitance probe signal (holdup); iii) pressure drop signal (ΔP); iiii) averaged γ-densitometric signal (averaged void fraction)

b are presented. The same data analyzed with diffusional analysis are presented in Fig. 5a, case i and Fig. 5b, case i. If results are compared, it can be observed that, for intermittent flow, the parameter $R(n)/S(n)$ shows a very complex behavior (line b in Fig. 4). It is hard to extract a definite power law scaling over a sufficiently broad time interval (one decade) from such behavior. The opposite is found applying diffusional analysis as shown in (Fig. 5b) case i, in which two clear cut slopes (in a log-log plot) can be identified.

4
On the choice of the appropriate experimental time series

4.1
The experimental signal

In principle, the same results should be obtained from the analysis of different experimental observables. However, due to the difference of measuring techniques, different levels of information may be retrieved from different observables. The large majority of the analyses of the features of experimental time series was performed using pressure drop fluctuations (Daw et al. 1990; Skrycke et al. 1993; Franca et al. 1991; Fan et al. 1993; Drahos et al. 1992). In the present work, pressure drop, void fraction and holdup were measured. This more complete monitoring of the flow enables a discussion about the usefulness of the information on flow regimes found by different measurements.

The information given by the pressure drop signal is intrinsically *non-local*: since the two probes are far apart, the signal is volume averaged. Capacitance probes and γ-densitometer give a section averaged density signal of higher quality when compared to the pressure drop. Furthermore, the γ-densitometer datum allows for the detection of small fluctuations characteristic of dispersed bubbles flow that may not be detected by capacitance probes. In this work, γ-densitometric signals were used for all of the flow regimes in order to ensure a consistent analysis of the different flow regimes.

The results of the analysis are shown in Fig. 5a and b, cases i, ii and iii (see again Fig. 3 for the data). The scaling behavior of $R_z^2(n)$ of the three time-series is almost identical within the range of experimental error (the error margin in the estimate of β_i is ± 0.06, at most). This strengthens the idea that the exponents β_i are intrinsic features of the flow regime.

When dealing with dispersed bubbles flow regime, though, diffusional analysis cannot be successfully applied to pressure drop and liquid holdup. In this regime, they both are practically constant in time and the small fluctuations are due to experimental errors. The conclusion is that diffusional analysis allows for the determination of the characteristics of the flow regime provided the measured variable actually carries the characteristics of the flow regime. Pressure drop and holdup values obtained by capacitance probes do not contain enough information for dispersed bubbles flow to be effectively analyzed.

4.2
Sensitivity to sampling time

Most of the signal analyses performed in previous works refer to pressure drop signals. Therefore, it seems appropriate to quantify the accuracy and the amount of information carried by pressure drop and γ-densitometric signals respectively. However, since the objective is to compare the quality of information from two different devices, that is all different kind of features of the flow regime which can be retrieved analyzing the two different signals, the volume over which the information is averaged should not affect the result. Therefore, in the following, diffusional analysis was applied to the pressure drop signal and to the signal which was derived averaging the void fraction over a control volume equivalent to the one over which the pressure drop was measured. Assuming that the flow pattern is stationary and fully developed, the volume average over the section S times the length L (the distance) between the pressure probes, is equivalent to the temporal average of a section-averaged quantity (i.e., the local density) over a time interval T_{av}. The time T_{av} is the propagation time of a fluid element along a length L, i.e. $T_{av} = L/j_T$, where $j_T = j_l + j_g$. The diffusional properties of the pressure drop signal have been compared with the γ-densitometric signal averaged over a number of samples $n_{av}=L/(j_T\Delta t)$. Therefore, if $\{\xi_i\}$ is the γ-densitometric output, the corresponding time averaged signal is given by $\{\xi_{av,i}\}$, $\xi_{av,i}=\sum_{j=i}^{i+n_{av}}\xi_j$.

Again in Fig. 5, which for the sake of completeness reports all analyses performed with diffusional analysis on available probes, the cases a iiii and b iiii show $R_z^2(n)$ for the time averaged density signal. These results may be compared with the others referring to time-series which were not processed. The averaged signal has an initial exponent $\beta_1=1.96$ greater than the slope of the unprocessed signal, $\beta_1=1.78$, and identical to that of the pressure drop signal. This effect was bound to occur, since the temporal average makes the time-series smoother and increases its coherence. Also, this result confirms the physical observation that pressure drop signal can be regarded as a temporal averaging of local fluctuations over a time-scale L/j_T. The other features of $R_z^2(n)$ are practically equivalent: the different asymptotic plateau level of the mean square displacements of Fig. 5 does depend on the Fourier component of the periodic signal. However, this is a characteristic property of the signal itself, not related to any physical property of the flow. The same comparison performed for other flow regimes gave the same results, as it is shown in Fig. 5b case iiii, where the situation for Intermittent flow regime is presented.

5 Quantitative characterization and fine structure of flow regimes

5.1 Slug frequency estimation

The characterization of the Intermittent flow regime requires a deeper understanding of the phenomena involved. Indeed, the alternation of large liquid slugs followed by elongated bubbles, makes the flow pattern highly non homogeneous, and a mathematical treatment very difficult to apply (Trapp and Mortensen 1993). In addition, average characteristics are needed experimentally to assess theoretical models. In particular, the slug frequency is a main parameter of this flow regime.

From diffusional analysis, one can directly obtain a quantitative estimate of the slug frequency as $f_c=1/T_c$ from the crossover behavior of $R_z^2(n)$. The crossover instant, n_c, is the value of n for which $R_z^2(n)$ begins to deviate from the initial slope (in a log-log plot). In this work, the deviation was considered to occur when the slope differed for more than 6% from the initial slope. The relation $T_c=4n_c\Delta t$ comes from the analysis of deterministic periodic signals and can be extended to stochastic time series possessing a deterministic component. It arises from the periodic signal being, on average, in phase for a time interval equal to one fourth of its period. That is, during one fourth of the period, the periodic signal acts as a directional bias (and, therefore, $\beta_2=2$).

In the analysis of experimental data, the value of β_1 deviates from the theoretical prediction $\beta_1\in[1.5–2]$. This is an effect of the complex slug length statistics: the motion of slugs is not strictly periodic. It has been shown (Giona et al. 1994b) that quasiperiodic signals exhibit a value of β_1 slightly less than 2, depending on the complexity of the frequency spectrum and on both number and intensity of the independent modes.

Figure 6 shows the value of the slug frequency, f_c, obtained from the analysis of $R_z^2(n)$, compared with the values of the frequency of the greater spectral component obtained with Fourier decomposition of the autocorrelation function. The agreement between these two estimates is rather satisfactory. It can be observed that slug frequencies estimated by means of diffusional analysis vary with j_g in a way smoother than the one exhibited by the frequencies obtained with Fourier analysis. This behavior is not yet completely understood. Nevertheless, the quantitative agreement in the estimate of slug frequency is a first important assessment of the versatility of diffusional analysis in extracting statistical and quantitative information from experimental observations of multiphase flow.

5.2 Fractal dimension of multiphase flow

In other works (e.g., Sæther et al. 1990; Franca et al. 1991; Daw et al. 1990) where the R/S technique is employed, the fractal dimension of the signal is used to characterize the signal behavior with a global feature. In Sæther et al. (1993) the fractal dimension of oil–water interfaces, which was obtained with image-analysis techniques, versus the mixture velocity is presented. It is found that the fractal dimension of the interface increases with the mixture velocity and with the complexity of the interface. In cases like the one treated here, the fractal dimension of the signal may not be easily related to the complexity of the interface except in stratified (or annular) flow. Indeed, in this regime, the density of the mixture in the measuring section can be related to the structure of the in terface and, therefore, the fractal dimension of the density signal might be interpreted as a measure of the complexity of the interface.

With diffusional analysis, the fractal dimension of the signal can be calculated from the exponent β_1, if the time series is regarded as the discretized representation of the graph of a function which is continuous but almost nowhere differentiable. The fractal dimension D_0 of the graph is given by

$$D_0=2-\frac{\beta_1}{2}. \tag{7}$$

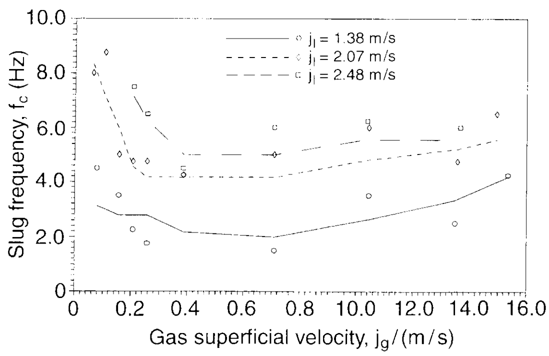

Fig. 6. Behavior of the slug frequency versus the gas superficial velocity at different liquid superficial velocity. Comparison of the frequency values from void fraction data obtained using diffusional analysis (lines), and Fourier decomposition of the autocorrelation function of the signal (symbols)

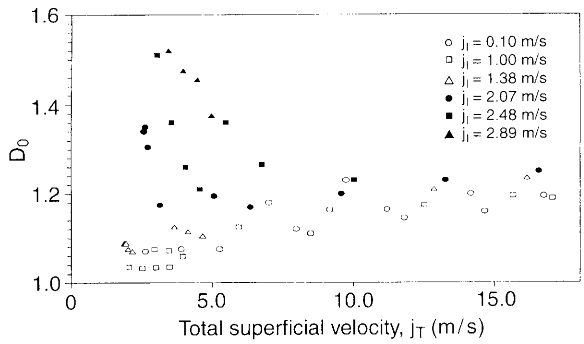

Fig. 7. Fractal dimension of the void fraction signals: Eq. (7) versus $j_T = j_g + j_l$, for different values of the superficial liquid velocity j_l

Figure 7 shows the behavior of D_0 versus the total superficial velocity $j_T = j_l + j_g$, for different values of j_l. The fractal dimension is related to the function ξ_i versus i and not to the properties of the interface. An examination of the results by Sæther et al. (1993), who applied the box counting method (Hao 1989) to the digitized image of an oil-water interface, leads to the following conclusion: the values of D_0 are much smaller ($D_0 \in [1, 1.2]$) in air-oil system than in water-oil system ($D_0 \in [1, 1.5]$); the fractal dimension D_0 for low liquid velocities in the Stratified region exhibits an almost monotonous trend with respect to the total superficial velocity j_T (see in Fig. 7 data for $j_l = 0.10$ and $j_l = 1.0$ m/s). However, the comparison with results by Sæther et al. (1993) is limited, given the physical differences of the interface in gas–liquid and liquid–liquid systems.

6
Summary and conclusions

A set of experimental data obtained for light oil and air flowing concurrently in a horizontal pipeline was analyzed using diffusional analysis. The scaling of $R_z^2(n)$ appears to be an adequate tool for the identification and characterization of multiphase flow regimes. Different flow regimes may be characterized on the basis of the values of their time scaling exponents, β_1 and β_2. A comparison between diffusional analysis and the R/S technique demonstrates that diffusional analysis can give better and more complete information about the investigated phenomenon. In particular, when the time series is constituted by a superposition of stochastic and periodic fluctuations, as in the case of intermittent flow, diffusional analysis is capable of providing information about the features of the periodic oscillations, slug frequency and, straightforwardly, slug length, and on the characteristics of the random fluctuations which are related to the dynamics of small bubbles.

For the intermittent flow regime, the results obtained by diffusional analysis have been compared also with a standard Fourier decomposition of the autocorrelation function of the signal with good agreement.

Particularly interesting, from an experimental point of view, a comparison of the amount information related to the fluid dynamics features of the different regimes, which can be retrieved analyzing different experimental signals has been presented. Pressure drop analysis, although largely used in the past to characterize multiphase flows, seems to be inadequate when the fine structure of the flow is sought. Furthermore, if the dispersed bubbles flow regime is investigated, capacitance probes signal may not detect the small holdup fluctuations associated with the motion of the bubbles. The γ-densitometer output appears to give more complete information for all flow regimes.

To obtain a global parameter characteristic of the signal, and, to a certain extent, of the flow regime, the fractal dimension of the signal has been calculated. The fractal dimension increases with the total superficial velocity.

The results presented in this work demonstrate that diffusional analysis may be regarded as an important tool to uncover fundamental features of multiphase fluid dynamics. The method is very simple and may be applied straightforwardly to the output of any experimental device. As a further development, it is suggested that diffusional analysis be used as a predictive tool for the identification of fluid dynamic regimes. For example, it may be a practical method to achieve *on-line* characterization of multiphase flow, applicable in monitoring and control units of industrial plant involving two and, eventually, three phase systems. It can be applied to time series coming from arbitrary probes, within the limitations on the experimental variable previously discussed.

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