Disturbance growth during sedimentation in dilute fibre suspension

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It is well known that dilute suspensions of fibres settling under gravity at low Reynolds number (*Re*) are unstable to density fluctuations. Koch and Shaqfeh (1989) predicted this instability in the absence of inertia and diffusive effects. The mechanism identified by them relies on the coupling between the orientation of the settling nonspherical particles and the flow field induced by the density fluctuations. Using a linear stability analysis in an unbounded homogeneous suspension they found that the normal mode density fluctuations with the maximum growth rate are those of infinite horizontal wavelength. In a bounded system, the container size limits the size of the largest wavelength. However, they didn't believe that such long wavelength fluctuations will in fact develop. The growth of a single streamer spanning the width of the box was reported in the simulations [Saintillan et al. (2005), Tornberg et al. (2006)] at short times in periodic suspensions, confirming that the longest wavelength is the most unstable in such systems. However, according to experiments of Herzhaft et al. (1998) and Metzger et al. (2007), wavelengths equal to the container dimension were not observed at early times.
Here, the early stage of sedimentation is studied using a linear stability analysis of Fokker-Planck equation with self-diffusion coupled to the Navier-Stokes equation with inertia, it is shown that inertia and self-diffusion damp long wavelengths and short wavelengths, respectively, leading to wavenumber selection. For small, but finite *Re* of the fluid bulk motion, the most unstable wavenumber is not zero any more and increases with *Re*. For wavenumber zero, the growth rate is zero instead of attaining its maximum as for zero *Re*. However, for small *Re* the influence of inertial effects on the dispersion relation is restricted to small wavenumbers. The velocity fluctuations of particles in dilute suspensions can lead to randomly fluctuating motions, which have a long time behavior characteristic of a diffusion process known as hydrodynamic self-diffusion. The effect of including self-diffusion is to damp the density fluctuations of the linear stability analysis only in the range of large wavenumbers. We also investigate the influence of the initial conditions. It is shown that just in the early development of the perturbation, the growth rates might differ from exponential growth of a single linear mode. Except this, the different initial conditions only have slight effect on the relationship between wavenumbers and growth rates.
Forthcoming work will focus on long times simulations of the non-linear development in bounded systems to estimate the effect of rotational diffusion and walls.

References

D. L. Koch and E. S. G. Shaqfeh, J. Fluid Mech. 209, 521(1989).
D. Saintillan, E. Darve, and E. S. G. Shaqfeh, Phys. Fluids 17, 033301(2005).
A.-K. Tornberg and K. Gustavsson, J. Comput. Phys. 215, 172 (2006).
B. Herzhaft and ÃĽ. Guazzelli, J. Fluid Mech. 384, 133 (1999).
B. Metzger, J. E. Butler, and ÃĽ. Guazzelli, J. Fluid Mech. 575,307 (2007).