EUROMECH Colloquium 513 on Non-Spherical Particles in Fluid Turbulence

April. 6-8, 2011, Udine

Two-way coupled simulation on ellipsoids suspensions in turbulent channel flow

Lihao Zhao & H. I. Andersson

Dept. of Energy and Process Engineering, NTNU, Norway

NTNU
Norwegian University of Science and Technology

Content

I. Research Background

II. Strategies and Governing Equations

III. Results

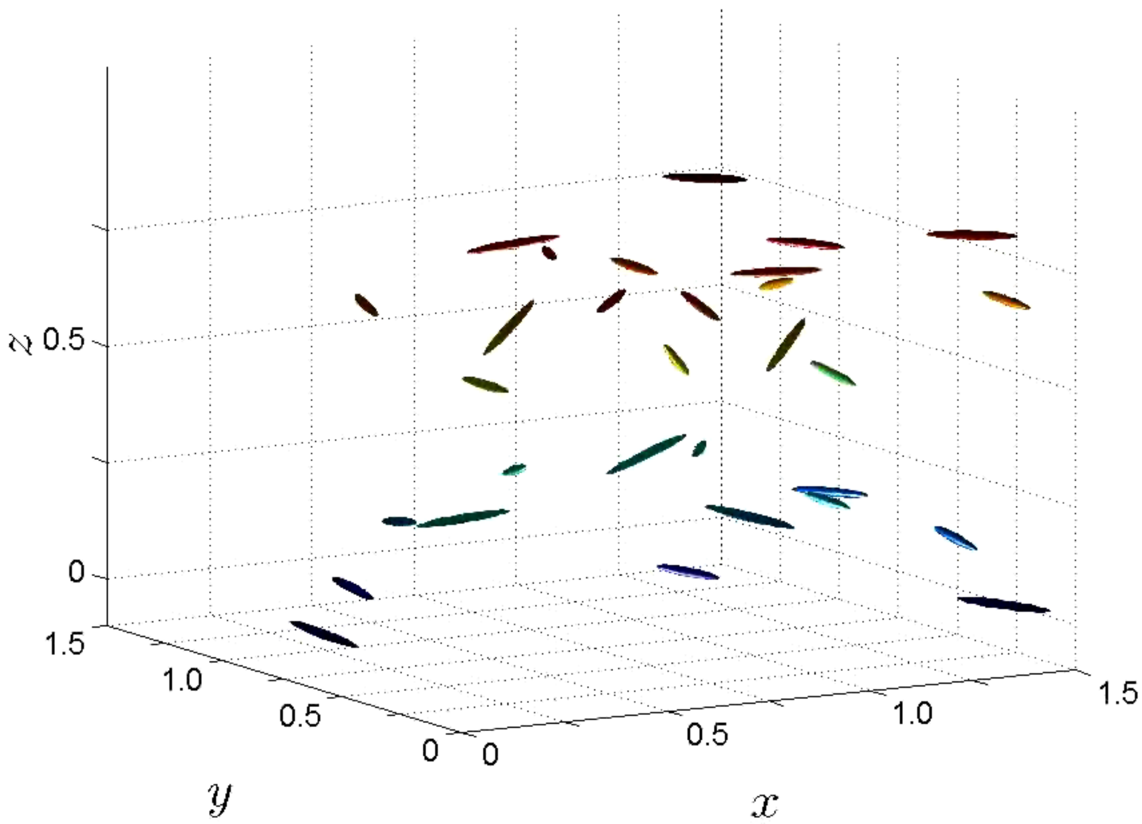
IV. Summary

NTNU
Norwegian University of Science and Technology

Background

- Ellipsoidal particles suspensions in turbulence is complicated

Animation of fiber suspensions in the channel flow



Background

- Background
  - Wood fibres in paper making industry
  - Drag reduction in the turbulent channel flow
  - Carbon nanotubes
- Approach of current work
  - Eulerian-Lagrangian point-particle approach with two-way coupling.
- Motivation of current work
  - Investigation on the mechanisms of interaction between ellipsoidal particles and fluid in the turbulent channel flow.

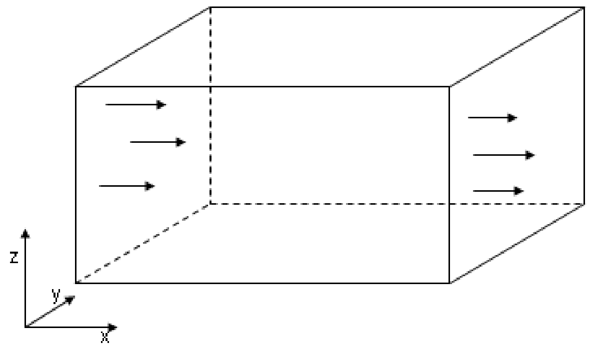


Fig. Physical model of the channel flow

NTNU
Norwegian University of Science and Technology

Strategies

- Fluid representation
  - Eulerian frame
  - Direct numerical simulation

- Particle representation
  - Lagrangian point particle approach with two-way coupling
    - Force-coupling
    - Torque-coupling *

(* Please refer to the presentation given by Dr. Barri
Fri. 09:00 – 09:20 M. Barri 'New scheme for torque coupling')

Eulerian fluid representation

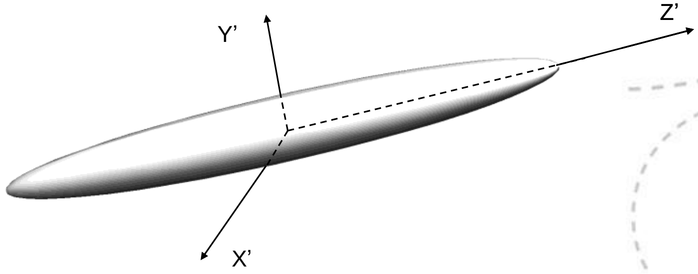
- Incompressible and isothermal Newtonian fluid.
- Frictional Reynolds number: $\mathrm{Re}_\tau = \dfrac{u_\tau h}{\nu}$
- Governing equations (non-dimensional):
  - Mass balance $\nabla \cdot \vec{u} = 0$
  - Momentum balance $\rho\left[\dfrac{\partial u}{\partial t} + (u \cdot \nabla) u\right] = -\nabla p + \mu \nabla^2 u + \boxed{F_p}$
- Direct numerical simulation (DNS), i.e. turbulence from first principles

NTNU
Norwegian University of Science and Technology

Lagrangian approach - Characteristic parameters

- Prolate ellipsoidal particles
  - a: radius in minor-axis
  - b: radius in major-axis

$$\lambda = \frac{b}{a}$$



- Lagrangian approach
  - Traking the location of each individual particle
- Finite number of particles
  - $\sim 10^6$
- Inertial
  - Particle response time
- Smaller than Kolmogorov length
  - Point force assumption
- Particle Reynolds number

$$\mathrm{Re}_p = \frac{|u_i - v_i| a}{\upsilon} < 1$$

NTNU
Norwegian University of Science and Technology

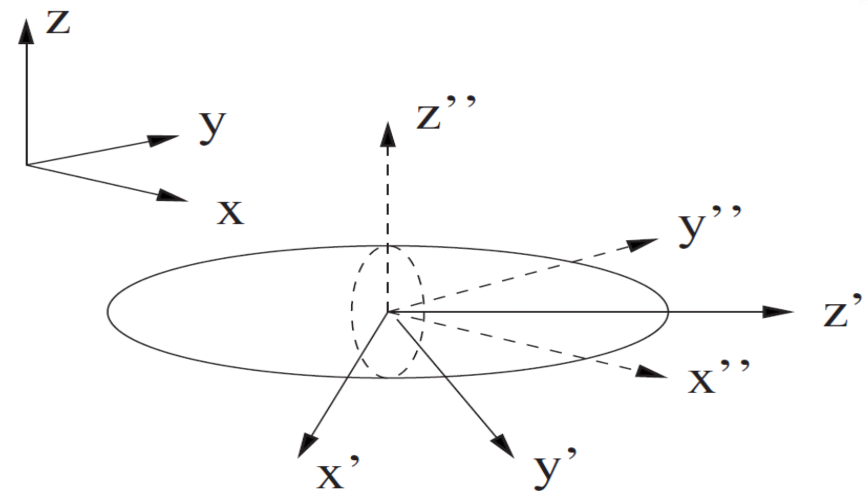
Lagrangian approach - Kinematics

Three different coordinate systems:

1. Laboratory system: [x, y, z]

2. Co-moving frame: [x”, y”, z”]

3. Particle frame: [x’, y’, z’]



Particle response time

- Ellipsoidal particle response time (by Shapiro and Goldenberg):
  - *the time required by the particles to respond to changes in the flow field.*

$$\tau_p^+ = \frac{2\lambda S a^{+^2}}{9\,\mathrm{Re}_\tau} \frac{\ln\left(\lambda + \sqrt{\lambda^2 - 1}\right)}{\sqrt{\lambda^2 - 1}}$$

  - *Density ratio:* $S = \dfrac{\rho_p}{\rho_f}$

NTNU
Norwegian University of Science and Technology

Hydrodynamic torque - Rotational motions

- The rotational motion in the particle frame is governed by

$$I'_{xx}\frac{d\omega'_x}{dt} - \omega'_y\omega'_z(I'_{yy} - I'_{zz}) = N'_x,$$

$$I'_{yy}\frac{d\omega'_y}{dt} - \omega'_z\omega'_x(I'_{zz} - I'_{xx}) = N'_y,$$

$$I'_{zz}\frac{d\omega'_z}{dt} - \omega'_x\omega'_y(I'_{xx} - I'_{yy}) = N'_z,$$

$$I'_{xx} = I'_{yy} = \frac{(1+\lambda^2)ma^2}{5}, \quad I'_{zz} = \frac{2ma^2}{5}.$$

Euler equations

$$N'_x = \frac{16\pi\mu a^3\lambda}{3(\beta_0 + \lambda^2\gamma_0)}[(1-\lambda^2)f' + (1+\lambda^2)(\xi' - \omega'_x)],$$

$$N'_y = \frac{16\pi\mu a^3\lambda}{3(\lambda^2\gamma_0 + \alpha_0)}[(\lambda^2 - 1)g' + (\lambda^2 + 1)(\eta' - \omega'_y)],$$

$$N'_z = \frac{32\pi\mu a^3\lambda}{3(\alpha_0 + \beta_0)}(\chi' - \omega'_z),$$

where f' and g' are the fluid rates of strain coefficients ξ', η', and χ' are the fluid rotation rate coefficients

Torque components for an ellipsoid subjected to linear shear under creeping flow by Jeffery 1922

Description of rotational motion

- Time evolution of Euler parameters

$$\begin{pmatrix} \dot{e}_0 \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e_0 & -e_1 & -e_2 & -e_3 \\ e_1 & e_0 & -e_3 & e_2 \\ e_2 & e_3 & e_0 & -e_1 \\ e_3 & -e_2 & e_1 & e_0 \end{pmatrix} \begin{pmatrix} 0 \\ \omega'_x \\ \omega'_y \\ \omega'_z \end{pmatrix}.$$

- Constraint

$$e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$$

NTNU
Norwegian University of Science and Technology

Hydrodynamic drag force (Brenner)- Translational motion

- The translational motion in the laboratory frame is governed by

$$m\frac{dv_i}{dt} = f_i(x_p)$$

- The drag force in creeping flow conditions is given by:

$$f_i(x_p) = \mu A^{-1} K_{ij}^{'} A(u_j - v_j) \qquad K_{ij}^{'} = \begin{pmatrix} K^{'}_{xx} & 0 & 0 \\ 0 & K^{'}_{yy} & 0 \\ 0 & 0 & K^{'}_{zz} \end{pmatrix}$$

  - Resistance tensor K ' is an intrinsic property of the particle. Particle orientation is absorbed in the resistance tensor.
  - A is the orthogonal transformation matrix comprised the direction cosines

Two-way coupled method (Force-coupling)

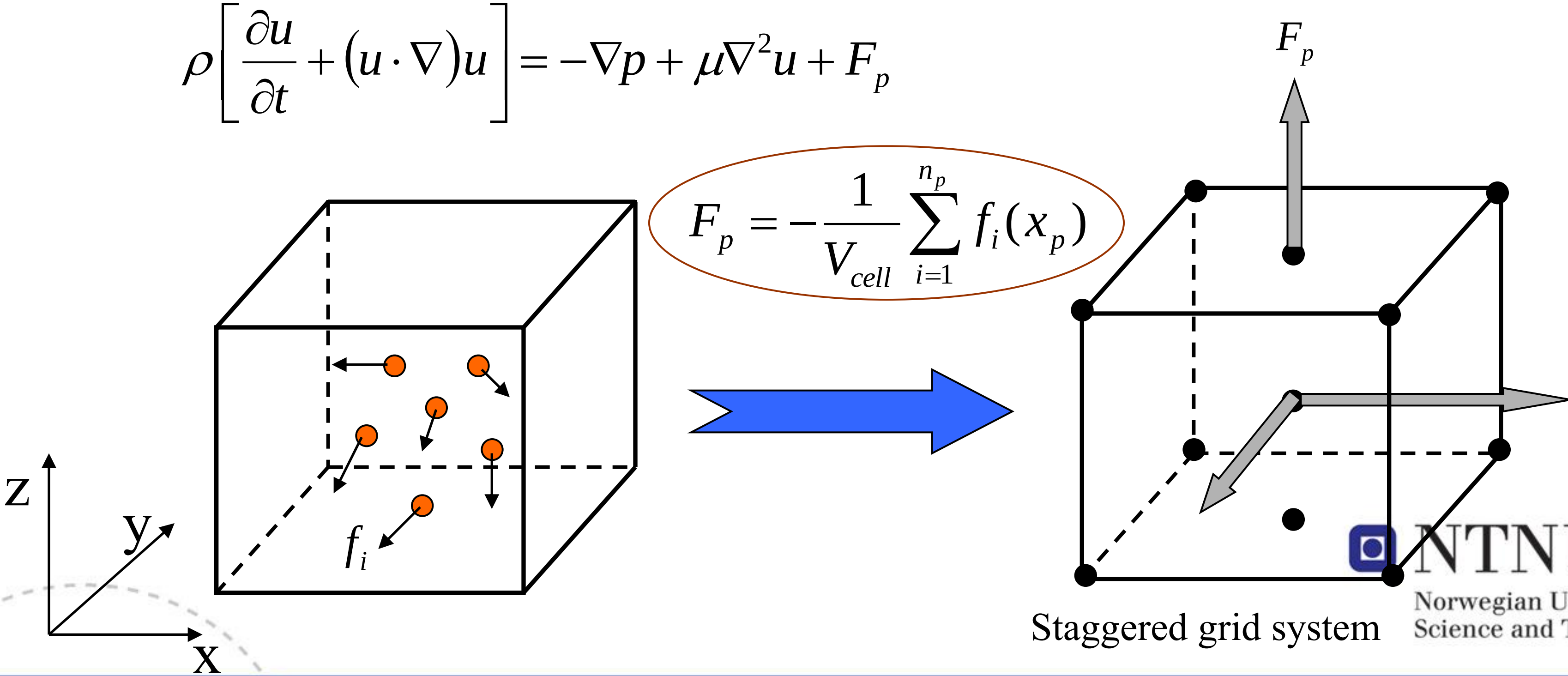
- Drag force is the point-force on the particle.

$$f_i(x_p) = \mu A^{-1} K_{ij}^{'} A(u_j - v_j)$$

- Volume force $\vec{F}_p$ in the momentum equation of fluid:

$$\rho\left[\frac{\partial u}{\partial t} + (u \cdot \nabla)u\right] = -\nabla p + \mu \nabla^2 u + F_p$$

$$F_p = -\frac{1}{V_{cell}} \sum_{i=1}^{n_p} f_i(x_p)$$



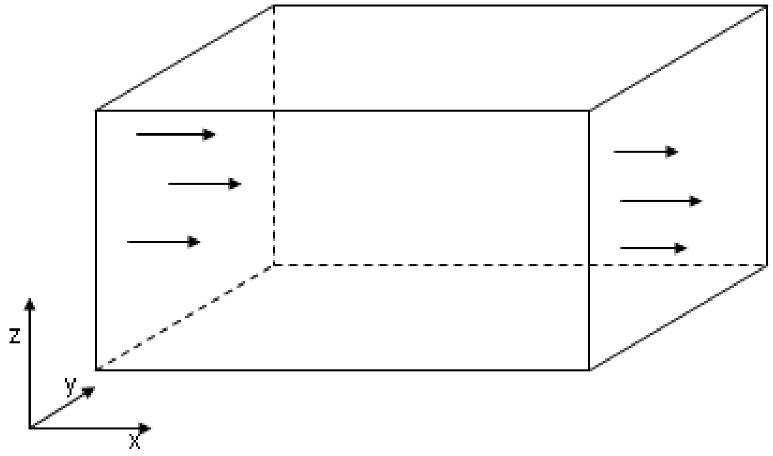
Staggered grid system

NTNU
Norwegian University of Science and Technology

Computation conditions

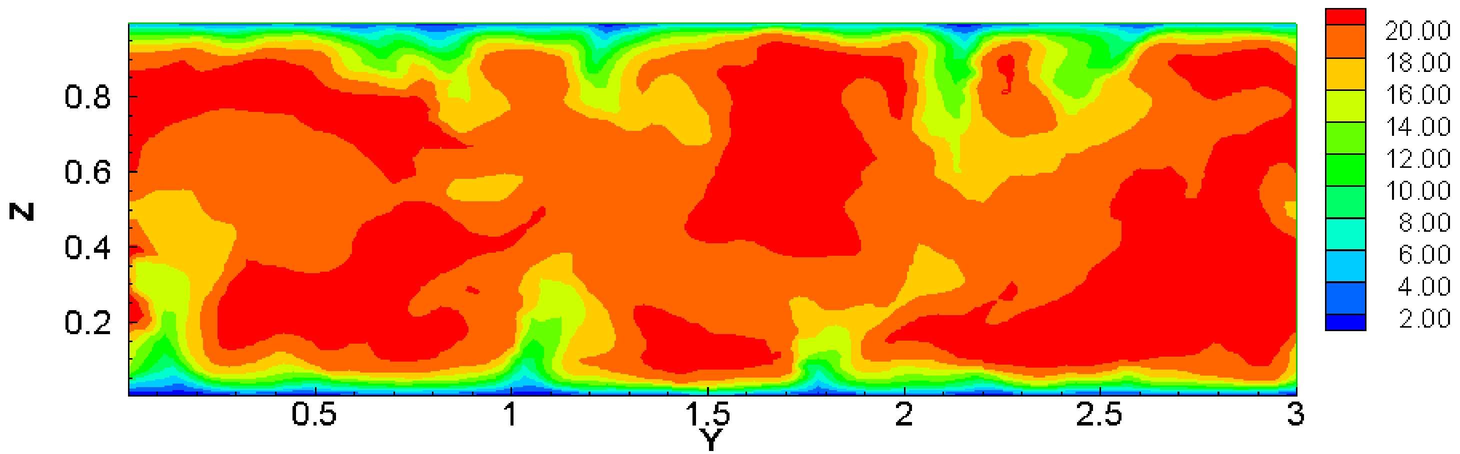
- Re_{τ} =360
- Channel model
  - 6h*3h*h (x*y*z)

- Particle number 4 million
- Radius a =0.001
- Aspect ratio 3
- Density ratio=557.1
- Translational response time 30

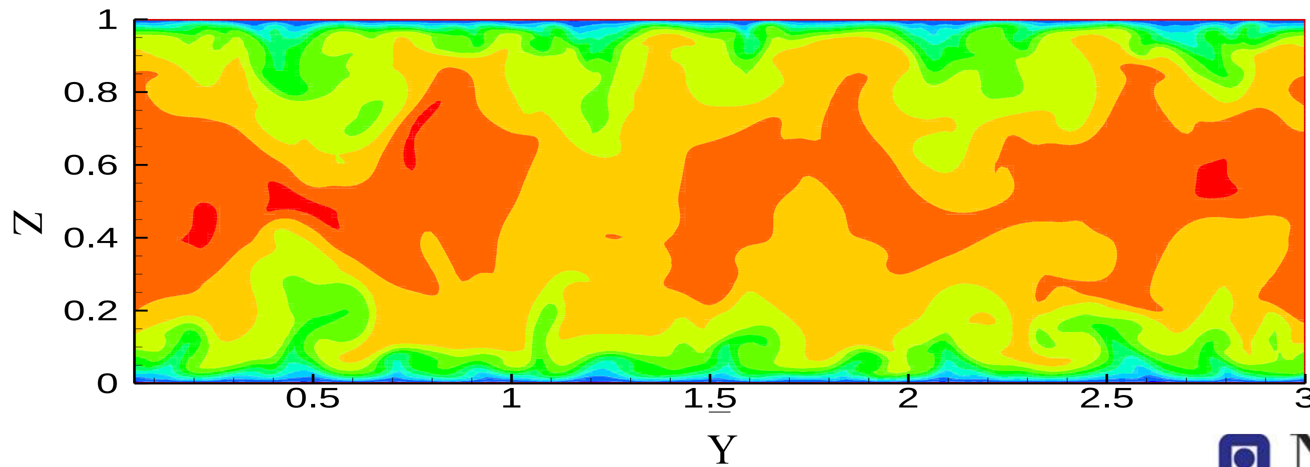


NTNU
Norwegian University of Science and Technology

Instantaneous contours

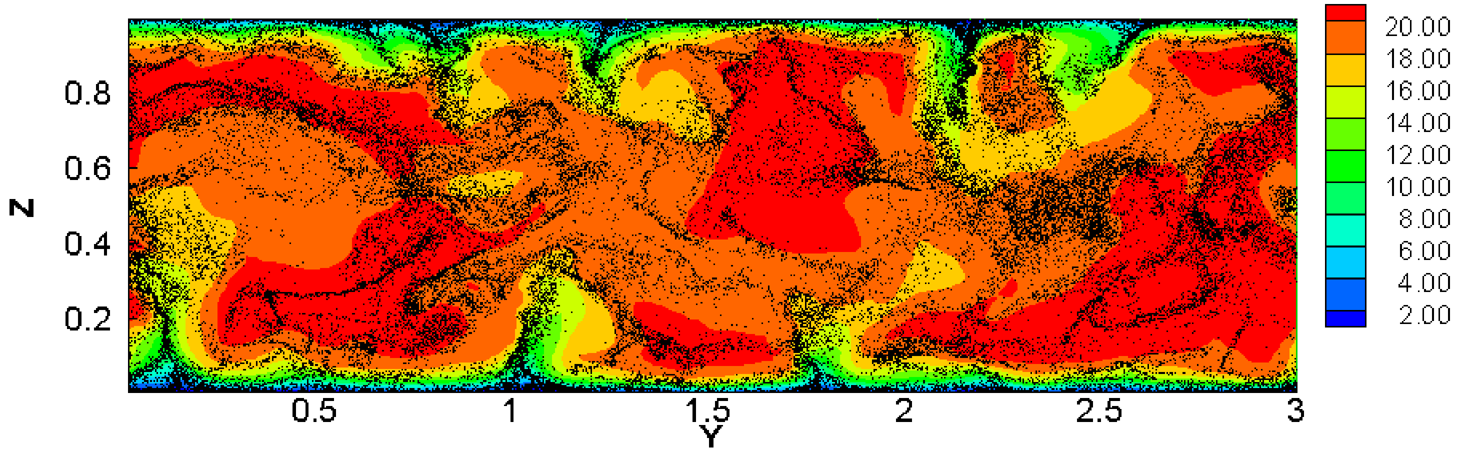


Streamwise velocity contour in YZ plane (With fiber)

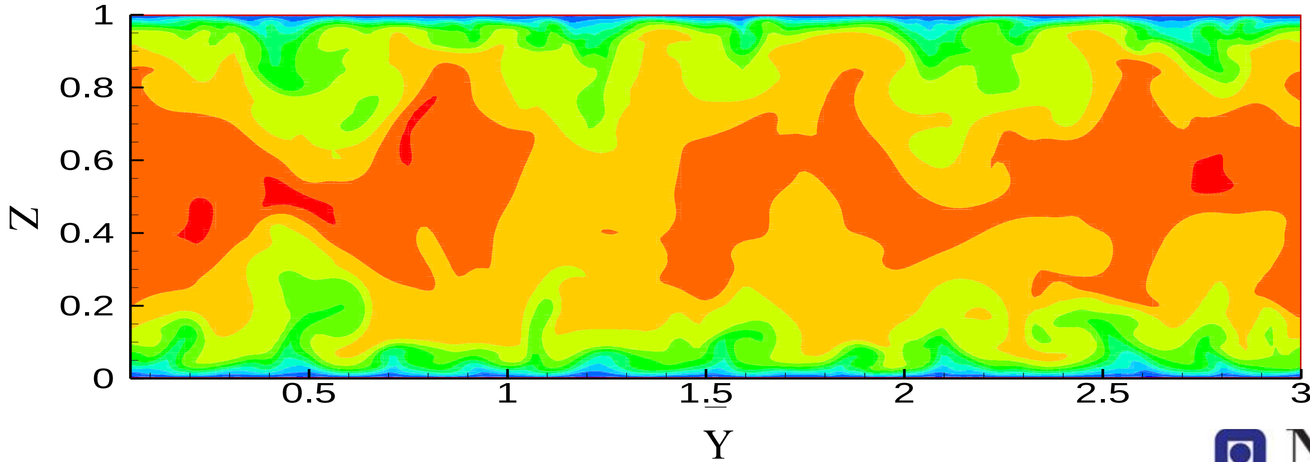


Streamwise velocity contour in YZ plane (Without fiber)

Instantaneous contours



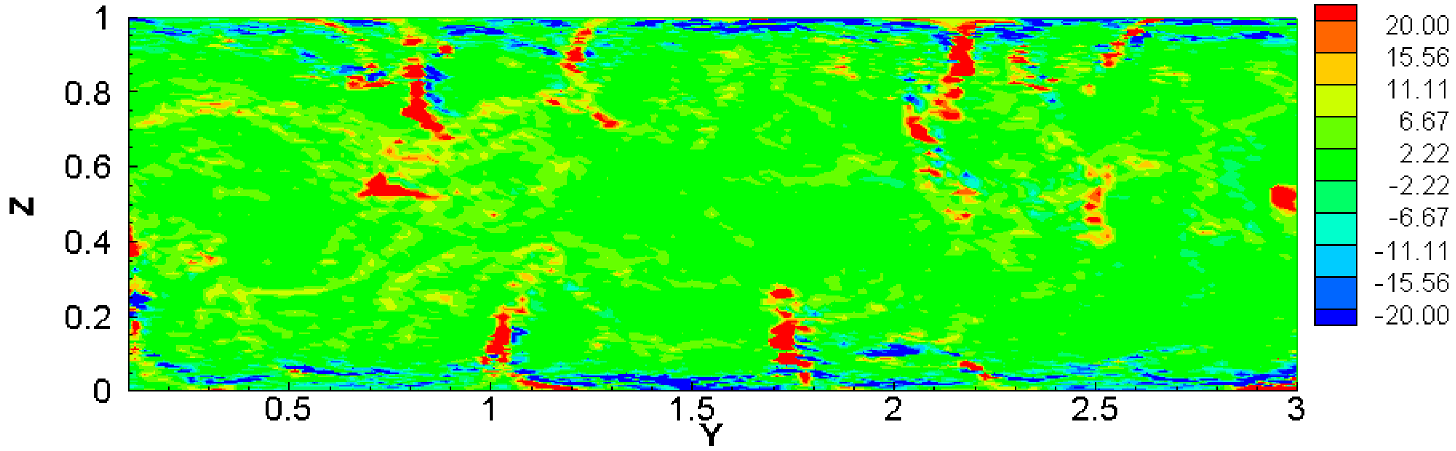
Streamwise velocity contour in YZ plane



Streamwise velocity contour in YZ plane (Without fiber)

NTNU
Norwegian University of Science and Technology

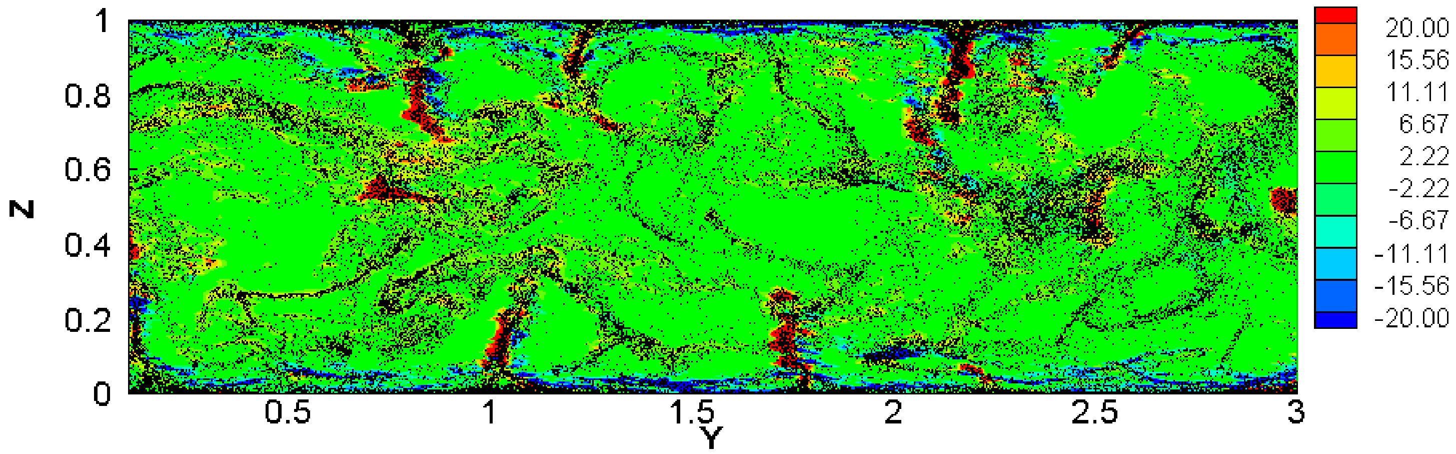
Instantaneous contours



Streamwise drag force (on the particle) contour in YZ plane

NTNU
Norwegian University of Science and Technology

Instantaneous contours



Streamwise drag force (on the particle) contour in YZ plane

Statistical results

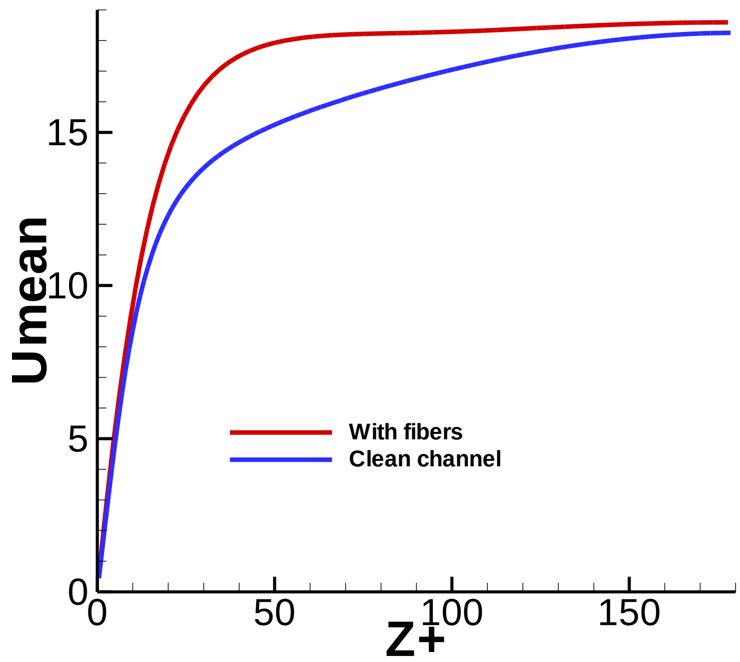


Fig. Comparison of mean streamwise velocity between fiber suspension flow and the flow without particles

Statistical results

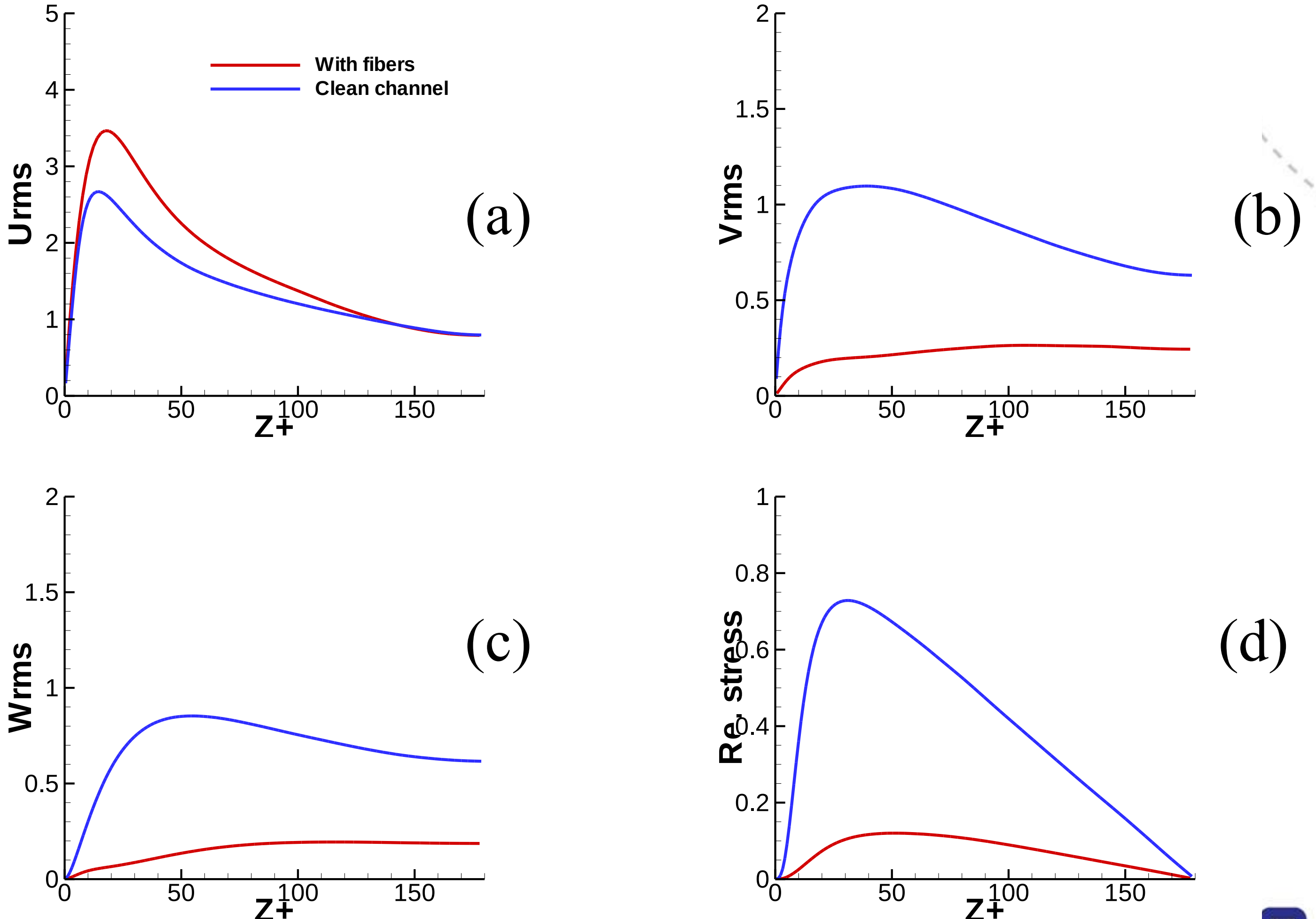


Fig. Comparison of streamwise, spanwise and wall-normal fluctuation velocities and Reynolds stresses between fiber suspension flow and the flow without particles

NTNU
Norwegian University of Science and Technology

Summary

- The force-coupling scheme was tested with a low aspect ratio case
- We found out the modulations on the turbulence in the channel flow by ellipsoids:
  - Drag reduction
  - Enhancement in the streamwise turbulence intensity
  - Damping effect on the spanwise, wall-normal directions turbulence intensities and also Reynolds stresses

NTNU
Norwegian University of Science and Technology

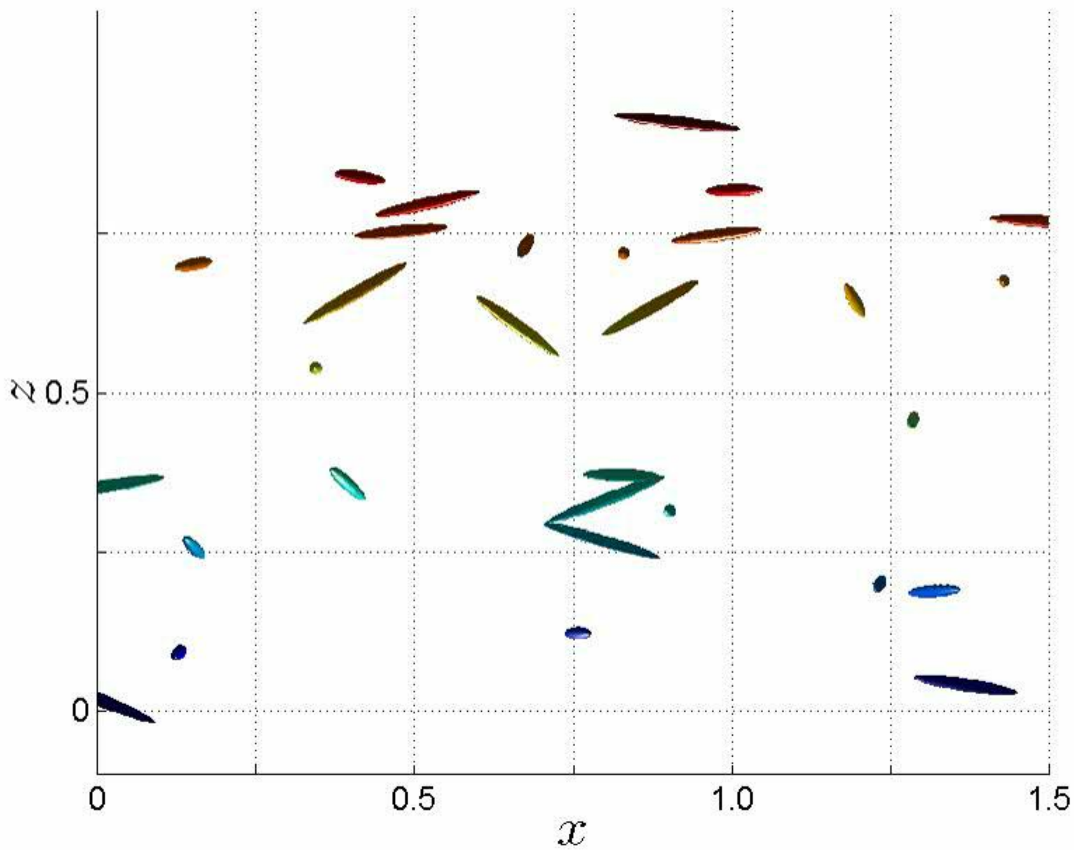
Some further directions

Improved physical realism by:

- Two-way coupling: torques *
- Implement additional force terms
- Improved wall-interaction modelling

(* Fri. 09:00 – 09:20 M. Barri ‘New scheme for torque coupling’)

Thank you!



Simulation procedure

1. Initial positions and orientations (Euler angles) of ellipsoids and initial velocity conditions for particle velocities and angular velocities are specified.
2. Euler parameters are evaluated.
3. Obtain transformation matrix A.
4. Calculate resistance tensor.
5. Solve equations for translational and rotational motion for calculating the new particle positions and Euler's parameters.
6. Return to step 3 and continue procedure until desired time period is reached.

$$\mathbf{F} = \mu \mathbf{A}^t \mathbf{K}' \mathbf{A}(\mathbf{u} - \mathbf{v}), \quad (6)$$

where $\mu = \rho\nu$ is the dynamic viscosity of the fluid. For an ellipse of revolution about the z'-axis, the resistance tensor $\mathbf{K}'$ is

$$\mathbf{K}' = \begin{pmatrix} k'_{xx} & 0 & 0 \\ 0 & k'_{yy} & 0 \\ 0 & 0 & k'_{zz} \end{pmatrix}, \quad (7)$$

where k'_{xx}, k'_{yy}, and k'_{zz} are the components along the x', y', and z' axes (principal directions), respectively, and are given as[31]

$$k'_{xx} = k'_{yy} = \frac{16\pi a(\lambda^2 - 1)^{3/2}}{(2\lambda - 3)\ln[\lambda + (\lambda^2 - 1)^{1/2}] + \lambda(\lambda^2 - 1)^{1/2}}, \quad (8)$$

$$k'_{zz} = \frac{8\pi a(\lambda^2 - 1)^{3/2}}{(2\lambda - 1)\ln[\lambda + (\lambda^2 - 1)^{1/2}] + \lambda(\lambda^2 - 1)^{1/2}}. \quad (9)$$

Rotation description of fibre

- Relation between Euler angles and Euler parameters

$$e_0 = \cos\frac{\phi+\psi}{2}\cos\frac{\theta}{2}$$
$$e_1 = \cos\frac{\phi-\psi}{2}\sin\frac{\theta}{2}$$
$$e_2 = \sin\frac{\phi-\psi}{2}\sin\frac{\theta}{2}$$
$$e_3 = \sin\frac{\phi+\psi}{2}\cos\frac{\theta}{2}$$

- Constraint

$$e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$$

NTNU
Norwegian University of Science and Technology

$$x' = Ax''$$

$$A^{-1}x' = A^{-1}Ax''$$

$$x'' = A^{-1}x'$$

$$\boxed{A^T = A^{-1}}$$

$$x'' = A^T x'$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{11} = e_0^2 + e_1^2 - e_2^2 - e_3^2, \quad a_{12} = 2(e_1e_2 + e_0e_3),$$

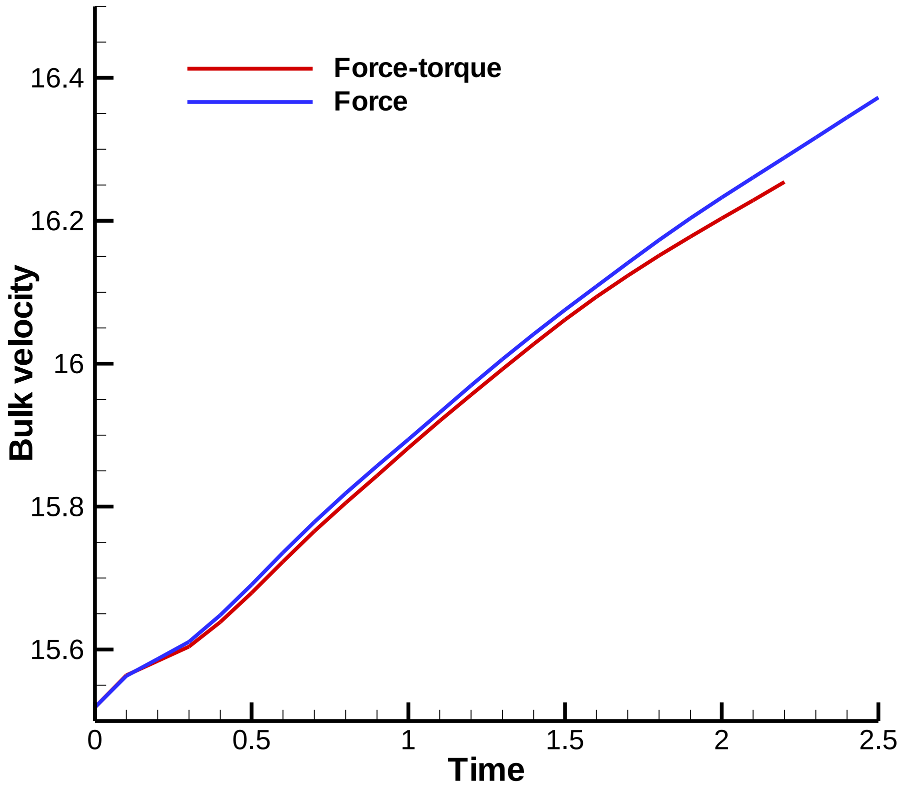
$$a_{13} = 2(e_1e_3 - e_0e_2), \quad a_{21} = 2(e_1e_2 - e_0e_3),$$

$$a_{22} = e_0^2 - e_1^2 + e_2^2 - e_3^2, \quad a_{23} = 2(e_2e_3 + e_0e_1),$$

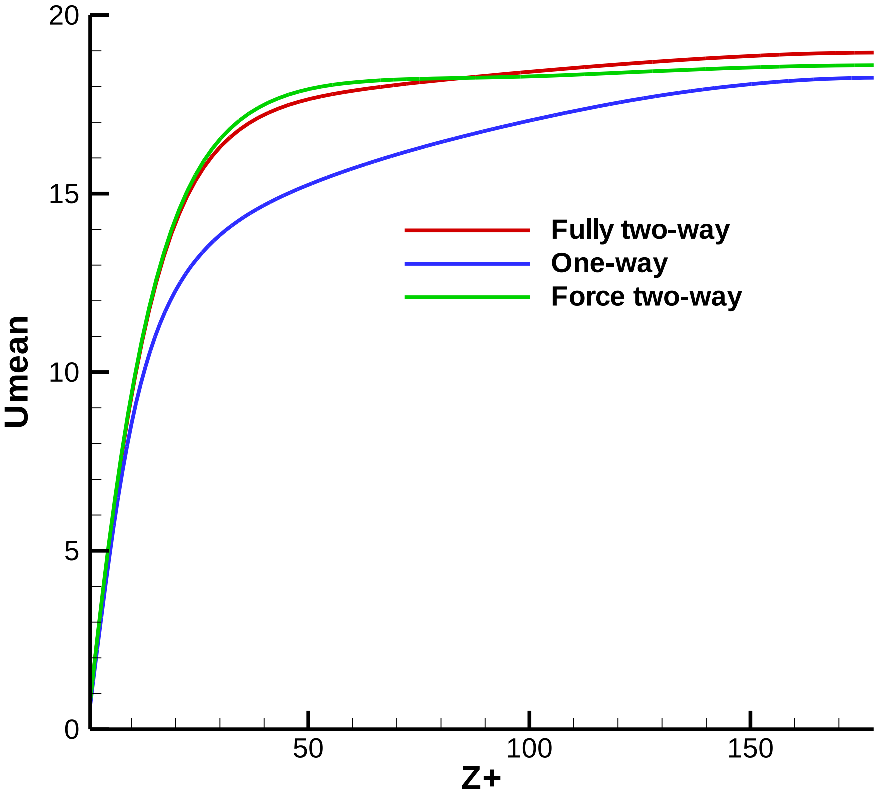
$$a_{31} = 2(e_1e_3 + e_0e_2), \quad a_{32} = 2(e_2e_3 - e_0e_1),$$

$$a_{33} = e_0^2 - e_1^2 - e_2^2 + e_3^2.$$

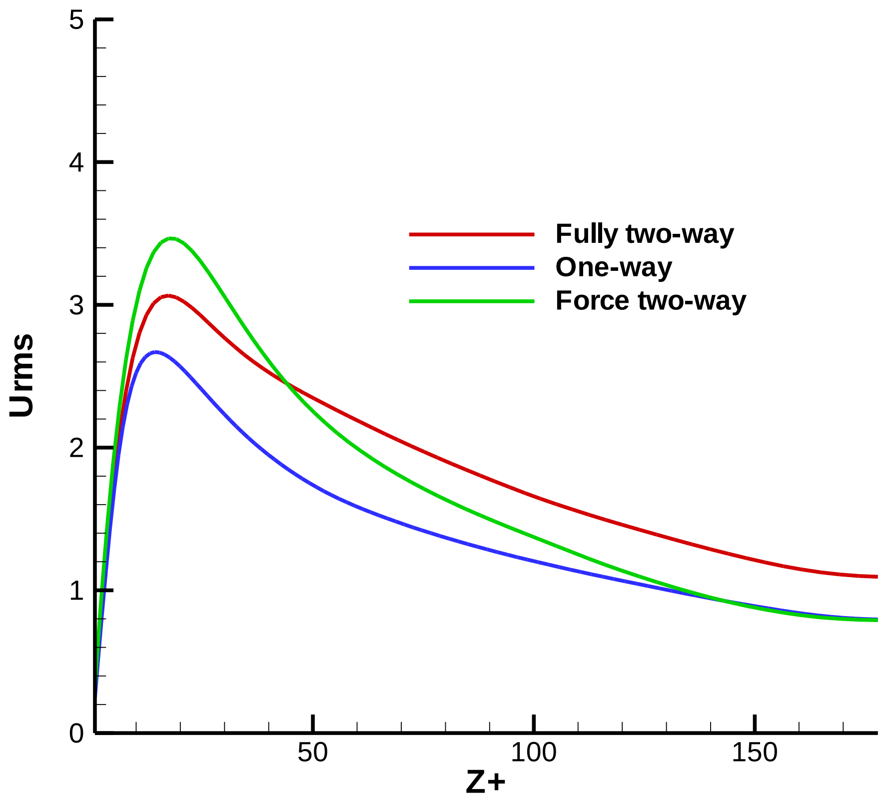
- A is the orthogonal transformation matrix comprised the direction cosines



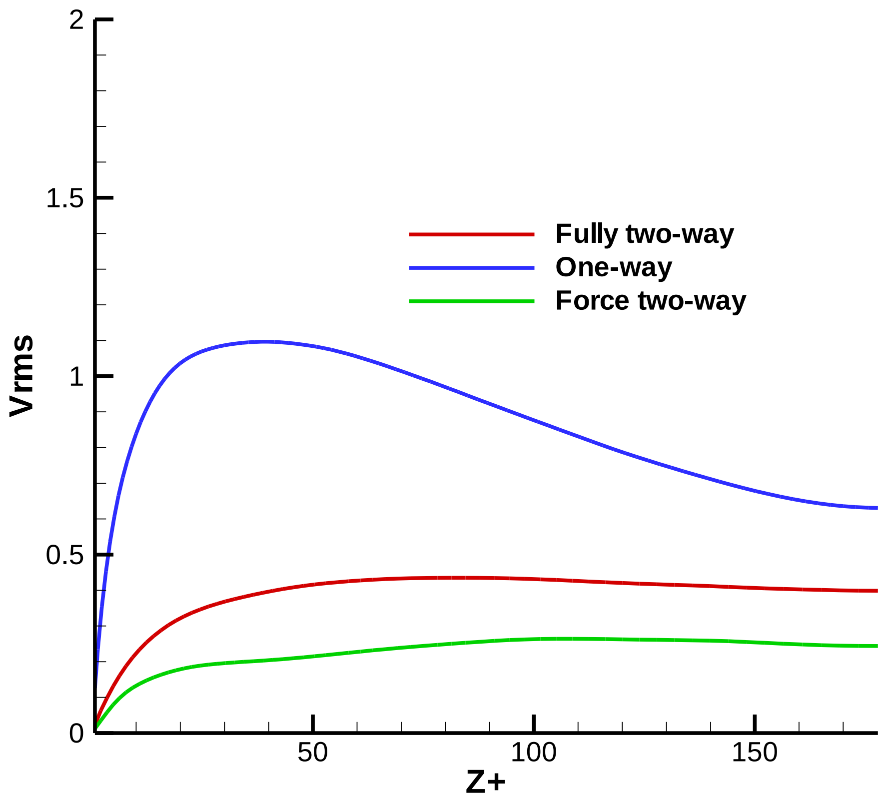
NTNU
Norwegian University of Science and Technology



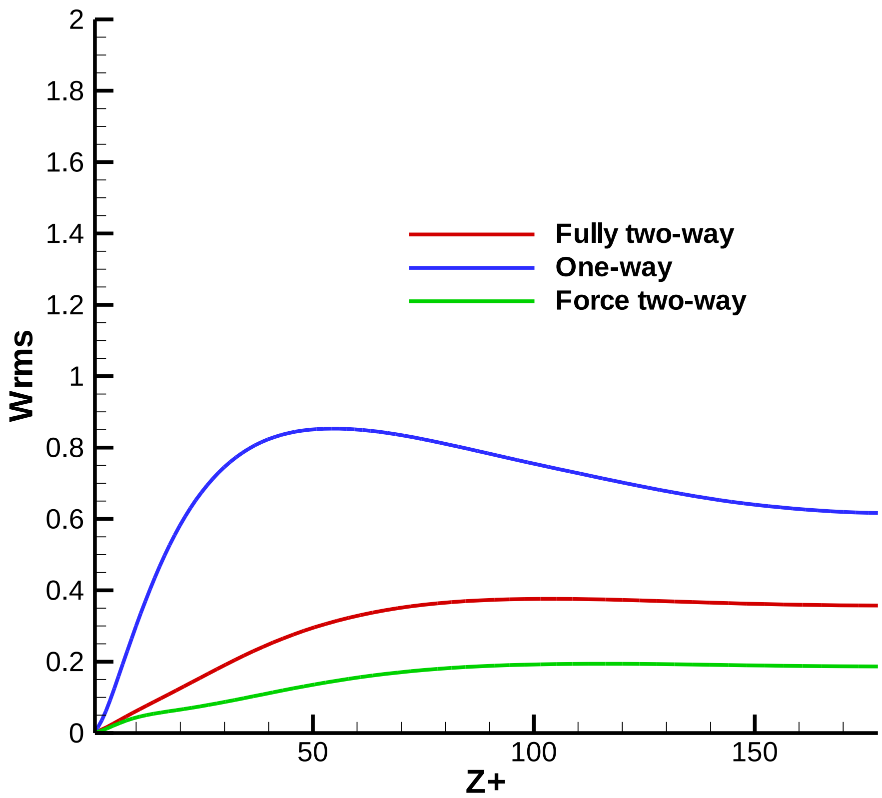
20
15
10
5
0
Umean
50
100
150
Z+
Fully two-way
One-way
Force two-way



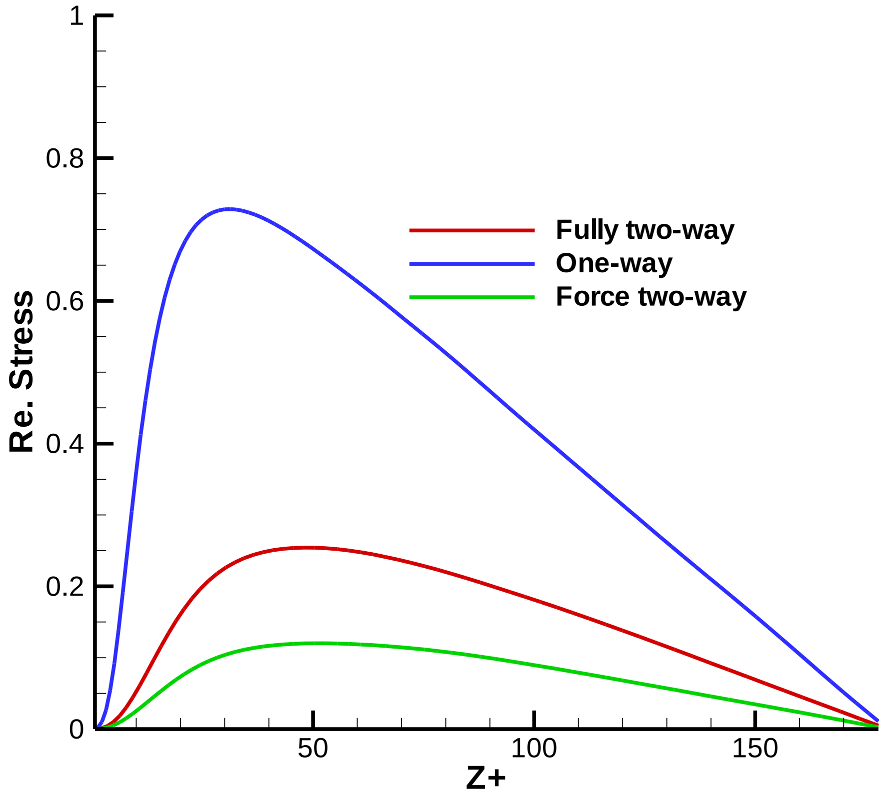
NTNU
Norwegian University of Science and Technology



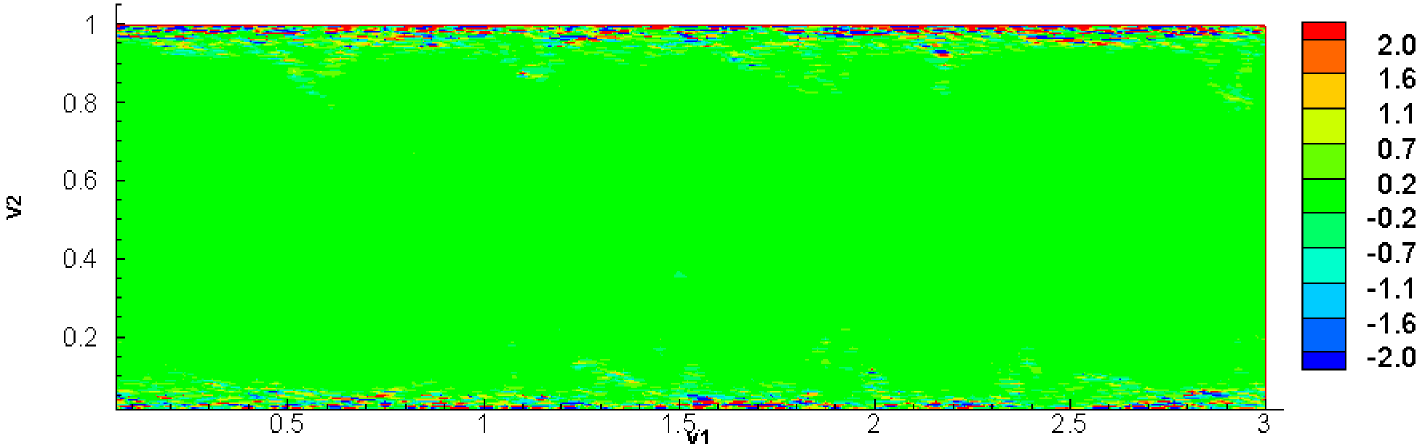
NTNU
Norwegian University of Science and Technology

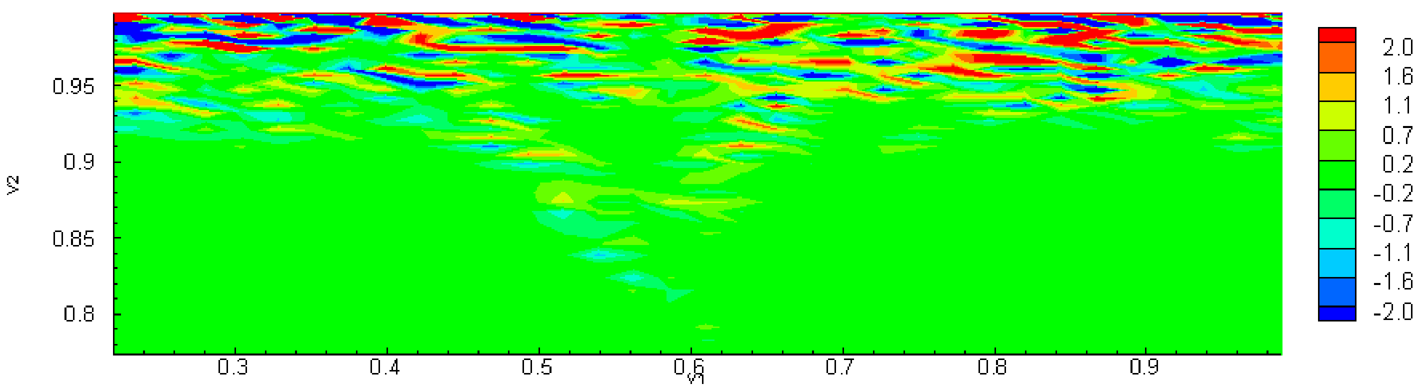


NTNU
Norwegian University of
Science and Technology



NTNU
Norwegian University of
Science and Technology





NTNU
Norwegian University of Science and Technology