Disturbance growth during sedimentation in dilute fibre suspension

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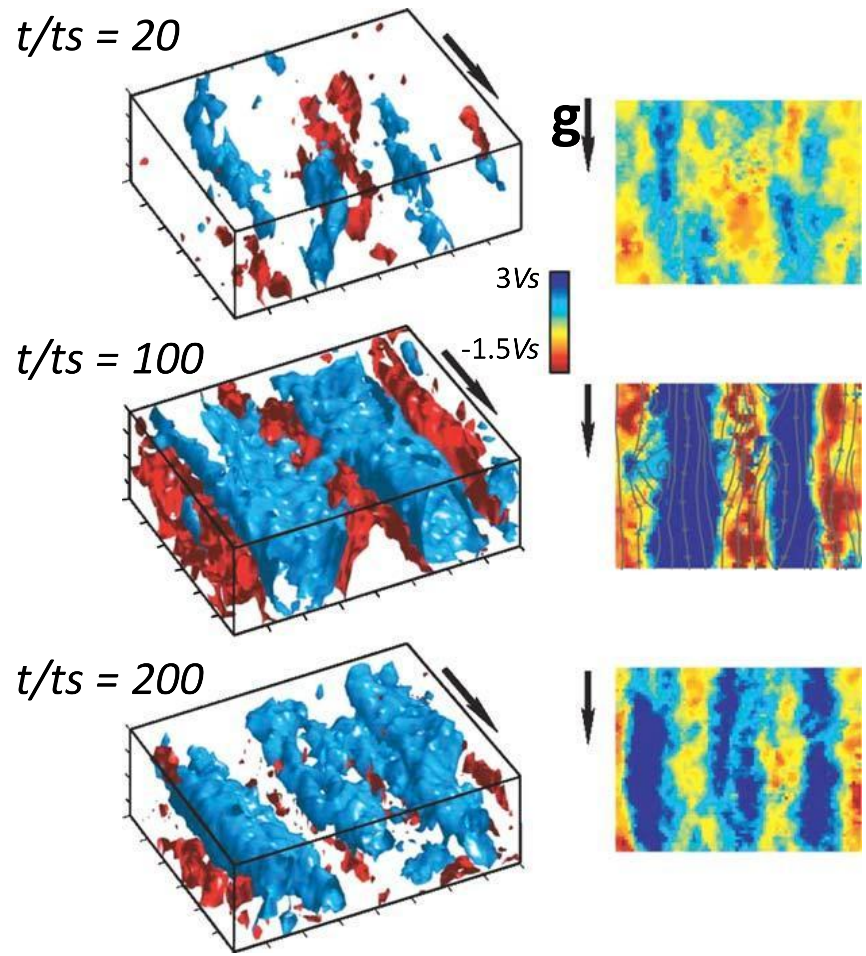
Sedimentation of fibres suspension

- instability : initially well-mixed suspension becomes inhomogeneous
- Clusters and streamers

- Experimental observation of the structure evolution
  – Blue: high density downwards streamer
  – Red: low density backflow

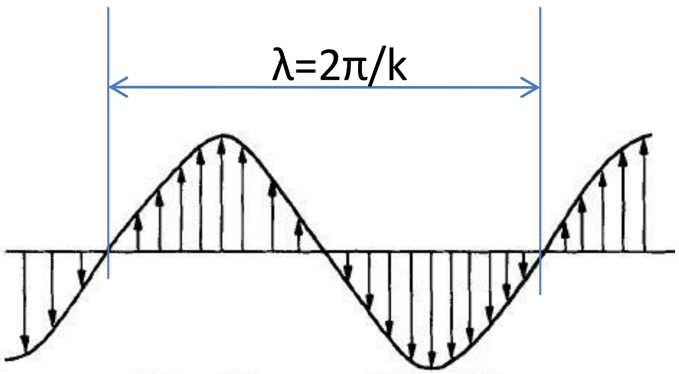
g : gravity
Vs : Stokes velocity



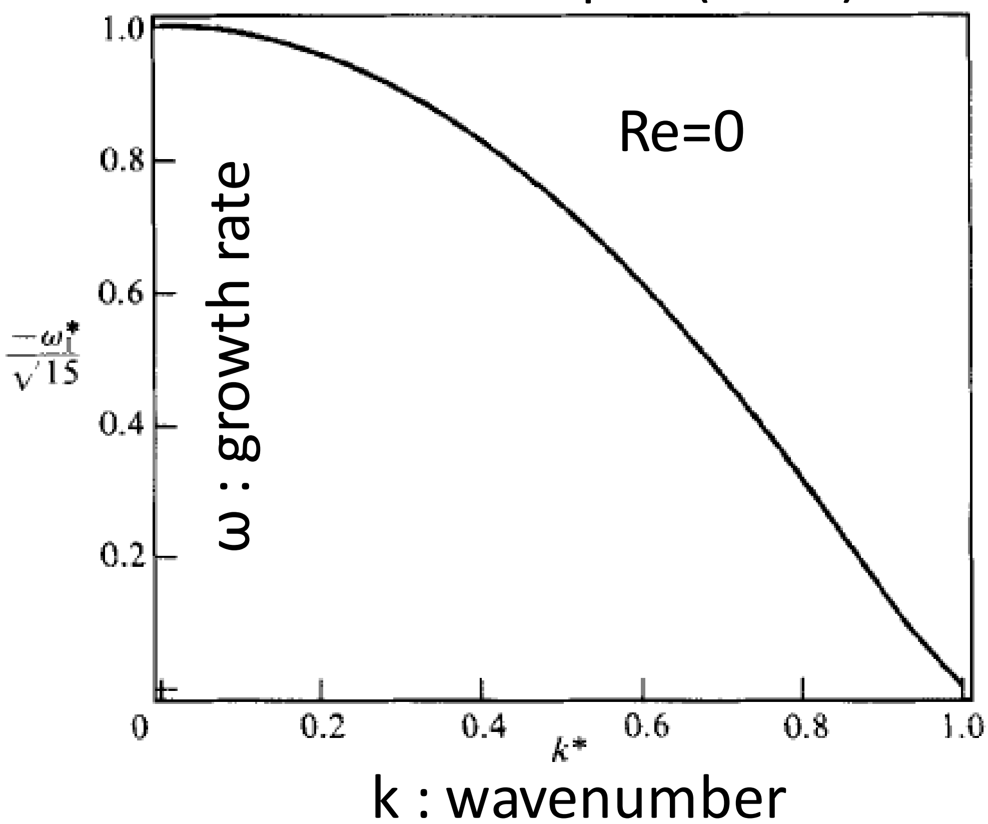
Metzger et al (2007)

Sedimentation of fibres suspension

Normal mode perturbation in particle number density



Koch & Shaqfeh (1989)



- normal mode density perturbation with the maximum growth rate are those of infinite horizontal wavelength.
- in the absence of inertia and diffusive effects.

Formulation

Fokker-Planck Equation

$$\frac{\partial \Psi}{\partial t} + \frac{\partial}{\partial \phi}\left(V_\phi \Psi - D_\phi \frac{\partial \Psi}{\partial \phi}\right) + \frac{\partial}{\partial y}\left[V_y \Psi - (D_s + D_\gamma)\frac{\partial \Psi}{\partial y}\right] = 0$$

- Ψ : bulk particle density in orientational space (Φ) and physical space (y)

V_ϕ : Rotational velocity

V_y : Linear velocity

D_ϕ : Orientation diffusion

D_s : Self-diffusion (linear)

D_γ : Shear-induced diffusion

Navier-Stokes Equation

$$\rho_{mix}\frac{\partial u}{\partial t} = \frac{\partial P}{\partial z} - \rho_{mix} g + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\mathrm{Re}_L = \frac{VL}{\nu}$$

normal mode particle number perturbation

$$\Psi(y,\phi,t) = 1 + \Psi'(k,\phi,\sigma)e^{\sigma t + ik \bullet y}$$

- σ : growth rate
- k : wavenumber

this drives a fluid velocity field

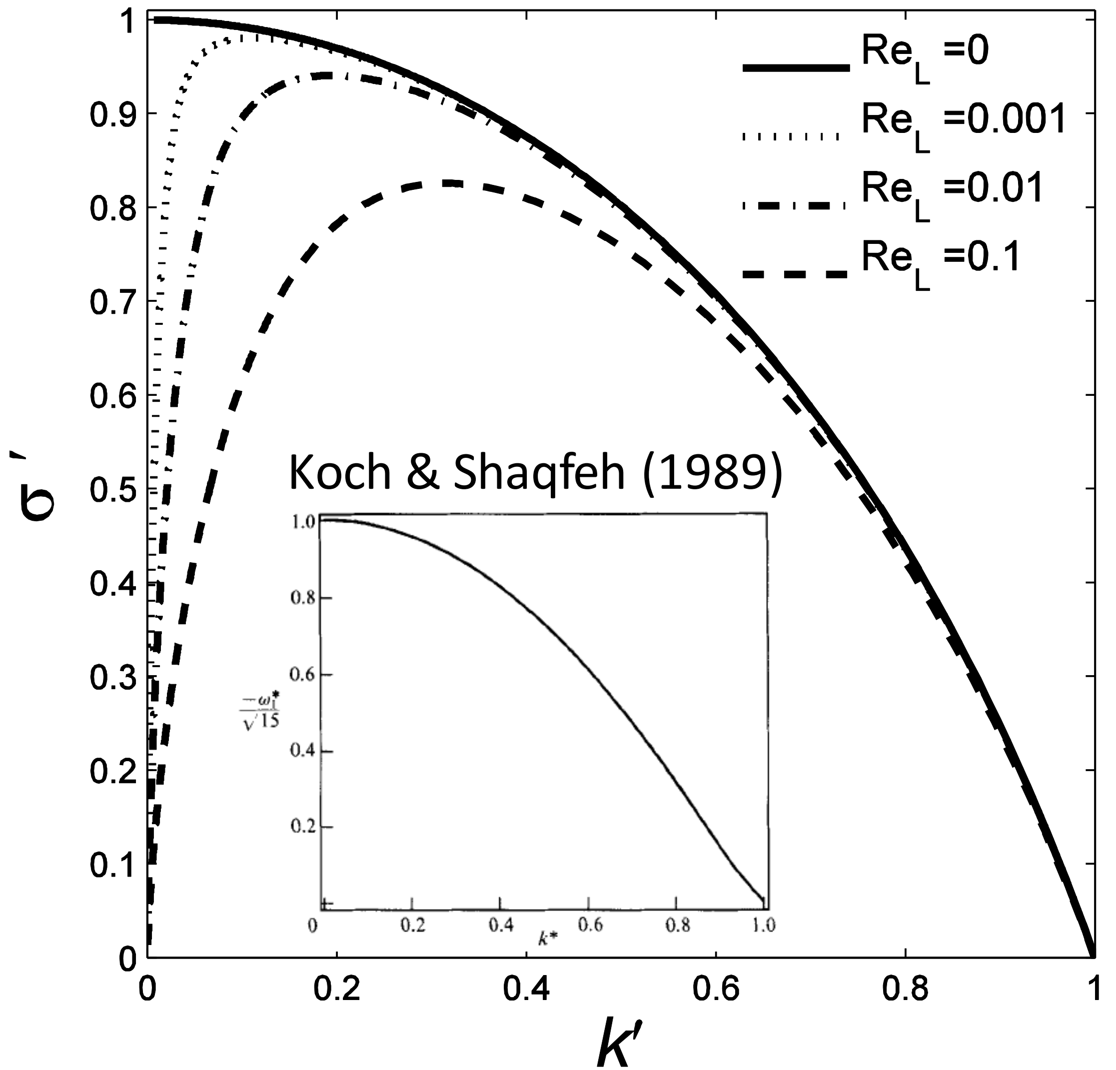
$$u(y,t) = u' e^{\sigma t + ik \bullet y}$$

Dispersion relation

$$1 + \frac{\hat{k}^2}{Re_L \hat{\sigma} + \hat{k}^2} \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin 2\phi \, \dot{\hat{y}}}{(\hat{\sigma} + \hat{D}_y^s \, \hat{k}^2)^2 + (\dot{\hat{y}} \, \hat{k})^2} \, d\phi = 0$$

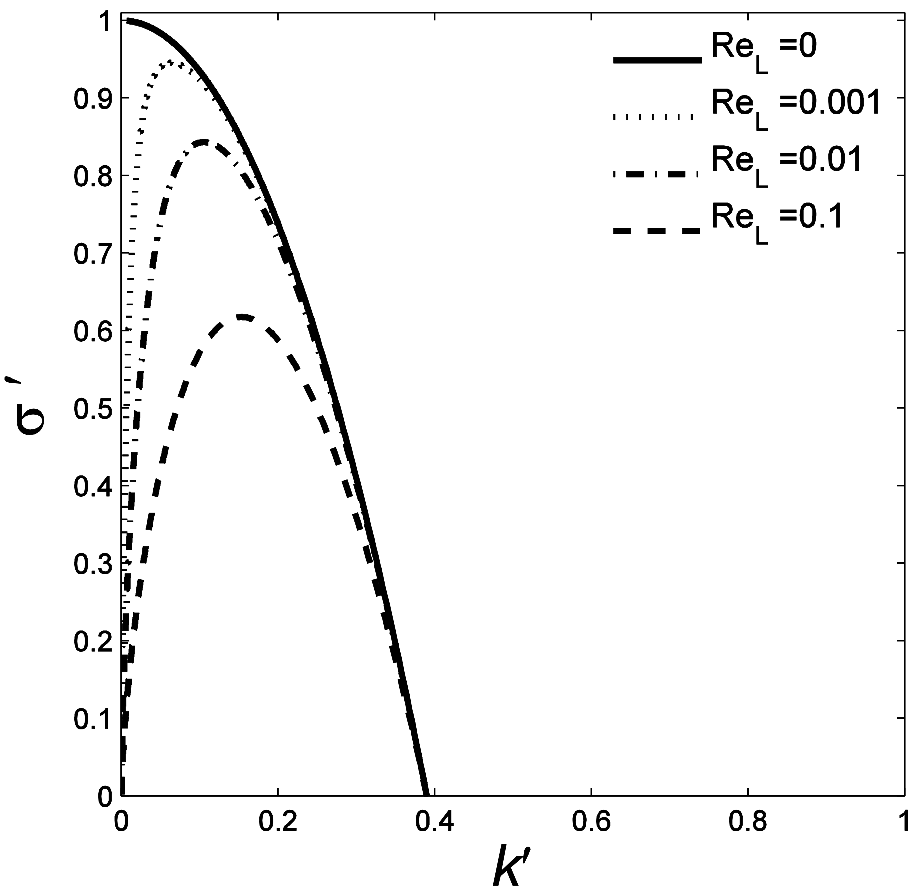
$\hat{D}_y^s$: nondimensional self-diffusion

Effect of the inertia (ReL)



The inertia rapidly decays the growth rate at low wavenumbers

Effect of the self-diffusion (Ds)



- self-diffusion reduces the density perturbation in the range of large wavenumbers
- it shrinks the range of unstable wavenumber.

coupling calculation

- Fokker-Planck Equation

$$\frac{\partial \Psi}{\partial t} + \frac{\partial}{\partial \phi}(V_\phi \Psi - D_\phi \frac{\partial \Psi}{\partial \phi}) + \frac{\partial}{\partial y}[V_y \Psi - (D_s + D_\gamma)\frac{\partial \Psi}{\partial y}] = 0$$

$\Psi(y)$

$\rho_{mix}(y)$

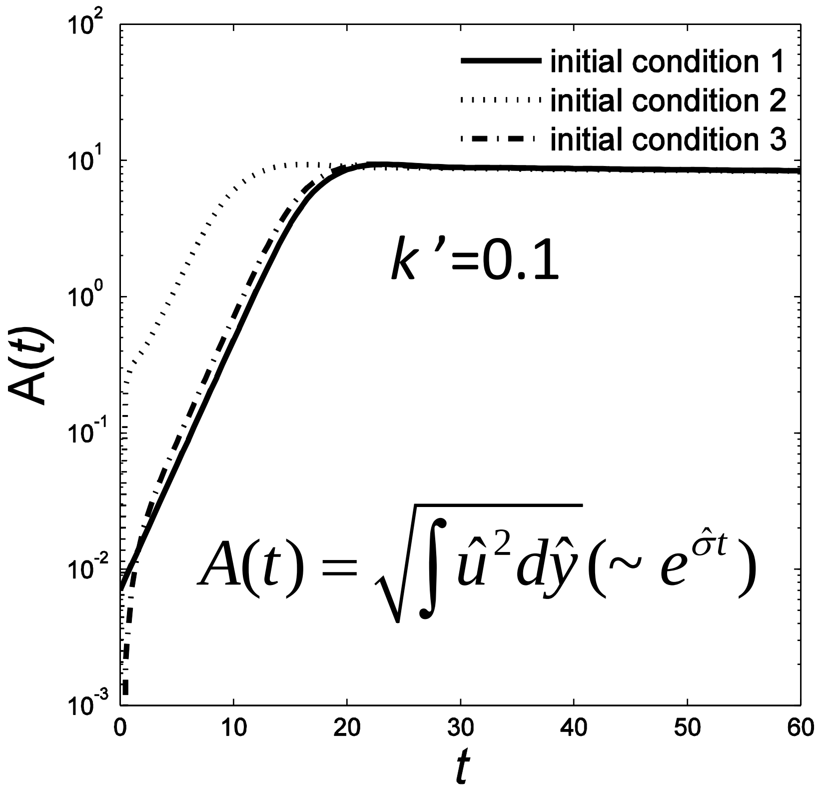
γ

u

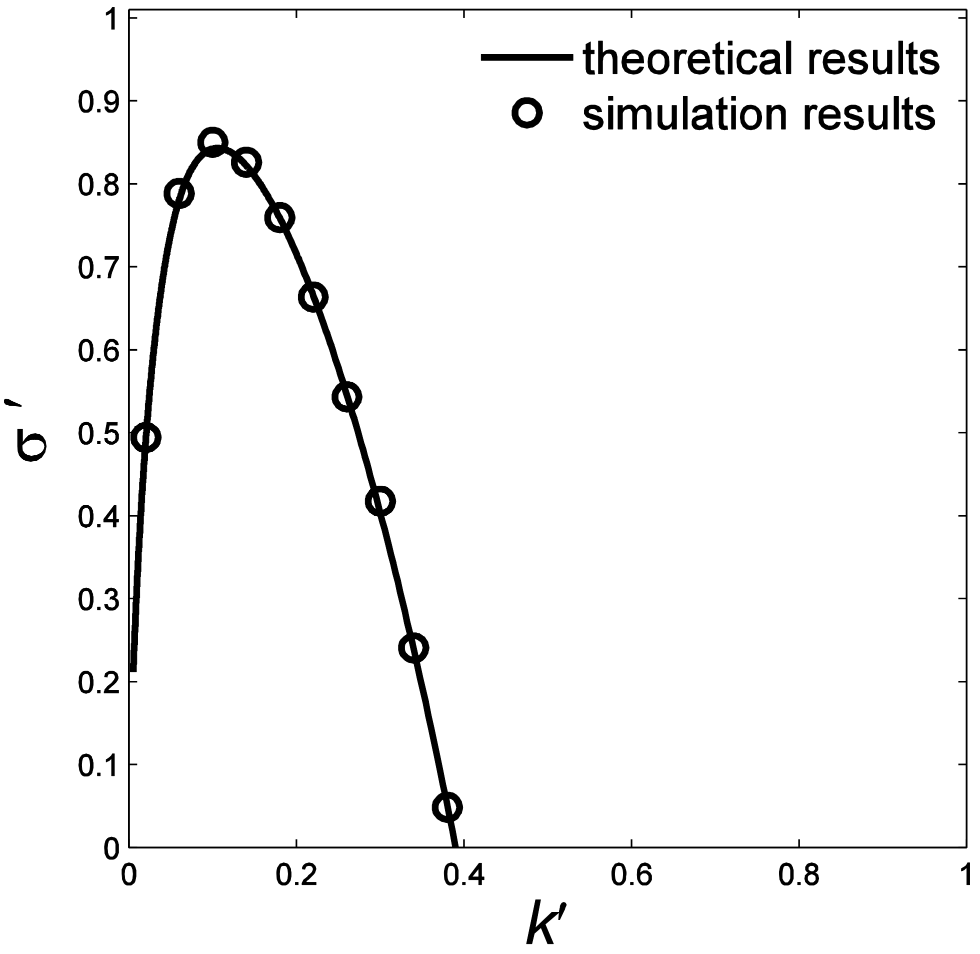
- Navier-Stokes Equation

$$\rho_{mix}\frac{\partial u}{\partial t} = \frac{\partial P}{\partial z} - \rho_{mix} g + \mu \frac{\partial^2 u}{\partial y^2}$$

Nonlinear evolution of the perturbation

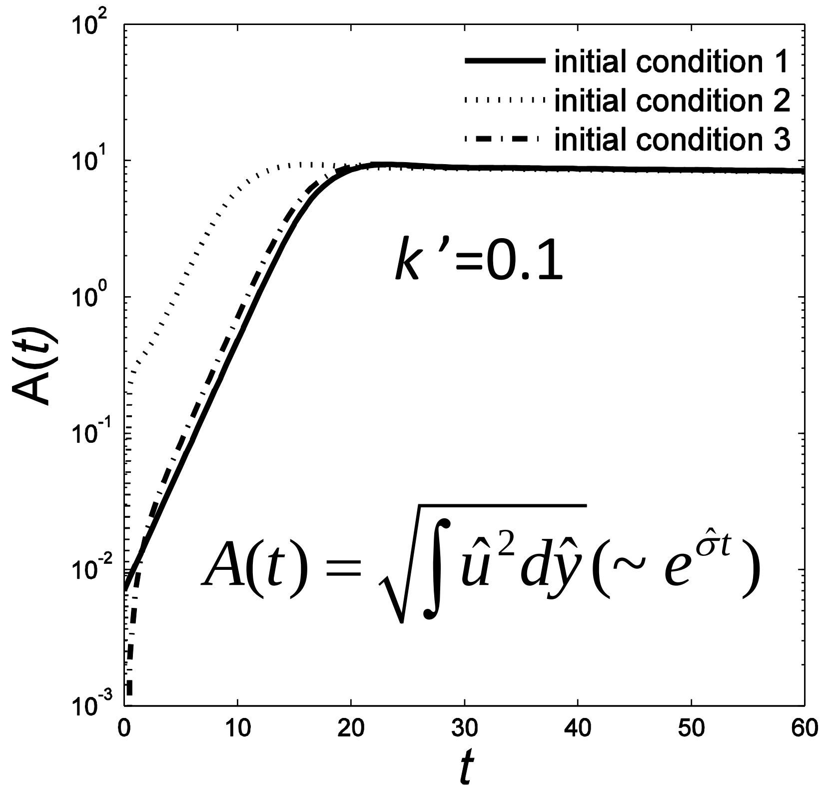


- the slopes of the lines are the growth rates
- The growth rates are independent of the ICs.



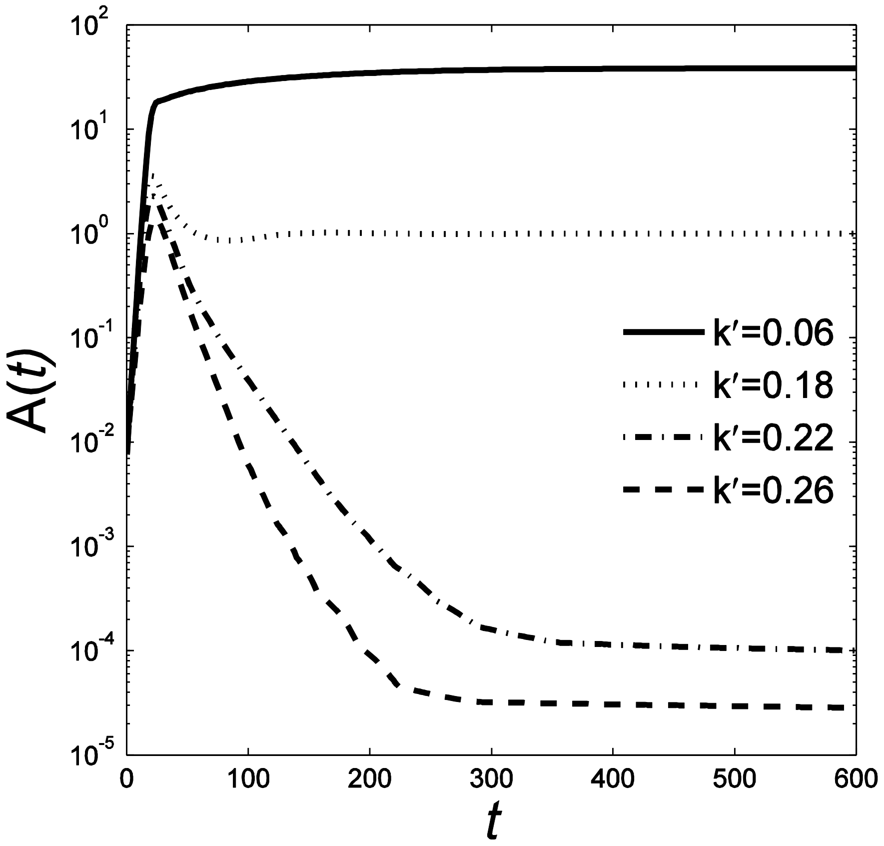
- At early stage, the linear terms dominate the evolution

Nonlinear evolution of the perturbation



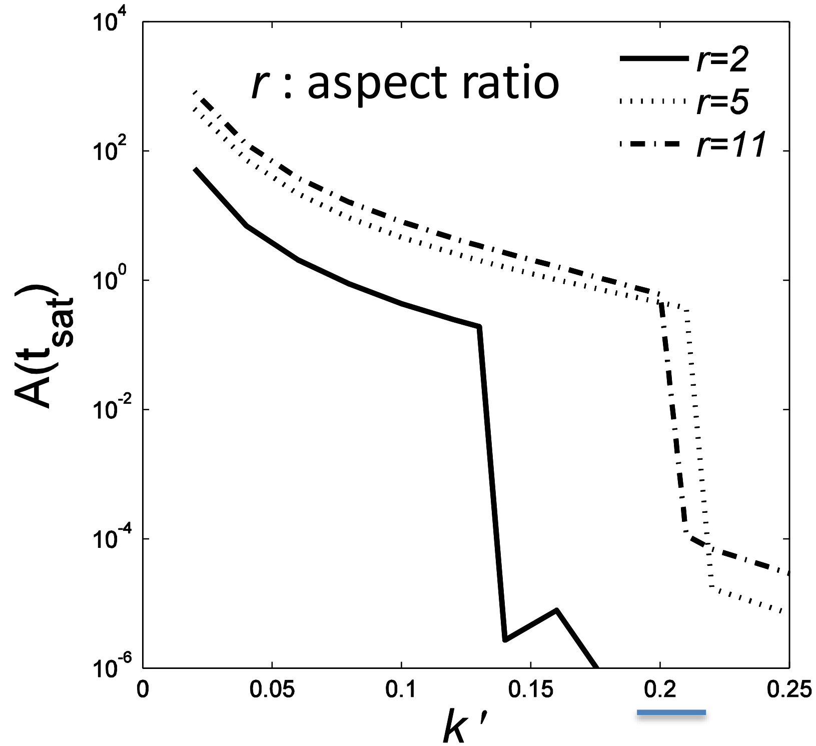
- The saturated values of the perturbation are independent of ICs.
- And Reynolds number *ReL*

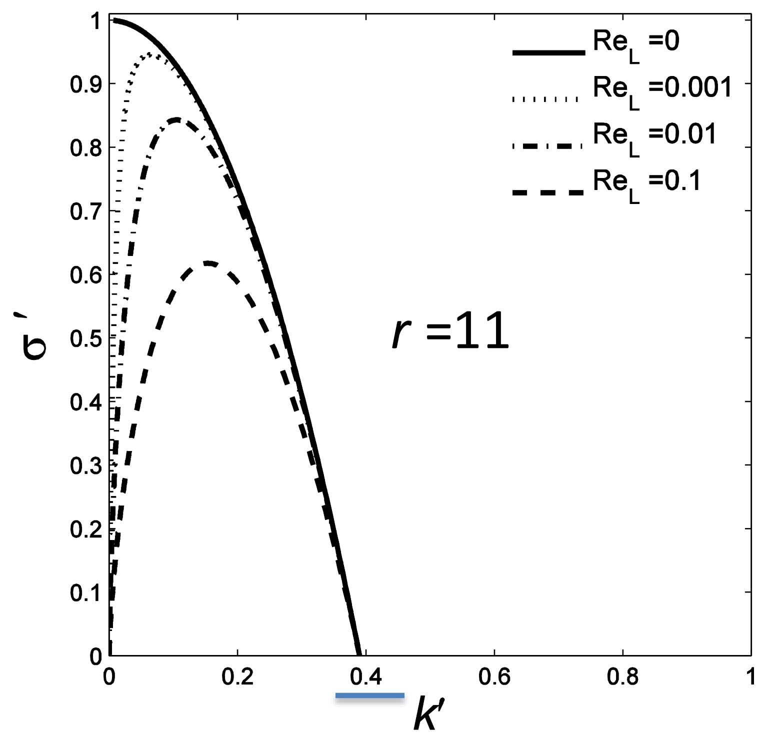
Steady state of the perturbation



- the steady state is always achieved.

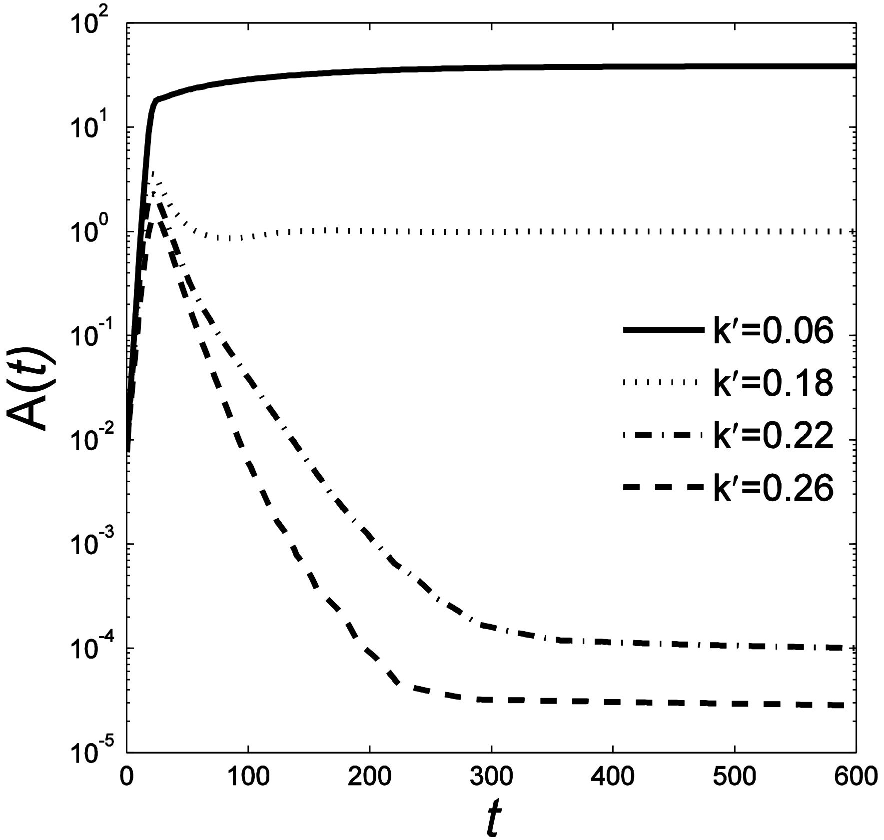
Saturated value - wavenumber





- A steep jump after a critical wavenumber
- nonlinear diffusivities also shrink the range of the unstable wavenumber.

Steady state of the perturbation



- For a certain range of wavenumbers, the perturbation grows first, but eventually disappears.

Conclusions

- Inertia and diffusions damp the perturbation at small wavenumbers and large wavenumbers, respectively, leading to a wavenumber selection.
- The steady state is always achieved and independent of the initical conditions and Reynolds number.
- For a certain range of wavenumbers, the perturbation grows first, but eventually disappears.

Thank you!