Wall turbulence with rod-like polymers

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in collaboration with

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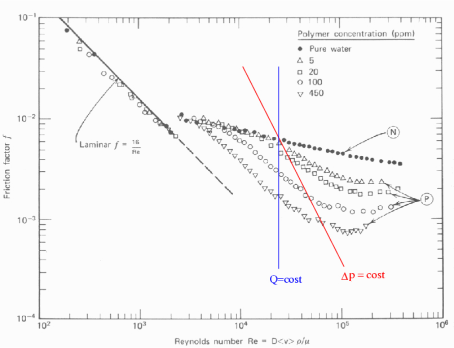
Introduction: drag reduction - I

(Virk, Sc.D. Thesis, MIT 1966)

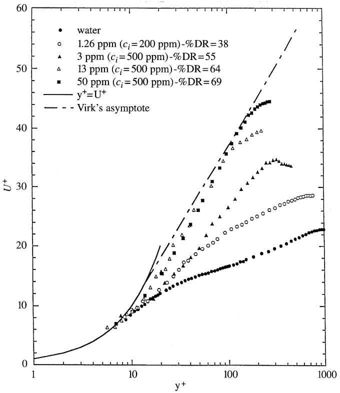
- Friction factor:
  $f \propto \Delta p / Q^2$
- Bulk Reynolds number:
  $\mathrm{Re_b} \propto Q$
- Wall friction:
  $\tau_\mathrm{w} = h \Delta p / L$
- Friction Reynolds number:
  $\mathrm{Re_w} = h \sqrt{\tau_\mathrm{w} / \rho_0} / \nu_0$



Introduction: drag reduction - II

(Warholic, Massah, Hanratty, Experiments in Fluids **27**, 1999)

- Friction velocity
  $u_\mathrm{w} = \sqrt{\tau_\mathrm{w}/\rho_0}$
- Mean velocity (viscous units)
  $U^+ = \langle u \rangle / u_\mathrm{w}$
- Wall normal distance (viscous units)
  $y^+ = y u_\mathrm{w} / \nu_0$
- MDR:
  $U^+ = 11.7 \log y^+ - 17.$



Prandtl-Karman plot

- Quantitatively drag reduction can be measured as the relative decrease of the friction factor

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho \overline{U}^2} = 2f$$

Introducing the viscous velocity, the decrease of f is described as

$$\frac{1}{\sqrt{f}} = \frac{\overline{U}}{u_\tau} = \overline{U}^+ = \frac{1}{Re_\tau}\int_0^{Re_\tau} \overline{u}^+ dy^+$$

- For a Newtonian flow, assuming the classical logarithmic law, provides in the limit of large Reynolds

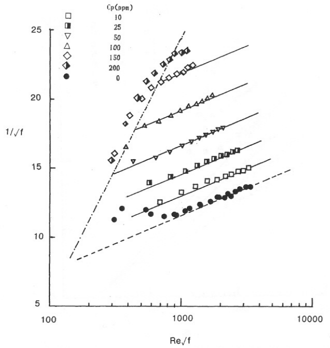
$$\frac{1}{\sqrt{f}} = const. + \frac{1}{\kappa}\log Re_\tau$$

- If the mean velocity profile changes also the P-K plot varies

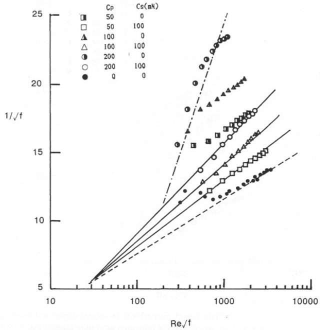
Flexible and Rodlike Polymers: Experiments

(Sasaki, Journal of the Physical society of Japan **60**, 1991).

- The addition of salt can help us to learn more about the different types of drag reduction



- - Polyelectrolicte



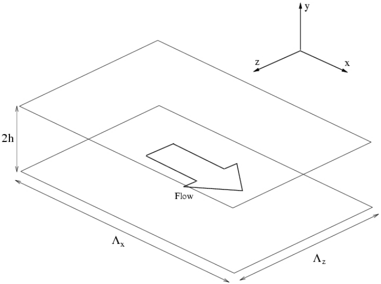
- - Polyelectrolicte at different concentrations plus salt

DNS of drag reducing wall bounded flows

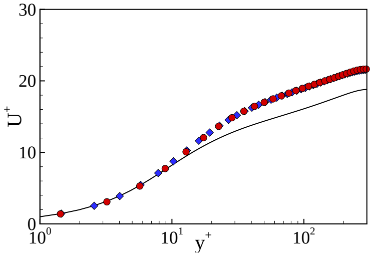
In principle, in such a problem much can be done via numerical simulations. The first computations have been done in a channel flow

- Flexible: Sureshkumar, Beris, Handler, Phys Fluids **9**, 1997
- Rodlike: Manhart, Journal of Non-Newtonian Fluid Mechanics **9**, 2002

Computational domain

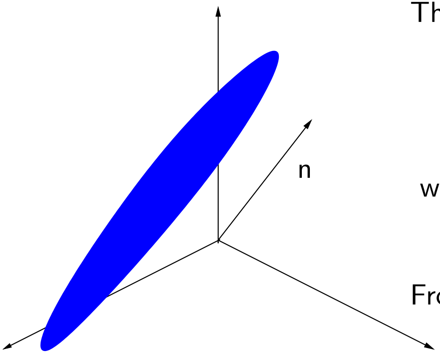


Mean profiles



Model for rodlike polymers

- The rheological properties of rodlike polymers can be assimilated to that of neutrally buoyant axisymmetric Brownian particles



The equation for a single particle is

$$\begin{aligned} \dot{\mathbf{n}} &= \Omega \cdot \mathbf{n} + \kappa \left[\mathbf{D} \cdot \mathbf{n} + \right. \\ &- \left. (\mathbf{n} \cdot \mathbf{D} \cdot \mathbf{n}) \, \mathbf{n} \right] + \Gamma(t) \end{aligned}$$

where κ is a geometric factor

$$\kappa = (r_e^2 - 1)/(r_e^2 + 1).$$

From the Fokker-Plank equation

$$\frac{\partial \Psi}{\partial t} + \frac{\partial}{\partial \mathbf{n}} (\dot{\mathbf{n}} \Psi) = \frac{\partial^2}{\partial \mathbf{n} \partial \mathbf{n}} D_r \Psi$$

Basic equations for rodlike polymers for $\kappa \to 1$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu_0 \nabla^2 \mathbf{u} + \nu_{\mathrm{p}} \nabla \cdot \mathbf{T}_{\mathrm{p}}$$

$$\frac{\partial \mathbf{R}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{R} = \mathbf{K}\mathbf{R} + \mathbf{R}\mathbf{K}^{\dagger} - \mathbf{E} : \mathbf{R}\mathbf{R} - 6\gamma_B \left(\mathbf{R} - \frac{\mathbf{I}}{3} \right)$$

$(\mathbf{R} =< \mathbf{nn} >, \ \mathbf{E} = 2\mathbf{D} = \mathbf{K} + \mathbf{K}^{\dagger})$

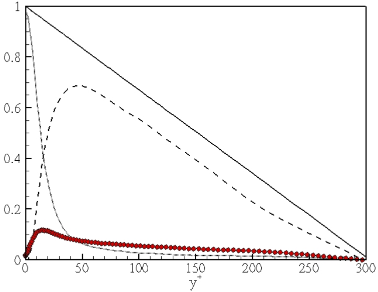
- Constitutive relations:

$$\mathbf{T}_{\mathrm{p}} = \mu_{\mathrm{p}} \left[\mathbf{E} : \mathbf{R}\mathbf{R} + 6\gamma_B \left(\mathbf{R} - \frac{\mathbf{I}}{3} \right) \right]$$

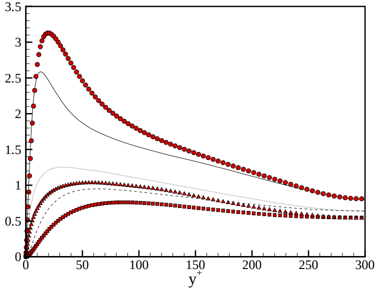
- In the limit where the Brownian diffusion can be neglected, i.e. $\gamma_B \to 0$, the control parameters are $\eta_p = \mu_p/\mu$ and $\mathrm{Re} = h u_{\mathrm{w}}/\nu$

Rodlike particles simulations

▷ Numerical simulations for rodlike polymers flows show drag reduction for a simulation with $\eta_p = 25$ and $Re_\tau = 300$. $2\pi h \times 2h \times \pi h$ for $128 \times 93 \times 64$



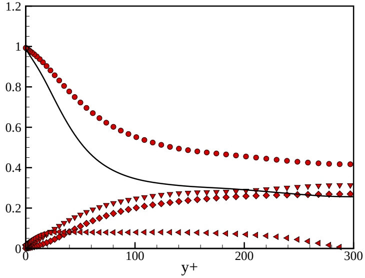
Total stress balance



Turbulence intensities

Conformation tensor

- The average values of the conformation tensor plotted as a function of the distance from the wall



The relevant component $R_{xx}, R_{yy}, R_{zz}, R_{xy}$ are plotted together with the order parameter

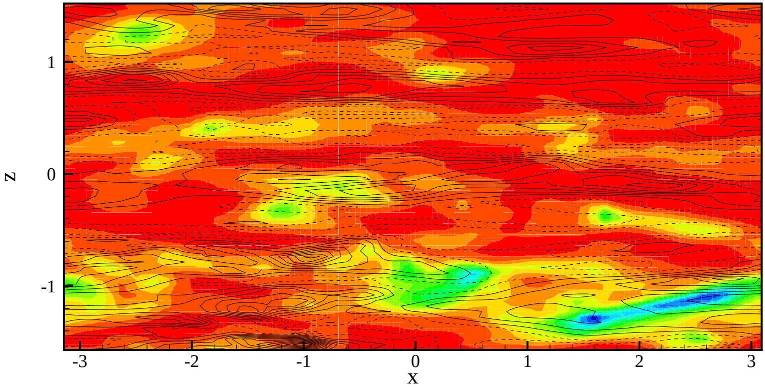
$$S^2 = \frac{3}{2} tr(\mathbf{S} \cdot \mathbf{S})$$

where $\mathbf{S}$ is the orientation tensor defined as

$$\mathbf{S} = \mathbf{R} - \frac{1}{3}\mathbf{I}$$

Order parameter

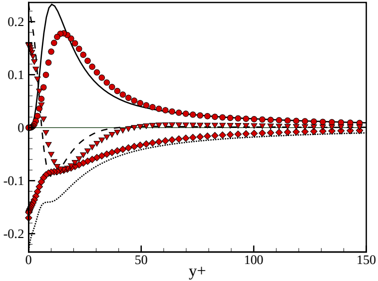
- ▷ Order parameter S^2 plotted on a plane at distance $y^+ = 6$ from the wall

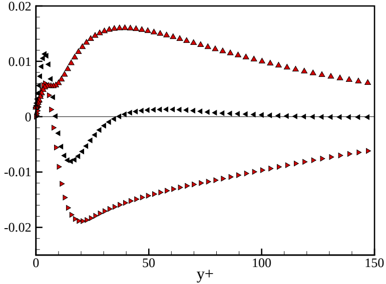


TKE budgets

- Single point statistics

$$\frac{d\Phi}{dy} + \frac{d\Phi_{\mathrm{p}}}{dy} = \pi(y) - < \epsilon_{\mathrm{N}}(y) > - < \psi_{\mathrm{p}}(y) >$$





- Production
- Newtonian dissipation
- Divergence of the flux

- Total particles contribution
- Particles spatial flux
- Particles "dissipation"

A phenomenological model for DR - I

(L'vov, Pomyalov, Procaccia, Tiberkevich, PRL **92**, 2004)

- Momentum equation close to the wall:

$$\tau_{\rm R}(y) + \tau_{\rm p}(y) \simeq h\frac{\Delta p}{L} \qquad \Rightarrow \tau_{\rm p}(y) \simeq h\frac{\Delta p}{L}$$

- Model equation for dissipation of TKE:

$$\nu_{\rm e}(y)\frac{u_{\rm rms}^2}{y^2} + \frac{u_{\rm rms}^3}{y} = \tau_{\rm R}\frac{dU}{dy} \qquad \Rightarrow \nu_{\rm e}(y)\frac{u_{\rm rms}^2}{y^2} \simeq \tau_{\rm R}\frac{dU}{dy}$$

- Closure assumptions:

$$\tau_{\rm p} = \nu_{\rm e}(y)\frac{dU}{dy} \qquad \tau_{\rm R} \simeq C^2 u_{\rm rms}^2$$

- $\nu_{\rm e} \propto y \Rightarrow dU/dy \propto 1/y$. Mixing length-like approach.

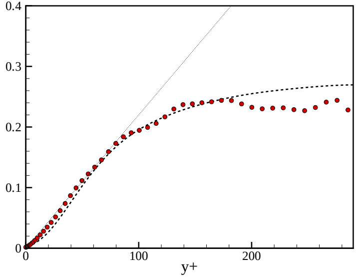
A phenomenological model for DR - II

(Benzi, Ching, De Angelis, Procaccia, PRE, 2008)

- An extension of the theory to rodlike polymers assumes that, as for the flexible ones, $\nu_{\rm e}$ depends only on the R_{yy} component of the conformation tensor, namely

$$\nu_{\rm e}(y) \propto \eta_p R_{yy}$$

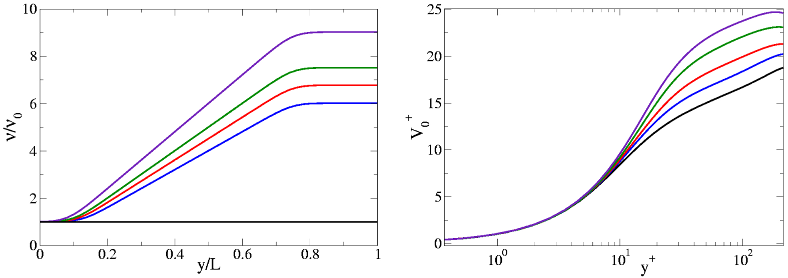
- This hypothesis is tested against the numerical results



The plot shows $< R_{yy} >$ (dashed line) and $< T_{xy} > / c\eta_p < dU/dy >$ (symbols). We find a linear behavior up the $y^+ = 80$

Linear viscosity simulations

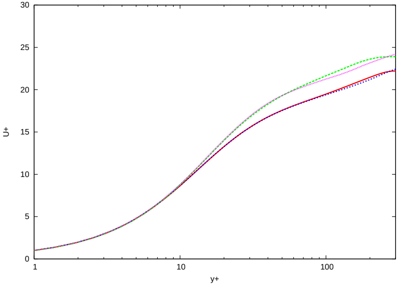
- A DNS of a Newtonian flow with $\mu_e(y)$ shows DR, the more the larger is the slope of the effective viscosity.

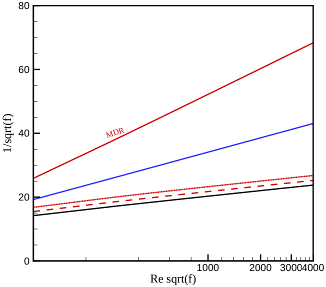


C	0	8	9	10	12
Re_τ	245	227	214	197	185
$DR\%$	–	13.8	21.6	36.9	42.0

A parametric study

- Increasing Reynolds number and the fibre concentration (Gillissen, Boersma, Mortensen, Anderson, JFM, **602**, 2008)





- For a given concentration a new log law appears independent on Reynolds number, the expected P-K plot is ladder-like

Final remarks and perspectives

- Simulations of flows for rodlike polymers can be achieved through a simplified rheological model
- The flow shows features similar to that of flexible polymers
- The results confirm recent theories on the effective nature of the rodlike polymers-turbulent interaction

(Virk, Wagger, Koury, ASME FED **237**, 1996).