Deformable Fibers and non-spherical particles in suspension flow

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• Effect of fiber deformation and orientation distribution on suspension rheology

• Fiber suspension in turbulent channel flow

• Deformation characterization in shear flow: rheology

• Lagrangian turbulence with deformable particles (red blood cells) and particle migration…
Experiment Setup

- PVC pipe
- Distributor
- Static pressure tappings, connecting to differential pressure transmitter
- Transducer, connecting to UVP
- Transducer adapter, connecting to positioner
- Plexiglas channel. 60 in. long, 2 in. wide, 0.65 in. high
- Ultrasonic gel
- Water

Xu, H., and Aidun, C.K., “Characteristics of Fiber Suspension Flow in a Rectangular Channel”
*Int. J. Multiphase Flow*, 3 1/3, pp. 318-336, 2005
Two parameters: (1) Location   (2) Velocity at the measuring point
Basic Principle of PUDV

(1) Doppler effect:

\[ f_d = \pm \frac{2V \cos \theta}{c} f_e \]

- \( f_d \): Frequency shift
- \( V \): Particle speed
- \( \theta \): Angle between ultrasound and particle direction
- \( f_e \): Frequency of Ultrasound
- \( c \): Speed of ultrasound
- \( D \): Distance between particle and transducer
- \( t \): Time between signal transmission and reception

(2) Echography

\[ D = \frac{c \cdot t}{2} \]
Internal architecture and measuring principle of PUDV

(a)

- Principle oscillator
- Synchronized demodulation
- Low-pass filter
- Doppler frequency estimation
- Emission amplifier
- Reception amplifier
- A/d converter
- High-pass filter
- Doppler frequency estimation
- US burst
- US echo
- Flow
- Wall
- Next burst
- Time
- Position
Sensitivity of PUDV measurement of fiber suspension

Flow rate: 65 GPM

Flow rate increase by 1.18%

Flow rate decrease by 1.03%

Fiber volume fraction: 0.5%
### Characterizations of fiber suspension flow

<table>
<thead>
<tr>
<th>Flow rate (Gal/minute)</th>
<th>Uavg (m/s)</th>
<th>Re</th>
<th>Volume concentration (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td>2.2</td>
<td>0.157</td>
<td>2050</td>
<td>✓</td>
</tr>
<tr>
<td>3.3</td>
<td>0.231</td>
<td>3025</td>
<td>✓</td>
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<td>36625</td>
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<td>59638</td>
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</tr>
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<td>5.60</td>
<td>73250</td>
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<tr>
<td>100</td>
<td>7.0</td>
<td>91510</td>
<td>✓</td>
</tr>
</tbody>
</table>
Velocity profile of water measured by PUDV at different flow rate

![Graph showing velocity profile](image_url)
Effect of concentration on velocity profile

Flow rate: 100 gpm

Re=91500

Concentration: 0.05%
Concentration: 0.10%
Concentration: 0.15%
Concentration: 0.30%
Concentration: 0.50%
Concentration: 0.75%
Concentration: 1.00%

Laminar flow
Turbulent flow
Effect of concentration on velocity profile

Flow rate: 65 gpm

Concentration: 0.05%
Concentration: 0.10%
Concentration: 0.15%
Concentration: 0.30%
Concentration: 0.50%
Concentration: 0.75%
Concentration: 1.00%

laminar flow
turbulent flow
Effect of concentration velocity profile

Flow rate: 40 gpm; Re=37000

- Concentration: 0.05%
- Concentration: 0.10%
- Concentration: 0.15%
- Concentration: 0.30%
- Concentration: 0.50%
- Concentration: 0.75%
- Concentration: 1.00%

Laminar flow
Turbulent flow
Effect of concentration on velocity profile

Flow rate: 2.2 gpm; Re=2000

Concentration: 0.10%
Concentration: 0.15%
Concentration: 0.30%
Concentration: 0.50%
Concentration: 0.75%
Concentration: 1.00%

Laminar flow
Turbulent flow
Mean velocity profile for fiber suspension in channel flow

\[ u^+ = \frac{1}{0.41} \ln(y^+) + 4.69 + \frac{\Pi}{0.41} \sin^2\left(\frac{y}{0.9b}\pi\right) \]

\[ \Pi = 0.98 \exp(0.14(nl^3) - 1.9 \times 10^{-5} \times \text{Re}) \]

Comparison of Experimental data and predicted velocity profile.

Xu, and Aidun; *Int. J. Multiphase Flow*, 3 1/3, pp. 318-336, 2005
**Rigid Fibers**

\[
\text{Pe} = \frac{\dot{\gamma} \mu L^3}{k_B T} \gg 1 \quad \text{Re} = \frac{\rho \dot{\gamma} L^2}{\mu} \ll 1
\]

\[
\eta = \frac{\mu_{\text{eff}}}{\mu} = 1 + F(c_{vf}, r_p; p(\phi)) \quad r_p \text{ aspect ratio;}
\]

\[
p(\phi) \text{ orientation dist.}
\]

\[
\sum = -P\mathbf{I} + 2\mu\mathbf{E} + \mu_f \left( \langle pppp \rangle - \frac{1}{3} I \langle pp \rangle \right) : \mathbf{E}
\]

\[
\mu_{\text{eff}} = \frac{\sum_{12}}{2\mu E_{12}} \quad \mu_f \text{ depends on } c_{vf}, p(\phi), r_p
\]

\[
p = p_x \mathbf{e}_x + p_y \mathbf{e}_y + p_z \mathbf{e}_z
\]

\[
N_1 = \sum_{xx} - \sum_{yy}
\]

\[
N_2 = \sum_{yy} - \sum_{zz}
\]
For deformable fibers, concept of *Bending Ratio* (BR)

Critical shear-induced buckling stress,

\[
(\mu\dot{\gamma})_{\text{crit}} = \frac{E_Y \left( \ln(2r_e) - 1.5 \right)}{2r_p^4}
\]

\[(\text{Forgacs and Mason, J. Col. Sci., 1959)}\]

- \(E_Y\), Young's Modulus
- \(L\), fiber length
- \(D\), fiber diameter
- \(r_p = L/D\)
- \(r_e\), effective aspect ratio
For deformable fibers, concept of *Bending Ratio* (*BR*)

Critical shear-induced buckling stress,

\[
(\mu\dot{\gamma})_{\text{crit}} = \frac{E_Y (\ln(2r_e) - 1.5)}{2r_p^4}
\]

(Forgacs and Mason, *J. Col. Sci.*, 1959)

\[
BR = \frac{E_Y (\ln(2r_e) - 1.5)}{2(\mu\dot{\gamma})r_p^4}
\]

\[
\eta = \frac{\mu_{\text{eff}}}{\mu} = 1 + F(c_{vf}, r_p, BR; p(\phi))
\]

- \(E_Y\), Young's Modulus
- \(L\), fiber length
- \(D\), fiber diameter
- \(r_p = L/D\)
- \(r_e\), effective aspect ratio
Lattice-Boltzmann Method

Discrete Boltzmann equation:

\[ f_i(r + e_i, t + 1) = f_i(r, t) - \frac{1}{\tau} \left( f_i(r, t) - f_i^{(eq)}(r, t) \right) \]

Macroscopic properties related to moments of distribution:

\[
\begin{align*}
\sum_{i=1}^{Q} f_i^{(eq)}(r, t) &= \rho \\
\sum_{i=1}^{Q} f_i^{(eq)}(r, t)e_i &= \rho u
\end{align*}
\]

\[
\sum_{i=1}^{Q} f_i^{(eq)}(r, t)e_i e_i &= c_s^2 \rho \mathbf{I} + \rho uu
\]

Symbols

- \( \mathbf{r} = \) position vector
- \( \mathbf{e}_i = \) lattice direction vector
- \( f_i = \) fluid distribution
- \( f_i^{(eq)} = \) equilibrium fluid distribution
- \( \rho = \) density
- \( \mathbf{x} = \) nodal displacement
- \( \mathbf{I} = \) identity tensor
- \( \mathbf{u} = \) fluid velocity
- \( \tau = \) relaxation time

Flexible fiber model

\[ F_i = \sum F_{in} \]

\[ F_{in}^{mov} = \frac{F_i}{(N + 1)} \quad \text{and} \quad F_{in}^{def} = F_{in} - F_{in}^{mov} \]

\[ dl_{in} = \frac{1}{E_y(\pi D^2/4)} \left[ p_{in} \cdot (F_{in}^{def} - F_{in-1}^{def}) \right] \]

Fiber boundary node

: position vector of \( n \)th hinge of fiber \( i \)

: unit vector parallel to axis of rod \( in \)

\[ p_{in} = \frac{r_{in} - r_{in-1}}{|r_{in} - r_{in-1}|} \]

Wu and Aidun, Int. J. Multiphase Flow, 2009
The no-slip boundary condition on the surface of the particle is satisfied by the requirement that the fluid velocity at the solid boundary node is equal to the solid velocity at that point.

\[ U_f(x^l, t) = \int u(x^e, t) D(x^e - x^l) \, dx^e \]

- \( x^e \): Eulerian position vector for fluid nodes
- \( x^l \): Lagrangian position vector for solid nodes
- \( u(x^e, t) \): discrete fluid velocity
- \( U_f \): fluid velocity at solid boundary nodes
- \( U_p \): solid velocity at solid boundary nodes
- \( D \): Dirac delta function

\[ F_{fsi}(x^l, t) = (\rho_f U_f(x^l, t) - \rho_f U_p(x^l, t))/\Delta t \]

Fiber deformation in shear; BR=40
Fiber deformation in shear; BR=0.36
Fiber deformation in shear; BR=0.18
Symbols are the experimental data of Forgacs and Mason (1959) for BR=0.5, 0.43 and 0.35. Lines are the corresponding LB-EBF simulation results (Wu and Adiun, 2010).
LB-EBF Simulation of deformable fibers

Fiber diameter $D = 0.12\text{mm}$, $r_p = 16 \mu = 13\text{Pa s}$

$BR >> 1$  \hspace{2cm}  $BR = 0.3$
Relative viscosity

Fiber diameter $D = 0.12\text{mm}$  
$c_{vf} = 0.0051 \sim 0.124$  
$E_Y = 3\text{GPa}$  
$\mu = 13\text{Pa s}$

Bibbo PhD thesis, MIT 1987
Petrich et al., J. Non-Newtonian F. M., 2000
Relative viscosity & orientation distribution

Fiber diameter $D = 0.12 \text{mm}$ $r_p = 16$ $\mu = 13 \text{Pa s}$

Bending ratio $BR$ versus viscosity $\eta$ and orientation distribution $p(\phi)$. Parameters include $r_p = 16$, $nL^3$, $c_v$, and $BR$ values.
Primary Normal stress difference, $N_1$

fiber diameter $D = 0.12\text{mm}$  $r_p = 16$  $\mu = 13\text{Pa s}$

\[
N_1 = \sum_{xx} - \sum_{yy}
\]
Batchelor’s relation: $N_1$

The stress in dilute suspension

$$N_1^B = \sigma_{xx}^B - \sigma_{yy}^B = \mu_{fiber} \dot{\gamma} \left( \langle P_x^3 P_y \rangle - \langle P_y^3 P_x \rangle \right) = -\frac{\mu_{fiber} \dot{\gamma}}{4} \left( \sin^4 \theta \sin 4\phi \right)$$
Batchelor’s relation: $N_1$

The stress in dilute suspensions

$$\sigma^B = 2 \mu E + \mu_{fiber} \left( \frac{1}{3} I_n \right) : E$$

$$N_1^B = \sigma^B_{xx} - \sigma^B_{yy} = \mu_{fiber} \dot{\gamma} \left( \langle p_x p_y \rangle - \langle p_y p_x \rangle \right) = -\frac{\mu_{fiber} \dot{\gamma}}{4} \left( \sin^4 \theta \sin 4\phi \right)$$

\[
\begin{align*}
\tan \theta &= \frac{C_j r_c}{\left( r_c^2 \cos^2 \phi + \sin^2 \phi \right)^{1/2}}
\end{align*}
\]
Relative Viscosity vs Shear rate

\[ \frac{AR}{c_{vf}} = 16 \]
\[ c_{vf} = 0.0529 \]
\[ D = 0.12 \text{mm} \]
\[ \mu = 13 \text{ Pa-s} \]

\[ \text{BR} = 0.3 \]

RIGID (BR \gg 1)

\[ \text{Re} = \frac{\dot{\gamma}LD}{\nu} \]

\[ \eta \]

\[ \text{Shear rate (1/s)} \]

\[ \text{(Re} \approx 10^{-5}) \quad \text{(Re} \approx 10^{-2}) \]

\[ \text{Georgia Tech} \]

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Conclusions

Relative viscosity increases as Bending Ratio decreases. This is due to broader fiber distribution, that is more fibers orienting in the compression and extension axes,

The primary normal stress difference seems to first decrease from stiff to slightly deformable fibers, and then increase sharply for highly deformable fibers. Orientation distribution is dominant....fiber-fiber contact less significant,

It appears that fiber deformation is the dominant factor in orientation distribution and relative viscosity as shear rate increases above 10, in the cases considered here;

Relative viscosity seems to be independent of shear rate for constant Bending Ratio

Wu and Aidun, *J. Fluid Mech.*, **662**, 122-33, 2010
Mubashar et al., in preparation, ....