NTNU

Joint 6th SIG43 –FP1005 Workshop on

Fiber Suspension Flow Modelling

Slip Velocity of Rigid Fibers in Wall-Bounded Turbulence

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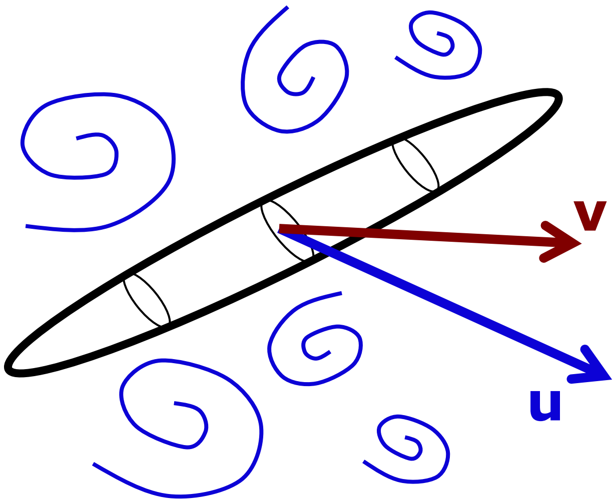
Motivation of the Study: Why are we looking at the slip velocity?

Slip velocity is a crucial variable in:

1. **Euler-Lagrange simulations:**
   - **One-way coupling: determines the drag experienced by fibers**
   - **Two-way coupling: determines reaction force from fibers on fluid**
2. **Two-fluid modeling of particle-laden flows**
   - **Modeling SGS fiber dynamics in LES flow fields**
   - **Crossing trajectory effects on time decorrelation tensor of u**



u: fluid velocity "seen"

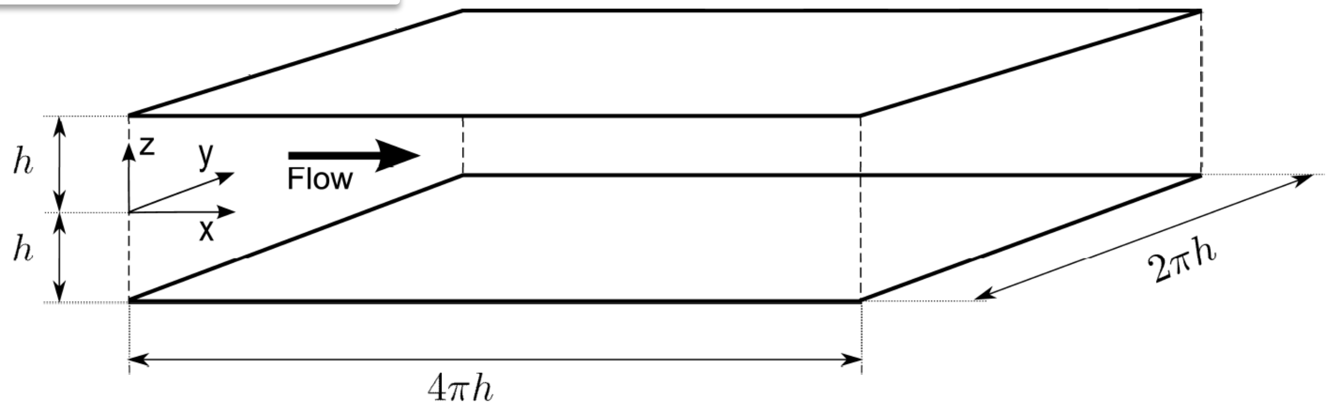
v: fiber velocity

Δu=u-v: slip velocity

⇨ **Aim of this study: statistical characterization of Δu at varying fiber inertia and elongation**

Methodology - Carrier Fluid

DNS of turbulent channel flow
@ $Re_\tau = u_\tau h/\nu =$ **150**, 180, 300



$$\begin{cases} \dfrac{\partial u_j}{\partial x_j} = 0 \\ \rho \left[\dfrac{\partial u_i}{\partial t} + u_j \dfrac{\partial (u_i)}{\partial x_j} \right] = -\dfrac{\partial P}{\partial x_i} + \mu \dfrac{\partial^2 u_i}{\partial x_j \partial x_j} \end{cases}$$

Pseudo-Spectral method: Fourier modes in *x and y*,
Chebyschev modes in *z.*

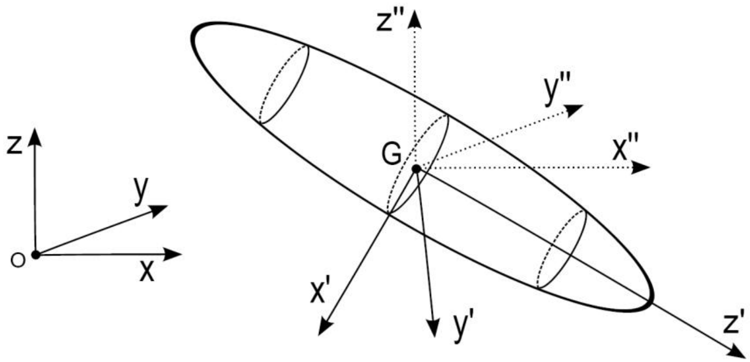
- **Examples: flow of air at 1.8 m/s in a 4 cm high channel**
  flow of water at 3.8 m/s in a 0.5 cm high channel

Methodology - Fibers

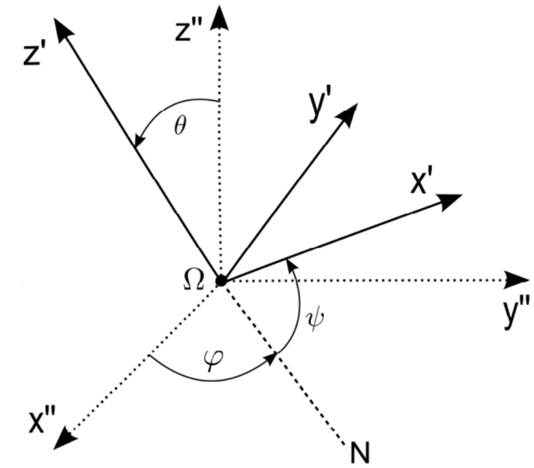
- Fibers are modelled as **prolate ellipsoidal particles**.
- **Lagrangian** particle tracking.
- Simplifying assumptions: dilute flow, **one-way coupling**, Stokes flow ($Re_P < 1$), pointwise particles (particle size is smaller than the smallest flow scale).
- Periodicity in *x* ed *y*, elastic rebound at the wall and **conservation of angular momentum**.
- **200,000 fibers** tracked, *random* initial position and orientation, linear and angular velocities equal to those of the fluid at fiber's location.

Methodology – Fiber Kinematics

Kinematics: described by (1) position of the fiber center of mass and (2) fiber orientation.



- ***x_G, y_G, z_G***
- **3 frames of reference (to define orientation)**
- **Euler angles: *φ, ψ, θ* (singularity problems)**



- **Euler parameters:** *e_0, e_1, e_2, e_3*

$$e_0 = \cos\left[\frac{1}{2}(\psi + \varphi)\right]\cos\left(\frac{\theta}{2}\right) \quad , \ldots$$

- **Rotation matrix:** $\mathbf{x}' = R_{Eul}\mathbf{x}''$

$$R_{eul} = \begin{bmatrix} e_0^2 + e_1^2 - e_2^2 - e_3^2 & 2(e_1e_2 + e_0e_3) & 2(e_1e_3 - e_0e_2) \\ 2(e_1e_2 - e_0e_3) & e_0^2 - e_1^2 + e_2^2 - e_3^2 & 2(e_2e_3 + e_0e_1) \\ 2(e_1e_3 + e_0e_2) & 2(e_2e_3 - e_0e_1) & e_0^2 - e_1^2 - e_2^2 + e_3^2 \end{bmatrix}$$

Methodology – Fiber Dynamics

Rotational dynamics: Euler equations with Jeffery moments.

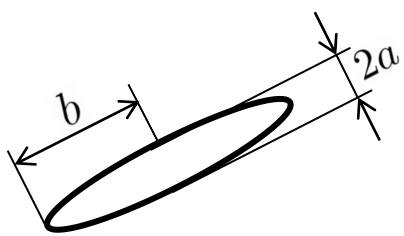
- **Euler Equations:** **(2nd cardinal law)**

$$\begin{cases} I_{x'x'}\dot{\omega}_{x'} + \omega_{y'}\omega_{z'}(I_{z'z'} - I_{y'y'}) = M_{x'}^{est} \\ I_{y'y'}\dot{\omega}_{y'} + \omega_{x'}\omega_{z'}(I_{z'z'} - I_{x'x'}) = M_{y'}^{est} \\ I_{z'z'}\dot{\omega}_{z'} + \omega_{x'}\omega_{y'}(I_{y'y'} - I_{x'x'}) = M_{z'}^{est} \end{cases}$$

(in the particle frame)

- **Jeffery moments:** ***(Jeffery, 1922)***

$$M_{x'}^{Jeff} = \frac{16\pi\mu a^3\lambda}{3(\beta_0 + \lambda^2\gamma_0)}\left[(1-\lambda^2)f' + (1+\lambda^2)(\xi' - \omega_{x'})\right]$$



Aspect Ratio

$$\lambda = \frac{b}{a}$$

- **Hence:** **Euler equations with Jeffery couples** → $\int\int(\ldots)dtdt$ → **e_0, e_1, e_2, e_3 (Euler parameters)**

$$R_{eul} = R_{eul}(e_0, e_1, e_2, e_3)$$

Methodology – Fiber Dynamics

Translational Dynamics: hydrodynamic resistance (Brenner, 1963).

- **First cardinal law:** $m_P \frac{d\mathbf{v}}{dt} = \sum_i \mathbf{F}_i = \mathbf{F}_{drag}$ **(inertia and drag only!!)**
- **Brenner's law: (form drag and skin drag)** $\mathbf{F}'_{drag} = \mu\pi a \bar{\bar{\mathbf{K}}}'(\mathbf{u}' - \mathbf{v}')$ **(in the fiber frame)**

Resistance Tensor

- **In the inertial (Eulerian) frame:**

$$\left.\begin{aligned} \mathbf{u}' &= R_{eul}\mathbf{u} \\ \bar{\bar{\mathbf{K}}}_{(\varphi,\theta,\psi)} &= R_{eul}^T \bar{\bar{\mathbf{K}}}' R_{eul} \end{aligned}\right\} \Longrightarrow \mathbf{F}_{drag} = \mu\pi a \bar{\bar{\mathbf{K}}}_{(\varphi,\theta,\psi)}(\mathbf{u} - \mathbf{v})$$

$$\begin{cases} m_P \dfrac{d\mathbf{v}}{dt} = \mu\pi a \bar{\bar{\mathbf{K}}}_{(\varphi,\theta,\psi)}(\mathbf{u} - \mathbf{v}) \\ \dfrac{d\mathbf{x}_{(G)}}{dt} = \mathbf{v} \end{cases} \Rightarrow \mathbf{v}(t) \quad \mathbf{x}_G(t)$$

(via numerical integration)

Once fiber orientation is known, fiber translational motion can be computed!

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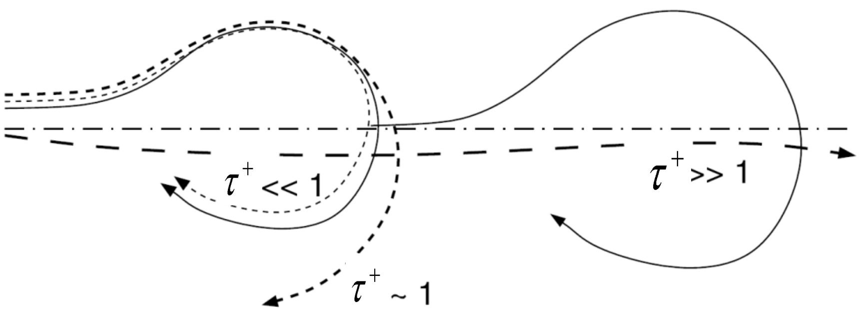
Relevant Parameters and Summary of the Simulations

The physics of turbulent fiber dispersion is determined by a small set of parameters

- **Aspect ratio:** $\lambda = \frac{b}{a}$ **(chosen values: $\underline{\lambda = 1.001, 3, 10, 50}$)**

- **Stokes number:** $St = \tau^+ = \frac{\tau_P}{\tau_F}$ **(chosen values: $\underline{\tau^+ = 1, 5, 30, 100}$)**

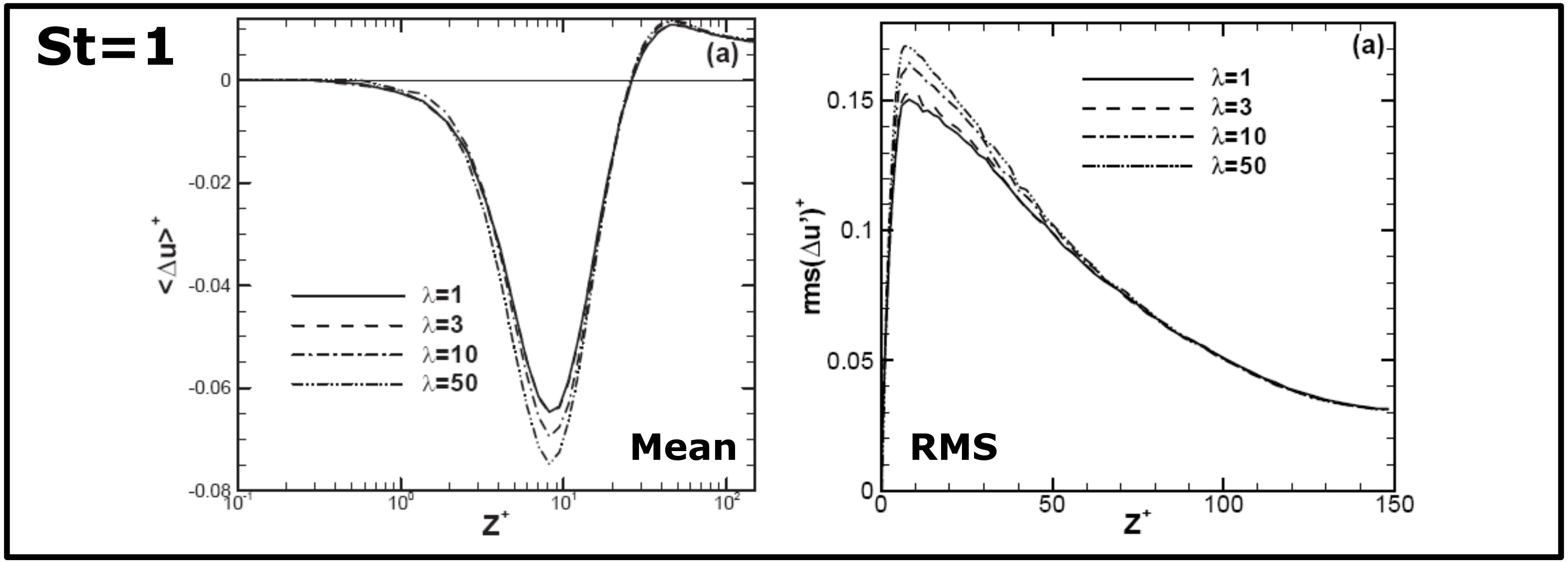


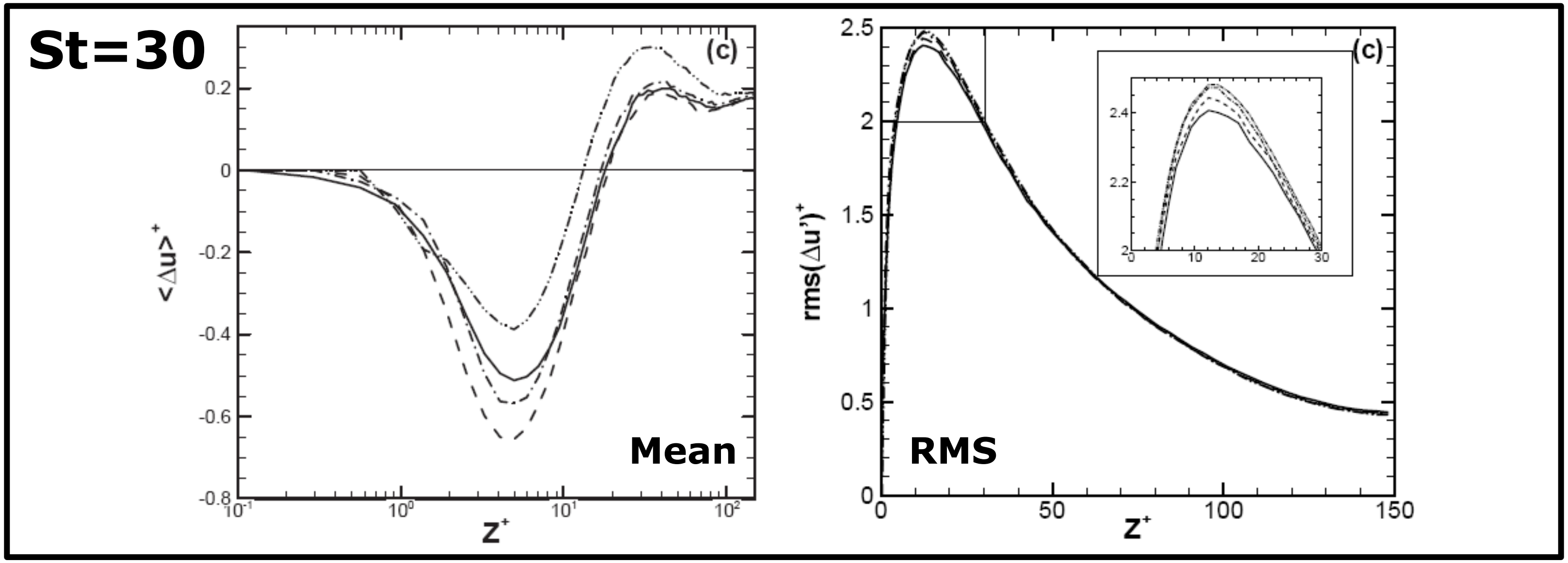
- $\tau^+ >> 1$: **large inertia ("stones")**
- $\tau^+ << 1$: **small inertia (tracers)**
- $\tau^+ \sim 1$: **preferential (selective) response to flow structures**

- **Specific density:** $S = \frac{\rho_P}{\rho_F}$

Input parameters: S, τ^+, λ

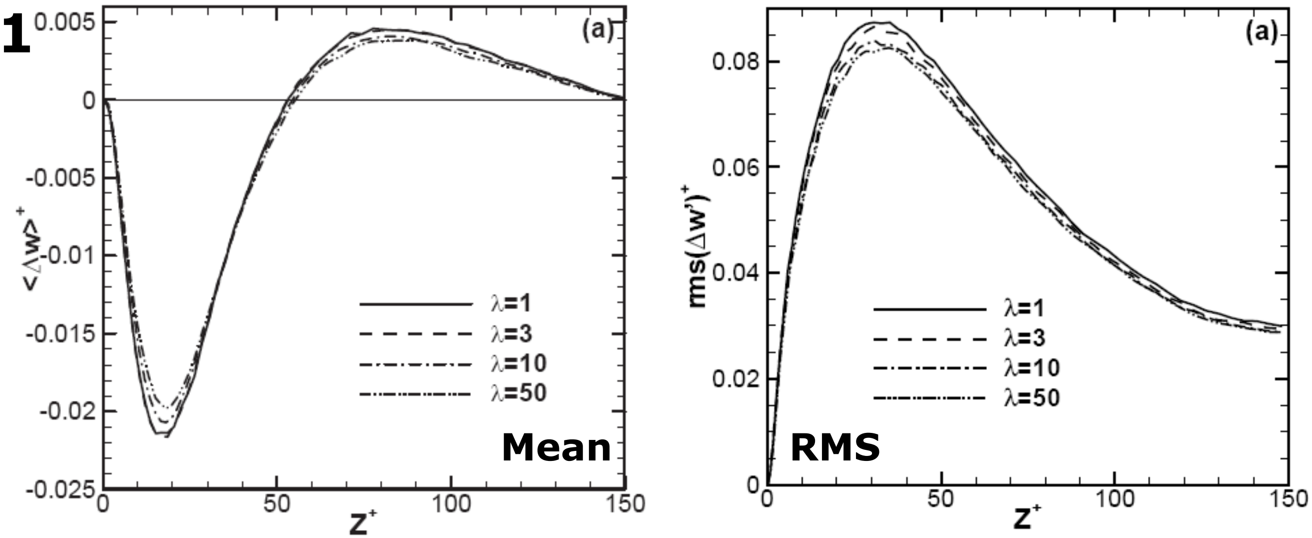
Results - Streamwise Slip Velocity: Mean and RMS values



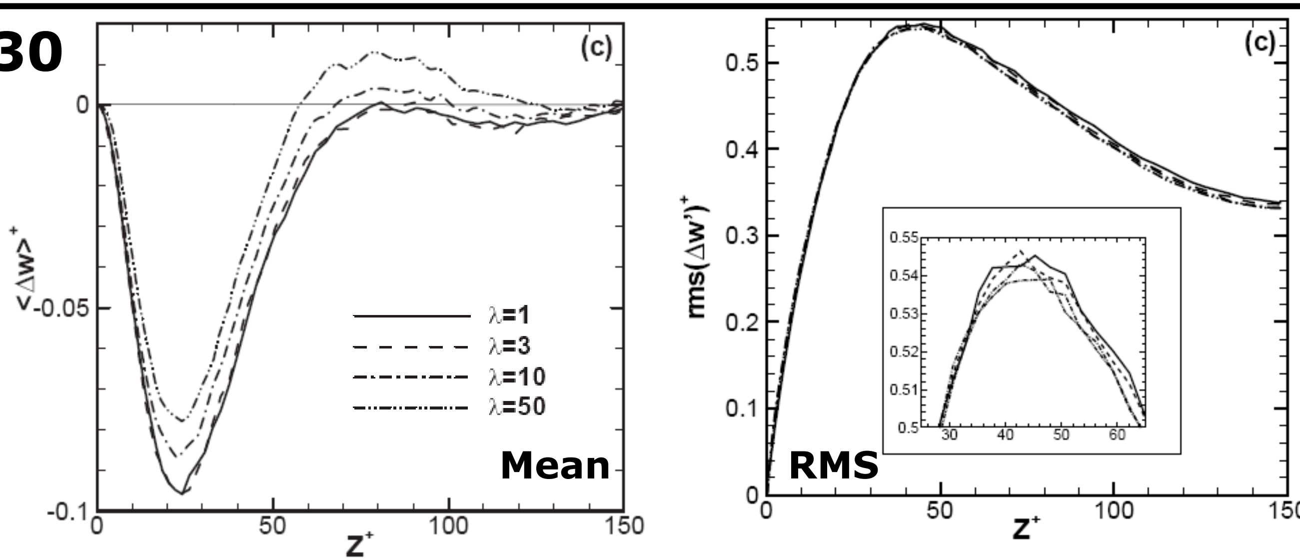


Results - Wall-Normal Slip Velocity: Mean and RMS values

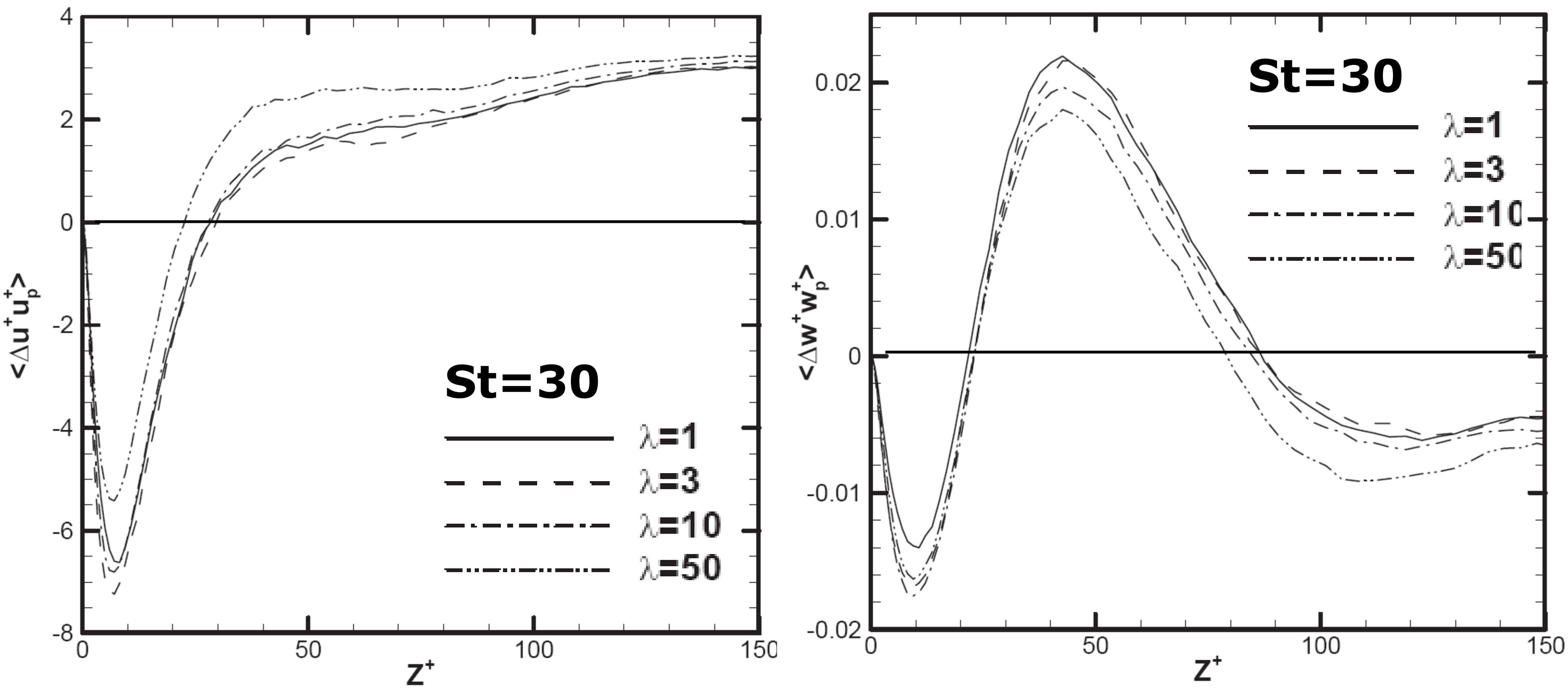
St=1



St=30



Results - Slip –Fibers Velocity Correlation: Mechanical work done by fluid on fibers

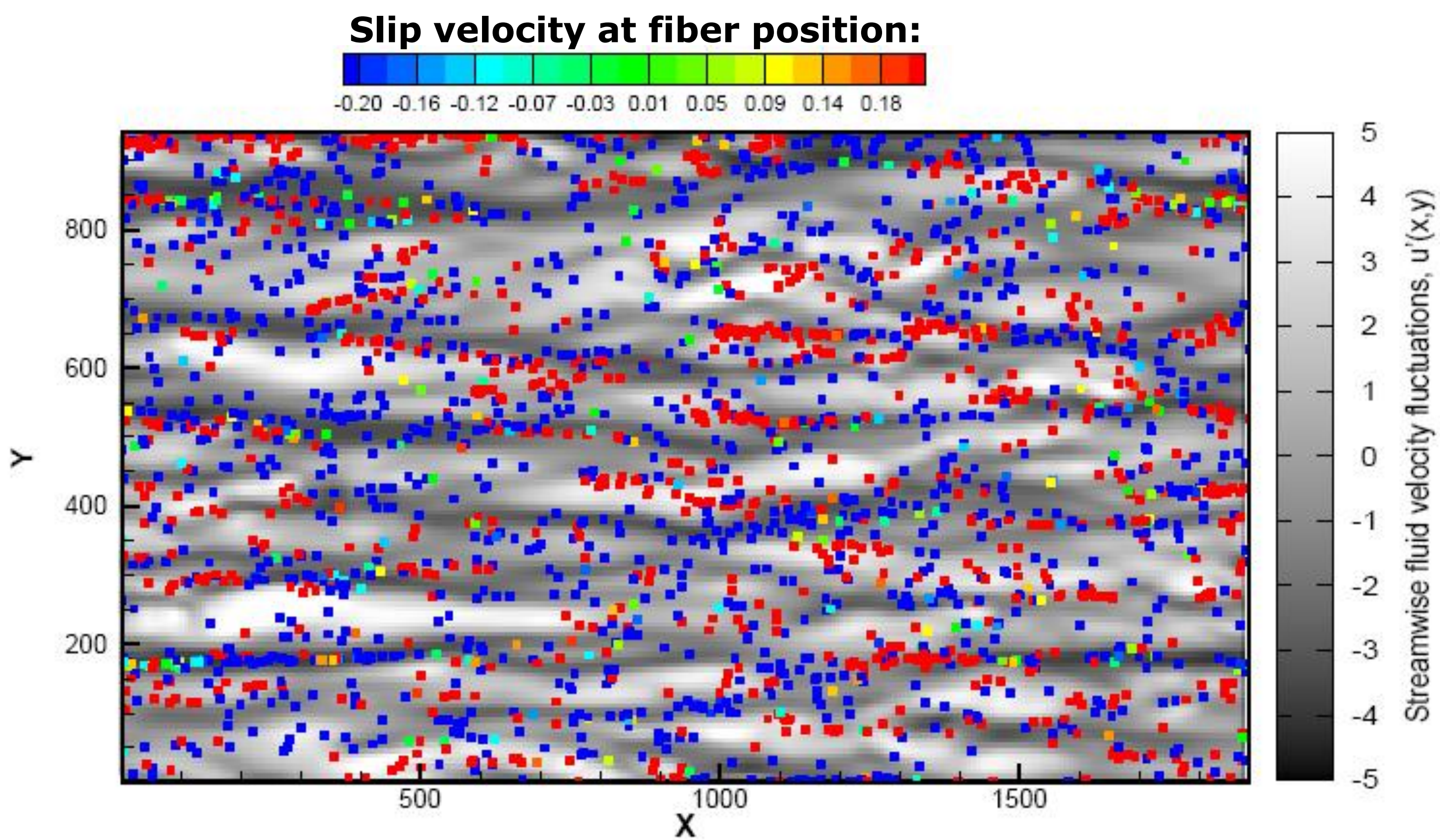


Negative $\langle \Delta u^+ u^+_p \rangle$: fibers exhert mechanical work on fluid

Positive $\langle \Delta u^+ u^+_p \rangle$: fluid exherts mechanical work on fibers

Results - Non-Homogeneity of Near-Wall Fiber Preferential Distribution

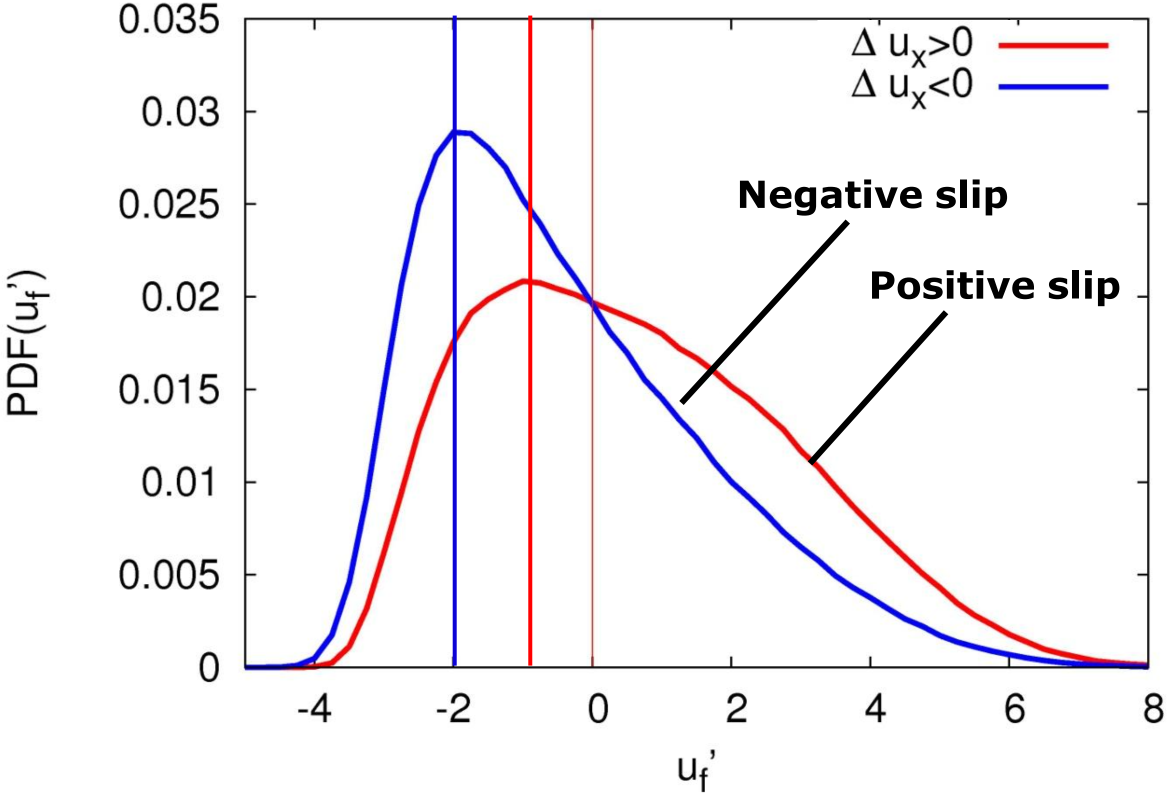
Top view: fibers accumulate in near-wall fluid Low-Speed Streaks (LSS)



Sample snapshot for τ^+=30, λ=50 fibers

Results - Using Slip Velocity to Analyze Fiber Accumulation in LSS

Effect of fiber elongation on conditioned PDF(u_f') – St=30

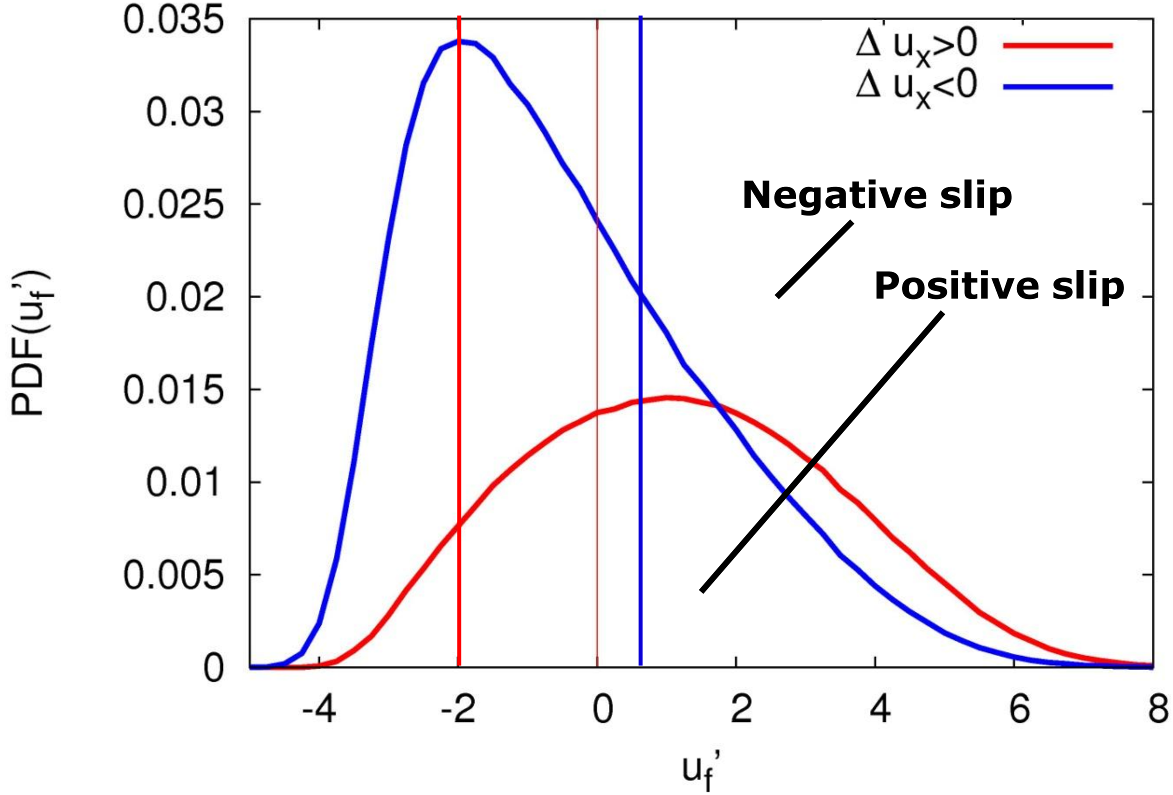


λ=50

The influence of *λ* is not dramatic: only a change in the peak values is observed (no PDF shape change)

Results - Using Slip Velocity to Analyze Fiber Accumulation in LSS

Effect of fiber inertia on conditioned PDF(u_f')



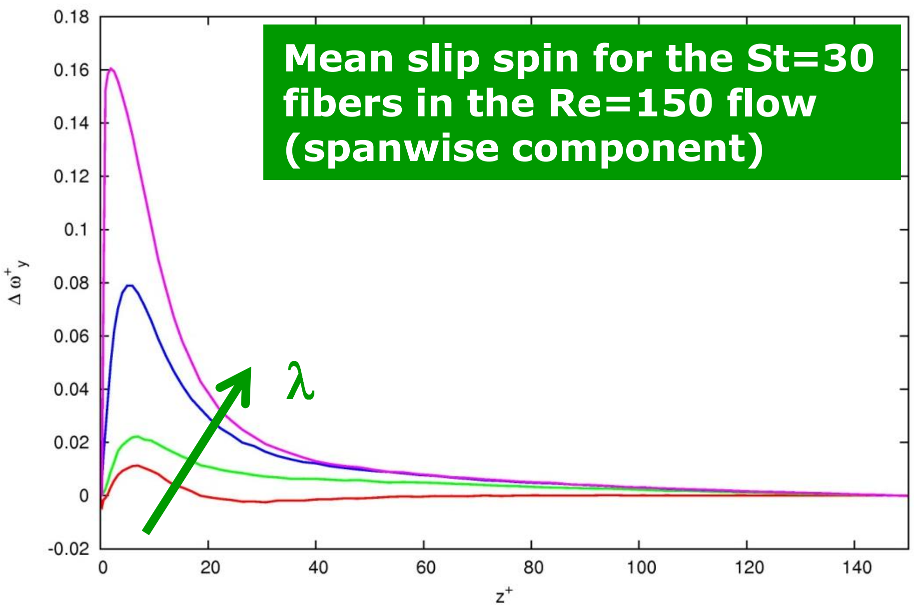
St=100

Significant PDF shape change with curve "inversion" between St=5 and St=30

Conclusions …

…and Future Developments

- Slip velocity is a useful measure of fibers-turbulence interaction in wall-bounded flows: its statistical characterization provides useful indications for modeling turbulent fiber dispersion
- Slip velocity statistics depend both on fiber elongation (quantitatively) and fiber inertia (also qualitatively!)
- RMS exceeds the corresponding mean value by roughly 3 to 5 times: the instantaneous slip velocity may thus frequently change sign

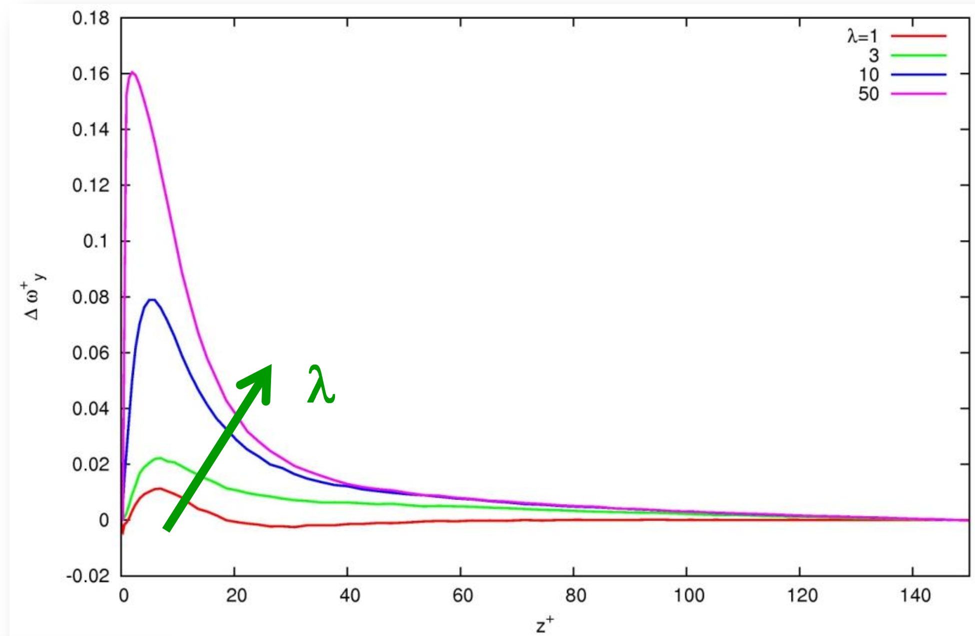


- Simulate more values of *St, λ* and *Re*
- Evaluate slip spin statistics

Outlook – Relative Fiber-Fluid Rotation (Slip Spin)

Evaluate slip spin statistics…

Relative spin between fluid and fibers determines fiber rotational dynamics…



Mean slip spin for the St=30 fibers in the Re=150 flow (spanwise component)

$\underline{Dynamics:}$

$$\frac{d\mathbf{v}^+}{dt^+} = \left(\frac{S-1}{S}\right)\mathbf{g}^+ + \frac{3}{4\lambda S {a^+}^2}\bar{\bar{\mathbf{K}}}_{(e_0,e_1,e_2,e_3)} \cdot (\mathbf{u}^+ - \mathbf{v}^+)$$

$$\frac{d\omega^+_{x'}}{dt^+} = \omega^+_{y'}\omega^+_{z'}\left(1 - \frac{2}{1+\lambda^2}\right) + \frac{20\left[(1-\lambda^2)f' + (1+\lambda^2)(\xi' - \omega^+_{x'})\right]}{(\alpha_0 + \lambda^2\gamma_0)(1+\lambda^2)S{a^+}^2}$$

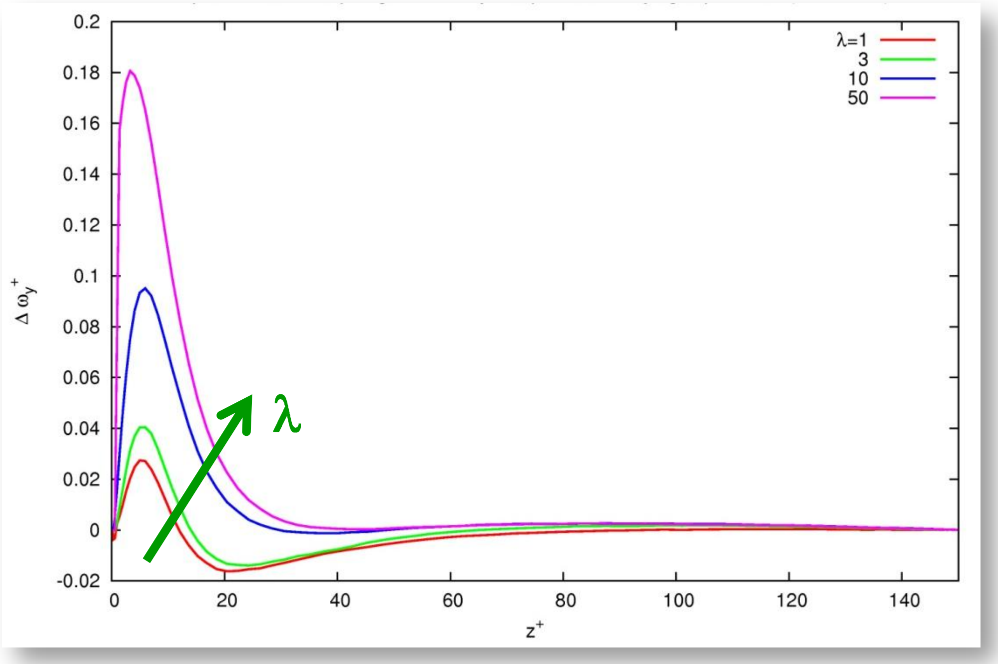
$$\frac{d\omega^+_{y'}}{dt^+} = \omega^+_{x'}\omega^+_{z'}\left(\frac{2}{1+\lambda^2} - 1\right) + \frac{20\left[(\lambda^2-1)g' + (\lambda^2+1)(\eta' - \omega^+_{y'})\right]}{(\alpha_0 + \lambda^2\gamma_0)(1+\lambda^2)S{a^+}^2}$$

$$\frac{d\omega^+_{z'}}{dt^+} = \frac{20}{(2\alpha_0)S{a^+}^2}(\chi' - \omega^+_{z'})$$

Outlook – Relative Fiber-Fluid Rotation (Slip Spin)

Evaluate slip spin statistics…

Relative spin between fluid and fibers determines fiber rotational dynamics…

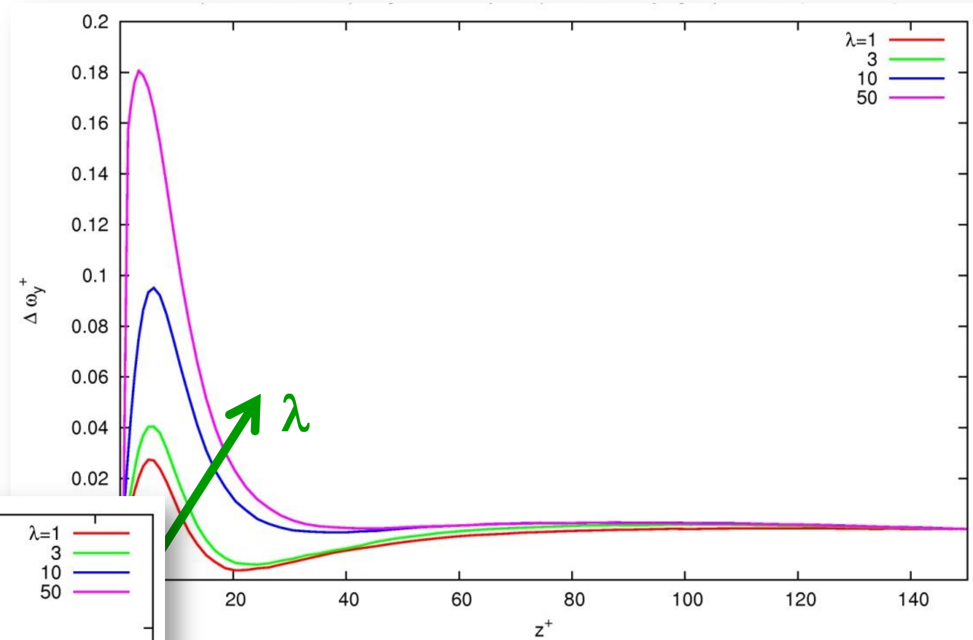


Mean slip spin for the St=100 fibers in the Re=150 flow (spanwise component)

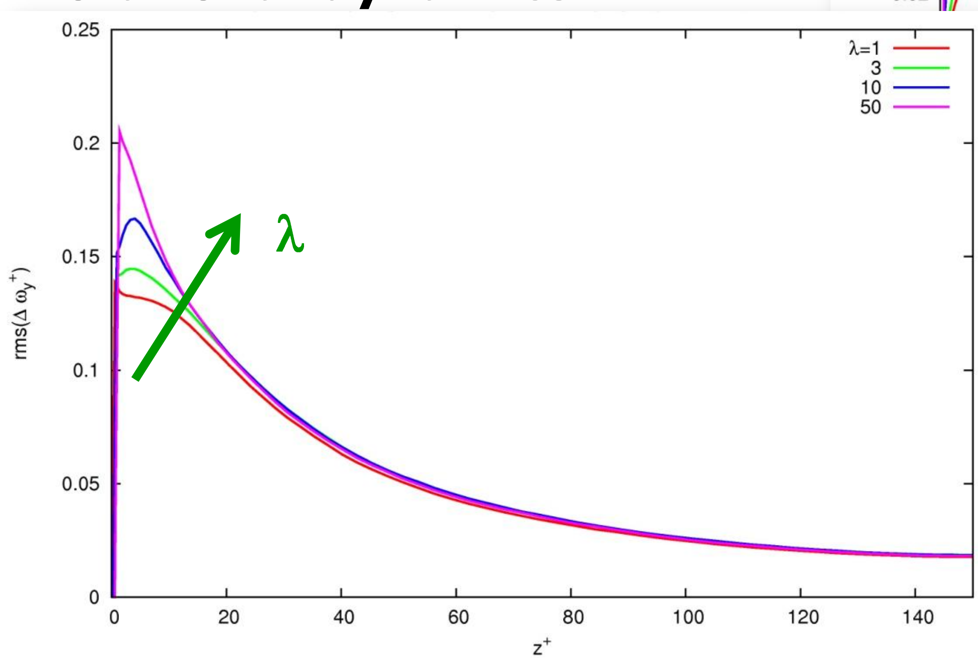
Outlook – Relative Fiber-Fluid Rotation (Slip Spin)

Evaluate slip spin statistics…

Relative spin between fluid and fibers determines fiber rotational dynamics…



Mean slip spin for the St=100 fibers in the Re=150 flow (spanwise component)



Slip spin rms for the St=100 fibers in the Re=150 flow (spanwise component)