Laboratory and numerical models for logjams

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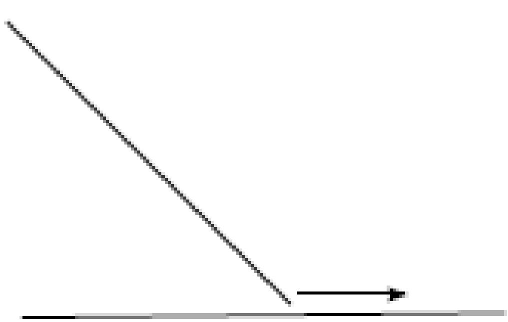
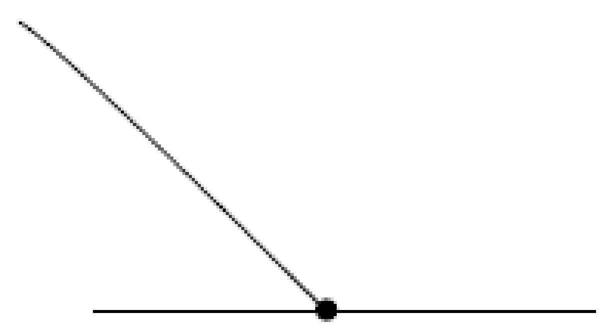
Steric interactions in fibre suspensions

Fibres cannot pass through each other. The contact force between two fibres may:

- Allow the point of contact to slide
- The point of contact may stick, as long as the fluid flow continues to push the particles together. The fibres pivot about their contact point.
- Fibres may bind irreversibly on contact (a less interesting case).





Logjams

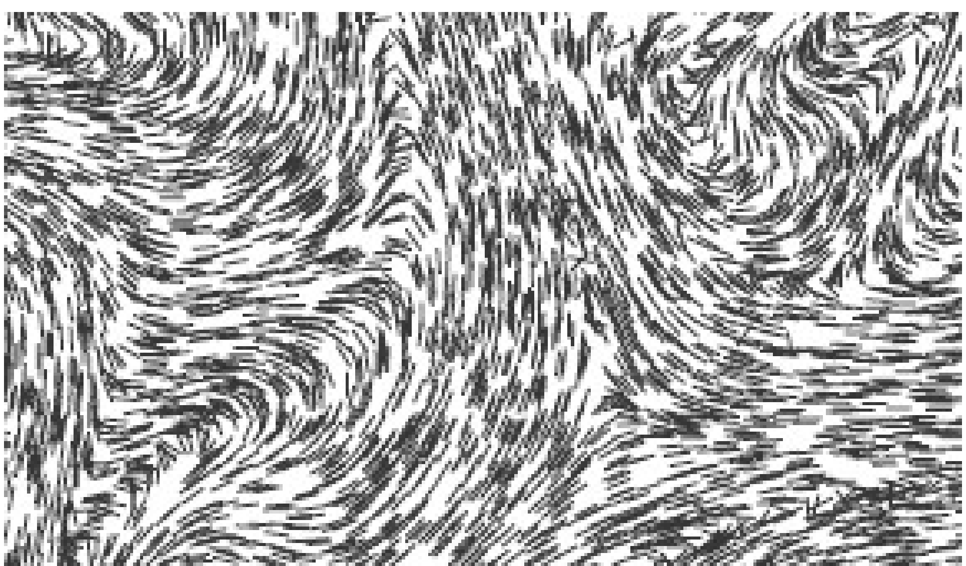
As well bas being worth studying in their own right, they are also a good model for steric interactions in fibre suspensions, because they are

- Macroscopic (hence readily controlled and imaged)
- Two-dimensional (hence simple).

Motivation

Studying logjams may provide an insight into 'elastic turbulence' (chaotic flows of polymer solutions at low Reynolds number). If logjams under shear show chaotic motion, this implies that polymer elasticity is not essential.

Also, logjams may break symmetry: either orientational isotropy of spatial homogeneity:



Miniature logjams

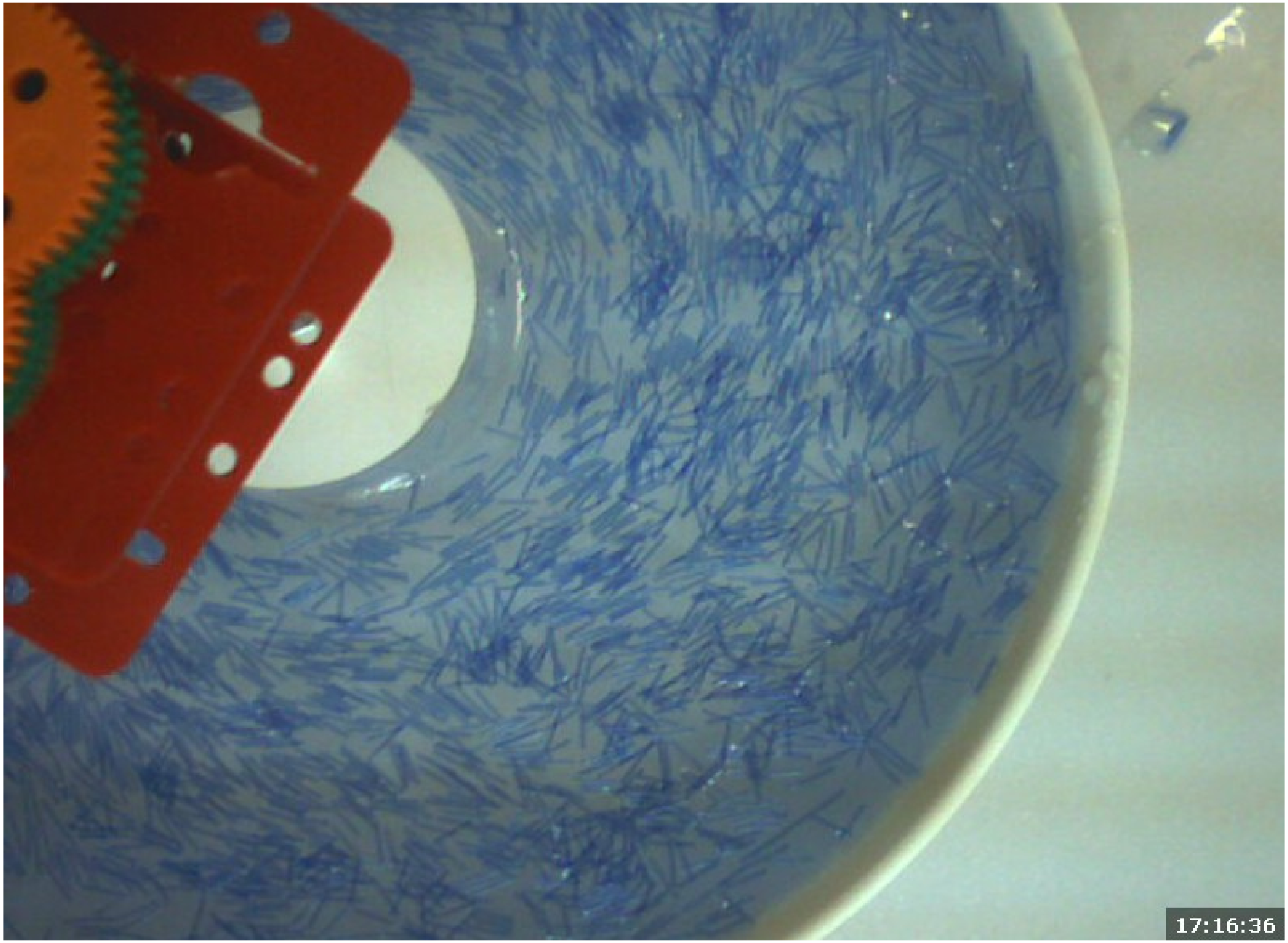
Logjams may be observed in laboratory scale experiments, using millimetre size rods. The interaction with the flow is dominated by viscous drag.

We made rods from nylon monofilament coloured fishing line (density 1.15), diameter 0.7mm, length 5mm.

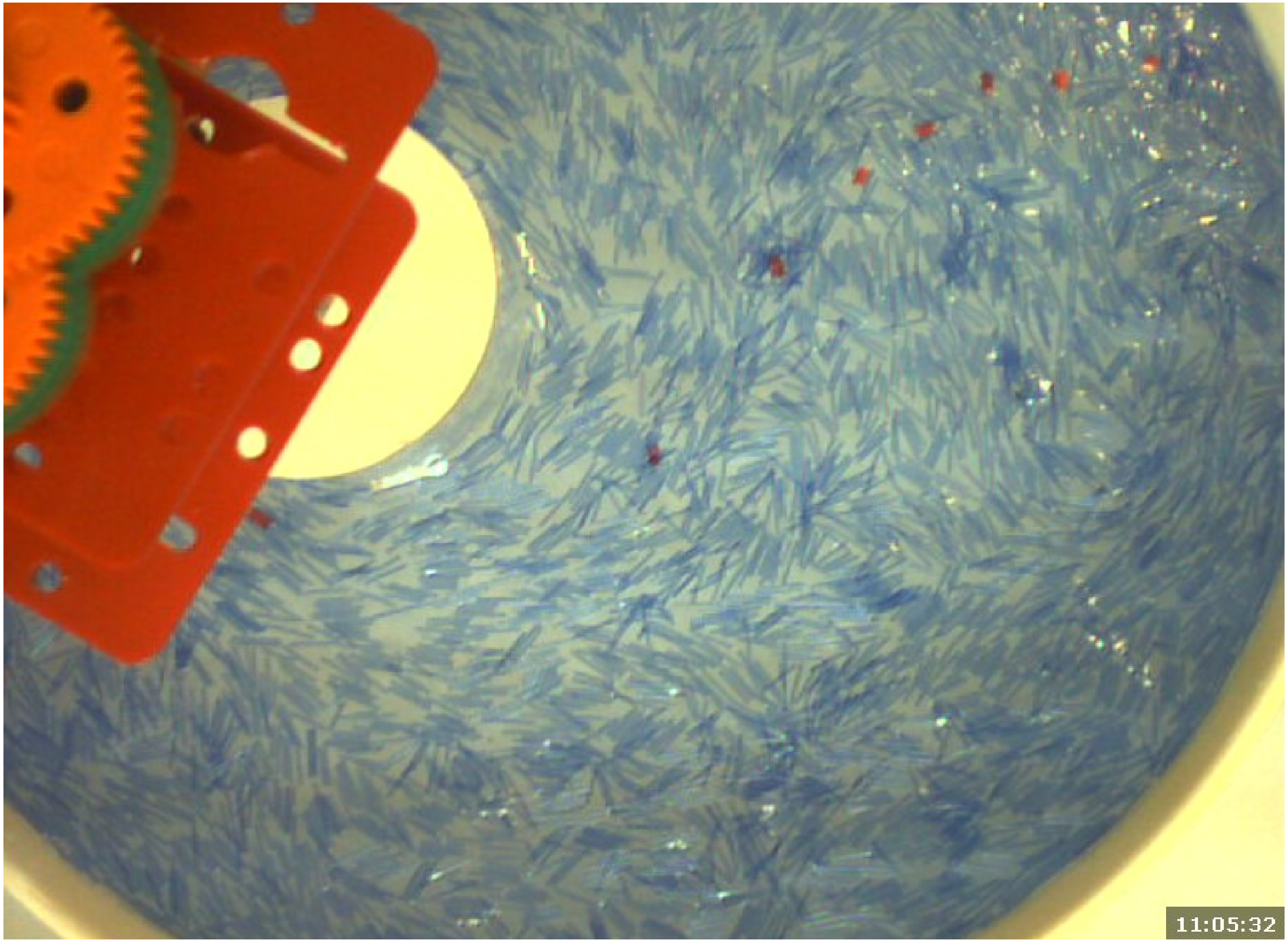
The fibres float on pure glycerol (density 1.26),which is very viscous, inexpensive, non-toxic.

The contact points appear to stick, rather than slide.

Simple experiment on coeutte flow



Journal bearing flow



Modelling logjams

It is sufficient to use a dumbbell model for interaction with the fluid, where the forces are mediated through the ends. There are four equations of motion per rod: two components of force, the torque, and the rate of change of length of each rod is zero.

$$\sum_k \mathrm{e}_1 \cdot \boldsymbol{F}_k = 0$$

$$\sum_k \mathrm{e}_2 \cdot \boldsymbol{F}_k = 0$$

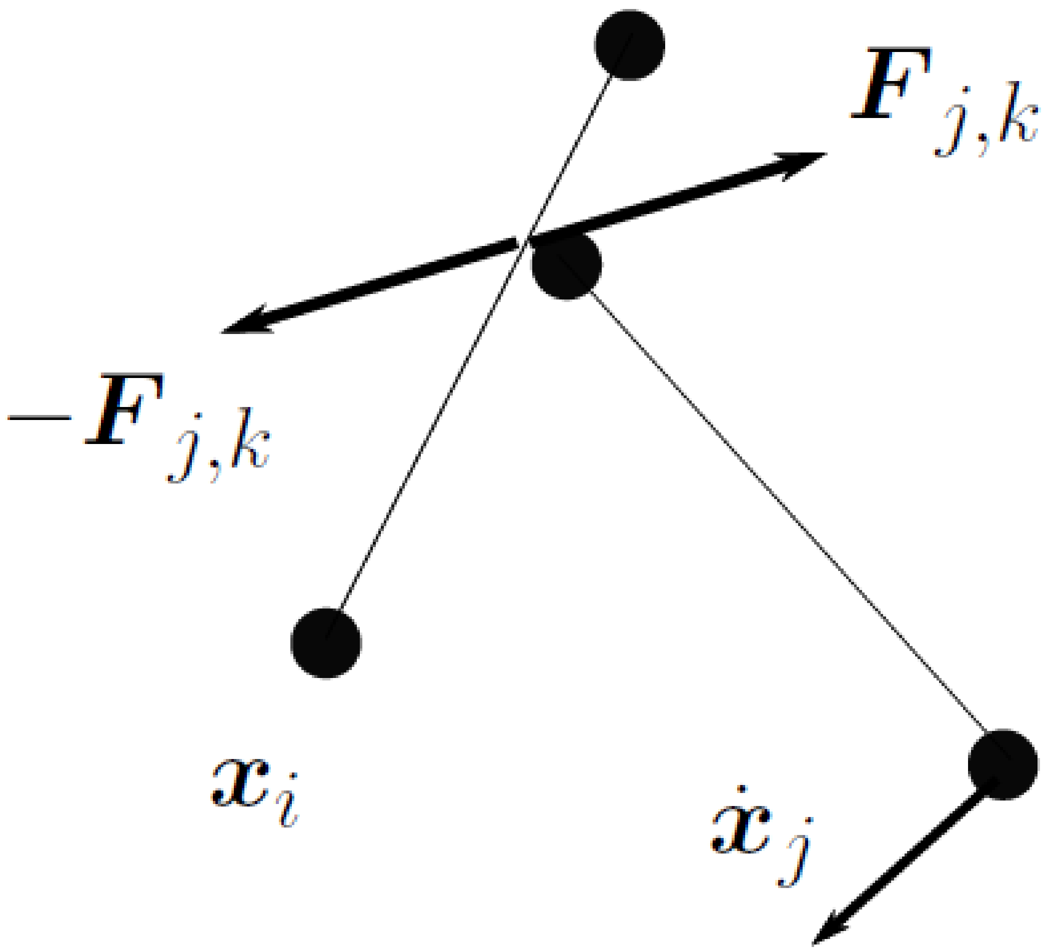
$$\boldsymbol{F}_{\mathrm{j}}^{(\mathrm{hyd})} = \boldsymbol{u}(\boldsymbol{x}_j, t) - \dot{\boldsymbol{x}}_j$$

$$\sum_k \boldsymbol{r}_k \wedge \boldsymbol{F}_k = 0$$

$$(\dot{\boldsymbol{x}}_{j+1} - \dot{\boldsymbol{x}}_j) \cdot (\boldsymbol{x}_{j+1} - \boldsymbol{x}_j) = 0$$

Contact forces

When there is a contact interation, there is an unknown contact force, and a mechanical constraint:



$$\boldsymbol{x}_i = w\boldsymbol{x}_j + (1-w)\boldsymbol{x}_{j+1}$$

Form of the equations of motion

Because the forces are linear, the equations of motion lead to a system of linear equations for the particle velocities.

$$\boldsymbol{F}_{\mathrm{j}}^{(\mathrm{hyd})} = \boldsymbol{u}(\boldsymbol{x}_j, t) - \dot{\boldsymbol{x}}_j$$

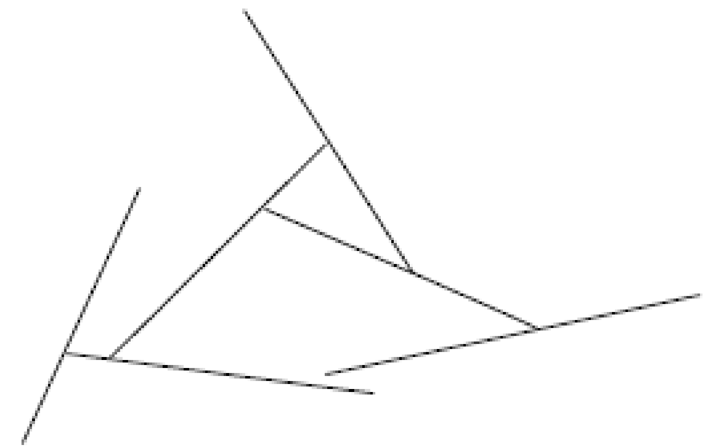
When there is a new contact, the number of degrees of freedom is reduced by two. However two components of the contact force appear as additional unknowns. The contacts are encoded in a matrix.

$$\boldsymbol{x}_i = w\boldsymbol{x}_j + (1-w)\boldsymbol{x}_{j+1}$$

$$c_{i,j} = w \ , \quad c_{i,j+1} = 1 - w$$

Complex contact interations

Contacts may form a complex network.



Want to express constrained coordinates in terms of unconstrained ones. Constraint matrix determined by summing an infinite series:

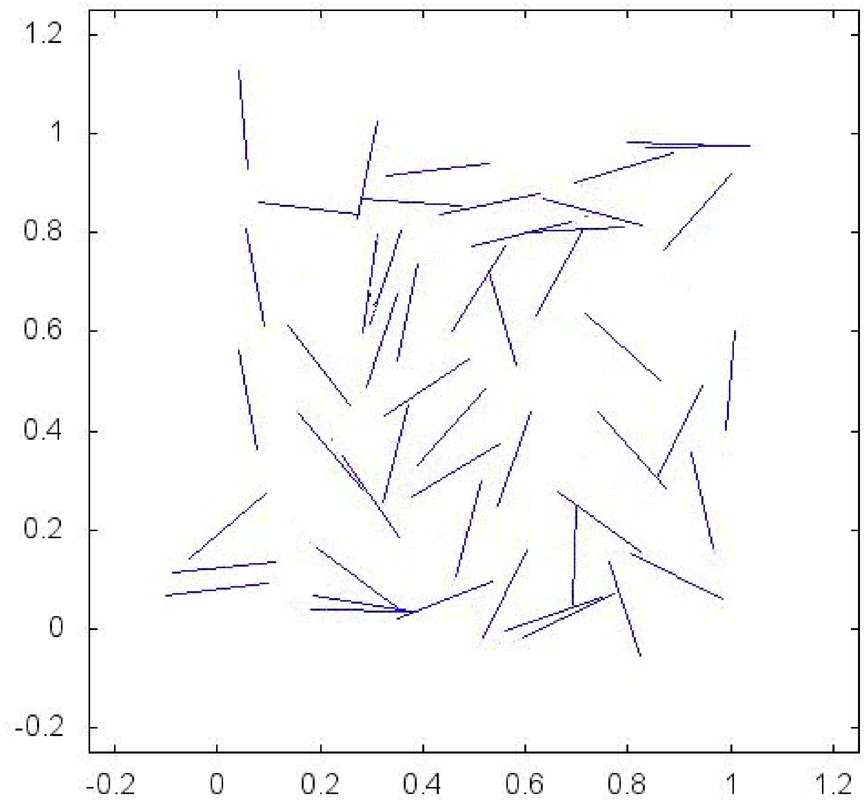
$$\mathbf{C} = \mathbf{C}_{\mathrm{cn}} + \mathbf{C}_{\mathrm{fr}}$$

$$\begin{aligned} \boldsymbol{X}_{\mathrm{cn}} &= \mathrm{C}_{\mathrm{fr}}\,\boldsymbol{X}_{\mathrm{fr}} + \mathrm{C}_{\mathrm{cn}}\,\boldsymbol{X}_{\mathrm{cn}} \\ &= \mathrm{C}_{\mathrm{fr}}\,\boldsymbol{X}_{\mathrm{fr}} + \mathrm{C}_{\mathrm{cn}}\mathrm{C}_{\mathrm{fr}}\,\boldsymbol{X}_{\mathrm{fr}} + \mathrm{C}_{\mathrm{cn}}^2\,\boldsymbol{X}_{\mathrm{cn}} \\ &= \left[\mathbf{I} + \mathrm{C}_{\mathrm{cn}} + \mathrm{C}_{\mathrm{cn}}^2 + \ldots\right] \mathrm{C}_{\mathrm{fr}}\,\boldsymbol{X}_{\mathrm{fr}} \end{aligned}$$

$$\boldsymbol{X}_{\mathrm{cn}} = \boldsymbol{\Gamma}\,\boldsymbol{X}_{\mathrm{fr}}$$

$$\boldsymbol{\Gamma} = [\mathbf{I} - \mathbf{C}_{\mathrm{cn}}]^{-1}\,\mathbf{C}_{\mathrm{fr}}$$

Simulation of logjam experiments



Conclusions

Contact between fibres is poorly understood.

Laboratory-scale logjams are the ideal laboratory to start to understand steric interactions.

I have described some preliminary laboratory and numerical investigations.

Further investigations:

- Look for instability (alternative mechanism for 'elastic turbulence').
- At low density, do the rods cluster together as rafts?
- At high density, is the rheology controlled by motion of topological defects.