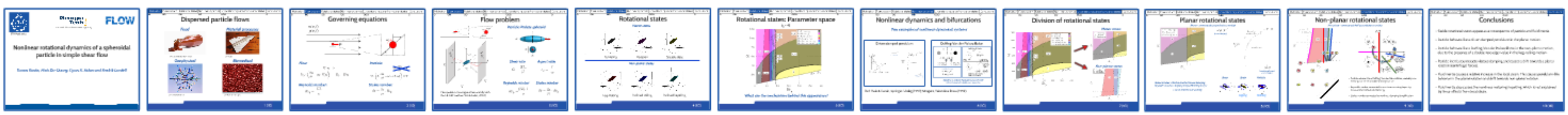
Thank you!


FLOW
Nonlinear rotational dynamics of a spheroidal particle in simple shear flow
Dispersed particle flows
Governing equations
Flow problem
Rotational states
Rotational states: Parameter space
Nonlinear dynamics and bifurcations
Division of rotational states
Planar rotational states
Non-planar rotational states
Conclusions

KTH Mechanics

G.W. Woodruff School of Mechanical Engineering

Nonlinear rotational dynamics of a spheroidal particle in simple shear flow

Tomas Rosén, Minh Do-Quang, Cyrus K. Aidun and Fredrik Lundell

Dispersed particle flows

Food

Ref: GorillaAttack/Shutterstock

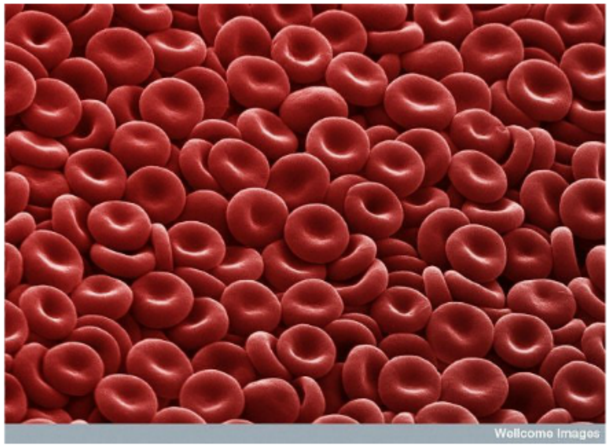
Material processes

Ref: Motif

Geophysical

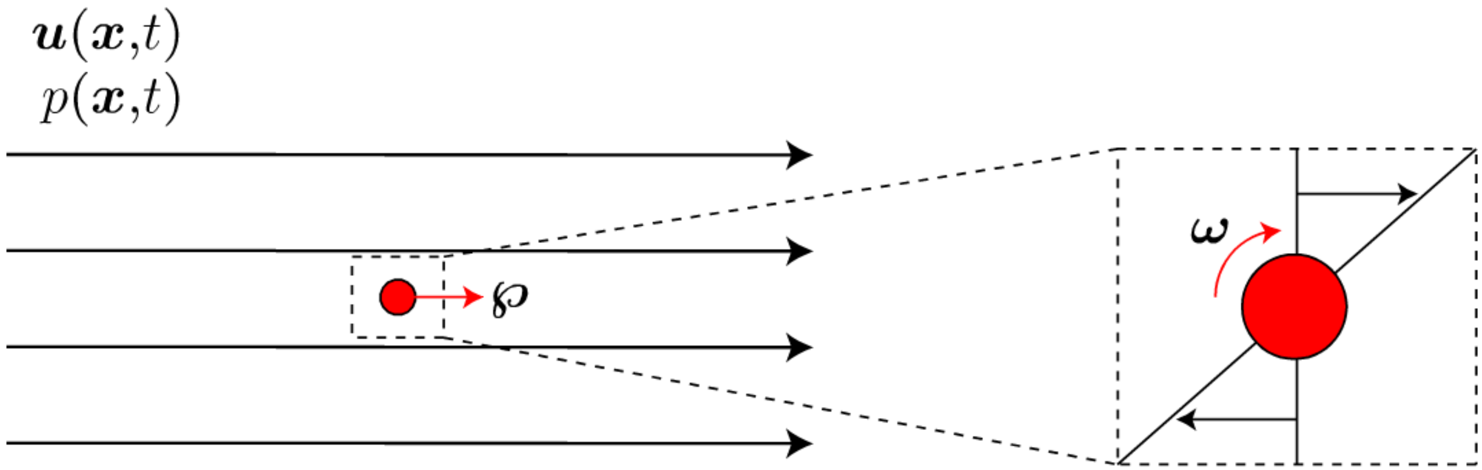
Ref: Snowcrystals.com

Biomedical



Ref: Wellcome Images

Governing equations



Flow:

$$\nabla \cdot \boldsymbol{u} = 0$$

$$Re_p \cdot \left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} \right) = -\nabla p + \nabla^2 \boldsymbol{u}$$

Particle:

$$St \cdot \frac{\partial \wp}{\partial t} = \boldsymbol{F}$$

$$St \cdot \left(\boldsymbol{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega}) \right) = \boldsymbol{T}$$

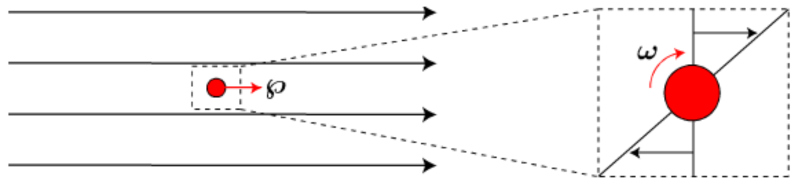
Reynolds number:

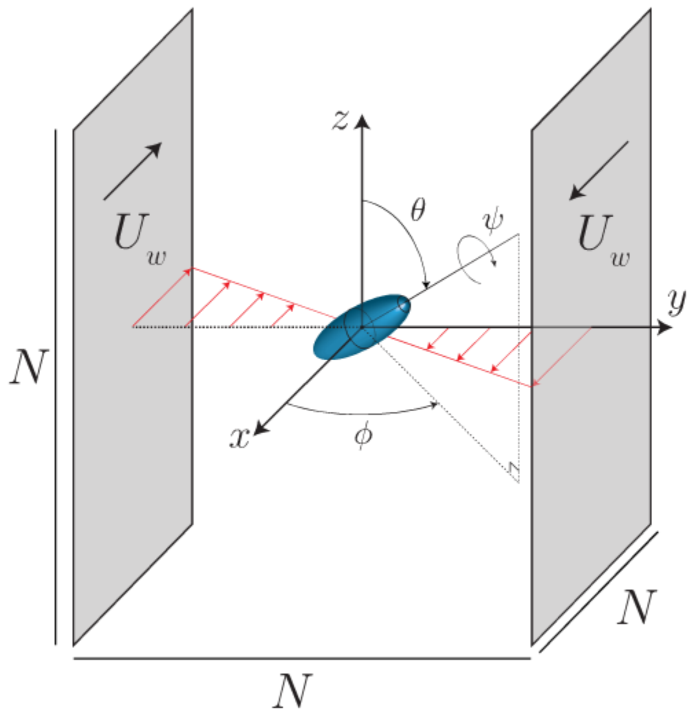
$$Re_p = \frac{UL}{\nu}$$

Stokes number:

$$St = \frac{\rho_p}{\rho_f} \cdot Re_p$$

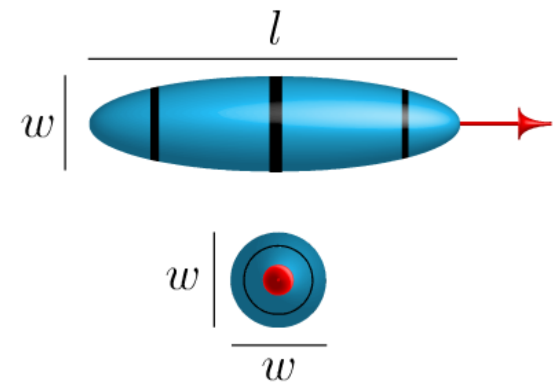
Flow problem





Flow problem investigated numerically with the LB-EBF method (Wu & Aidun, 2010)

Particle: Prolate spheroid



Shear rate

$$G = \frac{2U_w}{N}$$

Aspect ratio

$$r_p = \frac{l}{w}$$

Reynolds number

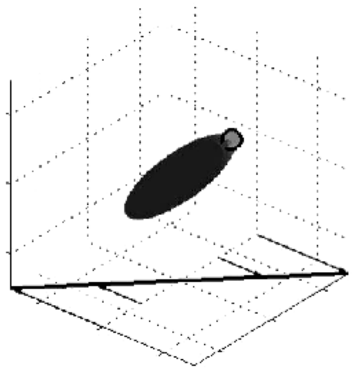
$$Re_p = \frac{(Gl)\cdot l}{\nu}$$

Stokes number

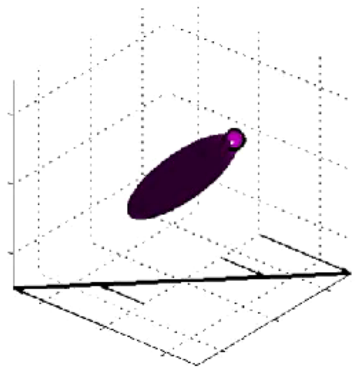
$$St = \frac{\rho_p}{\rho_f}\cdot Re_p$$

Rotational states

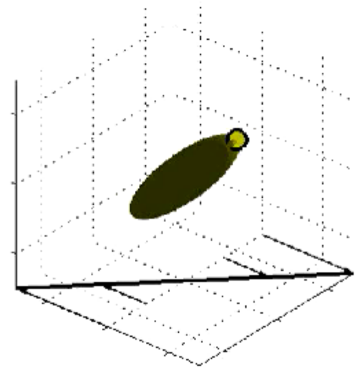
Planar states



Tumbling

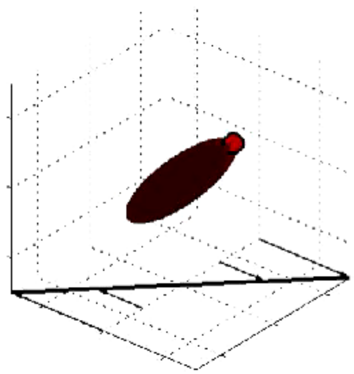


Rotation

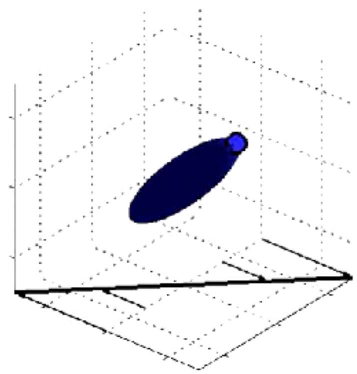


Steady state

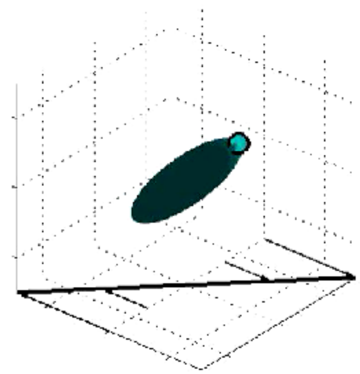
Non-planar states



Log-Rolling



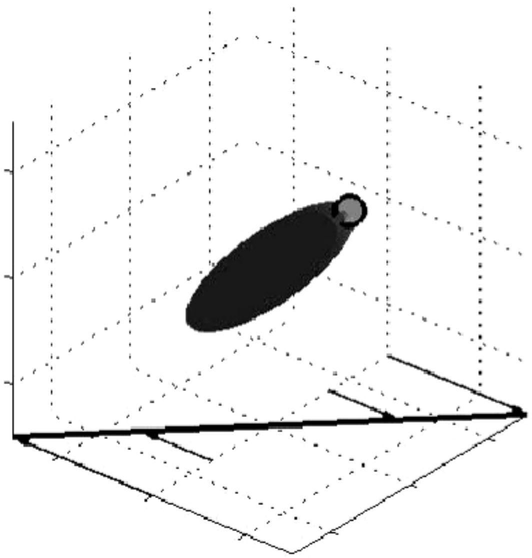
Inclined rolling



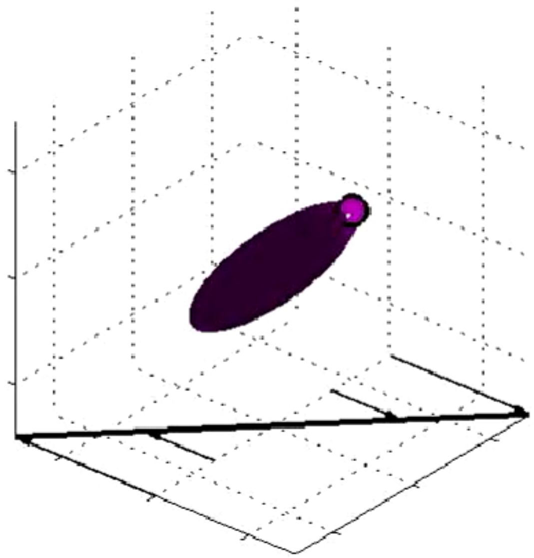
Inclined kayaking

Rotational states

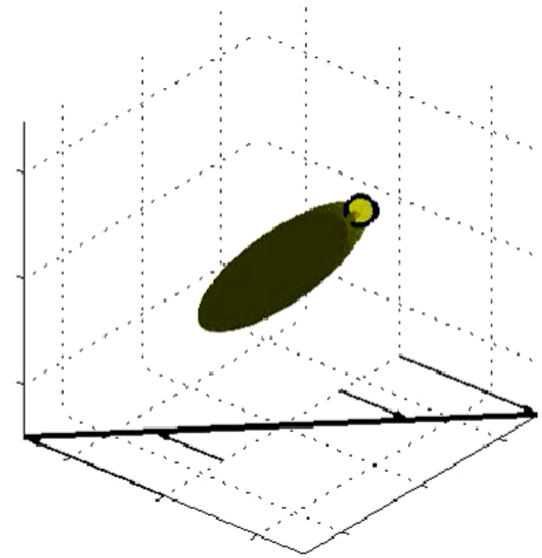
Planar states



Tumbling

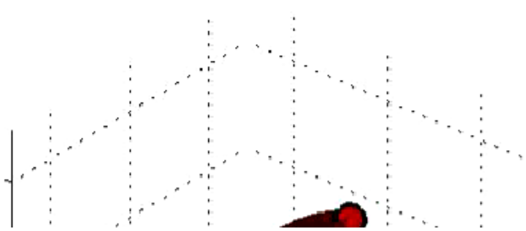


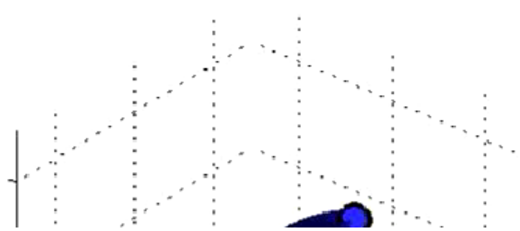
Rotation

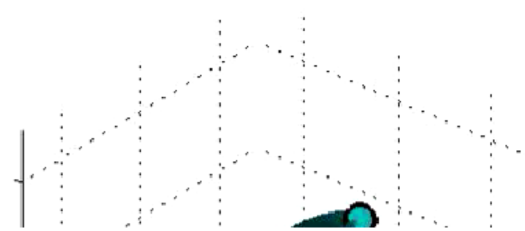


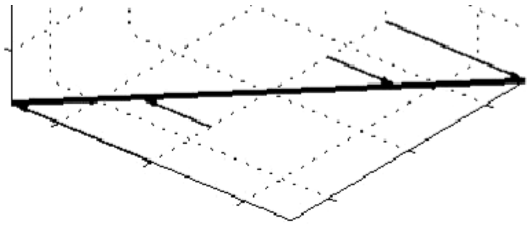
Steady state

Non-planar states

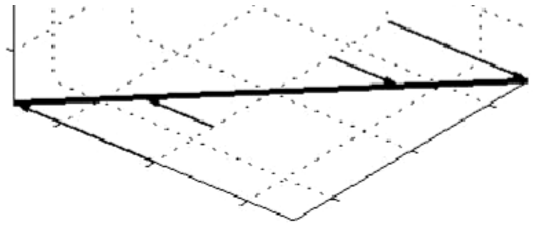




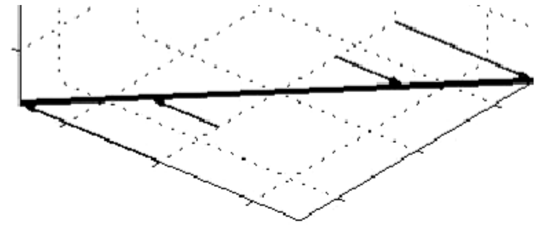




Tumbling

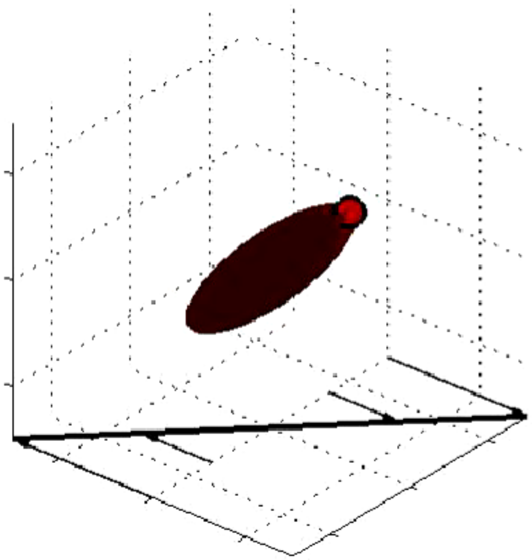


Rotation

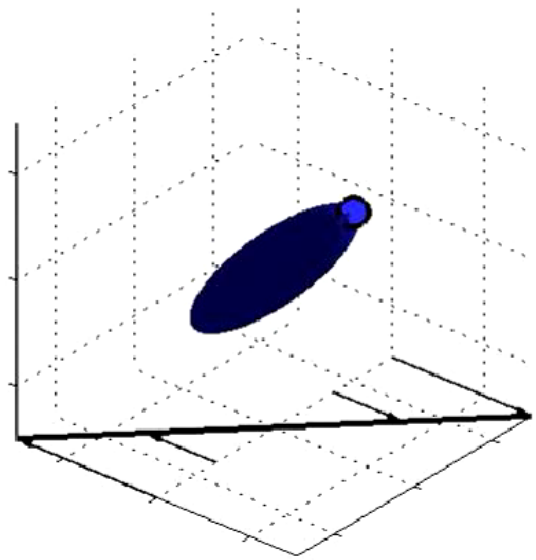


Steady state

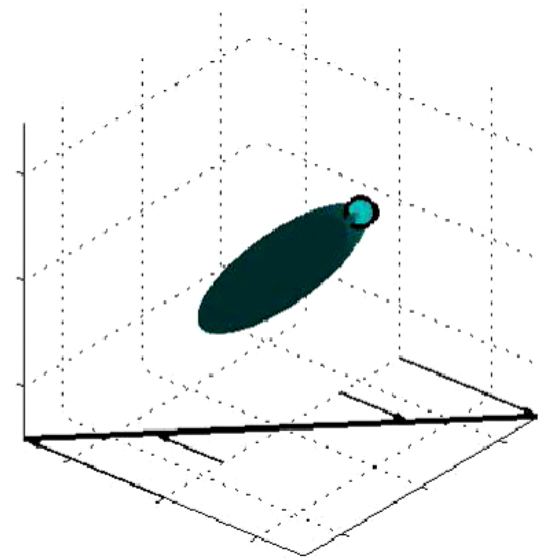
Non-planar states



Log-Rolling



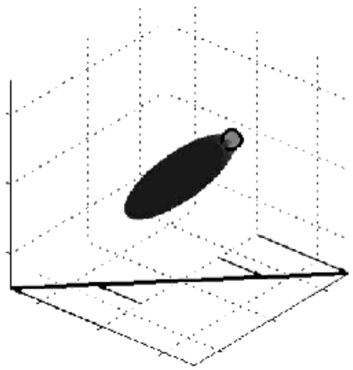
Inclined rolling



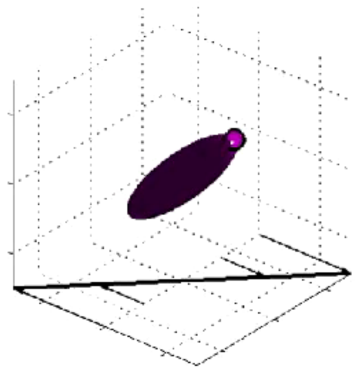
Inclined kayaking

Rotational states

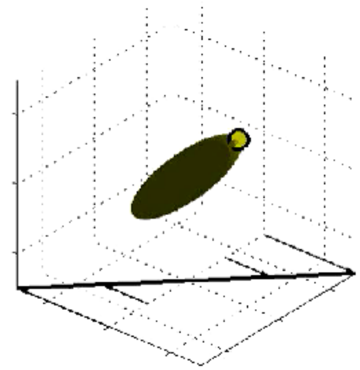
Planar states



Tumbling

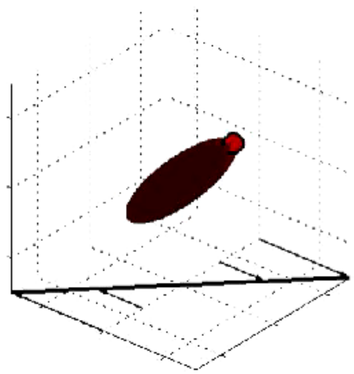


Rotation

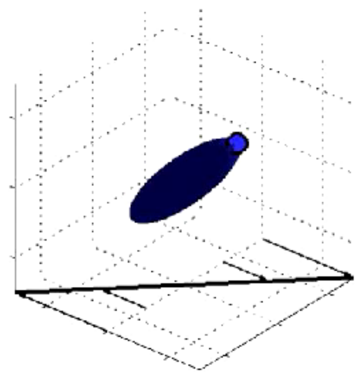


Steady state

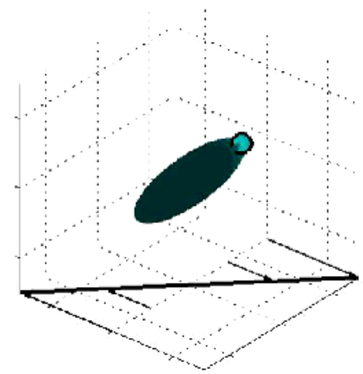
Non-planar states



Log-Rolling



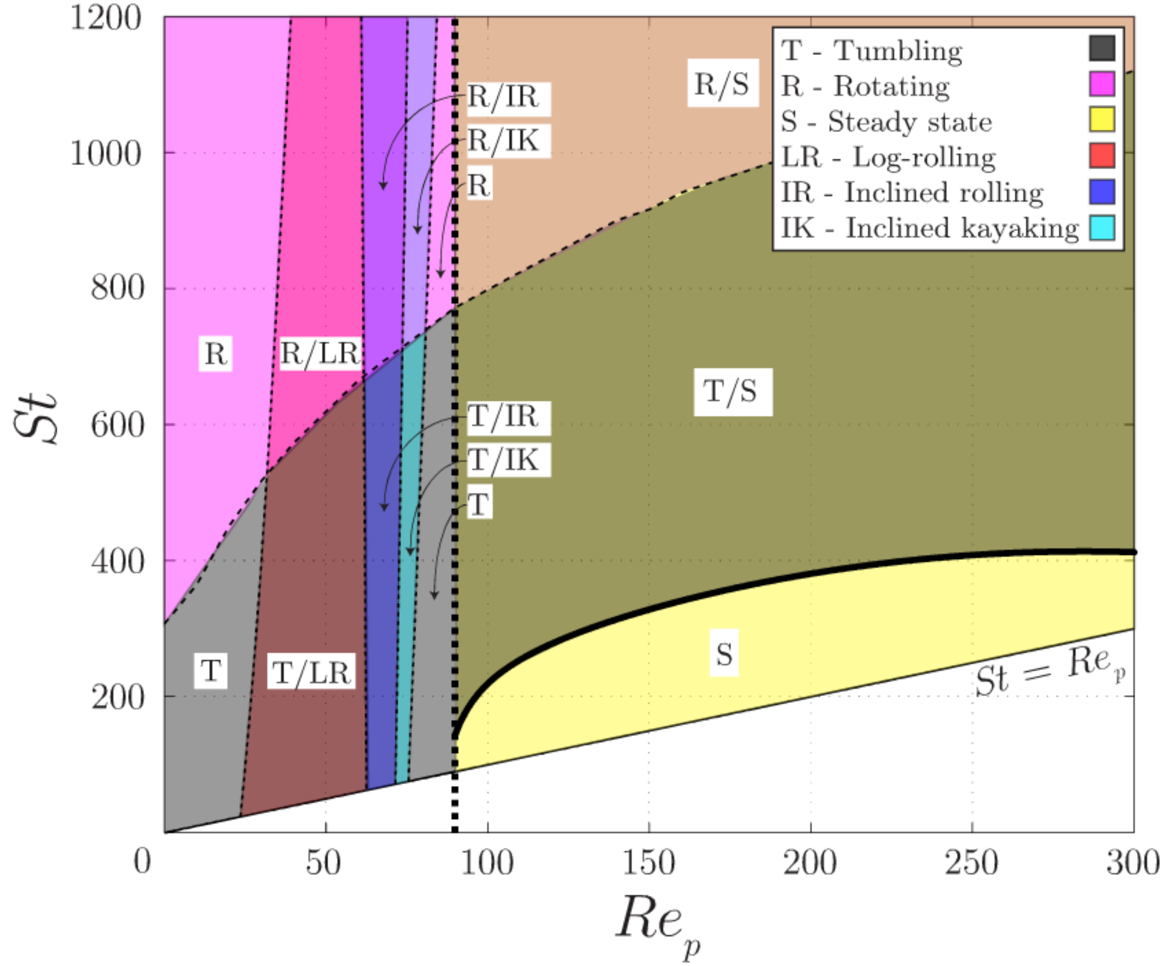
Inclined rolling



Inclined kayaking

Rotational states: Parameter space

($r_p = 4$)

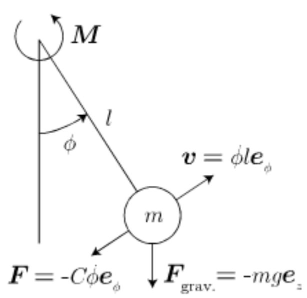


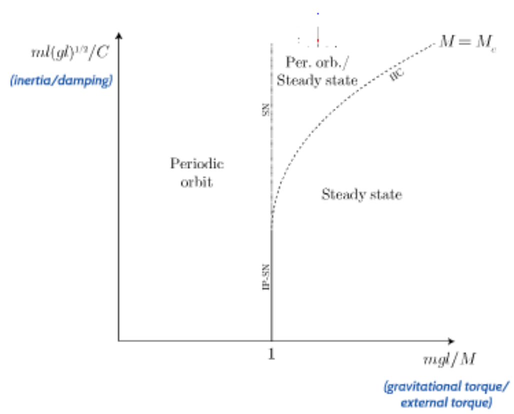
What are the mechanisms behind this appearance?

Nonlinear dynamics and bifurcations

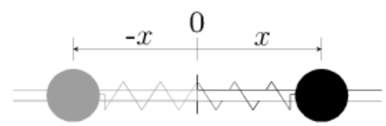
Two examples of nonlinear dynamical systems

Driven damped pendulum





Duffing-Van der Pol oscillator



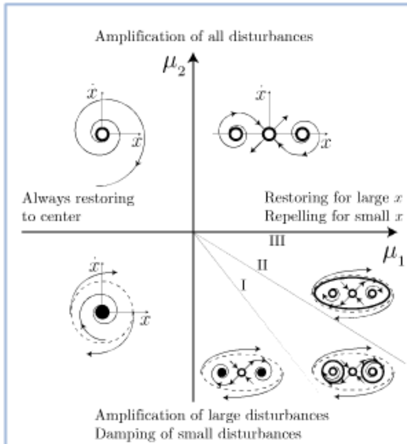
$$\ddot{x} = \alpha x + \beta \dot{x}$$

$\alpha < 0$ Restoring
$\alpha > 0$ Repelling

$\beta < 0$ Damping
$\beta > 0$ Amplifying

Consider the case:

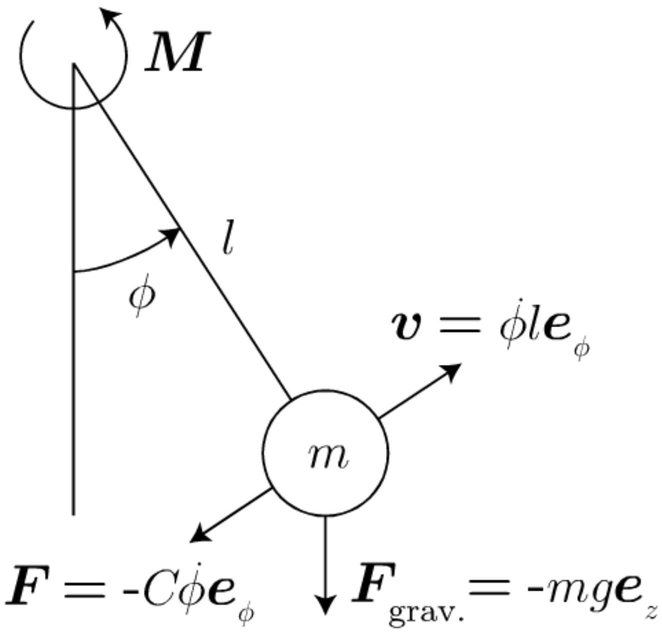
$$\alpha = \mu_1 - x^2$$
$$\beta = \mu_2 + x^2$$

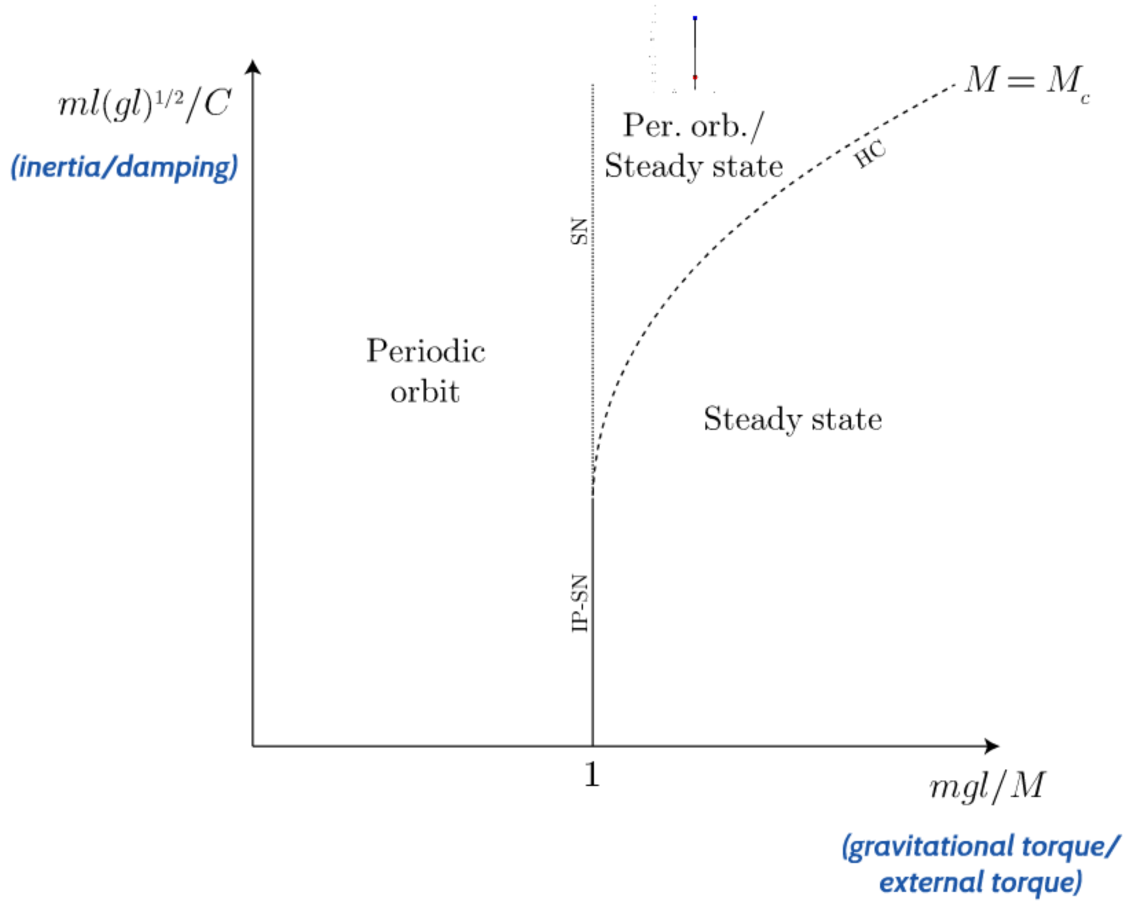


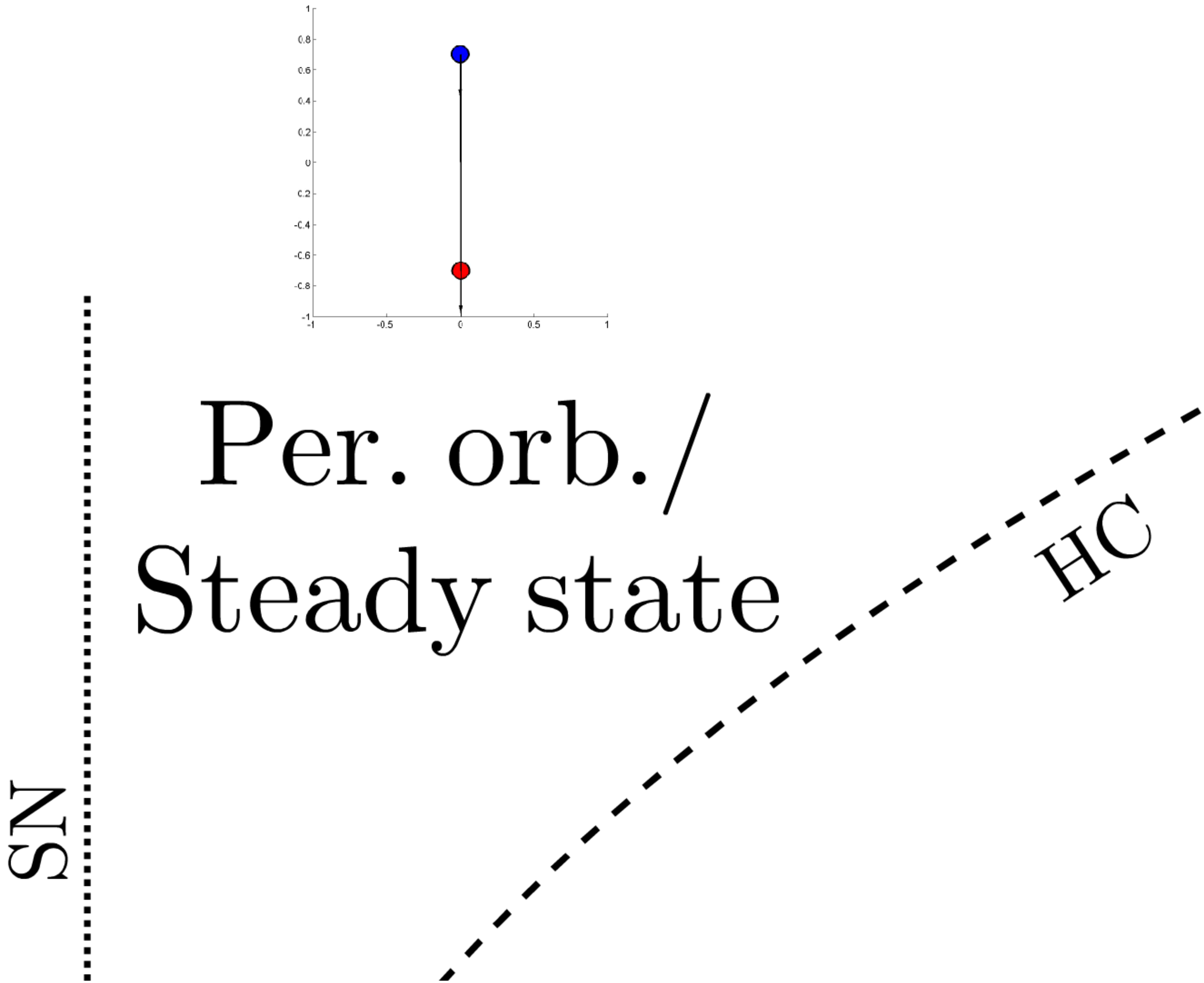
Typical for a nonlinear dynamical system with odd symmetry around a double-zero eigenvalue

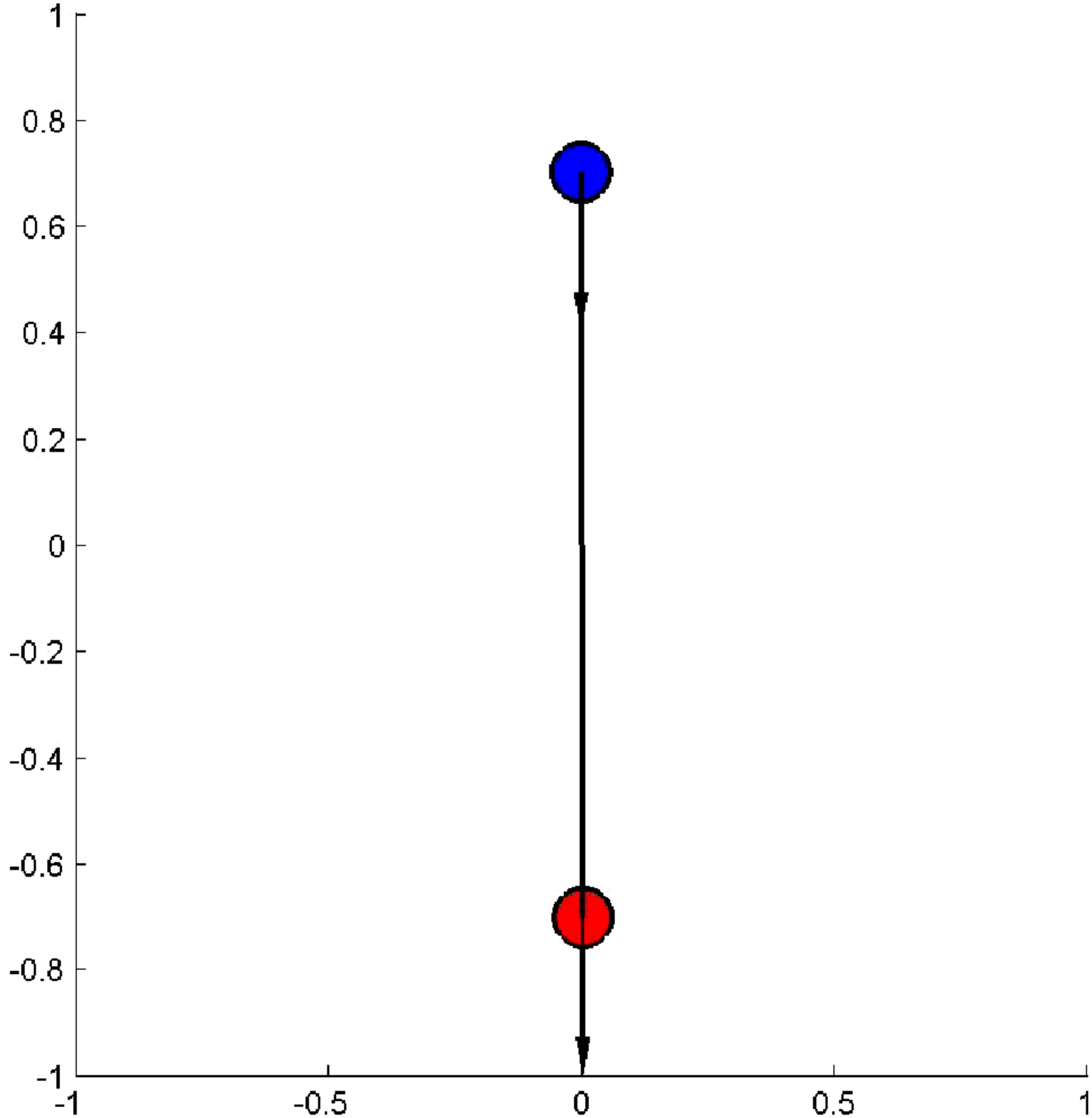
Ref: Hale & Kocak, Springer-Verlag (1991); Strogatz, Westview Press (1994).

Driven damped pendulum

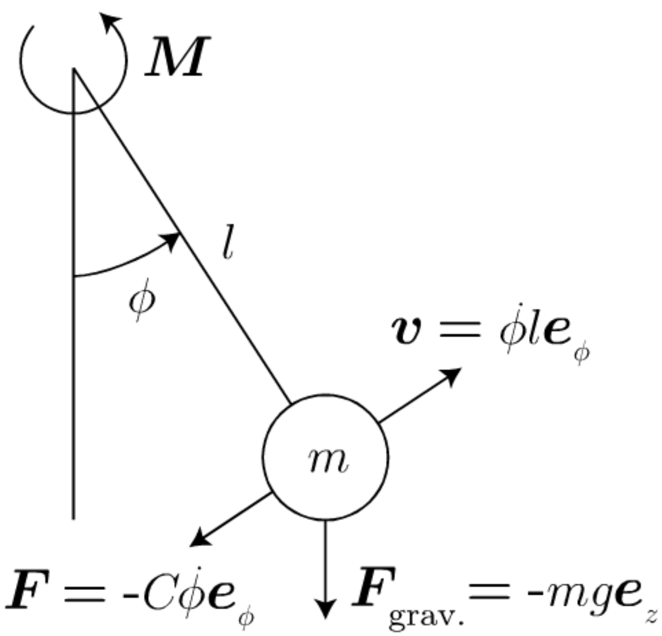


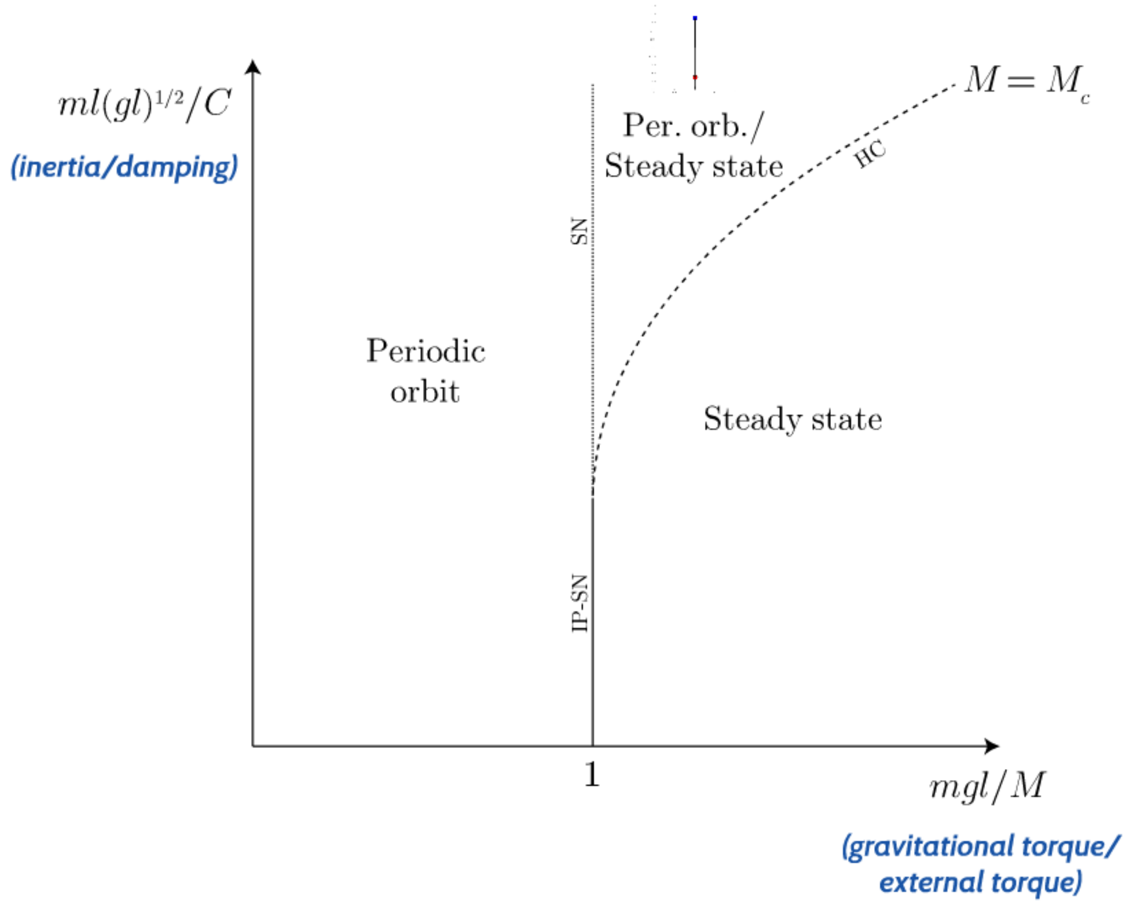



Per. orb./
Steady state
HC
SN

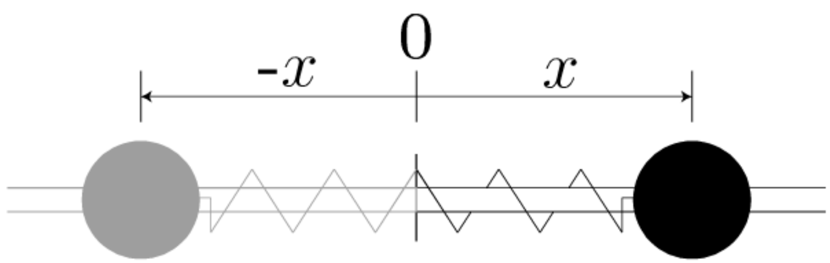

1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1

Driven damped pendulum





Duffing-Van der Pol oscillator



$$\ddot{x} = \alpha x + \beta \dot{x}$$

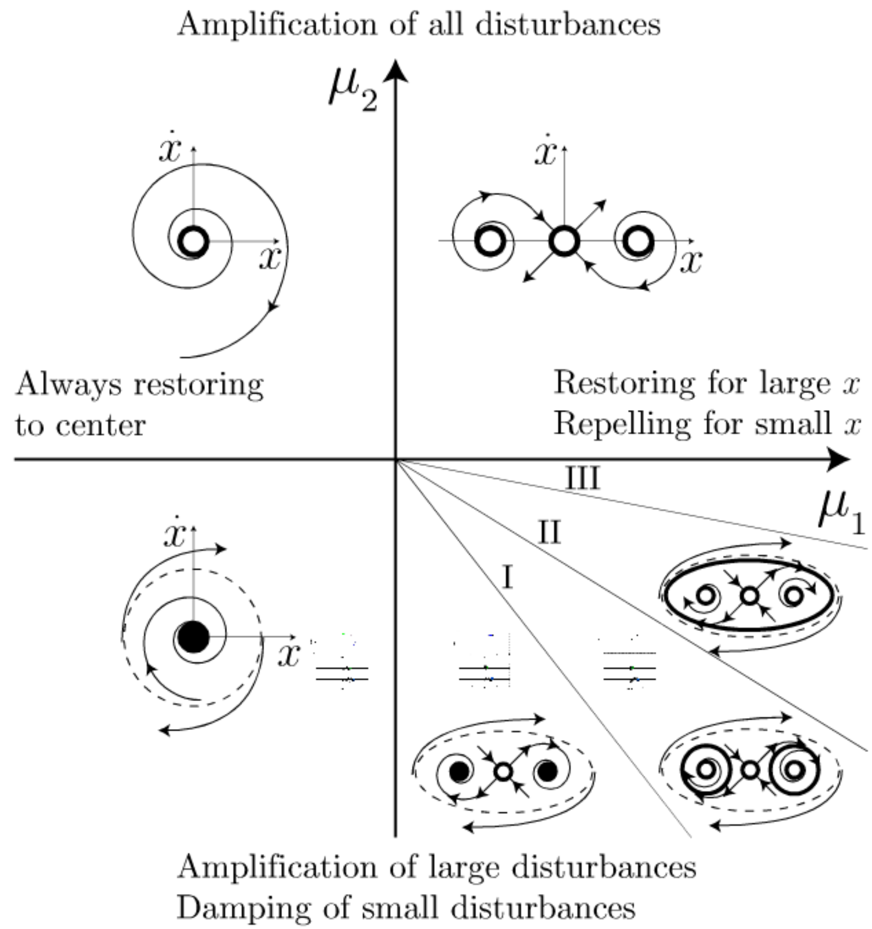
$\alpha < 0$ Restoring
$\alpha > 0$ Repelling

$\beta < 0$ Damping
$\beta > 0$ Amplifying

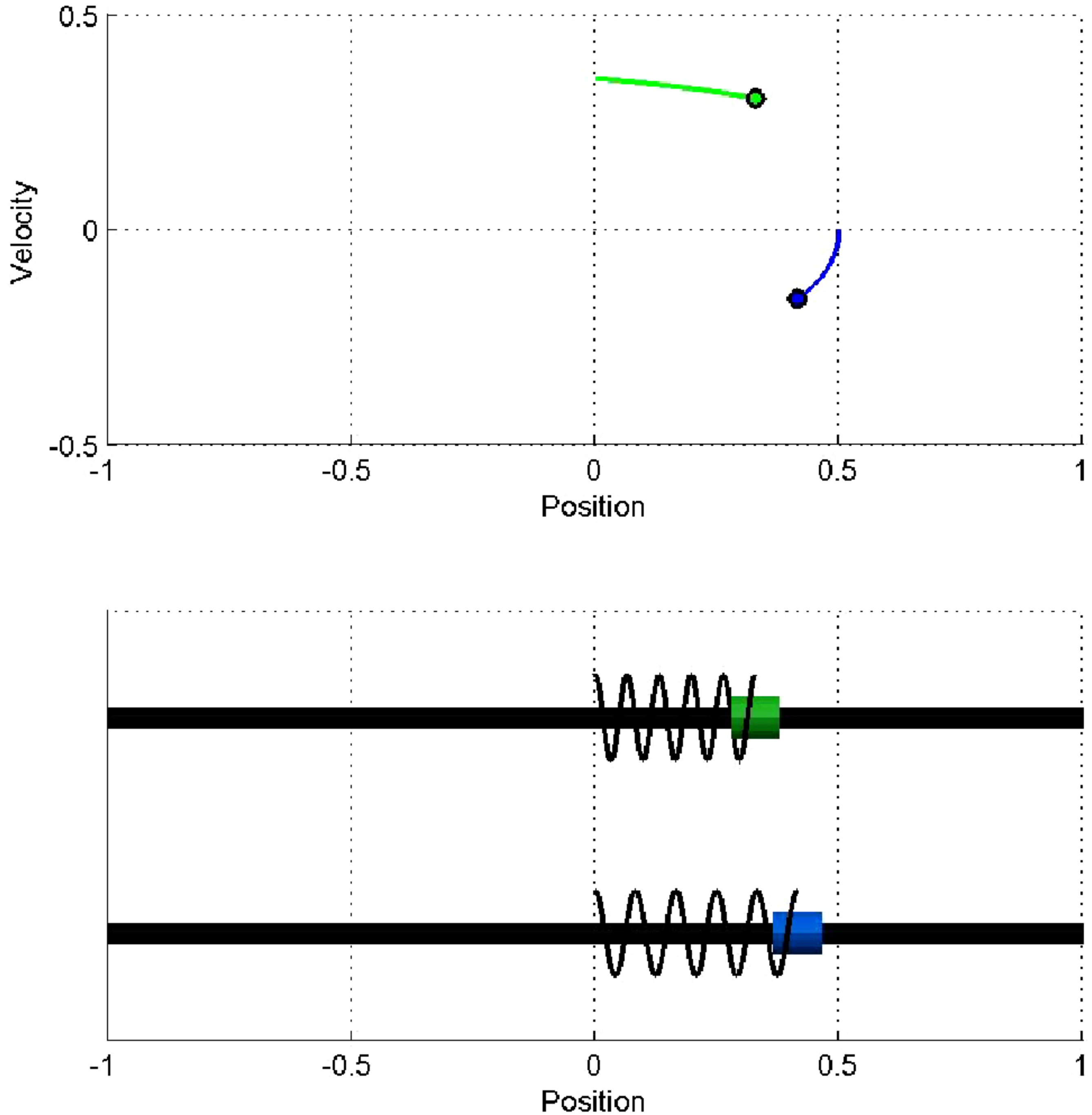
Consider the case:

$$\alpha = \mu_1 - x^2$$

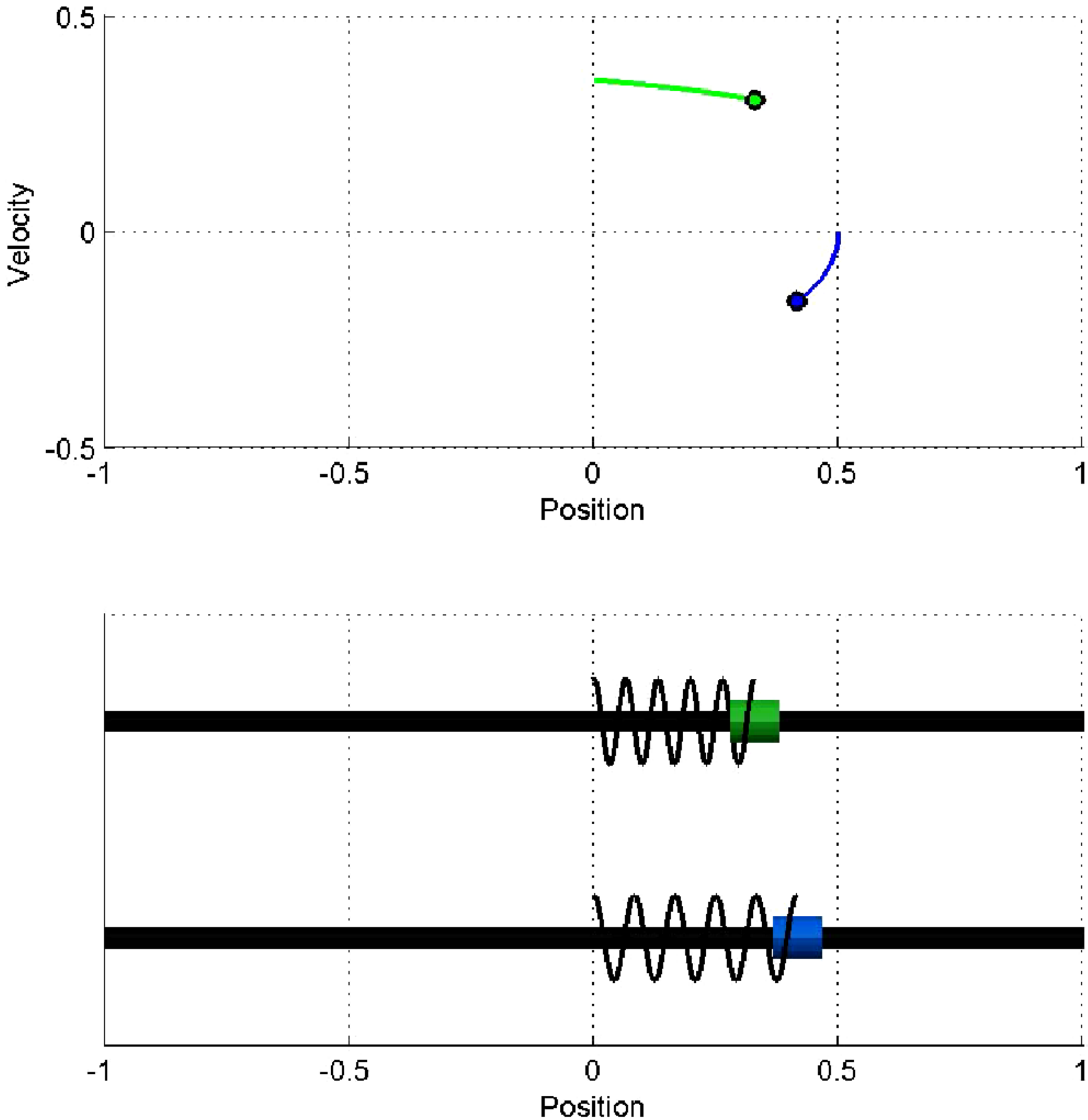
$$\beta = \mu_2 + x^2$$



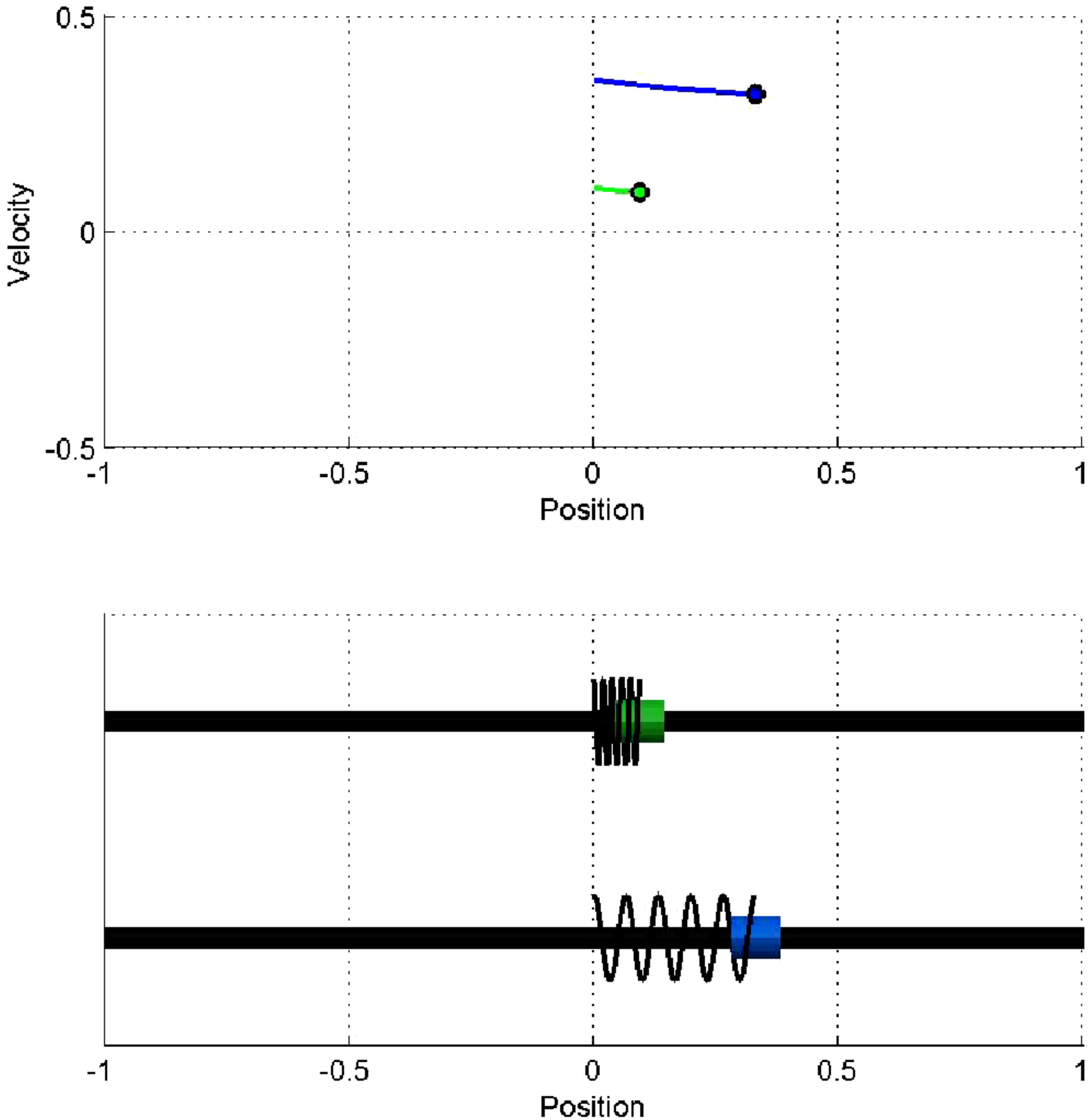
Typical for a nonlinear dynamical system with odd symmetry around a double-zero eigenvalue

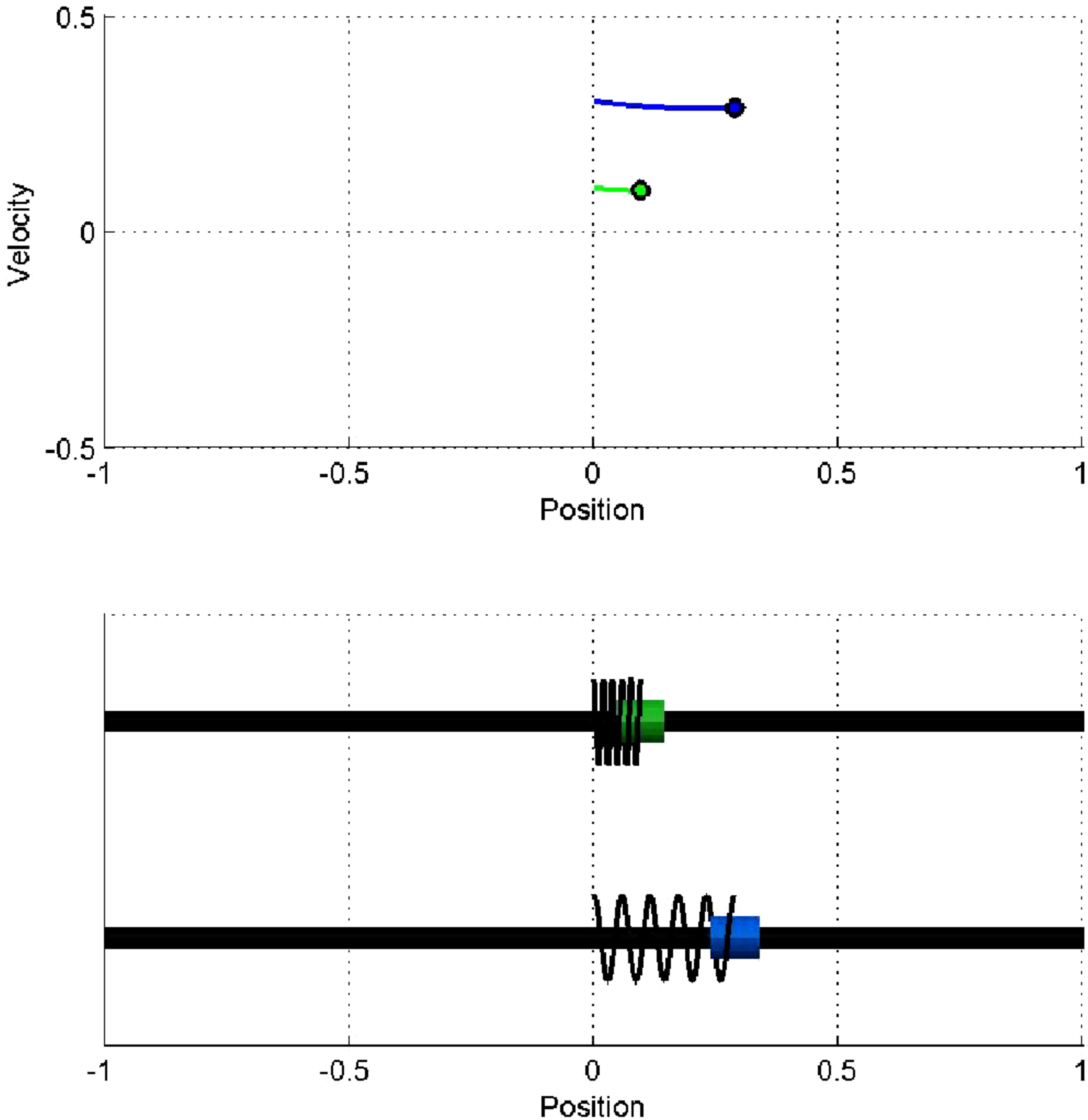


0.5
0
-0.5
Velocity
-1
-0.5
0
0.5
1
Position
-1
-0.5
0
0.5
1
Position

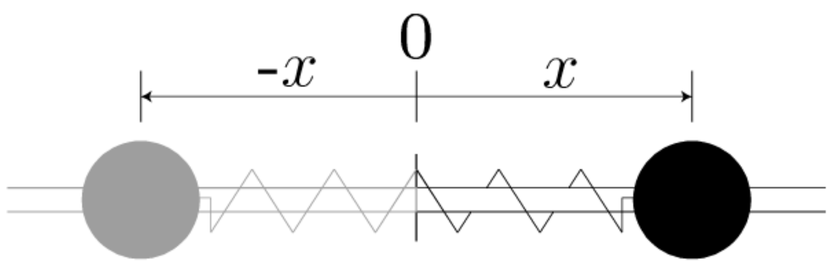


0.5
0
-0.5
Velocity
-1
-0.5
0
0.5
1
Position
-1
-0.5
0
0.5
1
Position


0.5
0
-0.5
Velocity
-1
-0.5
0
0.5
1
Position
-1
-0.5
0
0.5
1
Position


0.5
0
-0.5
Velocity
-1
-0.5
0
0.5
1
Position
-1
-0.5
0
0.5
1
Position

Duffing-Van der Pol oscillator



$$\ddot{x} = \alpha x + \beta \dot{x}$$

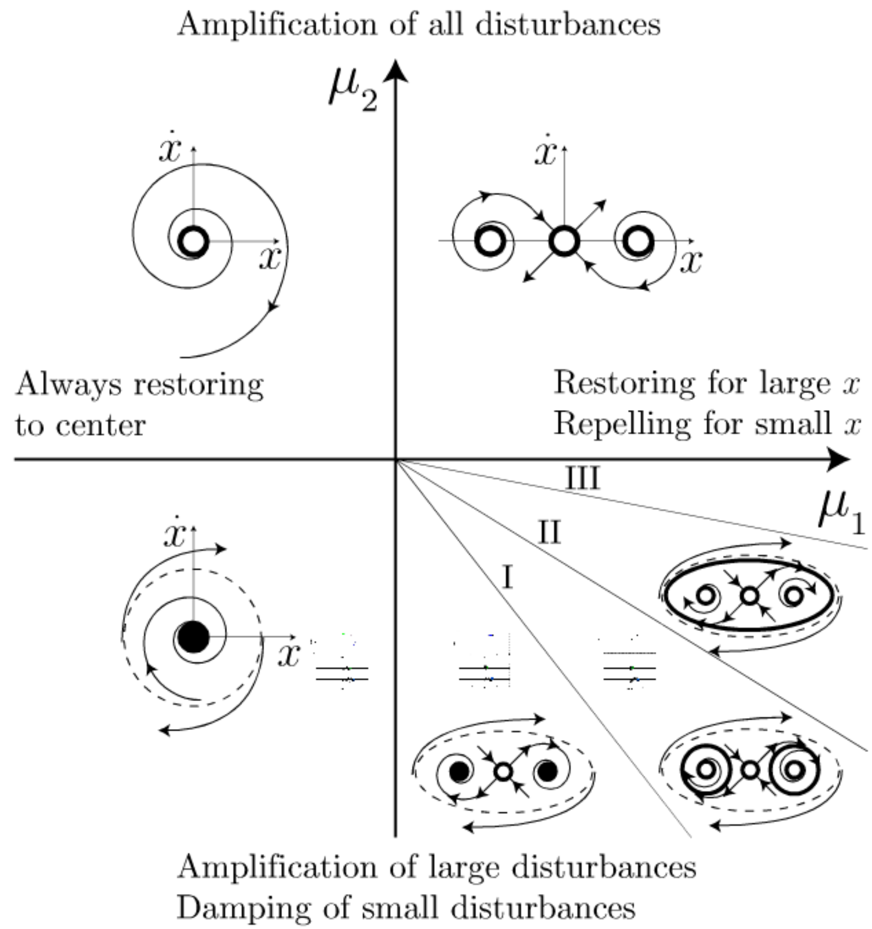
$\alpha < 0$ Restoring
$\alpha > 0$ Repelling

$\beta < 0$ Damping
$\beta > 0$ Amplifying

Consider the case:

$$\alpha = \mu_1 - x^2$$

$$\beta = \mu_2 + x^2$$

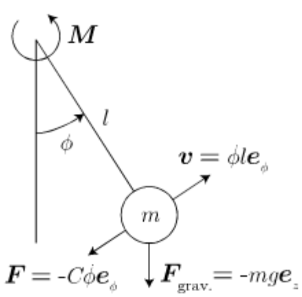


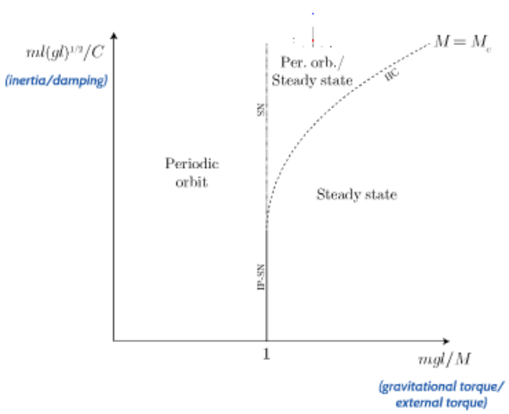
Typical for a nonlinear dynamical system with odd symmetry around a double-zero eigenvalue

Nonlinear dynamics and bifurcations

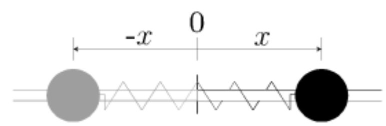
Two examples of nonlinear dynamical systems

Driven damped pendulum





Duffing-Van der Pol oscillator



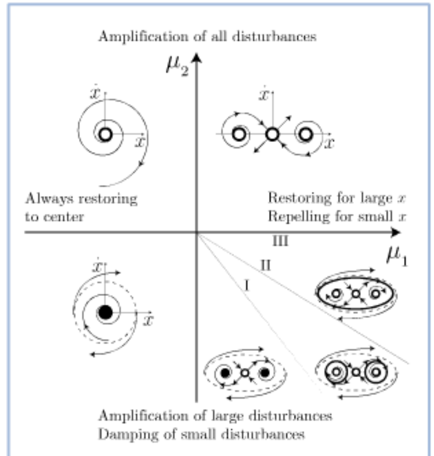
$$\ddot{x} = \alpha x + \beta \dot{x}$$

$\alpha < 0$ Restoring
$\alpha > 0$ Repelling

$\beta < 0$ Damping
$\beta > 0$ Amplifying

Consider the case:

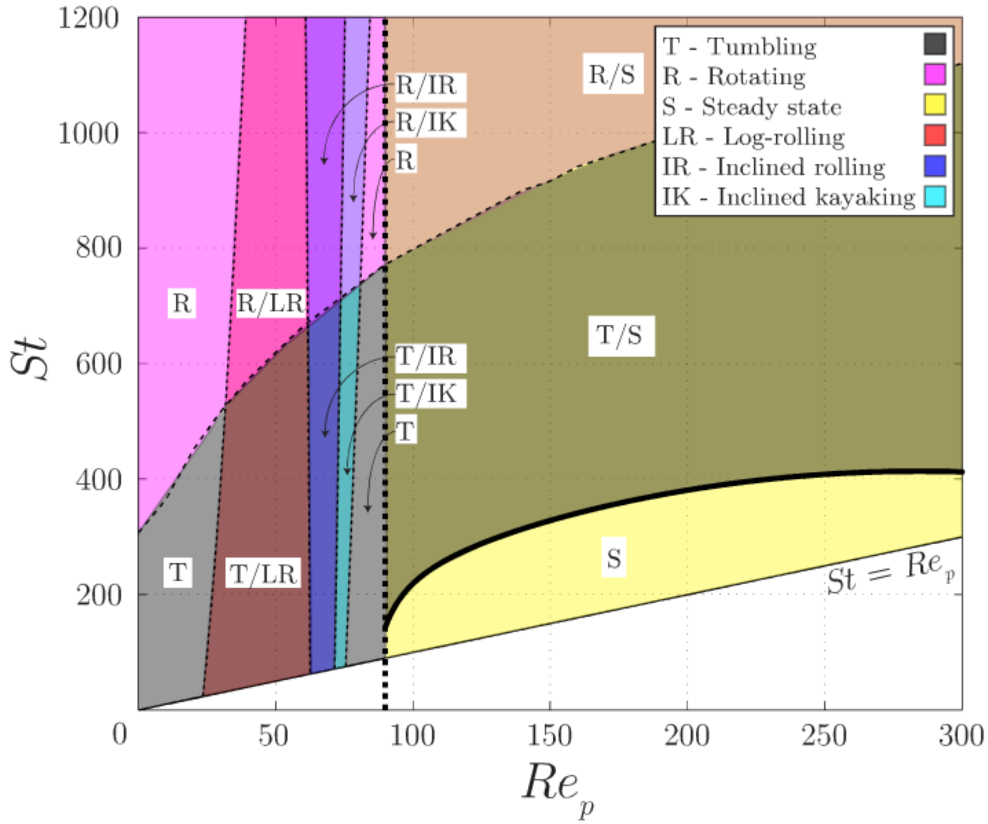
$$\alpha = \mu_1 - x^2$$
$$\beta = \mu_2 + x^2$$



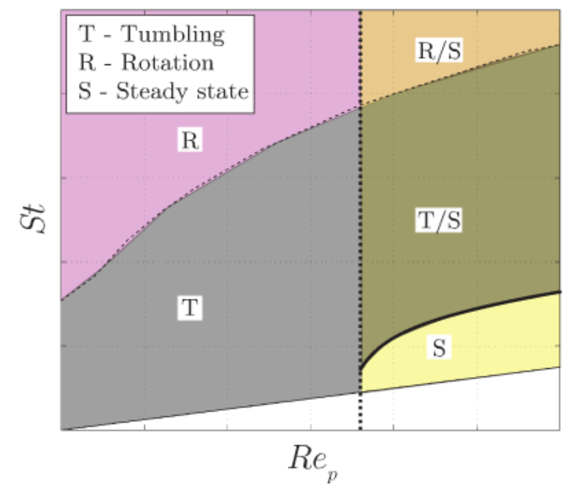
Typical for a nonlinear dynamical system with odd symmetry around a double-zero eigenvalue

Ref: Hale & Kocak, Springer-Verlag (1991); Strogatz, Westview Press (1994).

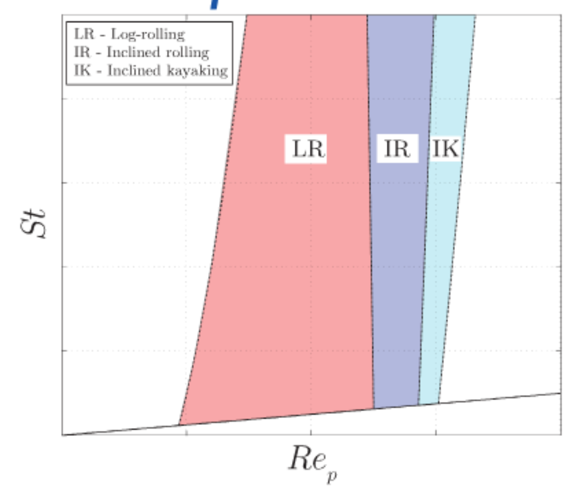
Division of rotational states



Planar states

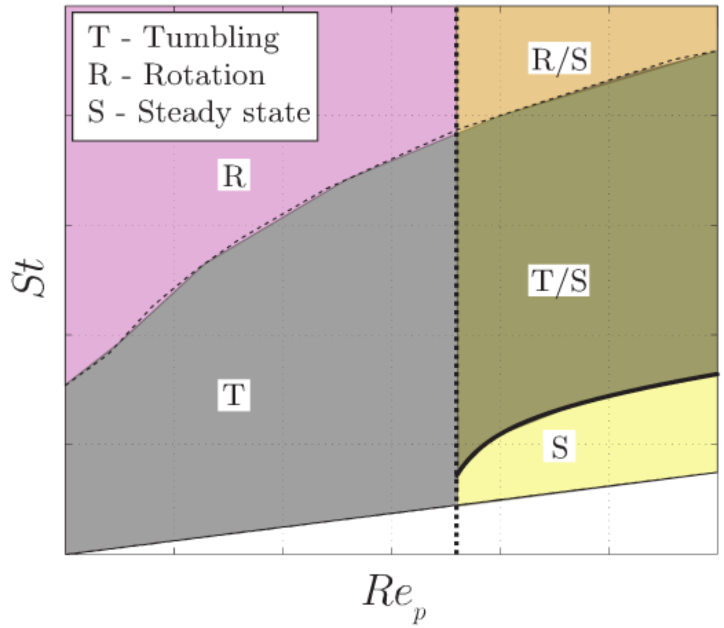


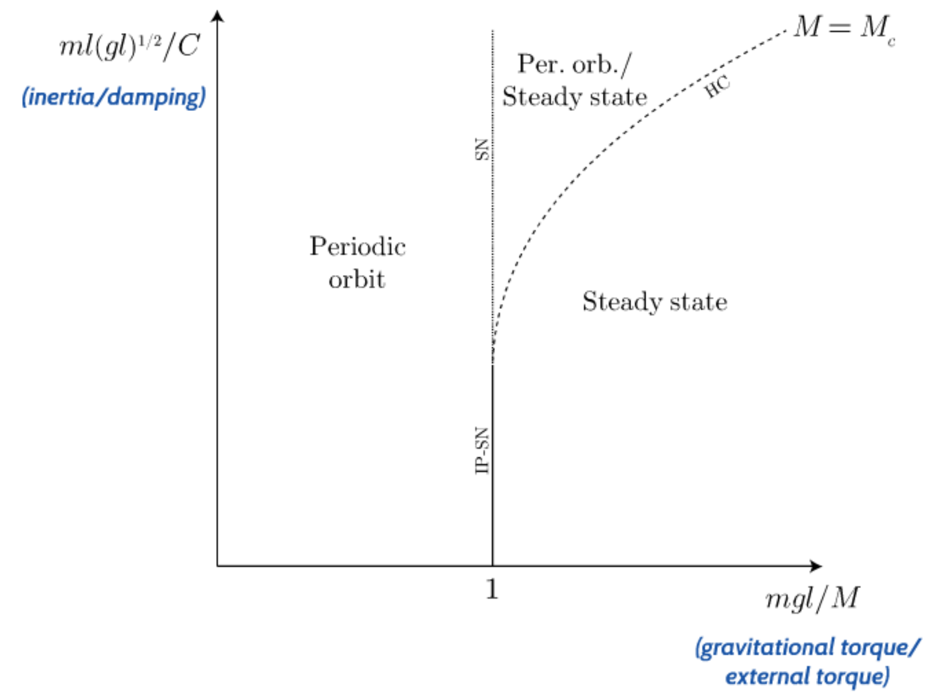
Non-planar states

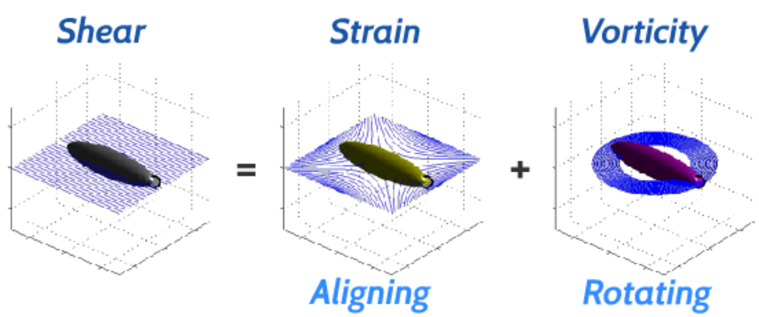


Planar rotational states

(Planar = symmetry axis perpendicular to vorticity)







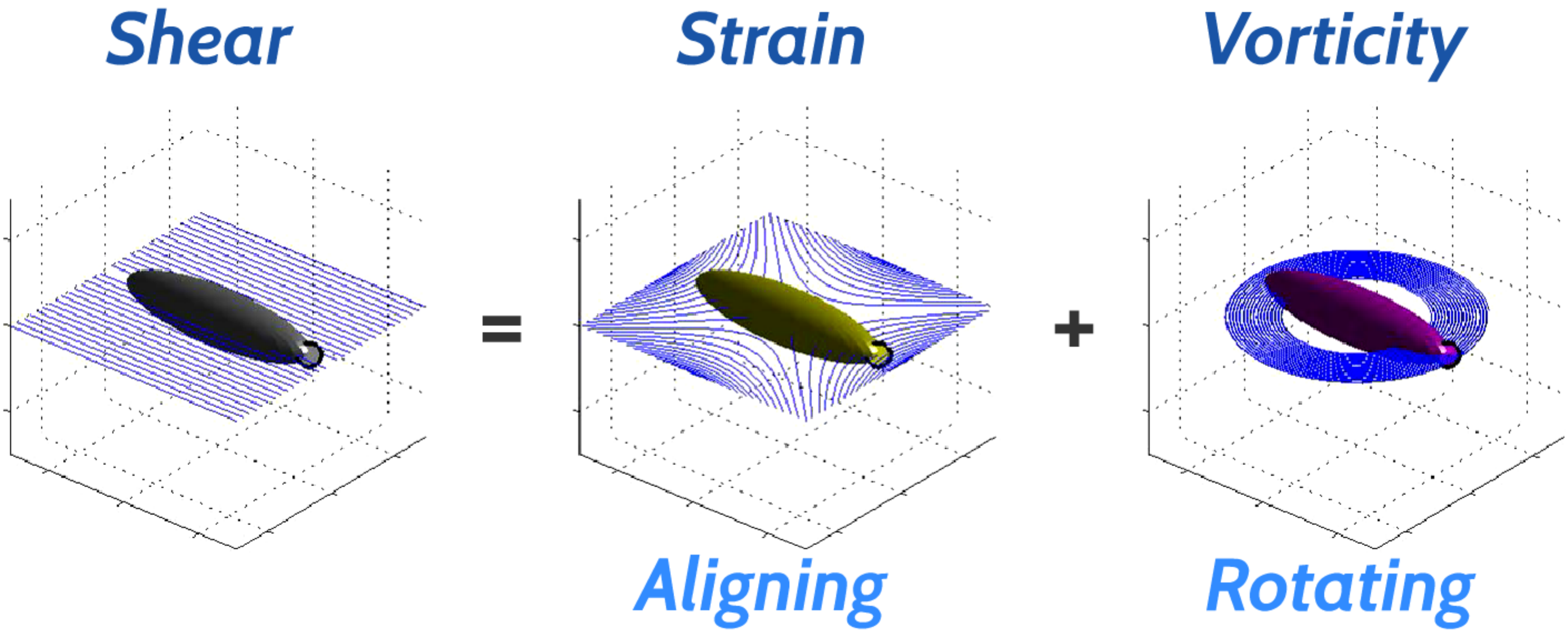
Stokes number = Particle inertia/viscous damping
Reynolds number = Aligning torque/Rotating torque

= Local strain/Local vorticity

1

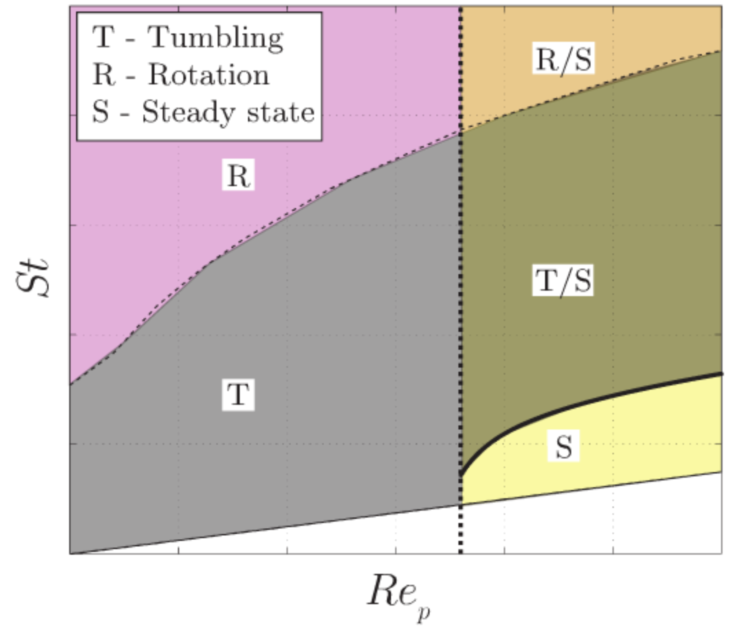
mgl/M

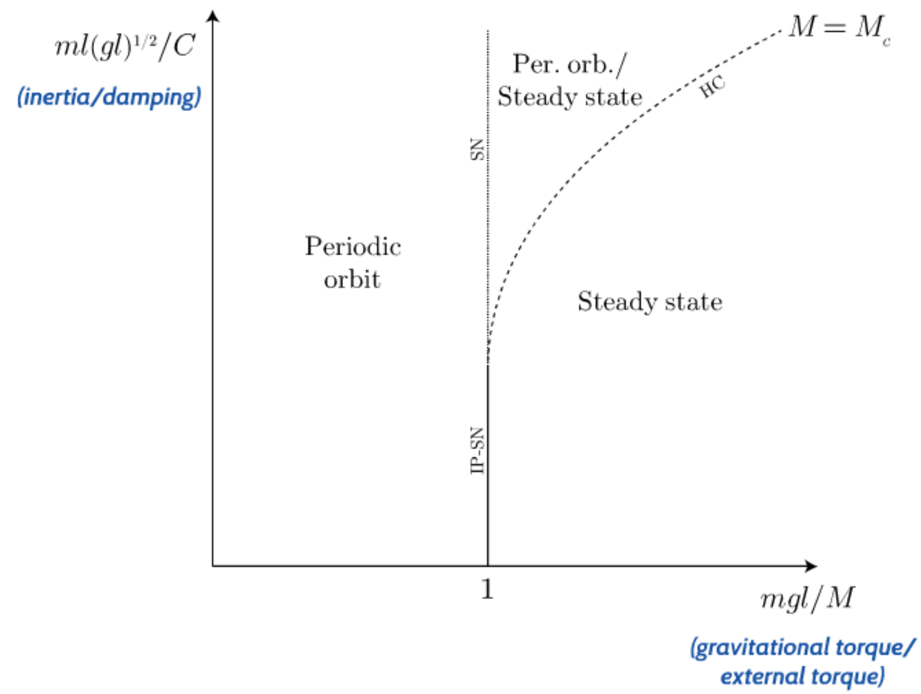
(gravitational tor
external torqu

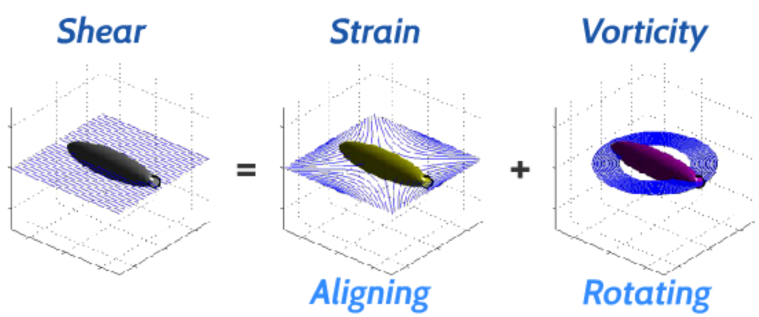


Planar rotational states

(Planar = symmetry axis perpendicular to vorticity)



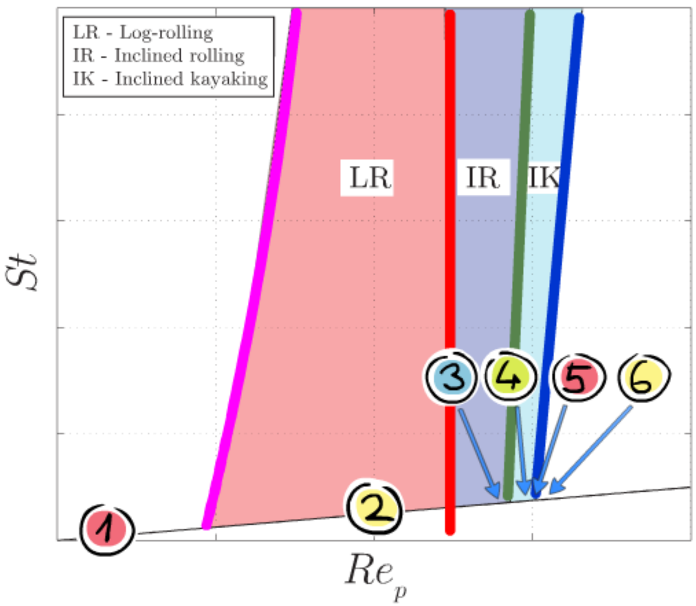


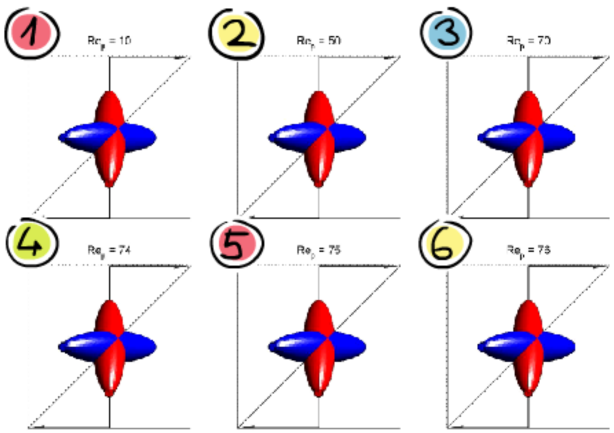


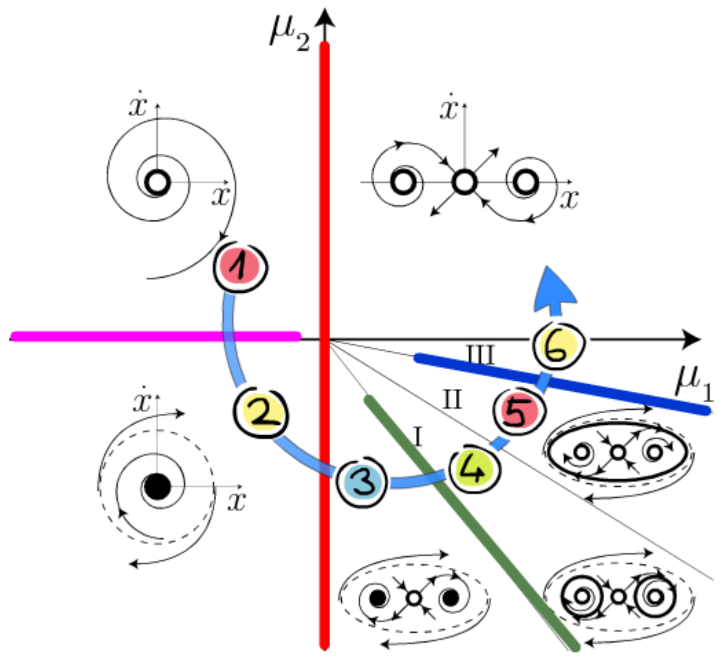
Stokes number = Particle inertia/viscous damping
Reynolds number = Aligning torque/Rotating torque
= Local strain/Local vorticity

Non-planar rotational states

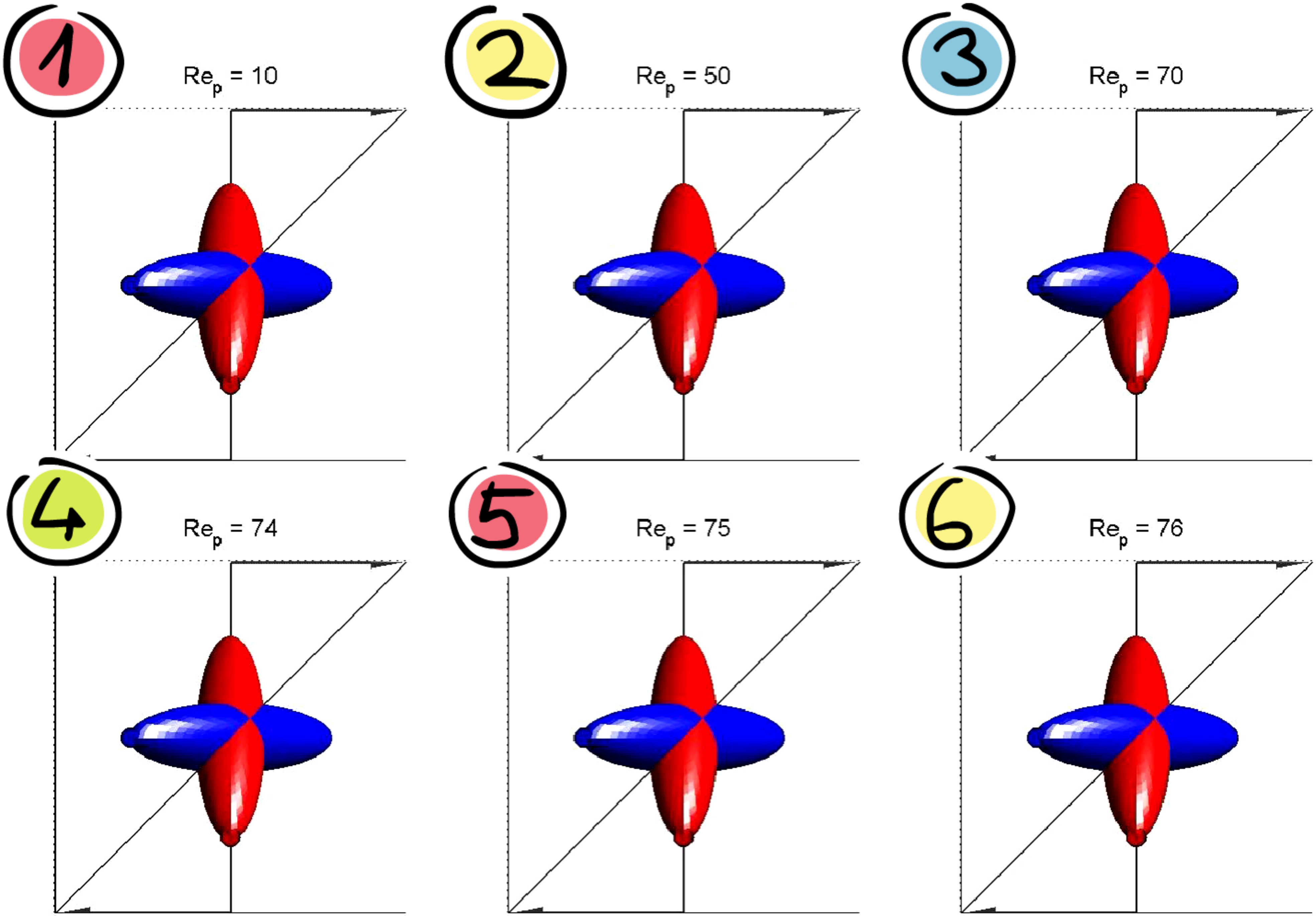
(Non-planar = symmetry axis NOT perpendicular to vorticity)

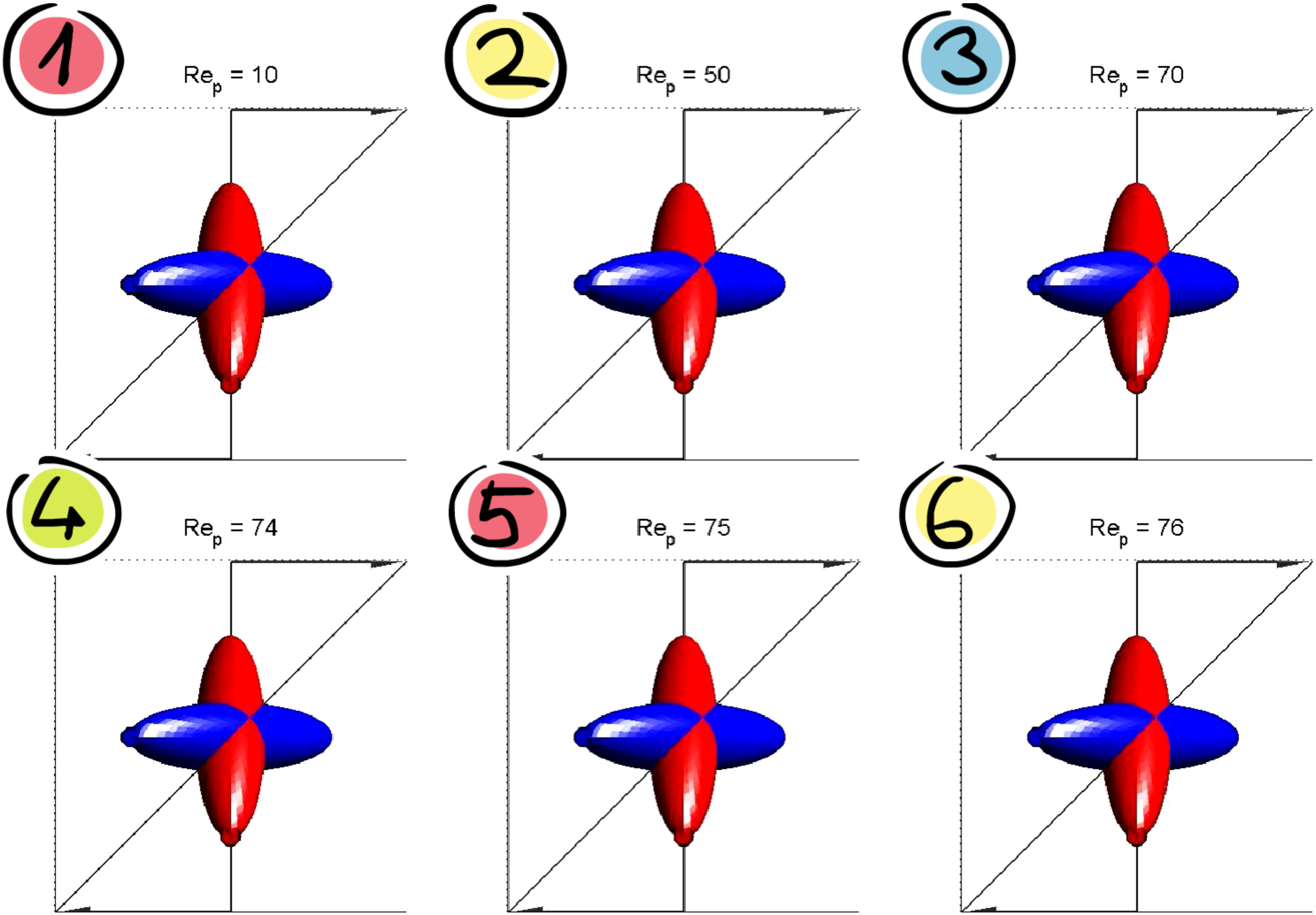






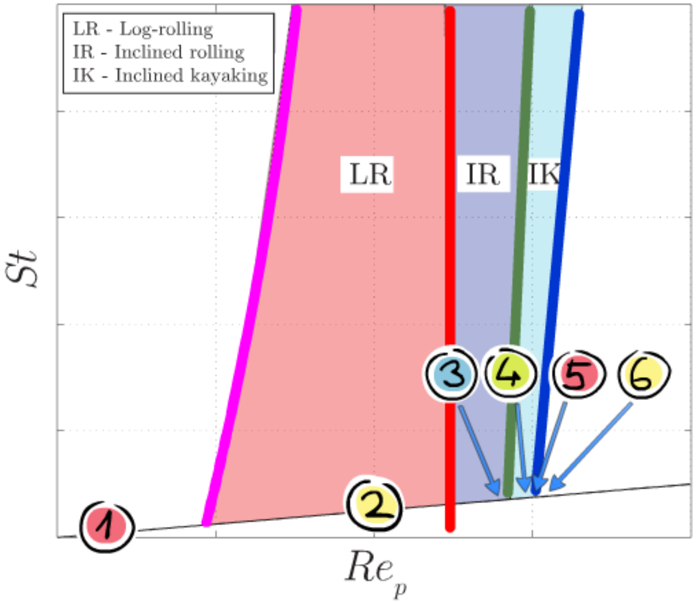
- Particle behaves like a Duffing-Van der Pol oscillator, probably due to the presence of a double zero eigenvalue.
- Reynolds number connected to nonlinear restoring/repelling forces and amplification/damping.
- Stokes number connected to nonlinear damping/amplification.

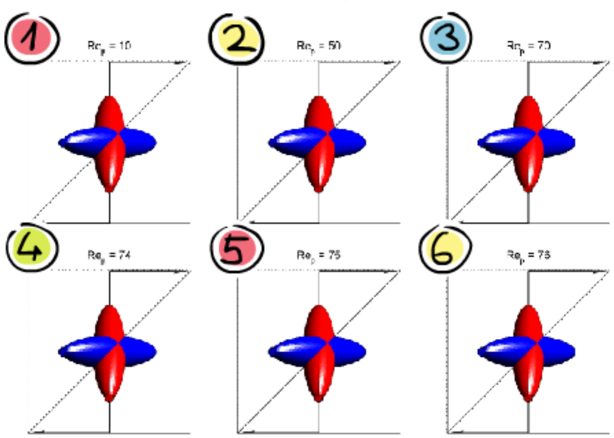

1
Re_p = 10
2
Re_p = 50
3
Re_p = 70
4
Re_p = 74
5
Re_p = 75
6
Re_p = 76

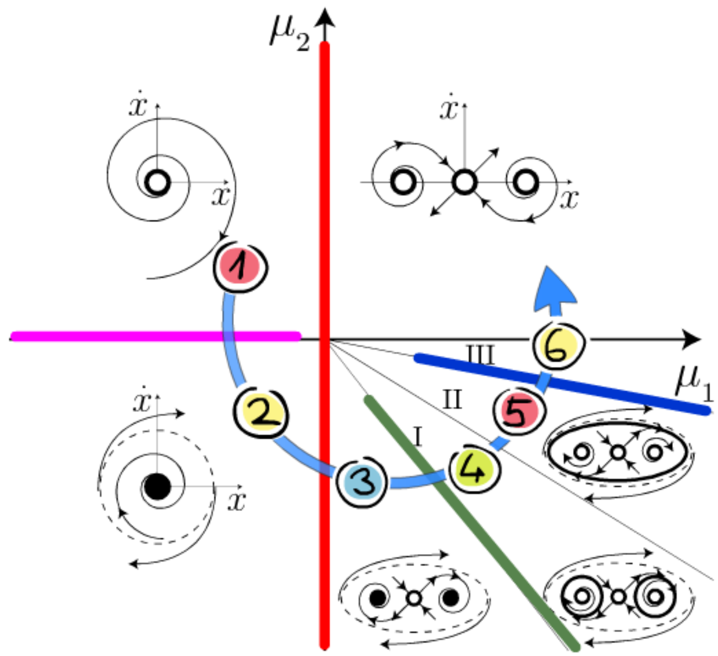

1
Re$_p$ = 10
2
Re$_p$ = 50
3
Re$_p$ = 70
4
Re$_p$ = 74
5
Re$_p$ = 75
6
Re$_p$ = 76

Non-planar rotational states

(Non-planar = symmetry axis NOT perpendicular to vorticity)





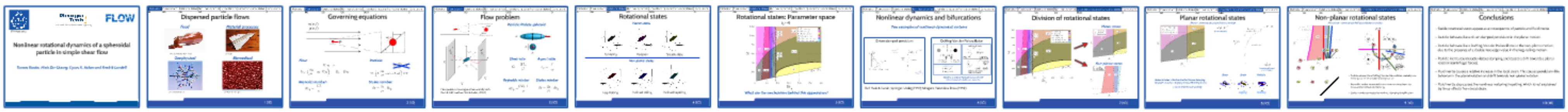


- Particle behaves like a Duffing-Van der Pol oscillator, probably due to the presence of a double zero eigenvalue.
- Reynolds number connected to nonlinear restoring/repelling forces and amplification/damping.
- Stokes number connected to nonlinear damping/amplification.

Conclusions

- Stable rotational states appear as a consequence of particle and fluid inertia.
- Particle behaves like a driven damped pendulum in the planar motion.
- Particle behaves like a Duffing-Van der Pol oscillator in the non-planar motion, due to the presence of a double zero eigenvalue in the Log-rolling motion.
- Particle inertia counteracts viscous damping and causes a drift towards a planar rotation (centrifugal forces).
- Fluid inertia causes a relative increase in the local strain. This causes pendulum-like behavior in the planar rotation and drift towards non-planar rotation.
- Fluid inertia also causes the nonlinear restoring/repelling, which is not explained by linear effects from local strain.

Thank you!



FLOW
Nonlinear rotational dynamics of a spheroidal particle in simple shear flow
Dispersed particle flows
Governing equations
Flow problem
Rotational states
Rotational states: Parameter space
Nonlinear dynamics and bifurcations
Division of rotational states
Planar rotational states
Non-planar rotational states
Conclusions