Joint 7th SIG43 –FP1005 Workshop on

Fiber Suspension Flow Modelling

On the relative motion between rigid fibers and fluid in turbulent channel flow

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Project supported by COST - European Cooperation in Science and Technology

Focus on two Observables: Relative (Slip) Velocity and Spin

Relative velocity (Δu)

Relative spin (Δω)

$$
\begin{cases}
\dfrac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \dfrac{3}{4\lambda Sa^2}\mathbf{K}\cdot\left(\mathbf{u}_{@p} - \mathbf{u}_p\right) \\
\dfrac{\mathrm{d}\Omega_{x'}}{\mathrm{d}t} = \Omega_{y'}\Omega_{z'}\left(1 - \dfrac{2}{1+\lambda^2}\right) + \dfrac{20\left[(1-\lambda^2)f' + (1+\lambda^2)(\xi' - \Omega_{x'})\right]}{(\beta_0 + \lambda^2\gamma_0)(1+\lambda^2)Sa^2} \\
\dfrac{\mathrm{d}\Omega_{y'}}{\mathrm{d}t} = \Omega_{x'}\Omega_{z'}\left(\dfrac{2}{1+\lambda^2} - 1\right) + \dfrac{20\left[(\lambda^2-1)g' + (\lambda^2+1)(\eta' - \Omega_{y'})\right]}{(\alpha_0 + \lambda^2\gamma_0)(1+\lambda^2)Sa^2} \\
\dfrac{\mathrm{d}\Omega_{z'}}{\mathrm{d}t} = \dfrac{20}{(\alpha_0 + \beta_0)Sa^2}(\chi' - \Omega_{z'})
\end{cases}
$$

Refs: Zhao et al. (2014) Phys. Fluids, In press
Marchioli et al. (2014) In preparation

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Motivation: Why are we looking at slip velocity and spin?

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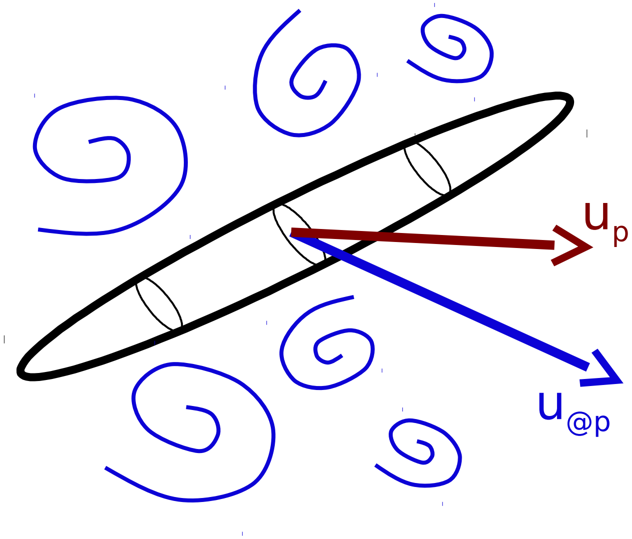
Slip velocity and spin are crucial in:

1. Euler-Lagrange simulations:
   - One-way coupling: determine the drag and torque experienced by fibers
   - Two-way coupling: determine reaction force/torque from fibers on fluid

2. Two-fluid modeling of particle-laden flows
   - Modeling SGS fiber dynamics in LES flow fields
   - Crossing trajectory effects on time decorrelation tensor of u



$\mathbf{u}_{@p}$: fluid velocity “seen”

$\mathbf{u}$p : fiber velocity

$\Delta\mathbf{u}=\mathbf{u}$@p- $\mathbf{u}$p : slip velocity

⇨ Aim of this study: statistical characterization of $\Delta\mathbf{u}$ and $\Delta\boldsymbol{\omega}$ at varying fiber inertia and elongation

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Problem: Dilute suspension of rigid fibers in turbulent channel

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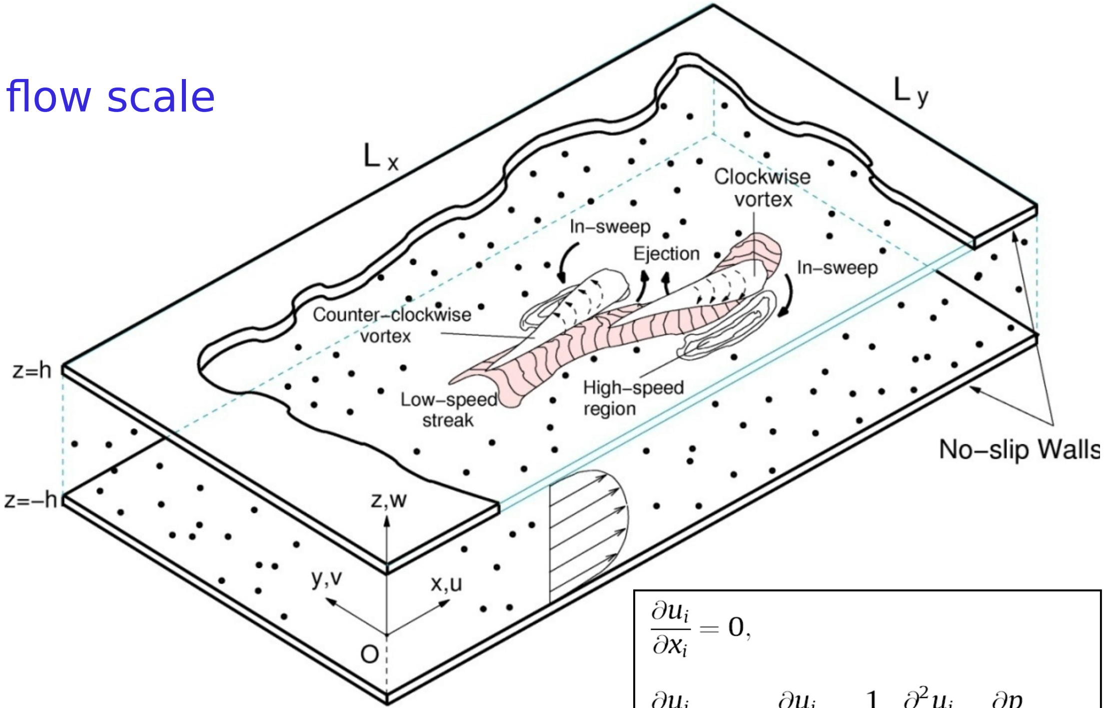
Fibers are modelled as non-deformable prolate ellipsoids evolving in 3D time-dependent fully-turbulent flow (e.g. Marchioli et al, 2010)

Assume:
- Fibers smaller than the smallest flow scale
  - > point-wise
  - > Stokes regime
- Dilute flow
  - > no turbulence modulation
  - > no collisions



$$\frac{\partial u_i}{\partial x_i} = 0,$$

$$\frac{\partial u_i}{\partial t} = -u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{Re_\tau} \frac{\partial^2 u_i}{\partial x_j^2} - \frac{\partial p}{\partial x_i} + \delta_{1,i}$$

Flow solver:

Time-dependent 3D turbulent flow

Channel size: L_{x} x L_y x L_z = $4\pi h$ x $2\pi h$ x $2h$

Pseudo-spectral DNS: Fourier modes (1D FFT) in the homogeneous directions,

Chebyshev coefficients in the wall-normal direction

Time integration: Adams-Bashforth (convective terms), Crank-Nicolson (viscous terms)

Modelling approach:
Fiber tracking

$$
\begin{cases}
\dfrac{\mathrm{d}\mathbf{x}_{p,(G)}}{\mathrm{d}t} = \mathbf{u}_p \\[2mm]
\dfrac{\mathrm{d}e_0}{\mathrm{d}t} = \dfrac{1}{2}(-e_1\Omega_{x'} - e_2\Omega_{y'} - e_3\Omega_{z'}) \\[2mm]
\dfrac{\mathrm{d}e_1}{\mathrm{d}t} = \dfrac{1}{2}(e_0\Omega_{x'} - e_3\Omega_{y'} + e_2\Omega_{z'}) \\[2mm]
\dfrac{\mathrm{d}e_2}{\mathrm{d}t} = \dfrac{1}{2}(e_3\Omega_{x'} + e_0\Omega_{y'} - e_1\Omega_{z'}) \\[2mm]
\dfrac{\mathrm{d}e_3}{\mathrm{d}t} = \dfrac{1}{2}(-e_2\Omega_{x'} + e_1\Omega_{y'} + e_0\Omega_{z'})
\end{cases}
$$

$$
\begin{cases}
\dfrac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \dfrac{3}{4\lambda S a^2}\mathbf{K}\cdot(\mathbf{u}_{@p} - \mathbf{u}_p) \\[2mm]
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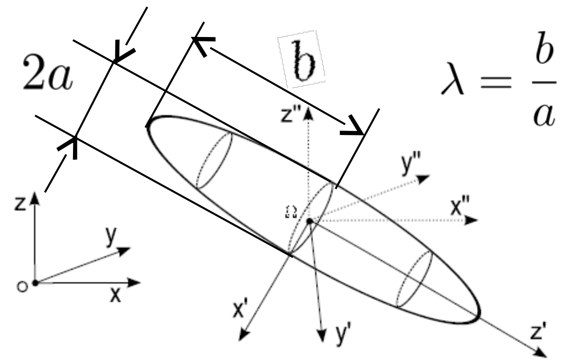
\end{cases}
$$



3 frames of reference:

Inertial, **X**=[x,y,z]

Particle, **X'**=[x',y',z']

Co-moving, **X"**=[x",y",z"]

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Problem: Dilute suspension of rigid fibers in turbulent channel

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Simulation parameters:

- Fluid

Re_τ	Fluid	$\rho_F\ [kg/m^3]$	$\nu\ [m^2/s]$	$h\ [cm]$	$u_\tau\ [m/s]$	$\overline{u_x}\ [m/s]$
150	Air	1.3	$1.57 \cdot 10^{-5}$	2.0	0.11775	1.77
150	Water	1000	$1.00 \cdot 10^{-6}$	0.5	0.03000	0.45

- Particles

Set	St	λ	S	$2b^+$	(μm)	(kg/m³)
F1-1	1	1.001	34.72	0.72	96.07	45.14
F1-3	1	3	18.57	2.16	287.93	24.14
F1-10	1	10	11.54	7.20	960.09	15.01
F1-50	1	50	7.54	36.00	4800.01	9.80
F5-1	5	1.001	173.60	0.72	96.07	225.68
F5-3	5	3	92.90	2.16	287.93	120.77
F5-10	5	10	57.70	7.20	960.09	75.01
F5-50	5	50	37.69	36.00	4800.01	49.00
F30-1	30	1.001	1041.70	0.72	96.07	1354.21
F30-3	30	3	557.10	2.16	287.93	724.23
F30-10	30	10	346.30	7.20	960.09	450.19
F30-50	30	50	226.15	36.00	4800.01	294.00
F100-1	100	1.001	3472.33	0.72	96.07	4514.03
F100-3	100	3	1857.00	2.16	287.93	2414.10
F100-10	100	10	1154.33	7.20	960.09	1500.63
F100-50	100	50	753.83	36.00	4800.01	979.98

$$St = \frac{2\lambda S a^2}{9\nu} f(\lambda)$$

$$f(\lambda) = \frac{\ln(\lambda + \sqrt{\lambda^2 - 1})}{\sqrt{\lambda^2 - 1}}$$

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Results: Near-wall fiber preferential distribution

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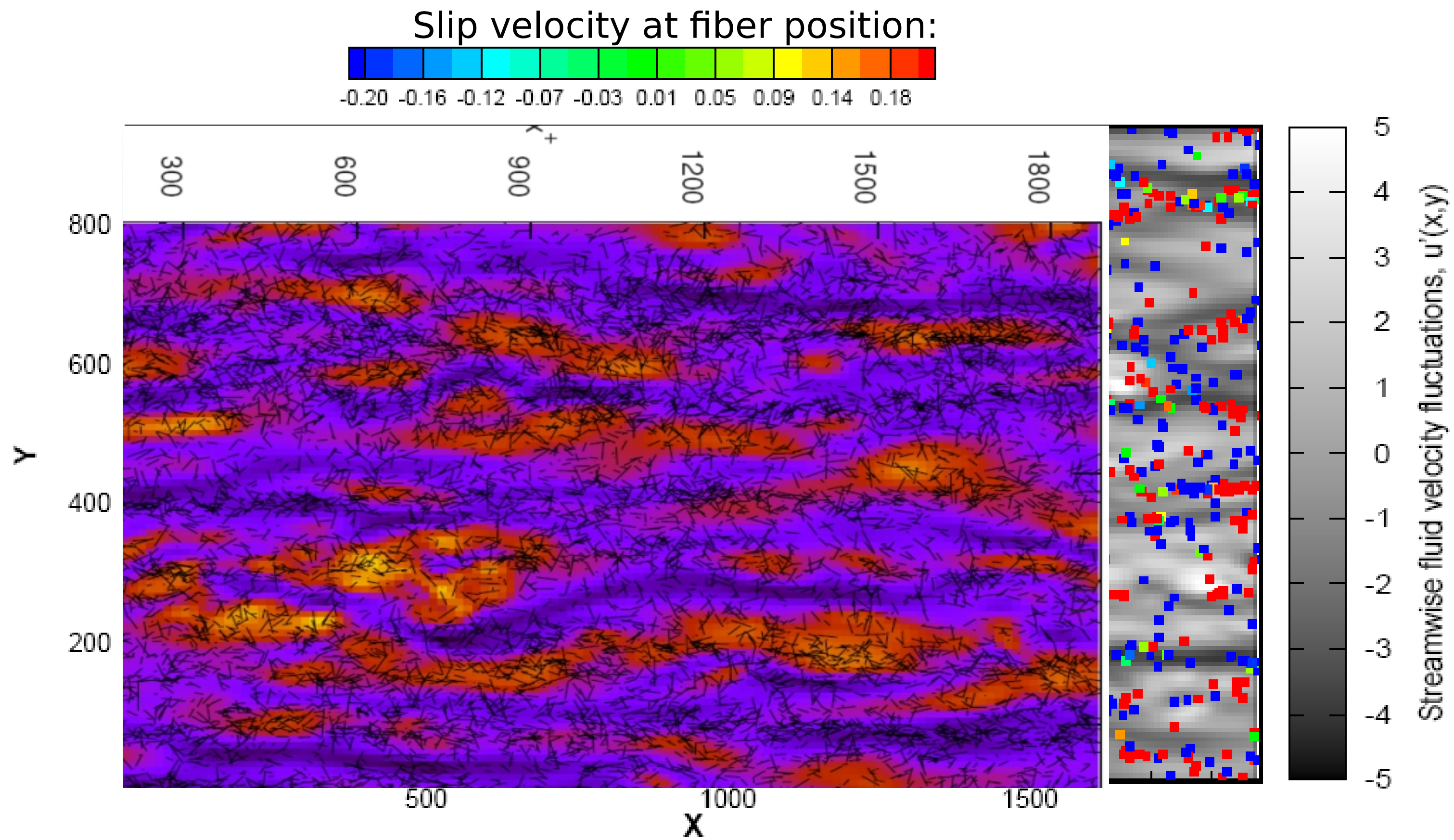
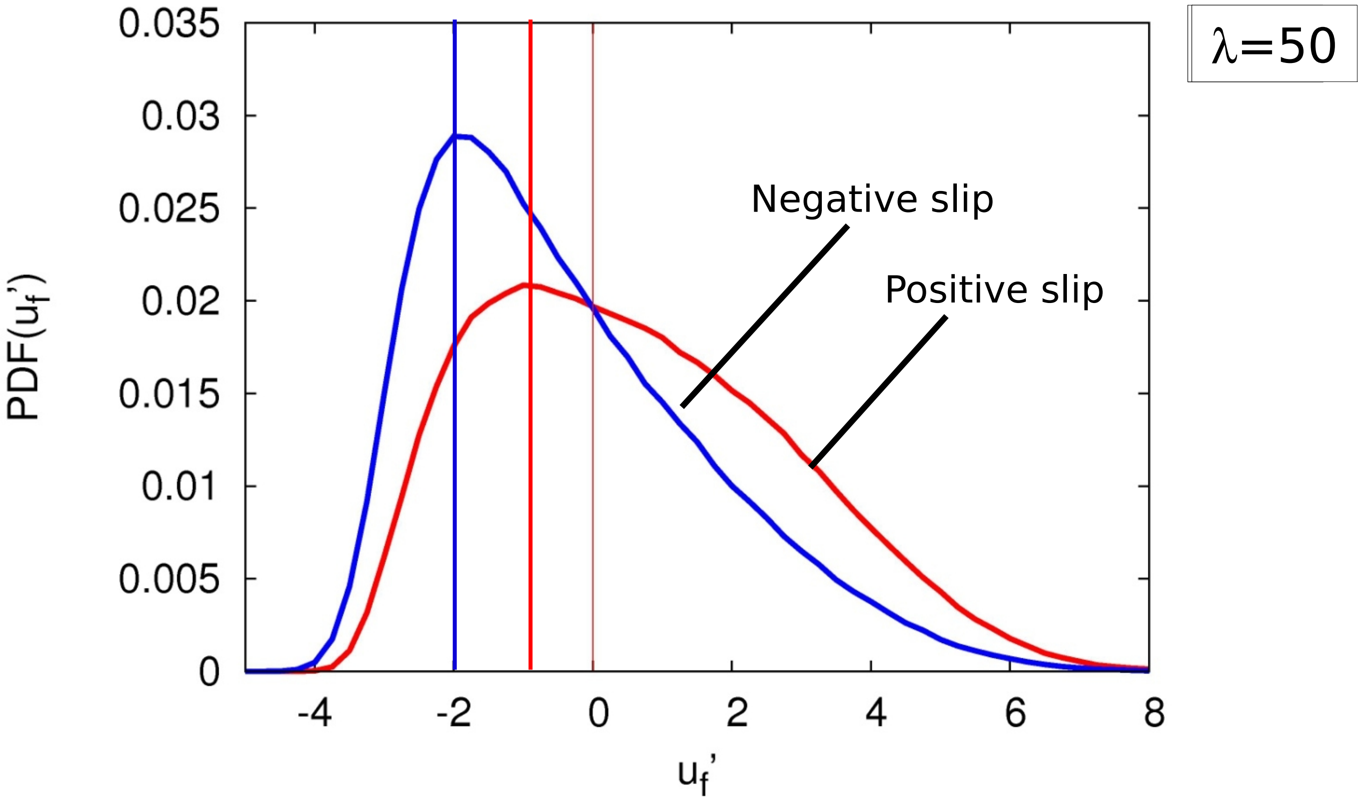
Top view: fibers accumulate in fluid Low-Speed Streaks



Sample snapshot for τ^+=30, λ=50 fibers

Results: Using slip velocity to examine fiber accumulation in LSS

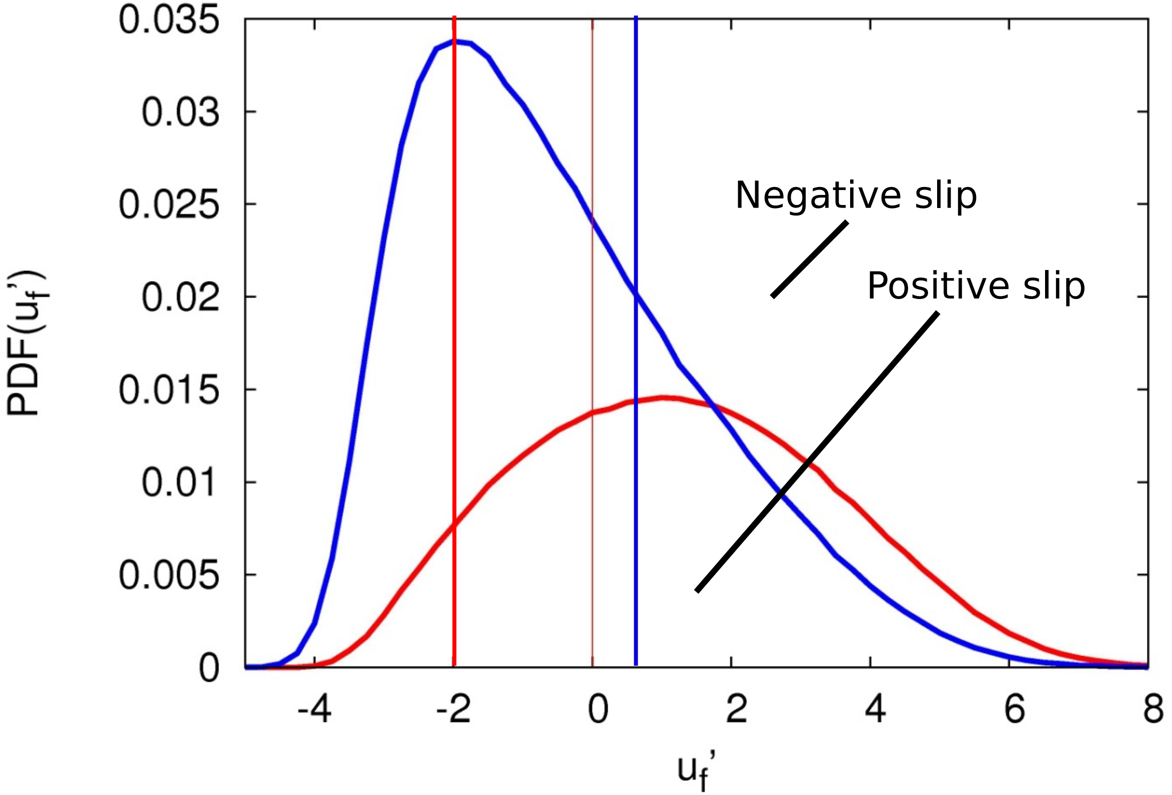
Effect of fiber elongation on conditioned PDF($u_{f'}$) – St=30



peak values is observed (no PDF shape change)

Results: Using slip velocity to examine fiber accumulation in LSS

Effect of fiber inertia on conditioned PDF($u_{f'}$)

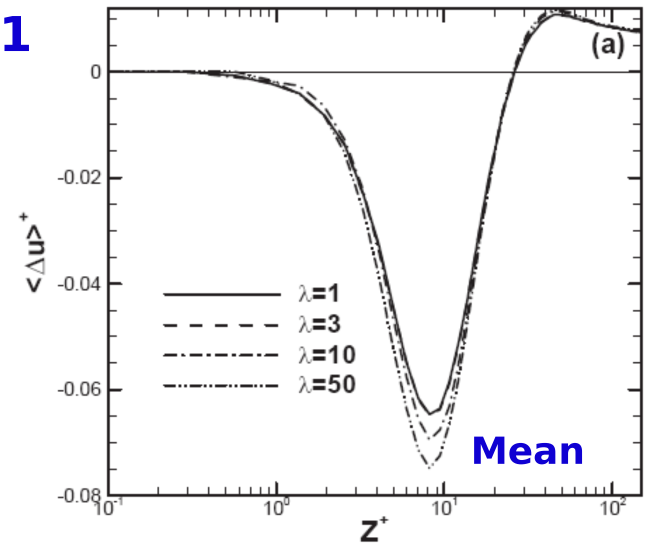


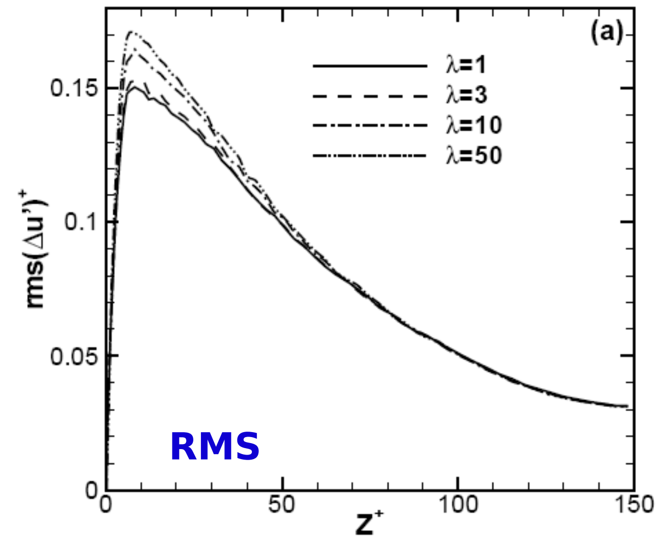
St=100

between St=5 and St=30

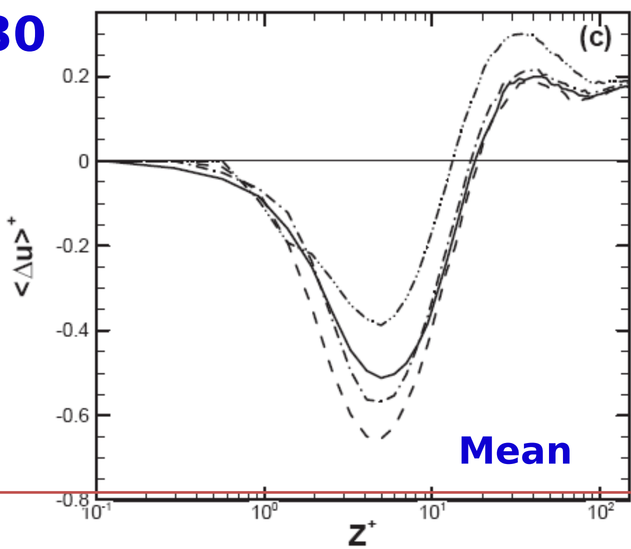
Results: Streamwise slip Velocity - Mean and RMS values

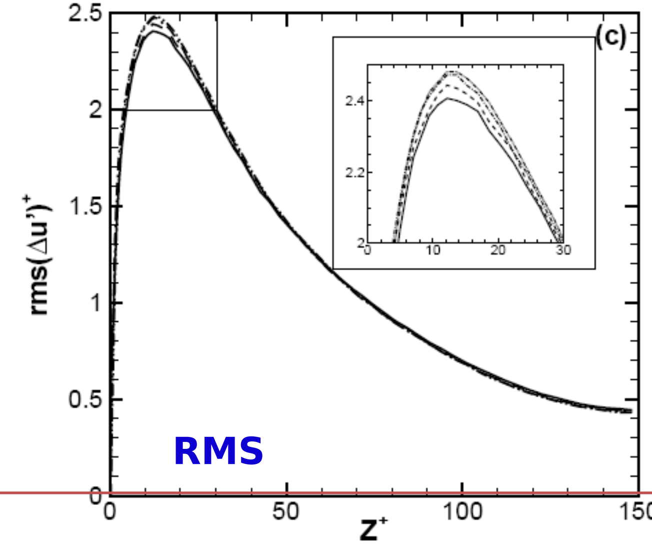
St=1





St=30





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Results: Wall-normal slip Velocity - Mean and RMS values

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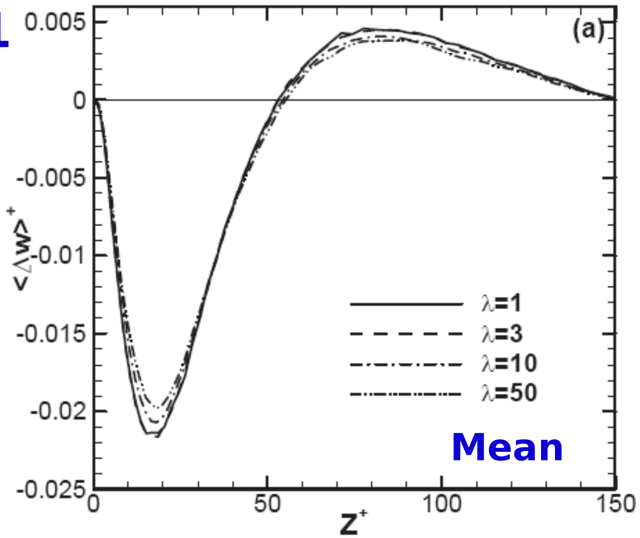
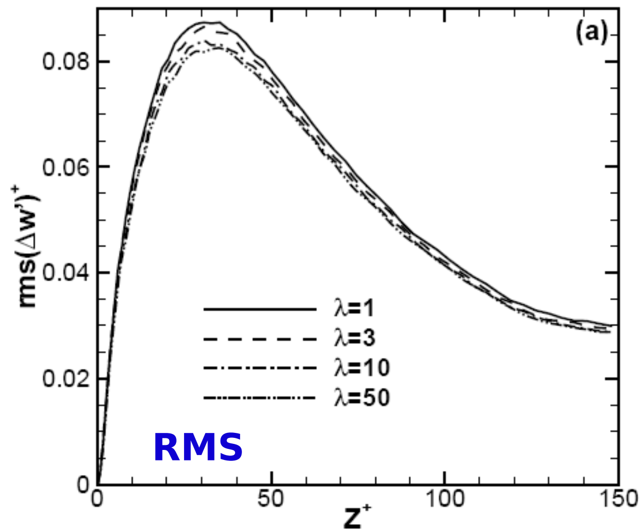
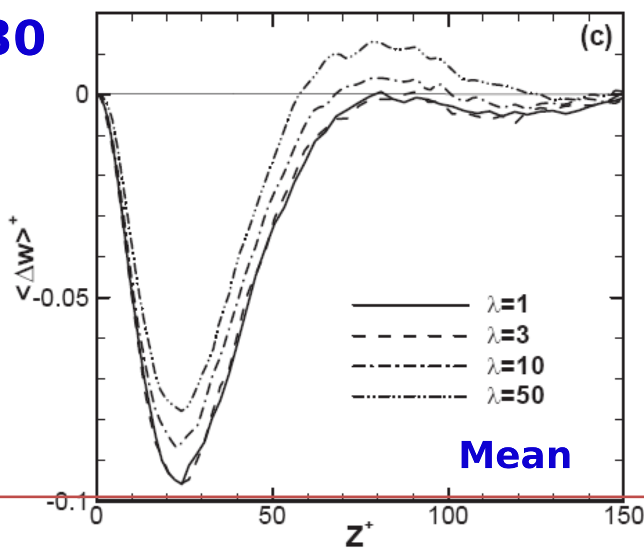
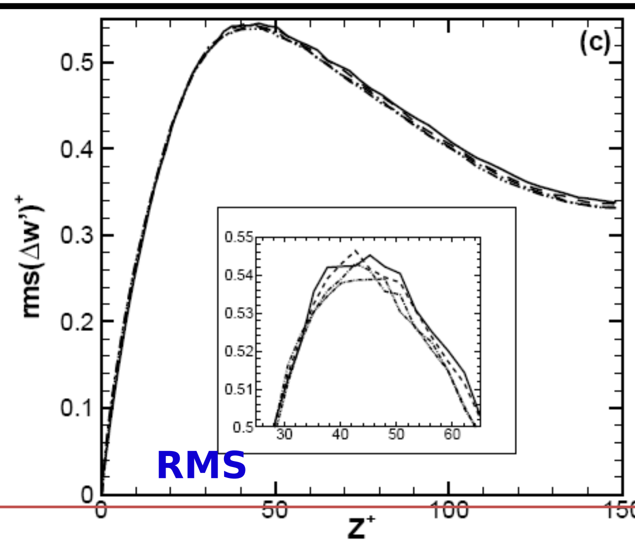
St=1



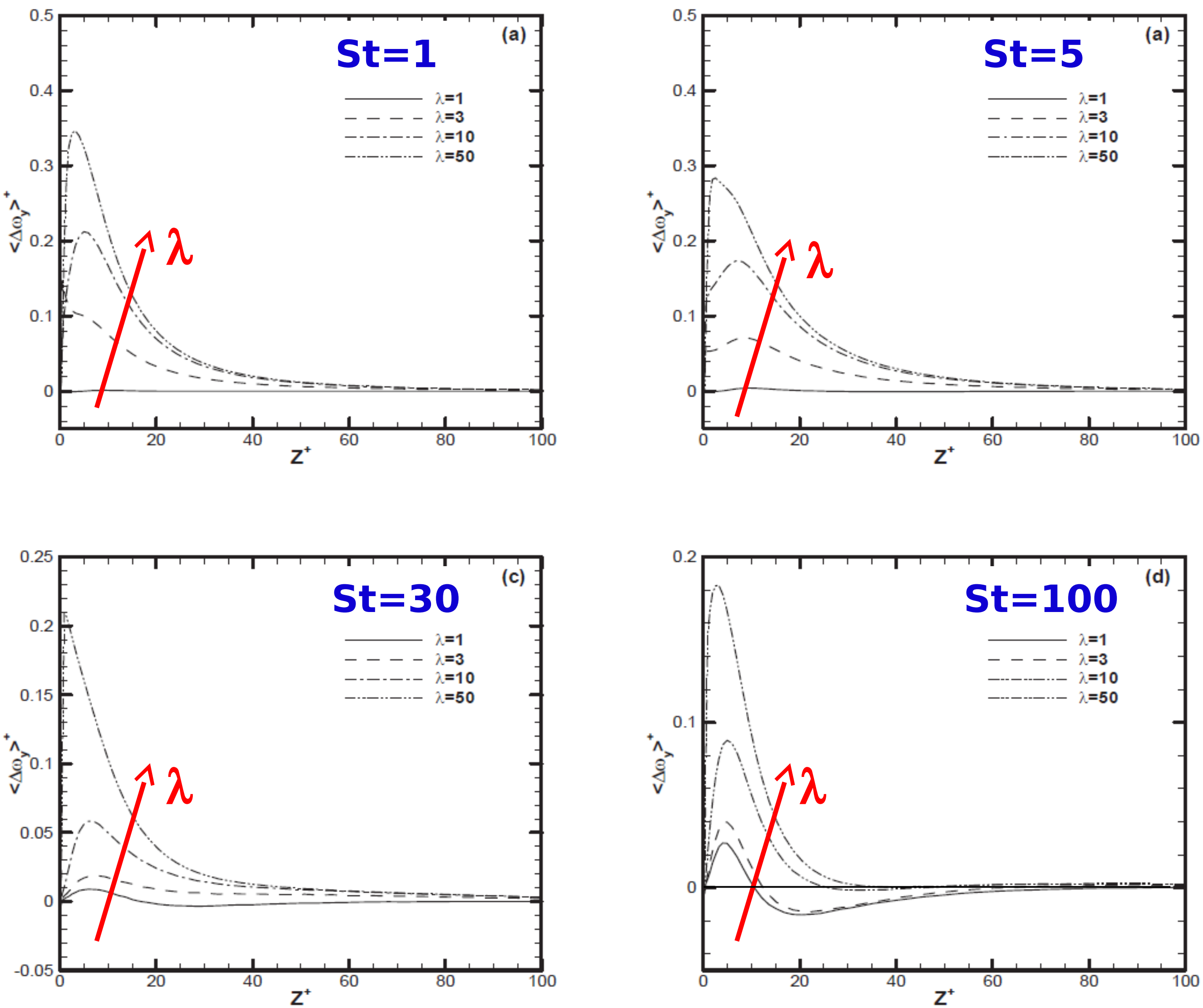


St=30

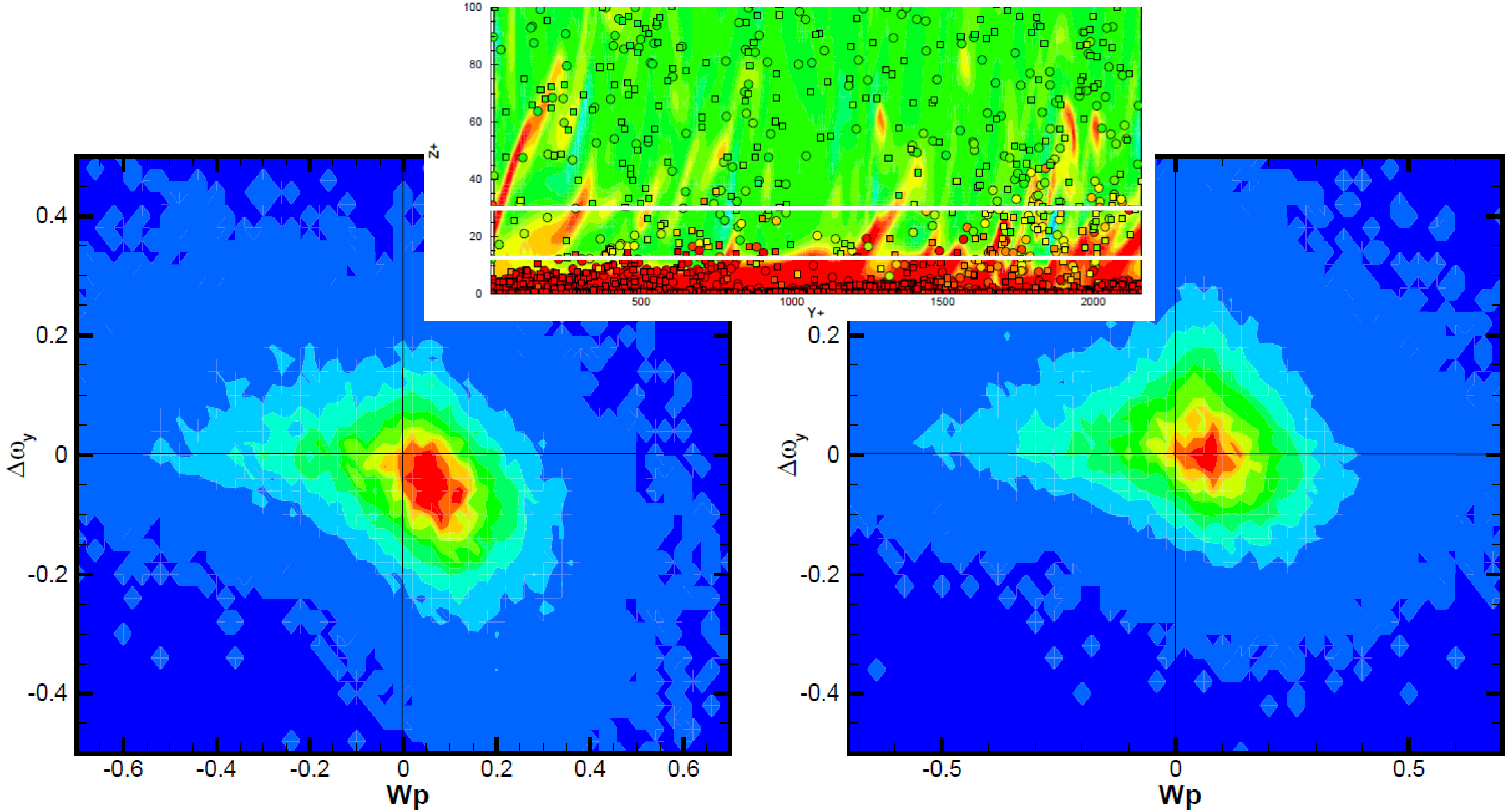




Results: Spanwise slip spin - Mean values

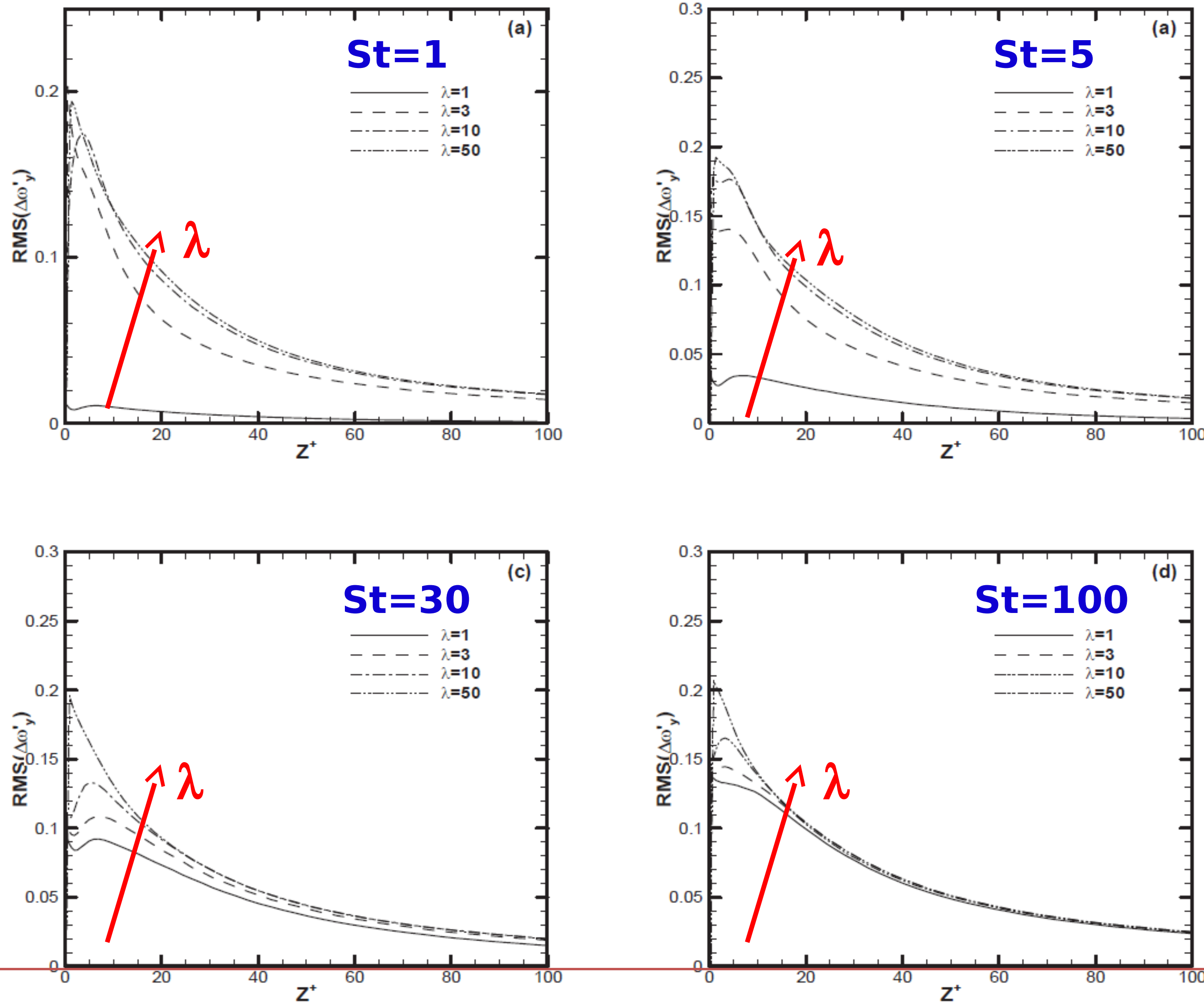


Results: Spanwise slip spin - Correlation w wall-normal vel.

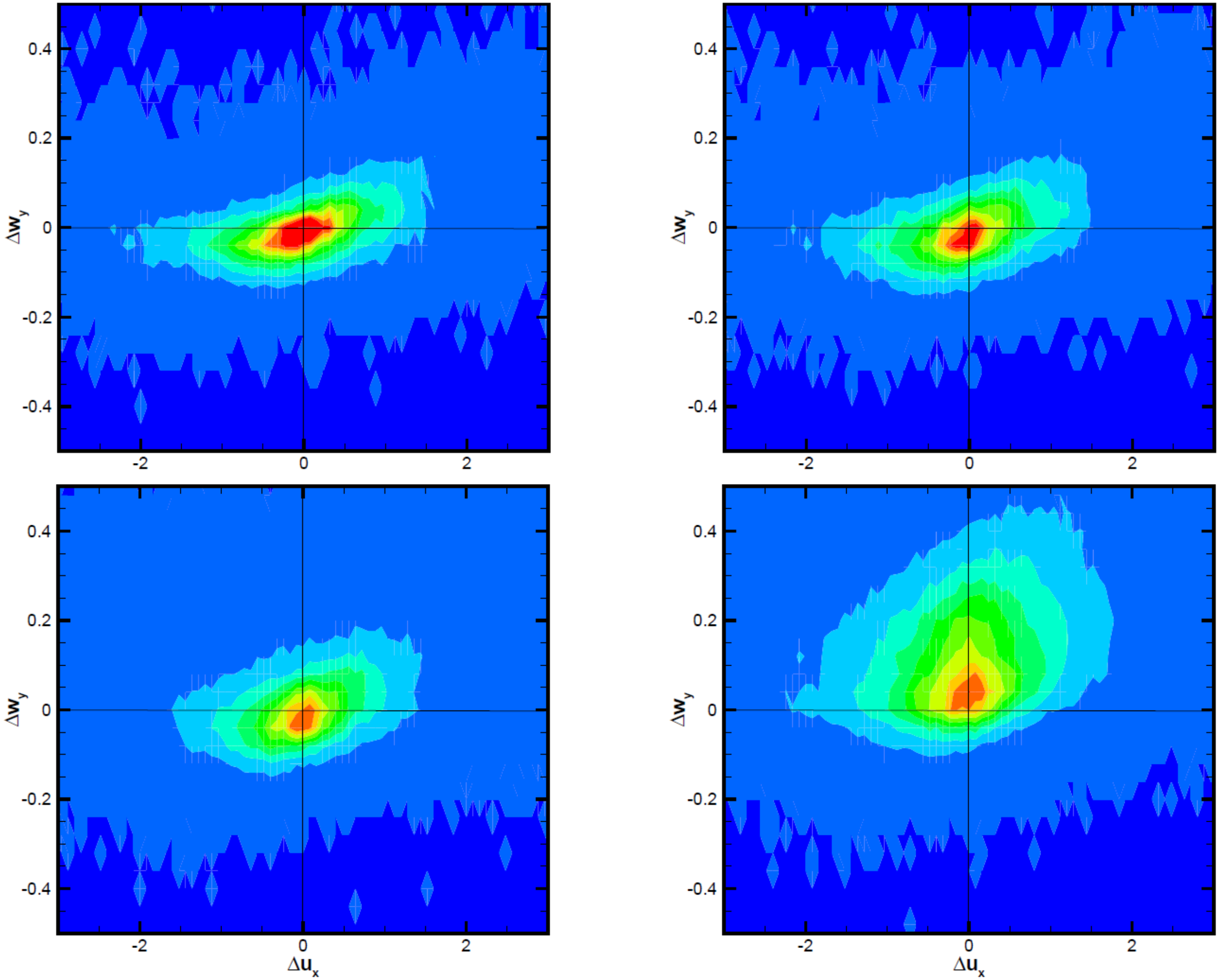


spanwise slip spin $\Delta\omega_y$ versus fiber wall-normal velocity Wp conditionally sampled at the position of the St=100 fibers in the region 10 < z+ < 30

Results: Spanwise slip spin - RMS values



Results: Spanwise slip spin vs streamwise slip velocity



spanwise slip spin $\Delta\omega_y$ versus streamwise slip velocity Δux conditionally sampled at the position of the St=30 fibers in the viscous region 3< z+ < 7.

Concluding remarks

Slip velocity and spin are useful measures of fibers-turbulence Interaction in wall-bounded flows: their statistics provide useful indications for modeling turbulent fiber dispersion

Slip velocity statistics depend both on fiber elongation (quantitatively) and fiber inertia (also qualitatively!)

RMS exceeds the corresponding mean value by roughly 3 to 5 times: the instantaneous slip velocity may thus frequently change sign

Slip spin is significantly influenced by fiber elongation ("more" than the slip velocity) but inertia has a relatively weak effect on it ("less" than the slip velocity)

The two quantities seem correlated only for small inertia (both translational and rotational)