FP1005 MC/WG Meeting, Stockholm, June 3-5, 2014

Numerical modeling of air–fiber flows

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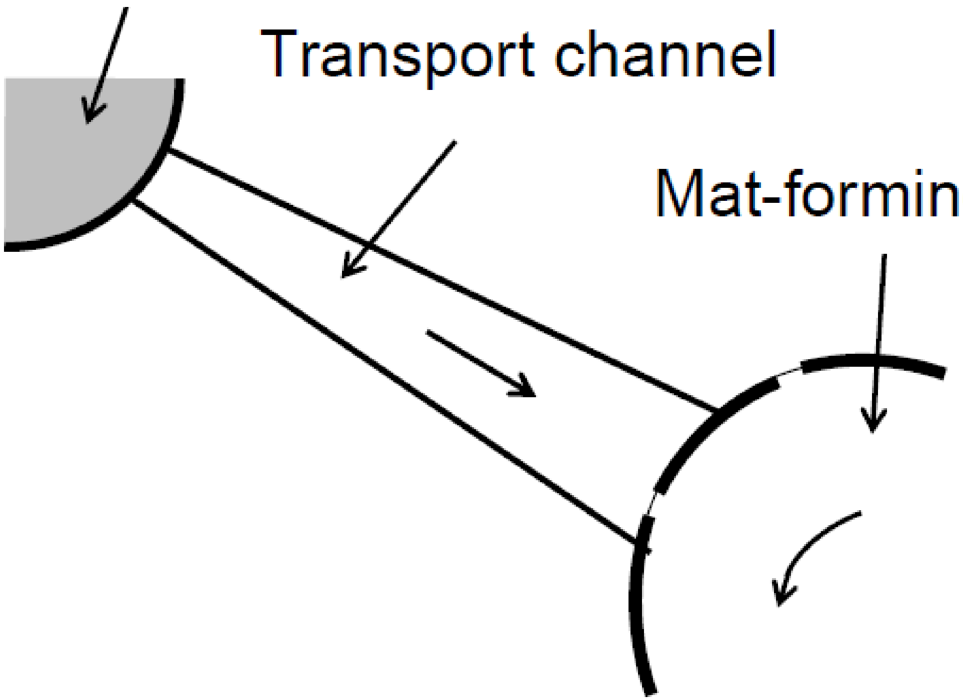
Industrial application

Absorbent hygiene products (diapers, sanitary napkins)

Core of product: Fibers

Fibers transported by flowing air and deposit on forming wheel



Uniform spatial fiber distribution on mat ensures product quality

Present work

Fundamental studies of fiber-flow dynamics

- Rheology of sheared fiber suspensions
- Fiber flocculation in turbulent flow field

Numerical method

Microhydrodynamics approach: Particle-level simulation technique

- Accounting for realistic fiber features: deformability, different shapes
- Inclusion of various forces: fiber-flow, fiber-fiber interactions
- Detailed information about fiber suspension: positions, orientations

Particle-level fiber model

- Chain of rigid cylindrical segments
- Each segment tracked individually – LPT
- Segment inertia taken into account
- Flexible and rigid fibers

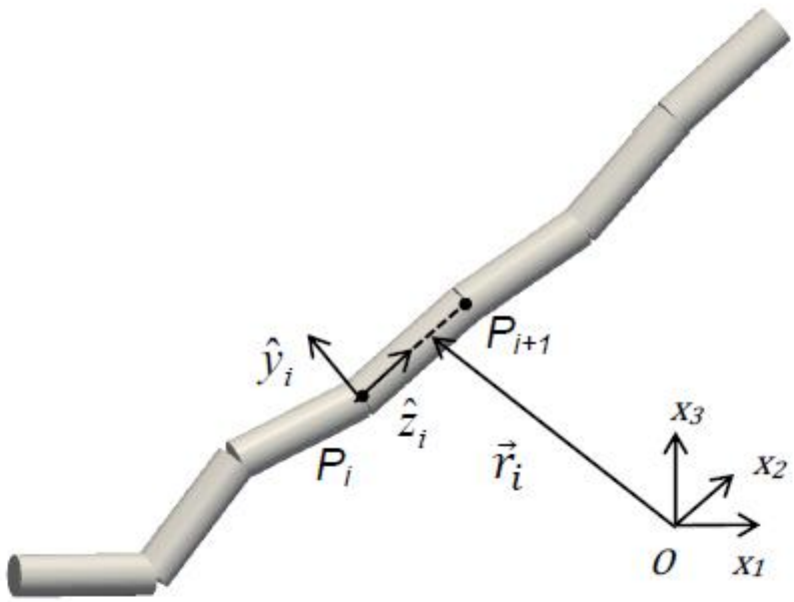
Schmid *et al. (J. Rheol.,2000)*

Lindström and Uesaka (*Phys. of Fluids*, 2007)

Fiber geometry

Chain of rigid cylindrical segments

Geometrical properties defined for **each segment**



- diameter
- length
- position vector
- unit direction vectors
- equilibrium shape

Flexible fiber equations of motion

Direct application of Euler's laws for **each fiber segment**

- Linear momentum equation

h: hydrodynamic

X: connectivity force

$$m_i \ddot{\boldsymbol{r}}_i = \boldsymbol{F}_i^h + \boldsymbol{X}_{i+1} - \boldsymbol{X}_i$$

- Angular momentum equation

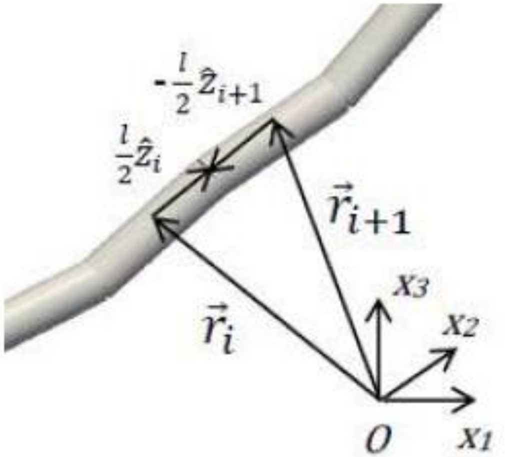
$$\frac{\partial(\boldsymbol{I}_i \cdot \boldsymbol{\omega}_i)}{\partial t} = \boldsymbol{T}_i^h + \boldsymbol{Y}_{i+1} - \boldsymbol{Y}_i + \frac{l_i}{2}\hat{\boldsymbol{z}}_i \times \boldsymbol{X}_{i+1} + \left(\frac{-l_i}{2}\hat{\boldsymbol{z}}_i\right) \times (-\boldsymbol{X}_i)$$

Y: sum of bending and twisting torques

Flexible fiber equations of motion (cont.)

- Connectivity constraint - end points of adjacent segments coincide

$$r_i + \frac{l_i}{2}\hat{z}_i = r_{i+1} - \frac{l_{i+1}}{2}\hat{z}_{i+1}$$



- Connectivity equation - time derivative of connectivity constraint

$$\dot{r}_{i+1} - \dot{r}_i = \frac{l_i}{2}\omega_i \times \hat{z}_i + \frac{l_{i+1}}{2}\omega_{i+1} \times \hat{z}_{i+1}$$

Connectivity force linear system

- Time discretization (implicit numerical scheme)
- Dimensionless equations to ensure numerical stability

$$\begin{bmatrix} S^*_{1,n-1} & T^*_{1,n-1} & 0 & 0 & 0 & 0 \\ Q^*_{2,n-1} & S^*_{2,n-1} & T^*_{2,n-1} & 0 & 0 & 0 \\ 0 & Q^*_{3,n-1} & S^*_{3,n-1} & T^*_{3,n-1} & 0 & 0 \\ 0 & 0 & Q^*_{4,n-1} & S^*_{4,n-1} & T^*_{4,n-1} & 0 \\ 0 & 0 & 0 & Q^*_{5,n-1} & S^*_{5,n-1} & T^*_{5,n-1} \\ 0 & 0 & 0 & 0 & Q^*_{6,n-1} & S^*_{6,n-1} \end{bmatrix} \cdot \begin{bmatrix} X^*_{2,n} \\ X^*_{3,n} \\ X^*_{4,n} \\ X^*_{5,n} \\ X^*_{6,n} \\ X^*_{7,n} \end{bmatrix} = \begin{bmatrix} V^*_{2,n-1} \\ V^*_{3,n-1} \\ V^*_{4,n-1} \\ V^*_{5,n-1} \\ V^*_{6,n-1} \\ V^*_{7,n-1} \end{bmatrix}$$

Linear system for connectivity forces for 7-segment fiber

- Solve velocities and angular velocities
- Evolve segment positions and orientations in time

Rigid fiber equations of motion

Formulated for fiber's center of mass (contributions summed over segments)

- Linear momentum equation

$$m\ddot{\vec{r}}_G = \sum_i \vec{F}_i^h$$

- Angular momentum equation

$$\bar{\bar{I}}_G \cdot \dot{\vec{\omega}} + \vec{\omega} \times (\bar{\bar{I}}_G \cdot \vec{\omega}) = \sum_i \{\vec{T}_i^h + \vec{r}_{Gi} \times \vec{F}_i^h\}$$

- Solve translational velocities and angular velocity
- Evolve segment positions and orientations in time

Rheology of sheared fiber suspensions

Understanding suspension structure

- Process optimization, product quality

Rigid, straight fibers

- Batchelor's theory – known fiber orientation
- Semi-dilute suspensions; hydrodynamic interactions

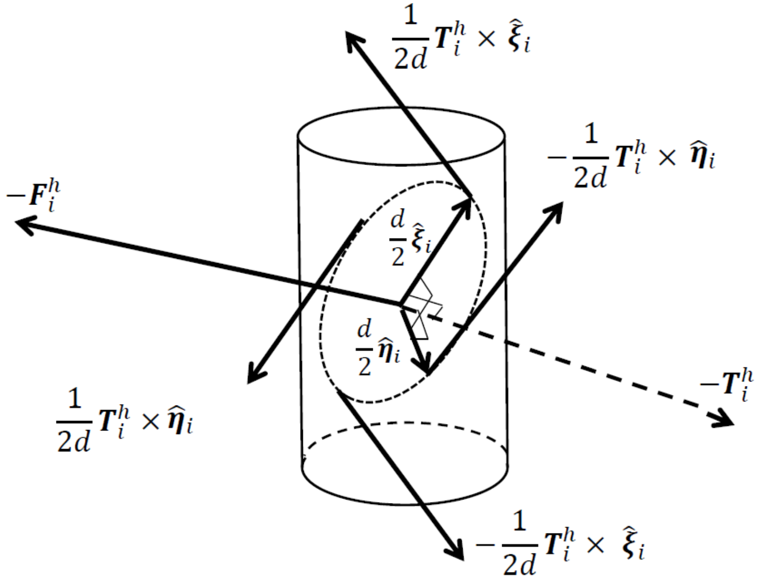
Flexible fibers, fibers with irregular equilibrium shapes

- No theoretical studies
- Few numerical studies – based on Batchelor's theory

Rheology of sheared fiber suspensions:
Direct method for deviatoric stress computation

- Dipole strength of single fiber
- Hydrodynamic forces and torques as localized tractions

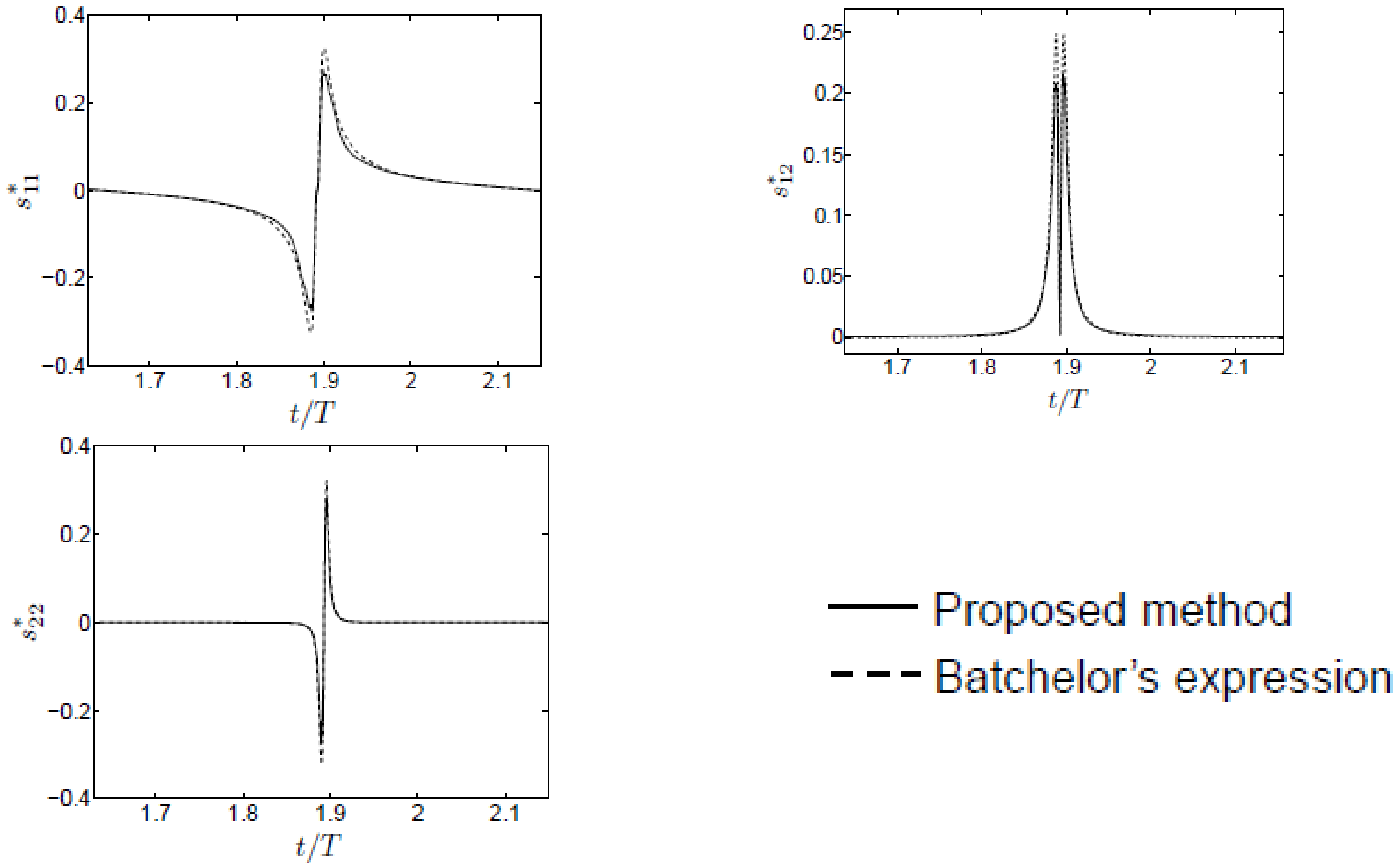
$$
\begin{aligned}
\boldsymbol{s}' &= \iint_A [\boldsymbol{\tau}\boldsymbol{r} - \frac{1}{3}(\boldsymbol{r}\cdot\boldsymbol{\tau})\boldsymbol{\delta}]dA \\
&= \sum_{i=1}^{N}[\boldsymbol{F}_i^h \boldsymbol{r}_i + \frac{1}{2}\boldsymbol{\epsilon}\cdot\boldsymbol{T}_i^h - \frac{1}{3}(\boldsymbol{r}_i\cdot\boldsymbol{F}_i^h)\boldsymbol{\delta}]
\end{aligned}
$$



- Flexible fibers
- Fibers with irregular equilibrium shapes

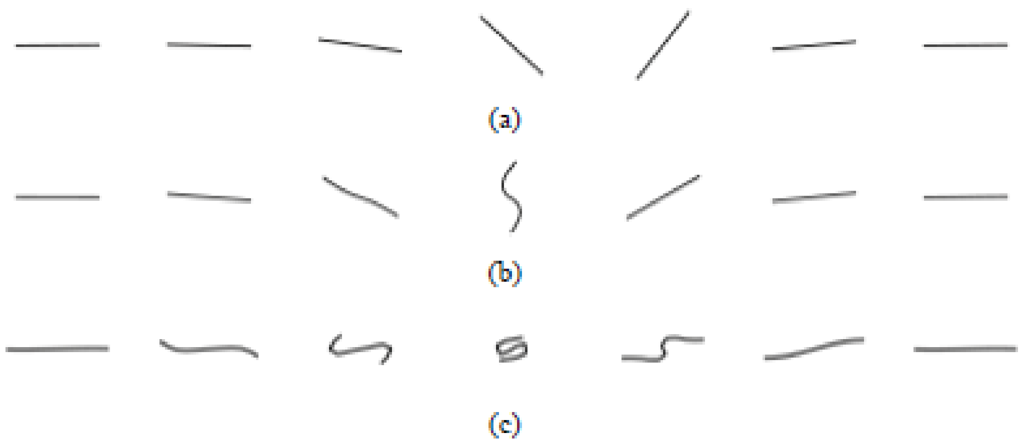
Rheology of sheared fiber suspensions: Dipole strength validation

Isolated, straight, rigid fiber – agreement with Batchelor's theory



Rheology of sheared fiber suspensions: Numerical experiments

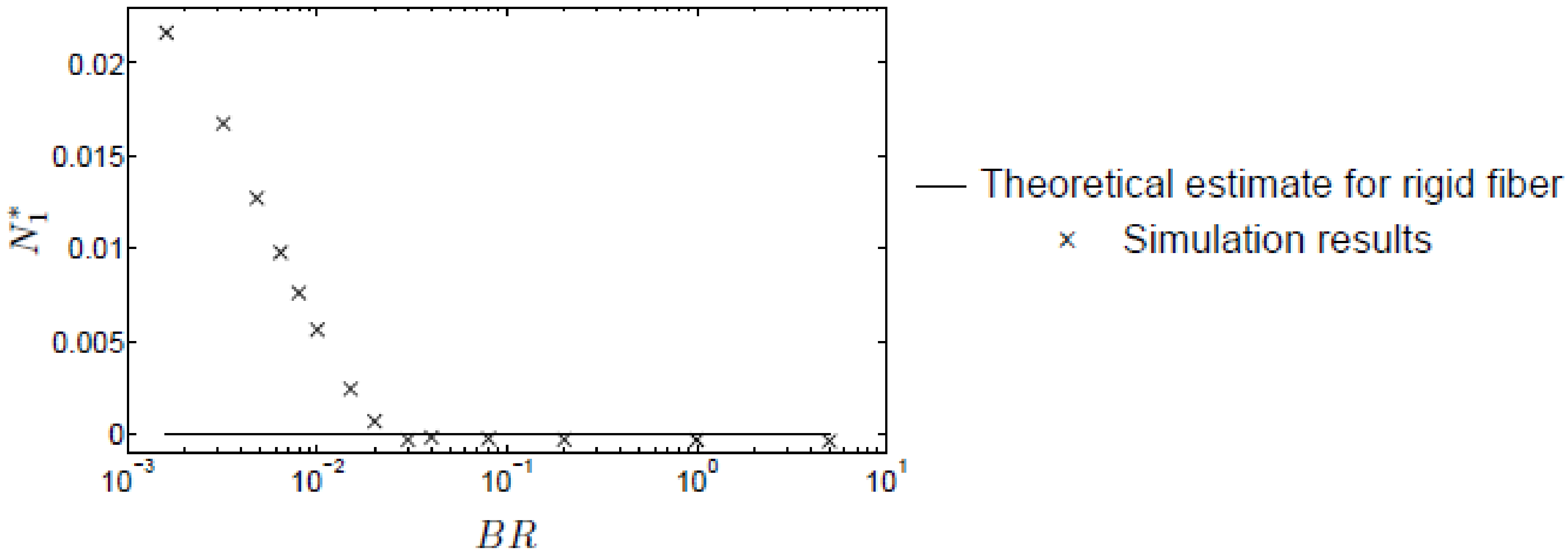
- Ergodicity assumption; Fibers initially in flow-gradient plane
- Initially straight fibers – S-shape deformations



Time–series of images from simulation for different bending ratio:
a) *BR*=1, b) *BR*=0.02, c) *BR*=0.0016.

Rheology of sheared fiber suspensions: Effects of fiber flexibility

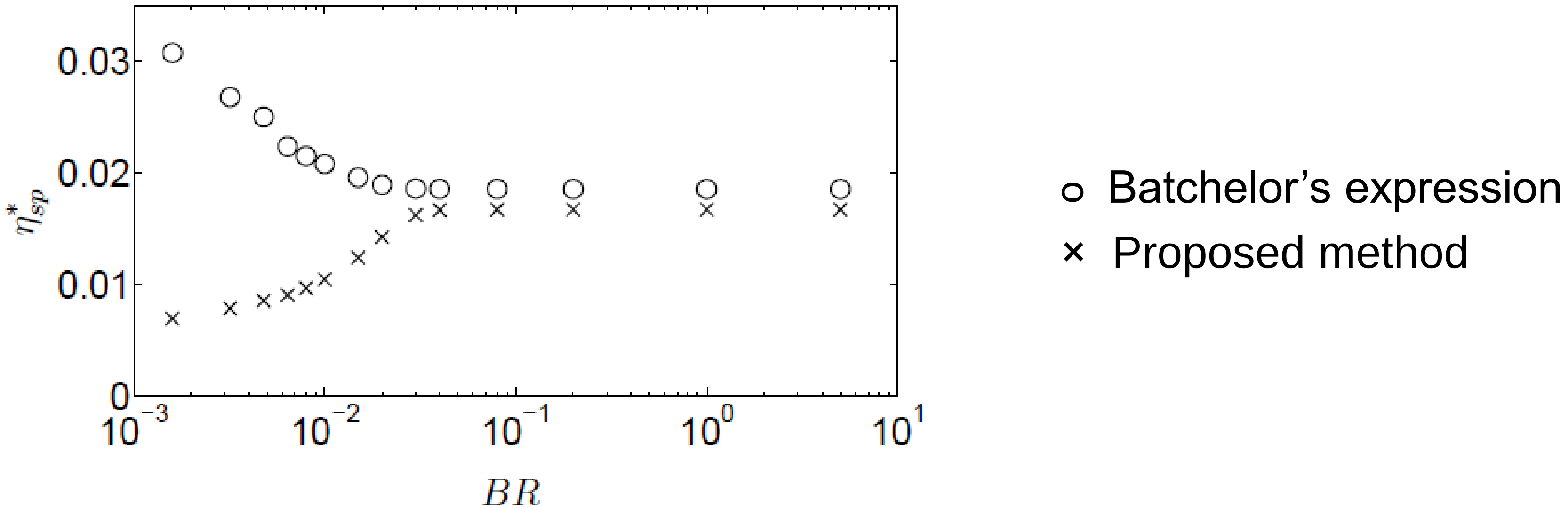
Relation between fiber bending ratio *BR* and first normal stress difference N_1^*



- Increases with fiber flexibility (lower *BR* values)

Rheology of sheared fiber suspensions: Effects of fiber flexibility

Relation between fiber bending ratio *BR* and specific viscosity (SV) η^*_{sp}



- Batchelor's theory: SV increases with fiber flexibility
  - Observed by other studies (Joung *et al.,J. Non-Newtonian Fluid Mech., 2001)*
- Proposed method: SV decreases with fiber flexibility
  - Reason: reduced viscous dissipation

Fiber flocculation in realistic flow fields

Geometries and flow conditions representative for industrial process

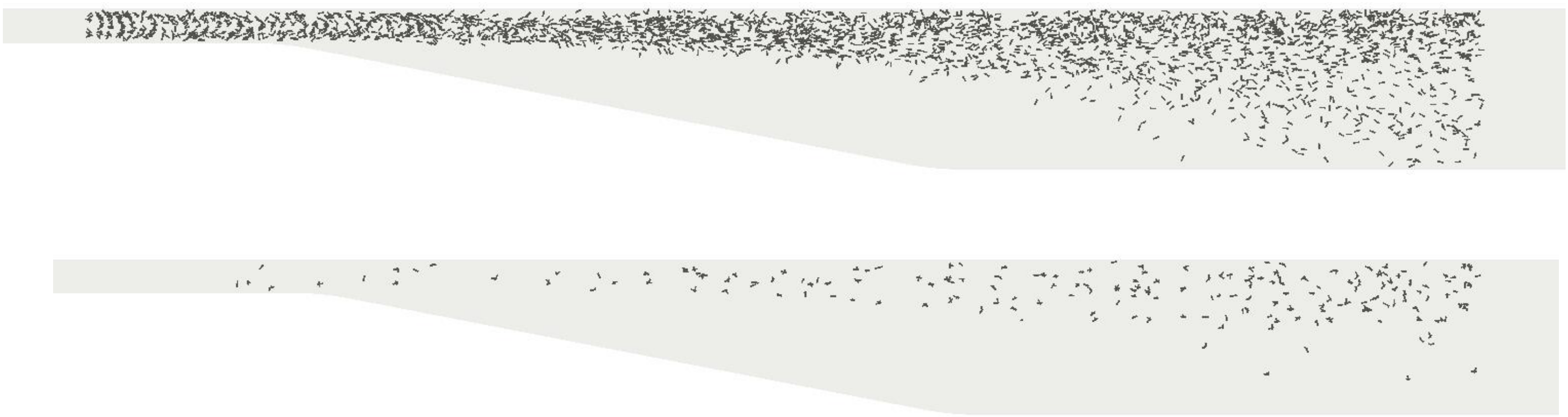
- Asymmetric planar diffuser
- High Reynolds number flow
- Non-creeping fiber-flow interactions

Fiber properties consistent with those of hardwood fibers

- Diameter, length, density

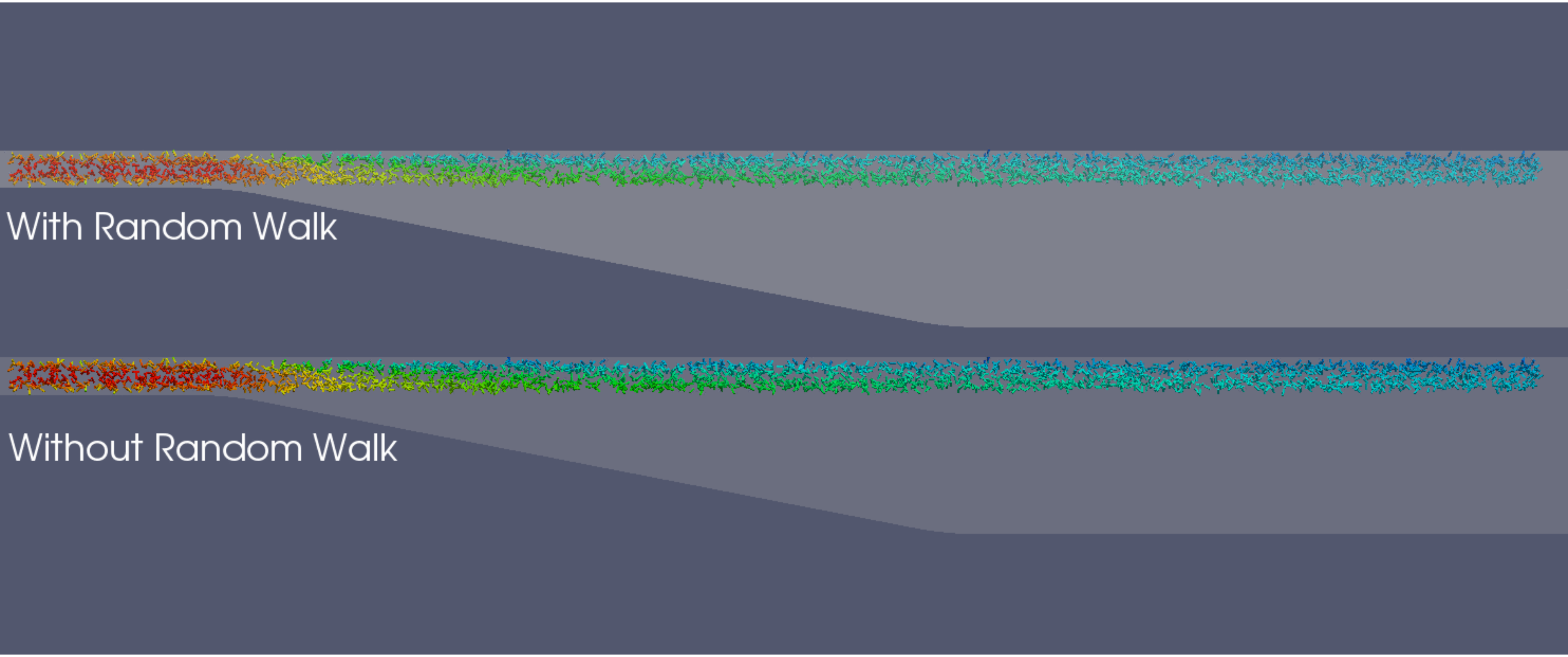
Fiber flocculation in realistic flow fields

- Dilute suspension of rigid, straight fibers
- Turbulent dispersion – stochastic model
- Short-range attractive forces – fibers interlock in flocs



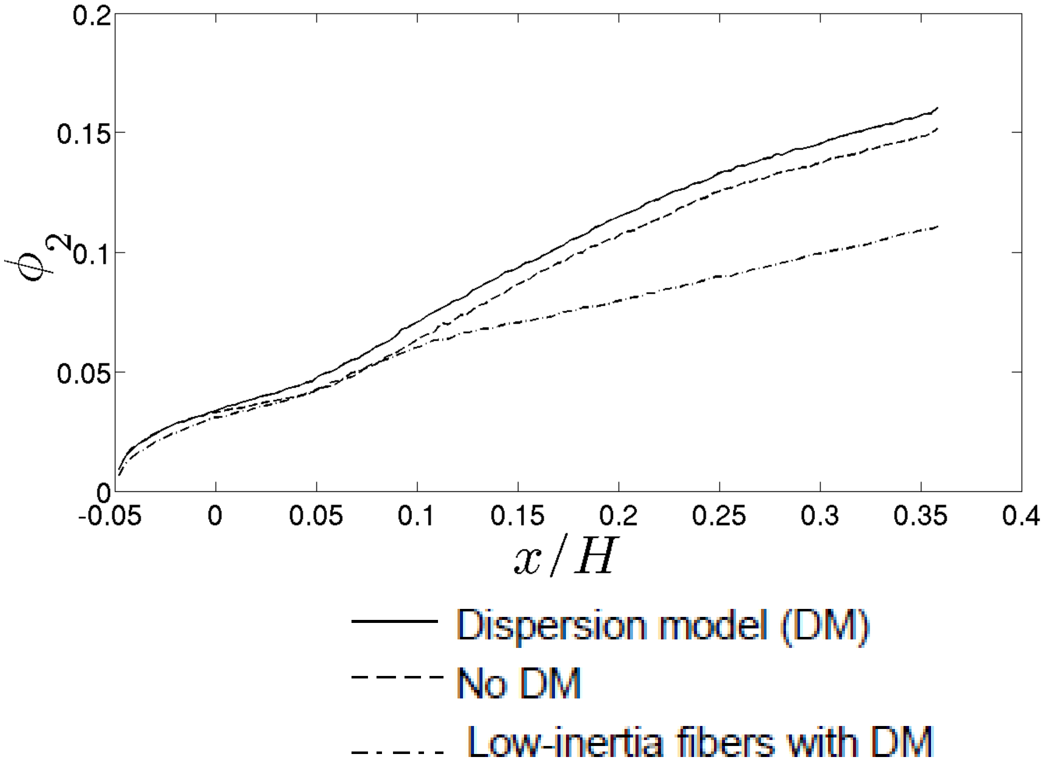
Fibers and fiber flocs – steady state solution

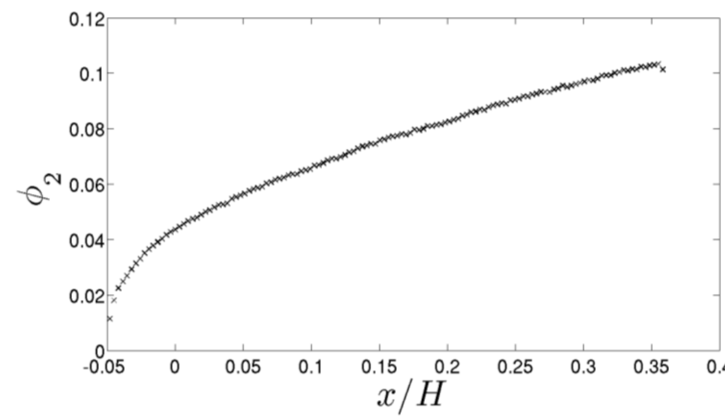
Top: all fibers; **Bottom:** fibers in flocs


With Random Walk
Without Random Walk

Fiber flocculation in realistic flow fields

Variation of flock mass fraction ϕ_2 along diffuser





Straight channel with DM

Darting fiber motion → enhances collisions → increase in flocculation rate

Conclusions

- Rheology of flexible fiber suspensions
  - Viscosity decreases with fiber flexibility
  - Important to properly account for fiber deformability
  - Experimentalíst – characterize fiber morphology

- Flocculation of rigid,straight fibers in turbulent flow
  - Darting fiber motion enhances collisions and flocculation rate

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Thank you!

Financially supported by Bo Rydin Foundation and SCA Hygiene Products AB