Joint 5th SIG43 – FP1005 Workshop on

Fiber Suspension Flow Modelling

Rotation Statistics of Fibers in Wall-Shear Turbulence

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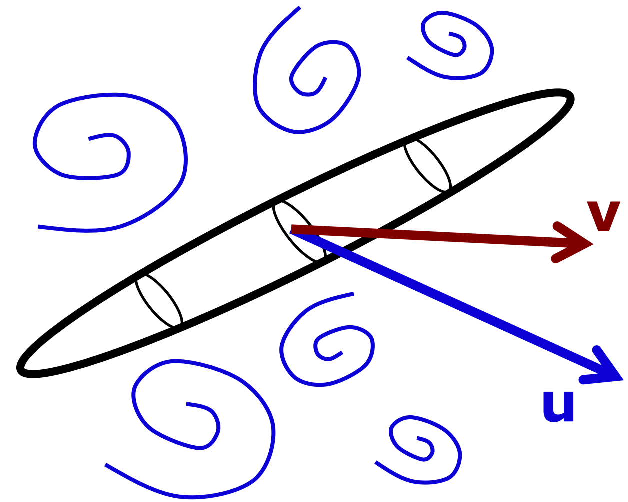
Motivation of the Study:
Why are we looking at these statistics?

Statistical characterization of fiber angular velocity is crucial to:

1. **Quantify rotational dispersion and rotation rates**

   Parameterization of rotation rates as a function of the ratio:

$$\frac{\text{fiber length}}{\text{Lagrangian integral flow scale}}$$



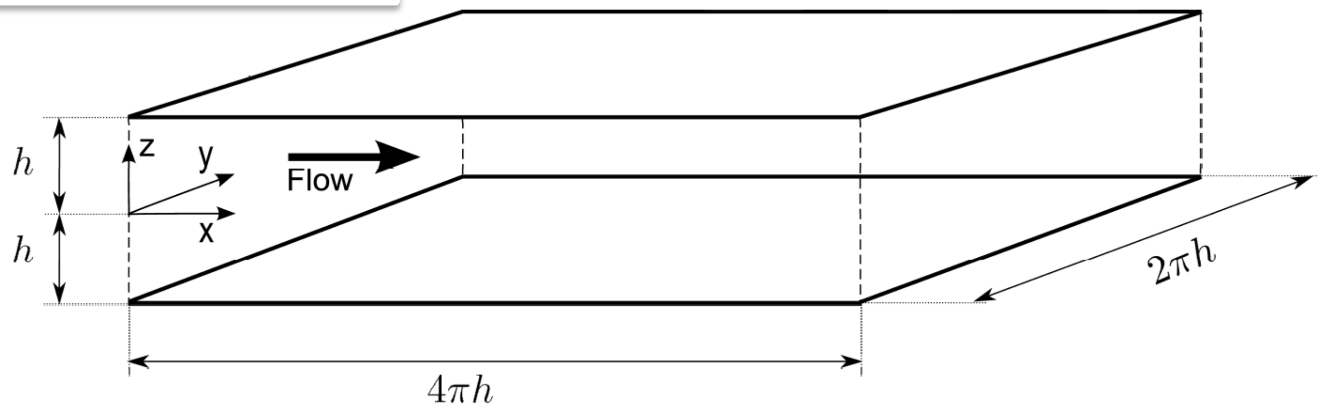
2. **Validate rheological models**

   How does the extra non-Newtonian stress, produced by the addition of fibers to the Newtonian carrier fluid, depend on fiber orientation distribution?

⇨ **Aim of the study: examine the effect of local shear and turbulence anisotropy on fiber rotation at varying fiber inertia/length**

Methodology - Carrier Fluid

DNS of turbulent channel flow
@ $Re_\tau = u_\tau\, h/\nu =$ **150**, 300



$$
\begin{cases}
\dfrac{\partial u_j}{\partial x_j} = 0 \\[2ex]
\rho \left[\dfrac{\partial u_i}{\partial t} + u_j \dfrac{\partial (u_i)}{\partial x_j} \right] = -\dfrac{\partial P}{\partial x_i} + \mu \dfrac{\partial^2 u_i}{\partial x_j \partial x_j}
\end{cases}
$$

<u>*Pseudo-Spectral*</u> method: Fourier modes in *x and y,*
Chebyschev modes in *z.*

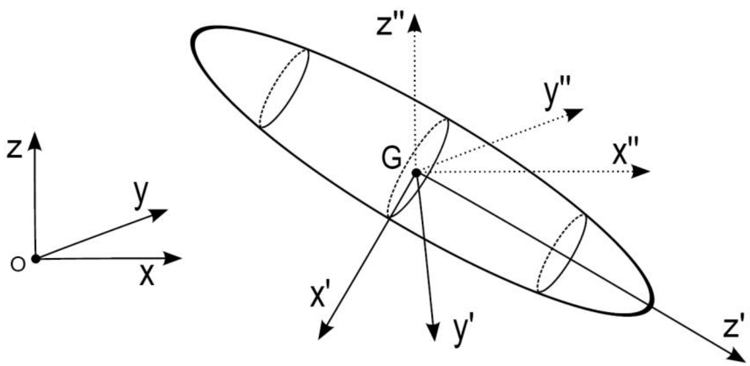
- **Examples: flow of air at 1.8 m/s in a 4 cm high channel**
  flow of water at 3.8 m/s in a 0.5 cm high channel

Methodology - Fibers

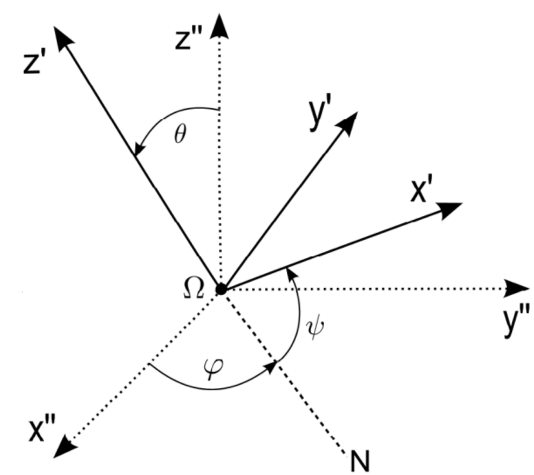
- Fibers are modelled as **prolate ellipsoidal particles**.
- **Lagrangian** particle tracking.
- Simplifying assumptions: dilute flow, **one-way coupling**, Stokes flow ($Re_p < 1$), pointwise particles (particle size is smaller than the smallest flow scale).
- Periodicity in *x* ed *y*, elastic rebound at the wall and **conservation of angular momentum**.
- **200,000 fibers** tracked, *random* initial position and orientation, linear and angular velocities equal to those of the fluid at fiber's location.

Methodology – Fiber Kinematics

<u>Kinematics</u>: described by (1) position of the fiber center of mass and (2) fiber orientation.



- x_G, y_G, z_G
- **3 frames of reference (to define orientation)**
- **Euler angles: φ, ψ, θ (singularity problems)**



- **Euler parameters:** e_0, e_1, e_2, e_3

$$e_0 = \cos\left[\frac{1}{2}(\psi + \varphi)\right]\cos\left(\frac{\theta}{2}\right) \quad , \ldots$$

- **Rotation matrix:** $\mathbf{x}' = R_{Eul}\mathbf{x}''$

$$R_{eul} = \begin{bmatrix} e_0^2 + e_1^2 - e_2^2 - e_3^2 & 2(e_1e_2 + e_0e_3) & 2(e_1e_3 - e_0e_2) \\ 2(e_1e_2 - e_0e_3) & e_0^2 - e_1^2 + e_2^2 - e_3^2 & 2(e_2e_3 + e_0e_1) \\ 2(e_1e_3 + e_0e_2) & 2(e_2e_3 - e_0e_1) & e_0^2 - e_1^2 - e_2^2 + e_3^2 \end{bmatrix}$$

Methodology – Fiber Dynamics

Rotational dynamics: Euler equations with Jeffery moments.

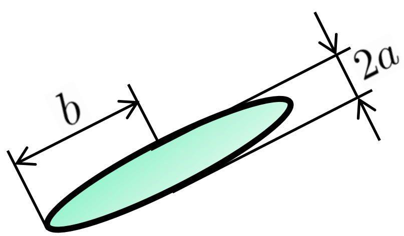
- **Euler Equations:** **(2nd cardinal law)**

$$\begin{cases} I_{x'x'}\dot{\omega}_{x'} + \omega_{y'}\omega_{z'}(I_{z'z'} - I_{y'y'}) = M_{x'}^{est} \\ I_{y'y'}\dot{\omega}_{y'} + \omega_{x'}\omega_{z'}(I_{z'z'} - I_{x'x'}) = M_{y'}^{est} \\ I_{z'z'}\dot{\omega}_{z'} + \omega_{x'}\omega_{y'}(I_{y'y'} - I_{x'x'}) = M_{z'}^{est} \end{cases}$$

(in the particle frame)

- **Jeffery moments:** ***(Jeffery, 1922)***

$$M_{x'}^{Jeff} = \frac{16\pi\mu a^3\lambda}{3(\beta_0 + \lambda^2\gamma_0)}\left[(1-\lambda^2)f' + (1+\lambda^2)(\xi' - \omega_{x'})\right]$$



Aspect Ratio

$$\lambda = \frac{b}{a}$$

- **Hence:** **Euler equations with Jeffery couples** → $\int\int(\ldots)dtdt$ → **e_0, e_1, e_2, e_3 (Euler parameters)**

$$R_{eul} = R_{eul}(e_0, e_1, e_2, e_3)$$

Methodology – Fiber Dynamics

Translational Dynamics: hydrodynamic resistance (Brenner, 1963).

- **First cardinal law:** $m_P \frac{d\mathbf{v}}{dt} = \sum_i \mathbf{F}_i = \mathbf{F}_{drag}$ **(inertia and drag only!!)**
- **Brenner's law: (form drag and skin drag)** $\mathbf{F}'_{drag} = \mu\pi a \bar{\bar{\mathbf{K}}}'(\mathbf{u}' - \mathbf{v}')$ **(in the fiber frame)**

  Resistance Tensor
- **In the inertial (Eulerian) frame:**

$$\left.\begin{aligned} \mathbf{u}' &= R_{eul}\mathbf{u} \\ \bar{\bar{\mathbf{K}}}_{(\varphi,\theta,\psi)} &= R_{eul}^T \bar{\bar{\mathbf{K}}}' R_{eul} \end{aligned}\right\} \Longrightarrow \mathbf{F}_{drag} = \mu\pi a \bar{\bar{\mathbf{K}}}_{(\varphi,\theta,\psi)}(\mathbf{u} - \mathbf{v})$$

$$\begin{cases} m_P \frac{d\mathbf{v}}{dt} = \mu\pi a \bar{\bar{\mathbf{K}}}_{(\varphi,\theta,\psi)}(\mathbf{u} - \mathbf{v}) \\ \frac{d\mathbf{x}_{(G)}}{dt} = \mathbf{v} \end{cases} \Rightarrow \mathbf{v}(t) \quad \mathbf{x}_G(t)$$

(via numerical integration)

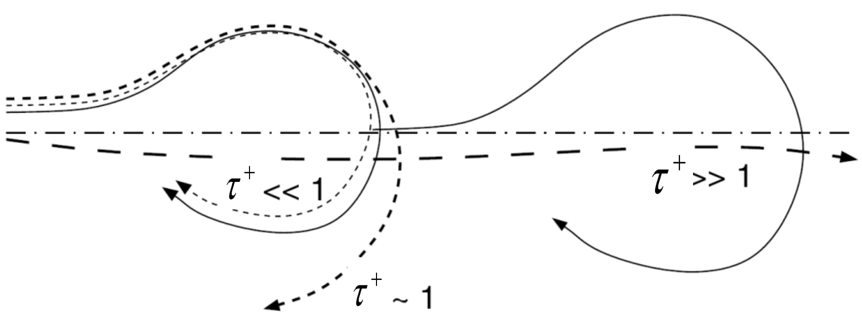
Once fiber orientation is known, fiber translational motion can be computed!

Relevant Parameters and Summary of the Simulations

The physics of turbulent fiber dispersion is determined by a small set of parameters

- **Aspect ratio:** $\lambda = \frac{b}{a}$ **(chosen values: $\underline{\lambda=1.001, 3, 10, 50}$)**

- **Stokes number:** $St = \tau^+ = \frac{\tau_P}{\tau_F}$ **(chosen values: $\underline{\tau^+=1, 5, 30, 100}$)**

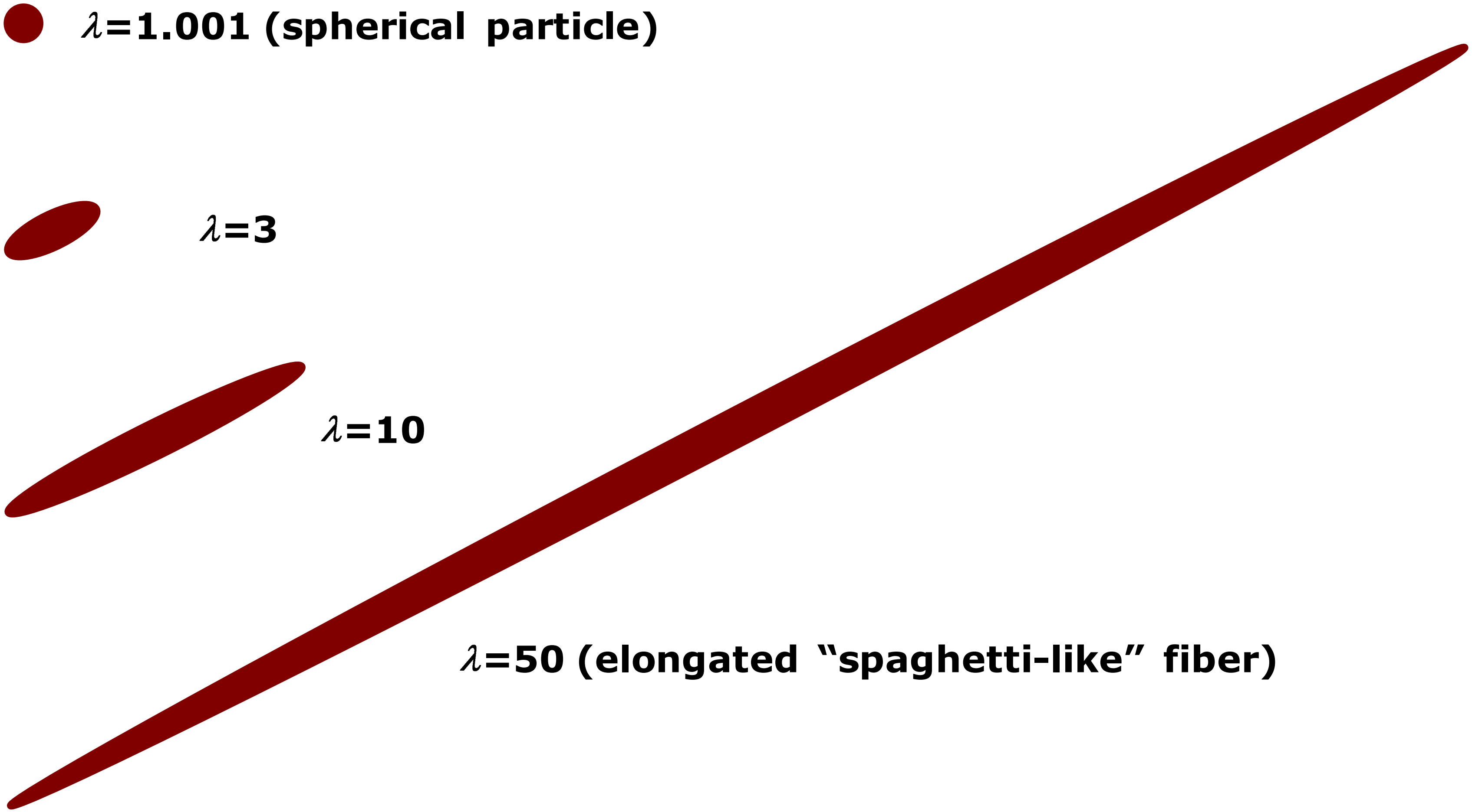


- $\tau^+ >> 1$: **large inertia ("stones")**
- $\tau^+ << 1$: **small inertia (tracers)**
- $\tau^+ \sim 1$: **preferential (selective) response to flow structures**

- **Specific density:** $S = \frac{\rho_P}{\rho_F}$

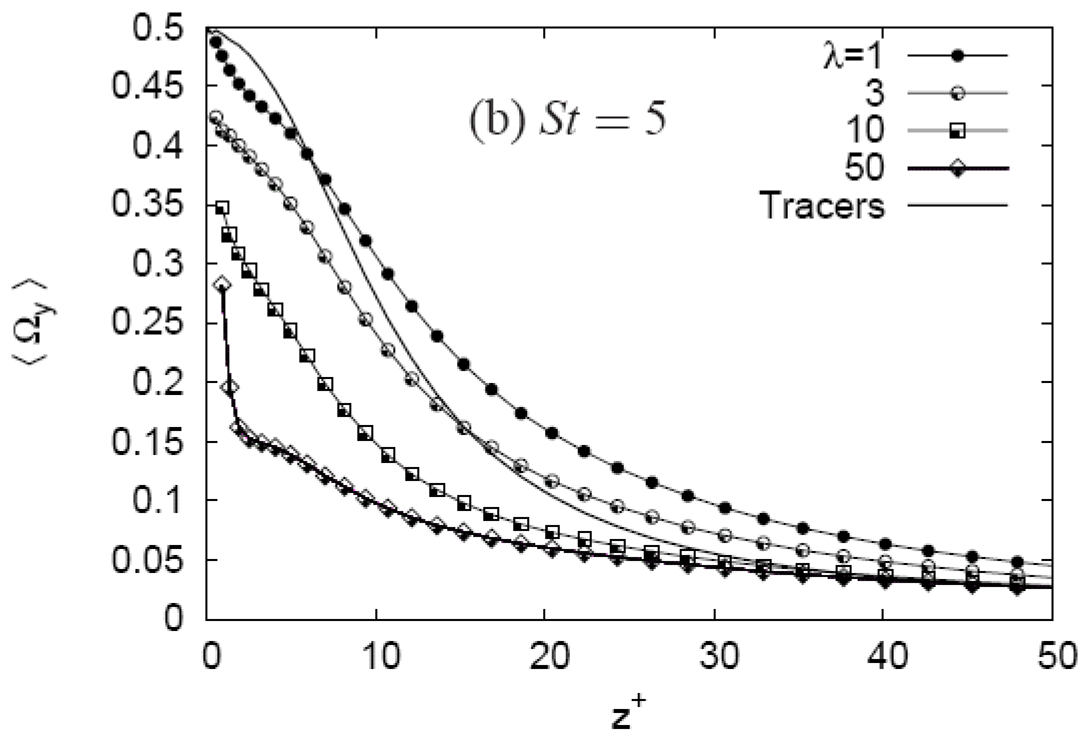
Input parameters: S, τ^+, λ

“Cartoon” of fiber’s elongation



Results - 1st/2nd Order Statistics of Fiber Angular Velocity

Mean (space & time-averaged) angular velocity in the near-wall region



Strong effect of fiber inertia and length near the wall, negligible away from it!

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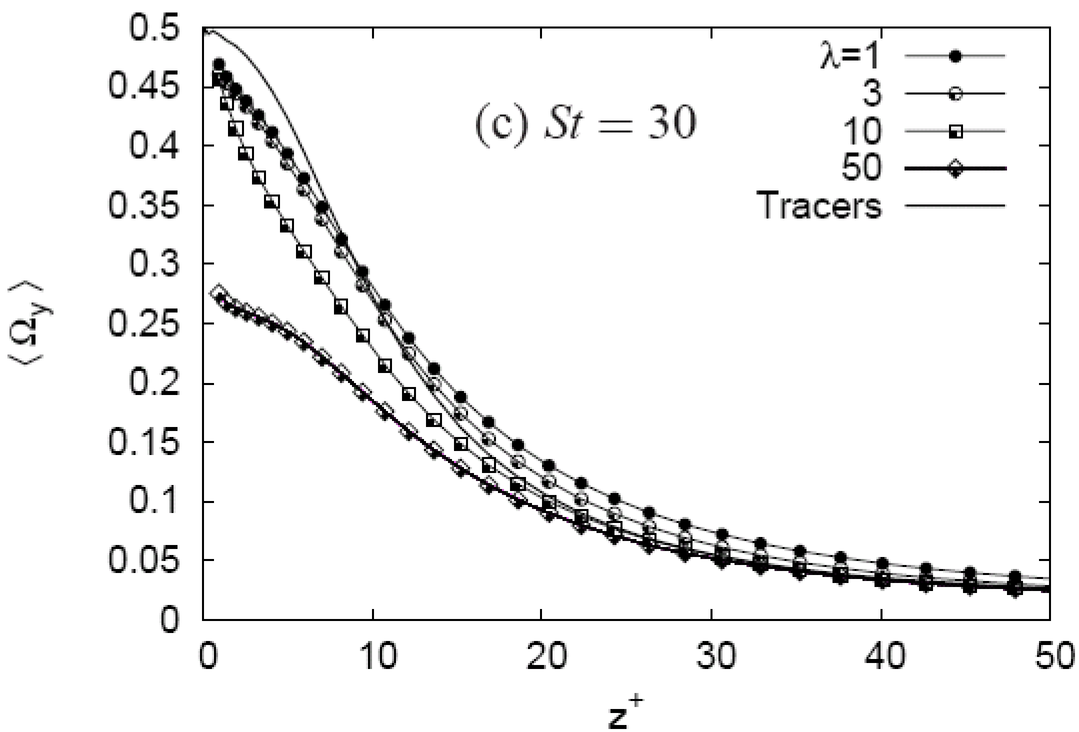

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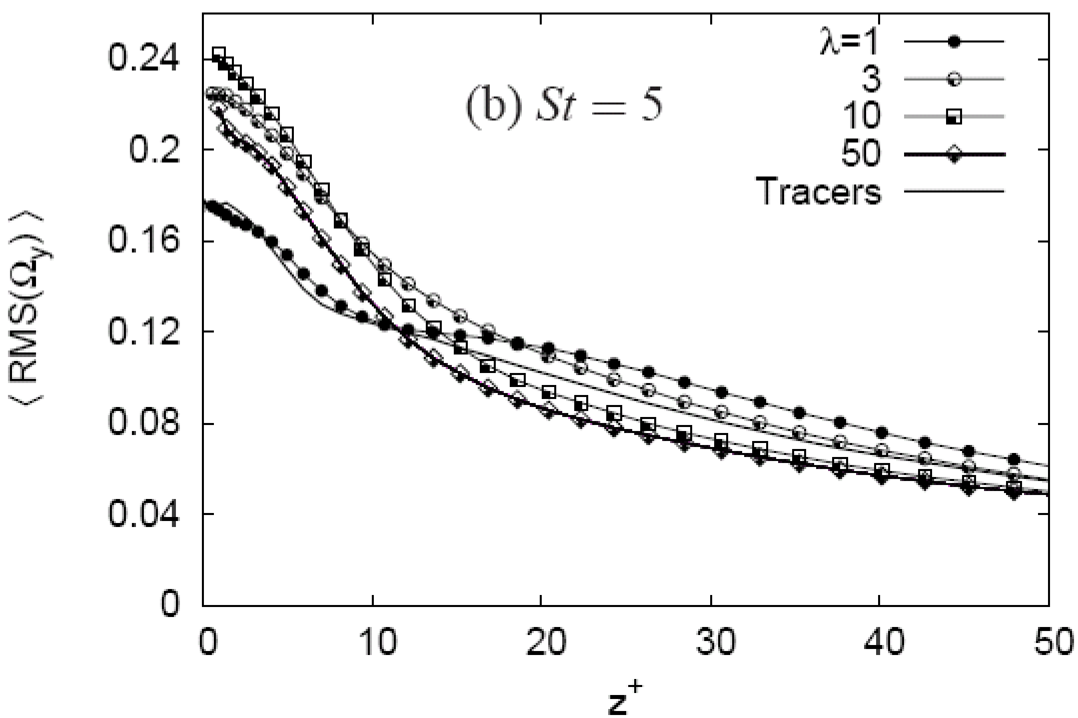
Root Mean Square of fiber angular velocities in the near-wall region



Strong effect of fiber inertia near the wall, length-related effects weaken!

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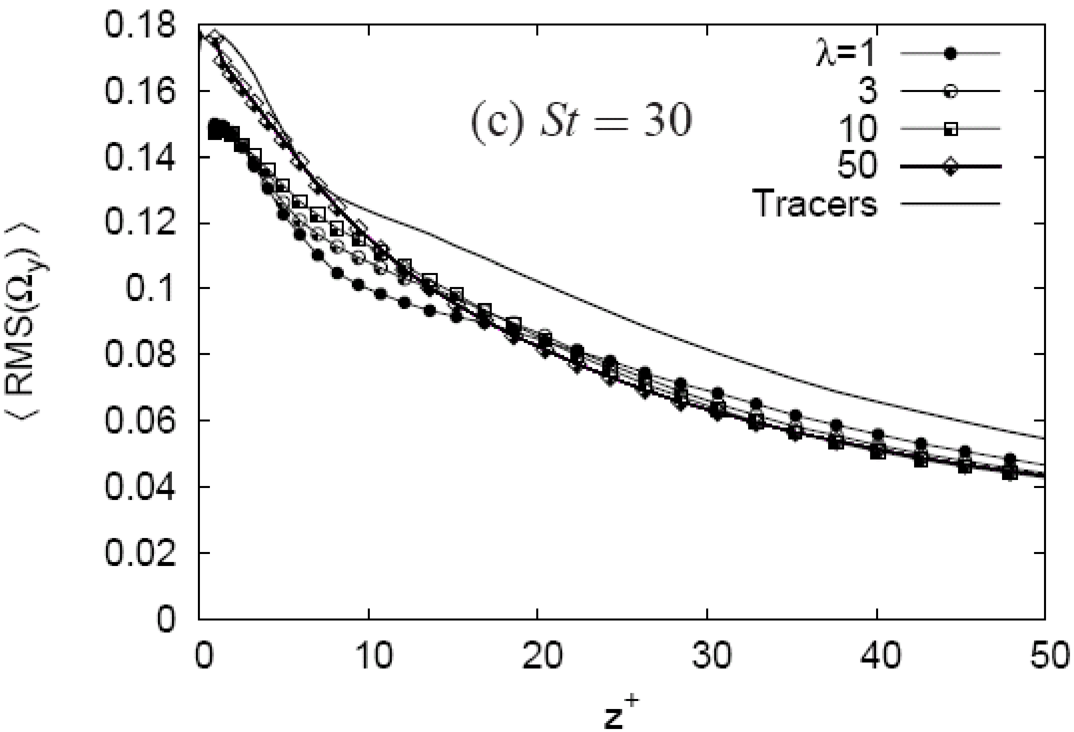

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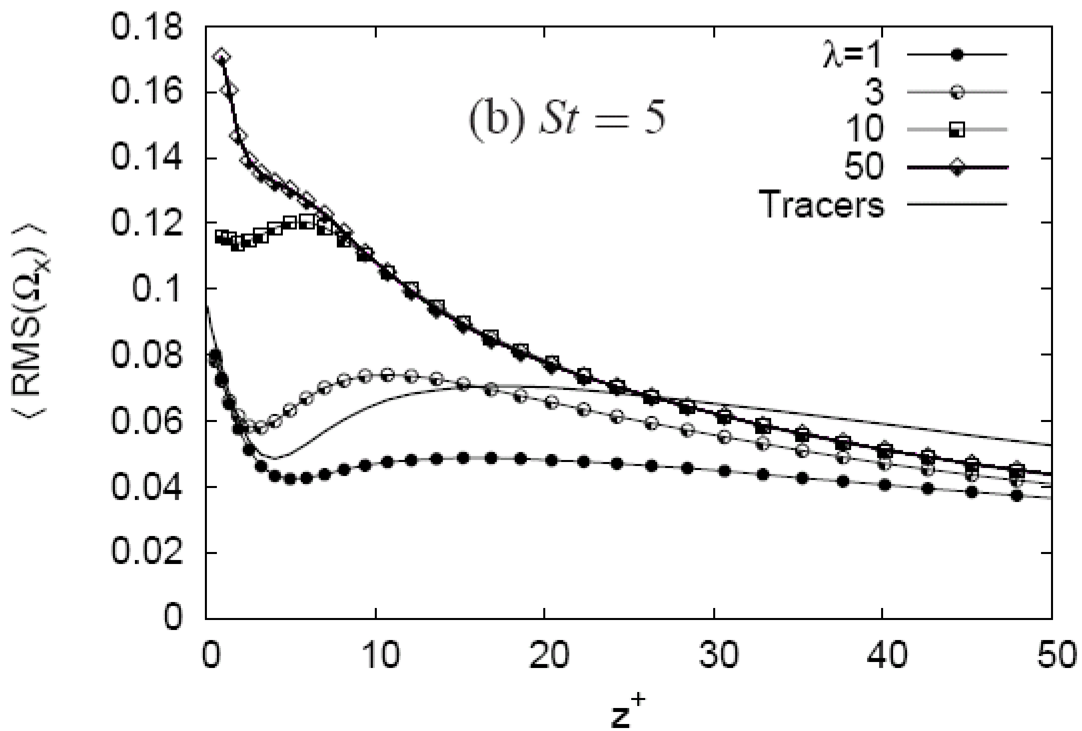

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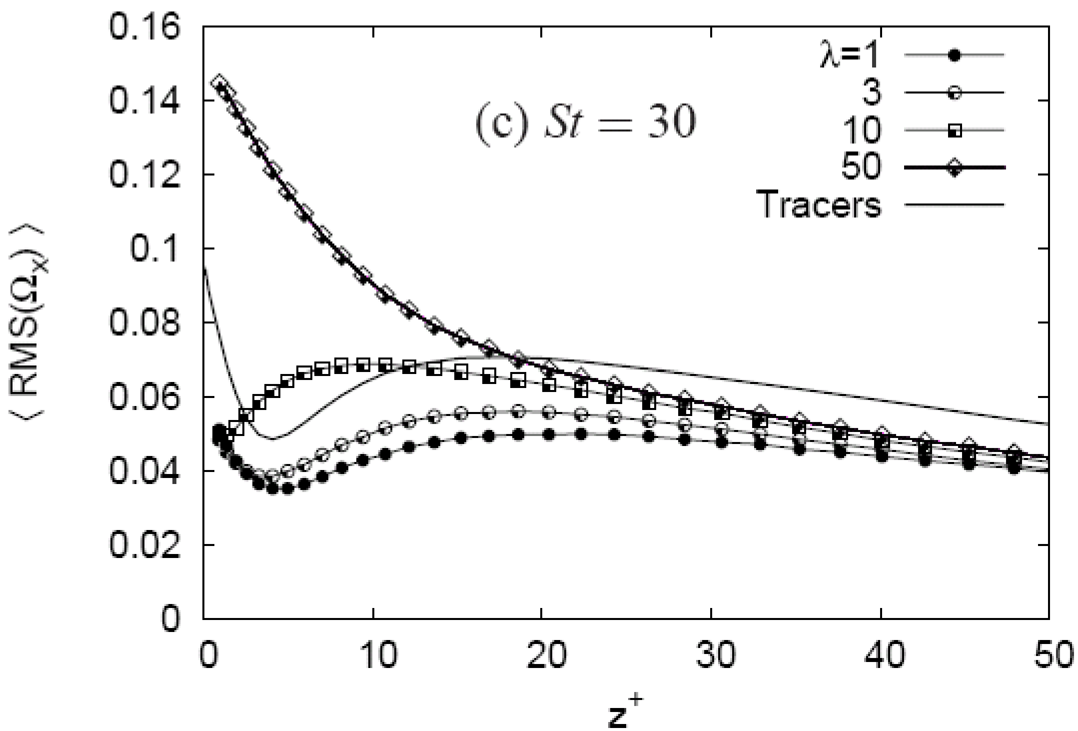

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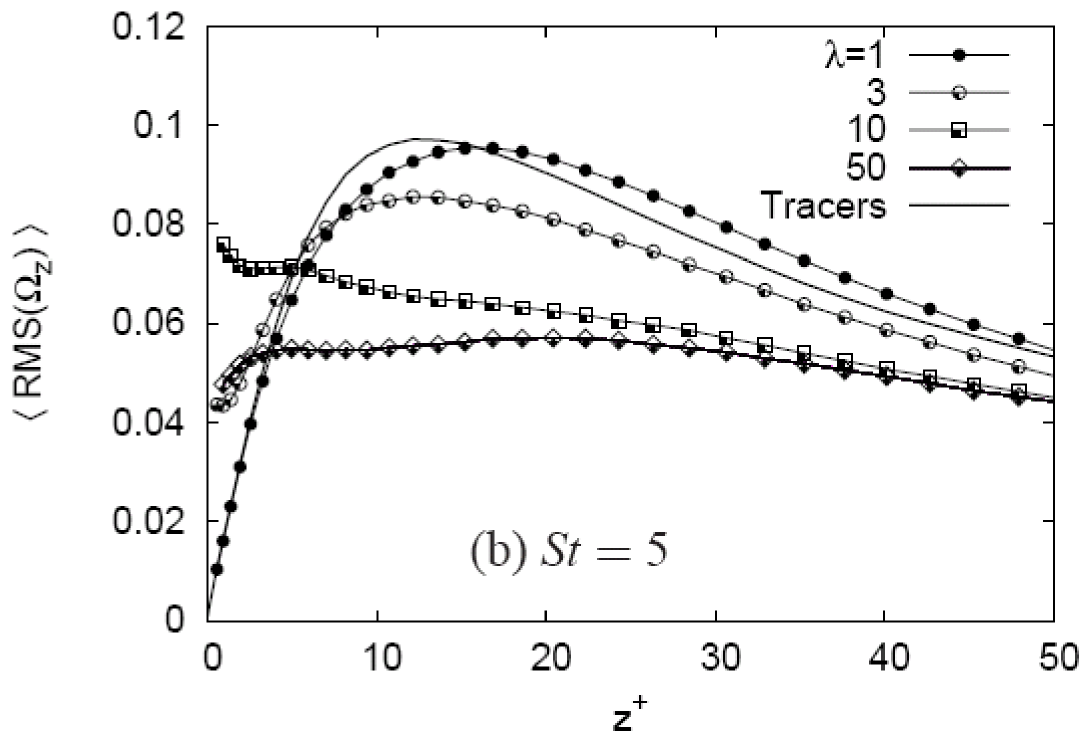

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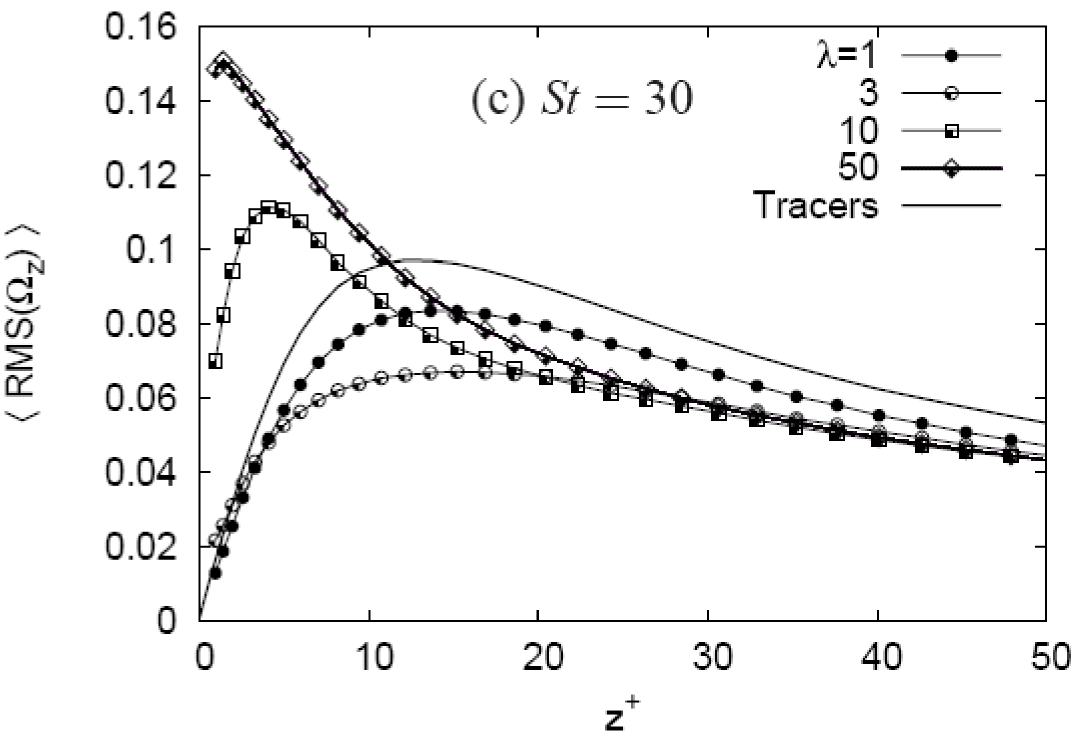

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Results - Characterization of the Fiber Rotation Process

Can fiber rotation be described within the theory of diffusion as a Ornstein-Uhlenbeck (OU) process?

The OU process is completely characterized by:

- **statistically-stationary Gaussian distribution**
- **exponentially decaying autocovariance**

$$R(\tau) = \alpha^2 \exp(-\tau / T_*)$$

where:

α^2 **variance of the Gaussian distribution**

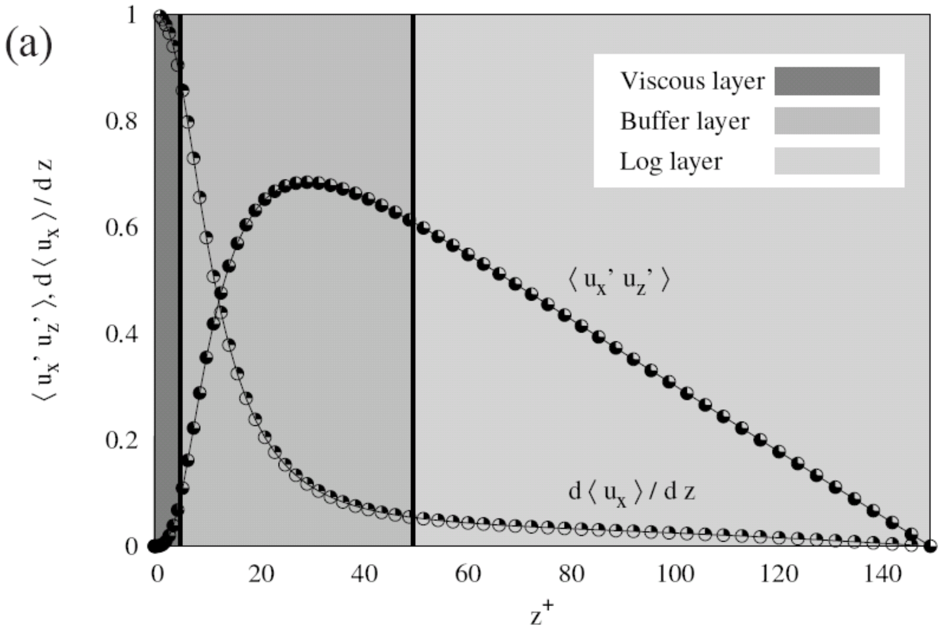
T_* **integral timescale of the process**

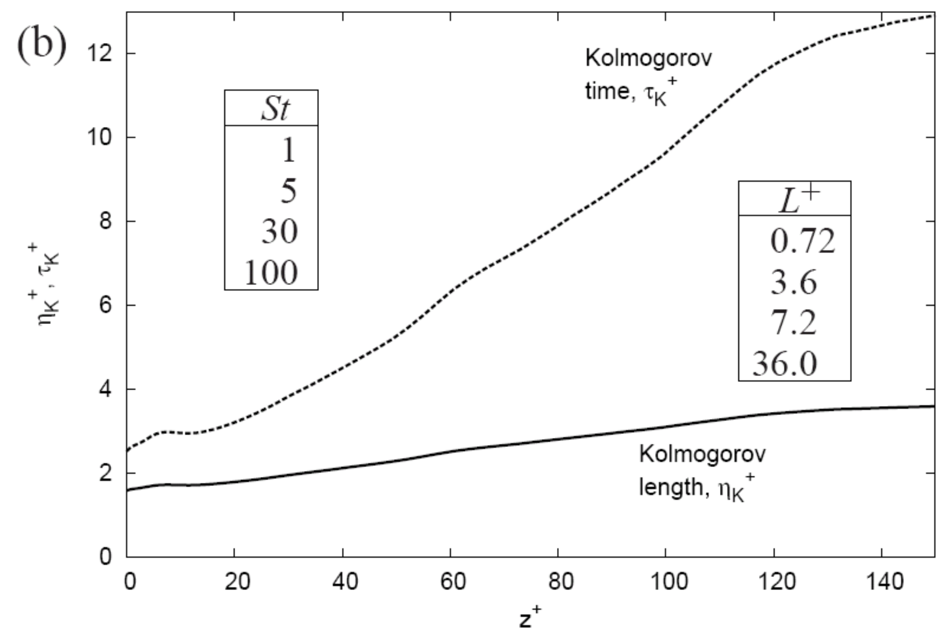
We aim at assessing the applicability of such description in wall-shear turbulence

Results - Characterization of the Fiber Rotation Process

We compute Lagrangian autocorrelations for fibers with different inertia and elongation conditioning the statistics to 3 specific regions of the flow

$$R_{\Omega_i,\Omega_i}(\tau) = \frac{\langle \Omega_i'(\mathbf{x}_p(t_0),t_0)\Omega_i'(\mathbf{x}_p(t_0+\tau),t_0+\tau)\rangle}{\langle \Omega_i'(\mathbf{x}_p(t_0),t_0)^2\rangle^{1/2}\langle \Omega_i'(\mathbf{x}_p(t_0+\tau),t_0+\tau)^2\rangle^{1/2}}$$





St
1
5
30
100

L^+
0.72
3.6
7.2
36.0

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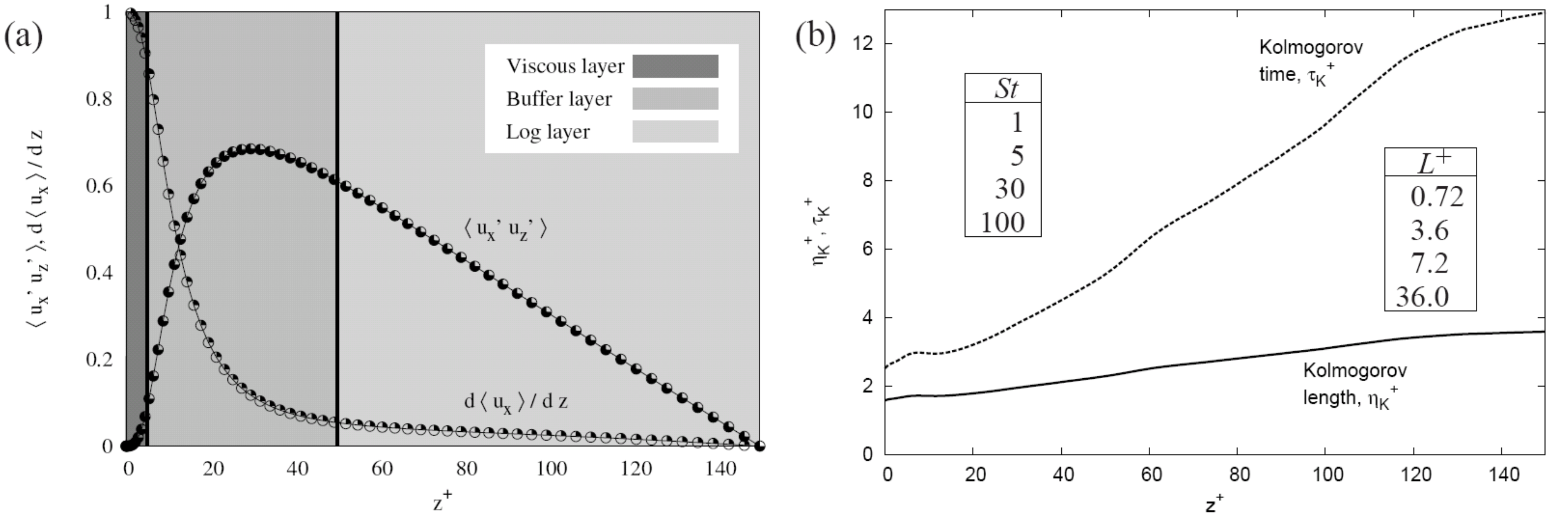
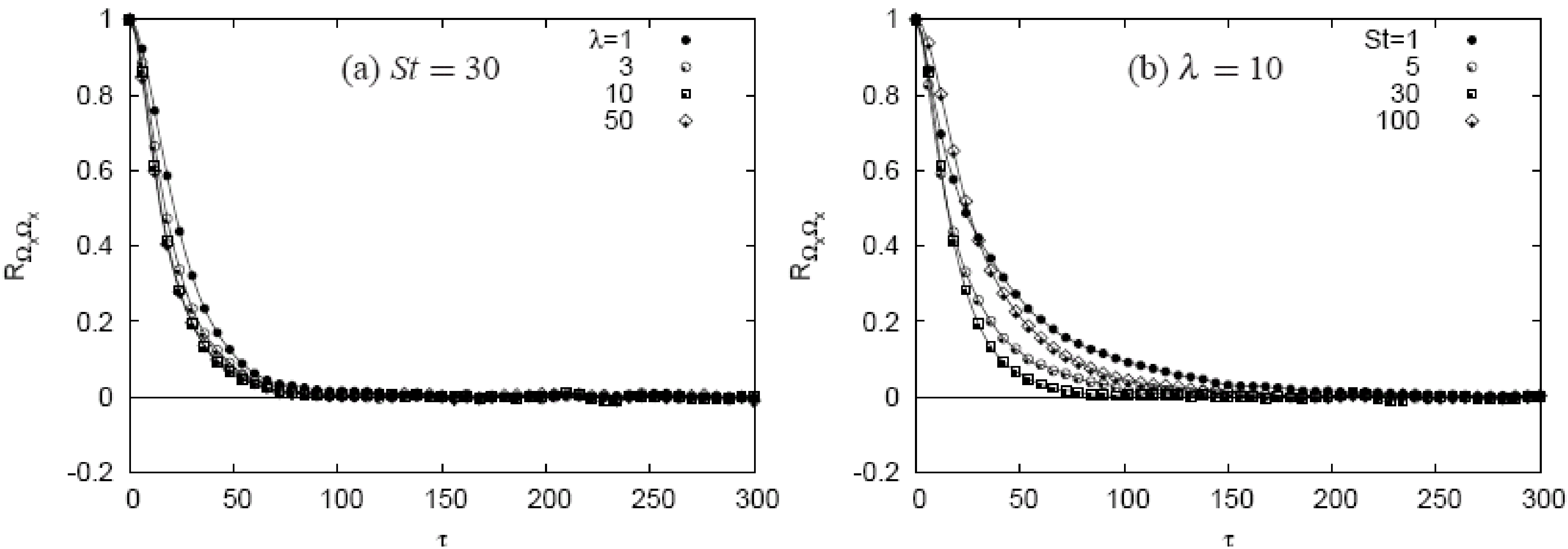


Results - Characterization of the Fiber Rotation Process

Conditioned **Lagrangian autocorrelation** curves in wall-shear turbulence

Log layer



In the log layer, autocorrelation curves display a neat exponential decay. This trend becomes less clear as the wall region is approached.

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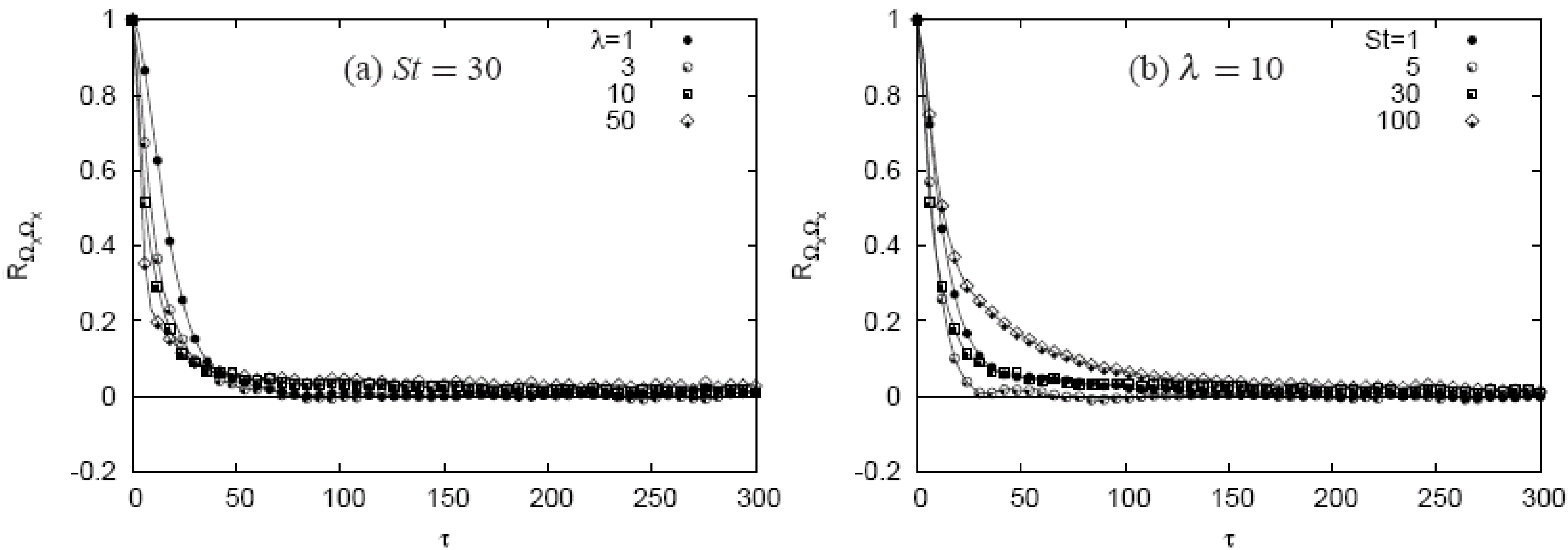
Log layer → **Buffer layer**



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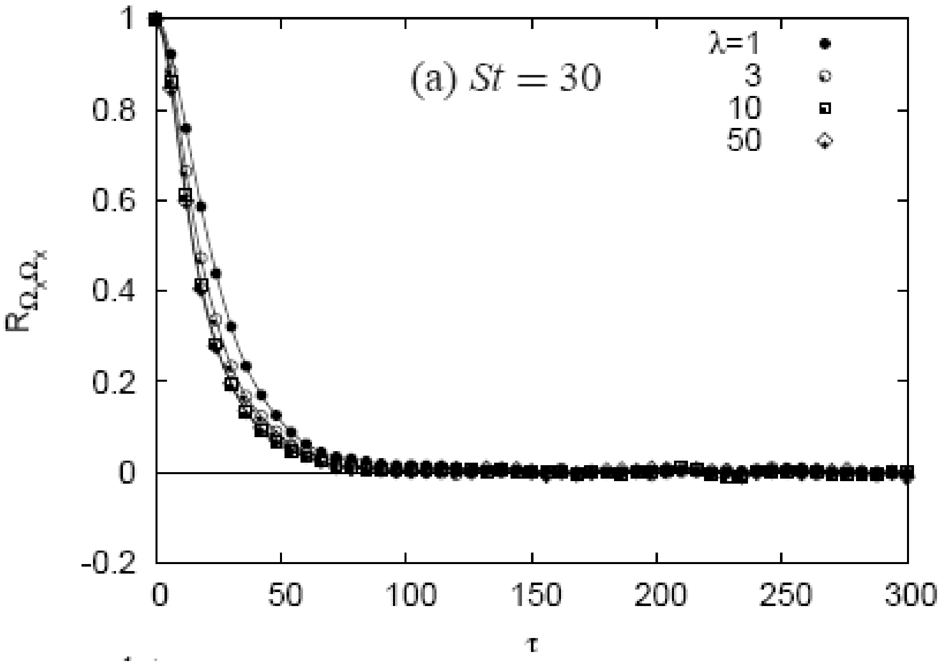
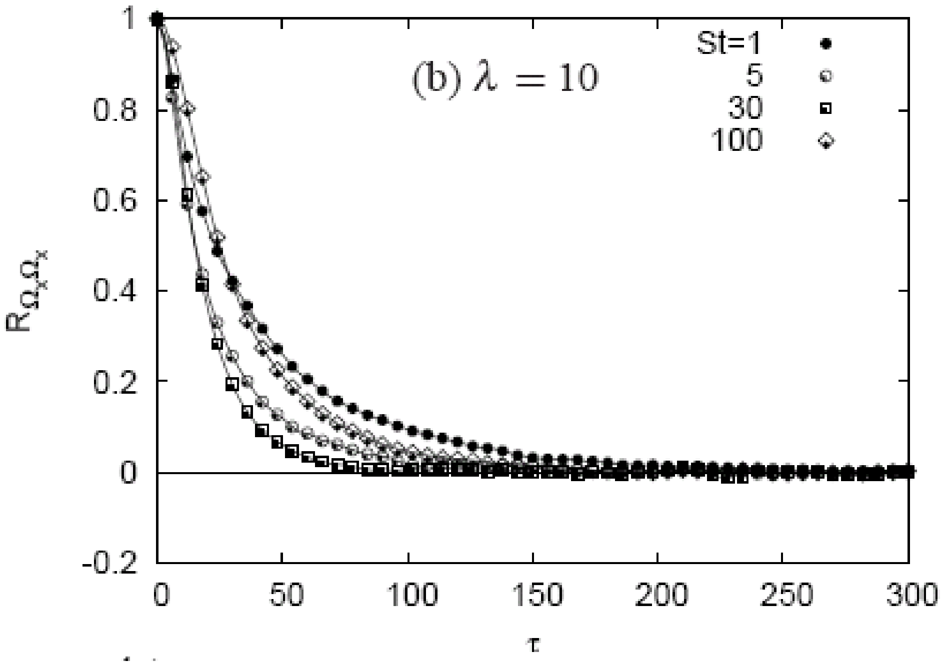
Results - Characterization of the Fiber Rotation Process

Conditioned **Lagrangian autocorrelation** curves in wall-shear turbulence

Log layer → **Buffer layer** → **Viscous layer**





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We further characterize the rotation process by computing the normalized turbulent **rotational diffusivity** conditioned to the same 3 regions

$$\Gamma_{\Omega}(\tau) = \int_0^{\tau} R_{\Omega_i,\Omega_i}(t)\mathrm{d}t$$

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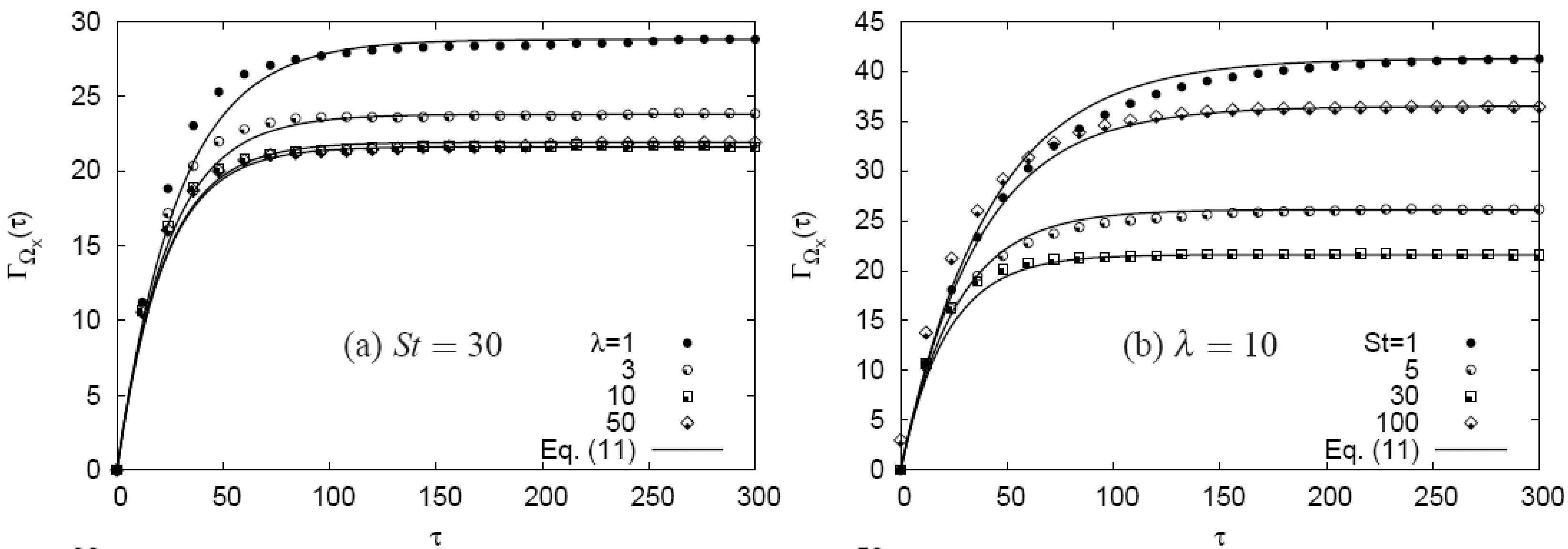

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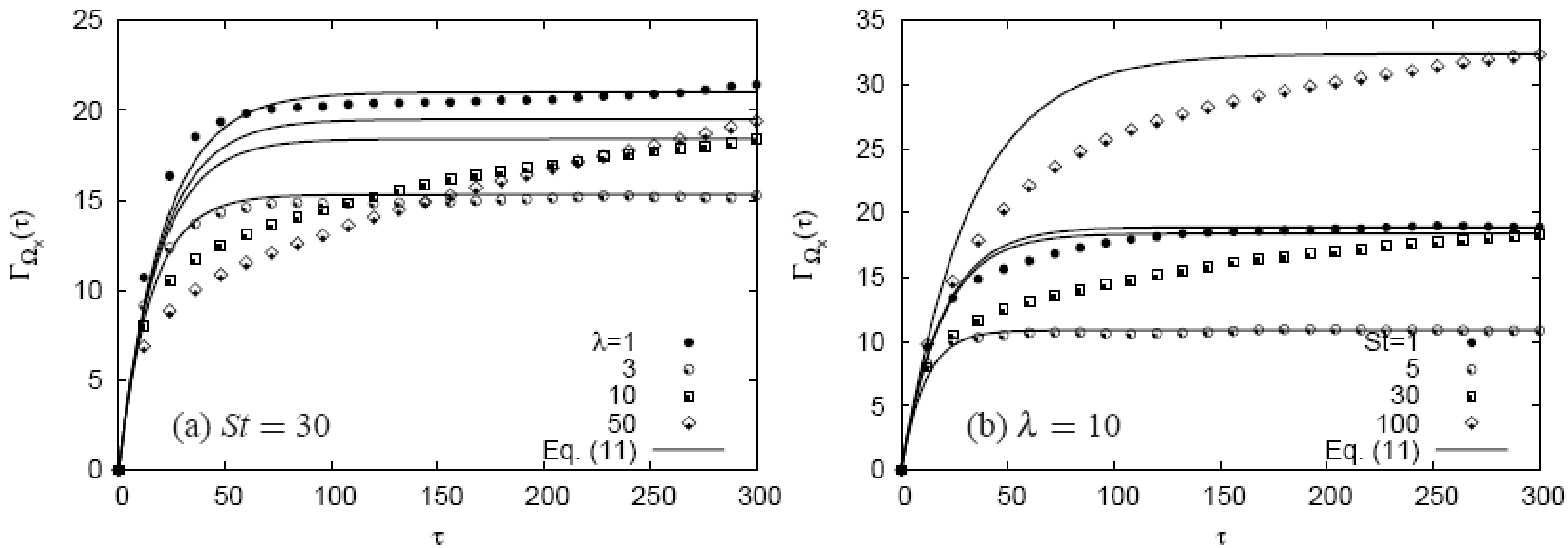

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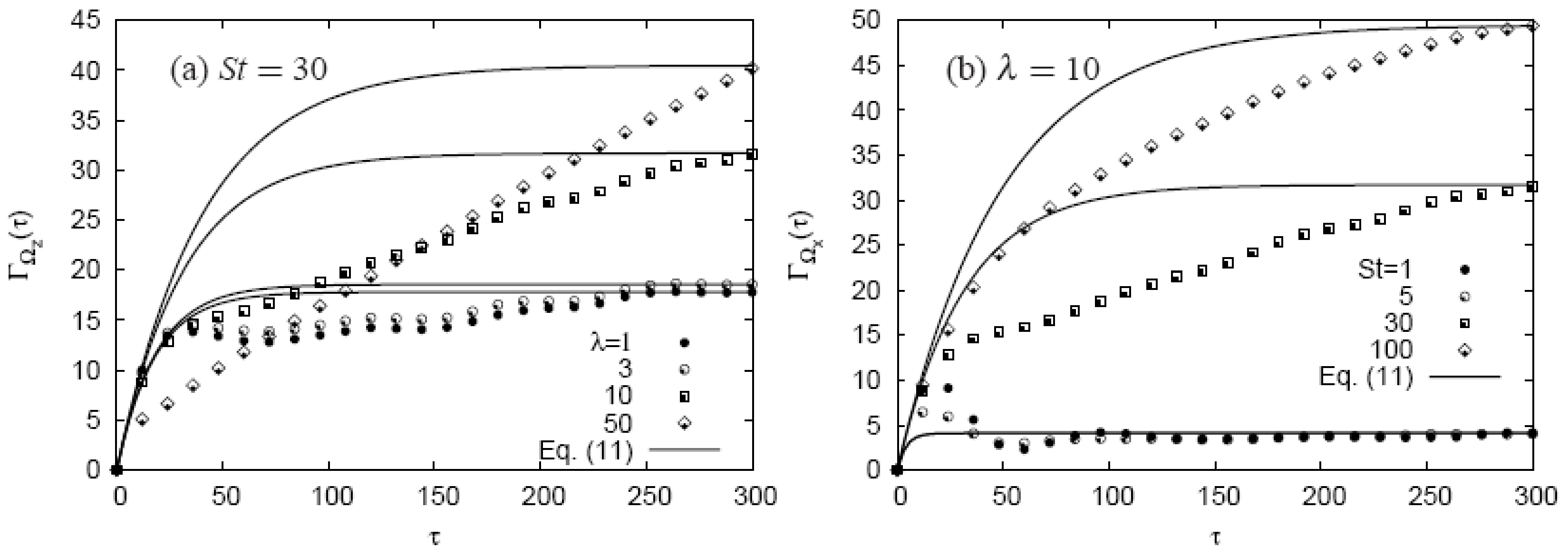
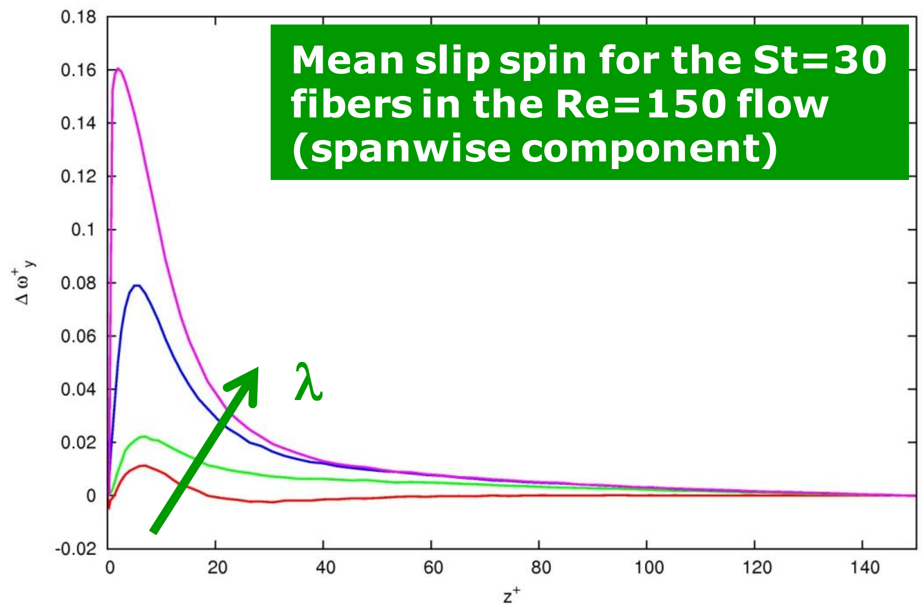


Conclusions . . .

. . . and Future Developments

- Fiber angular velocity is a useful measure of fibers-turbulence interaction in wall-bounded flows: its statistical characterization provides useful indications for modeling turbulent fiber dispersion
- Angular velocity statistics depend both on fiber elongation (quantitatively) and fiber inertia (also qualitatively!)
- Fiber rotation exhibits autocorrelation curves for the angular velocities that decay exponentially in the log layer (but not in the wall layer…)



- Simulate more values of *St*, *λ* and *Re*
- Evaluate slip angular velocity (spin) statistics