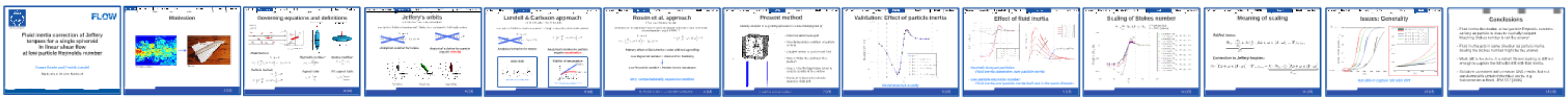
Thank you!


FLOW
Motivation
Governing equations and definitions
Jeffery's orbits
Lundell & Carlsson approach
Rosén et al. approach
Present method
Validation: Effect of particle inertia
Effect of fluid inertia
Scaling of Stokes number
Meaning of scaling
Issues: Generality
Conclusions

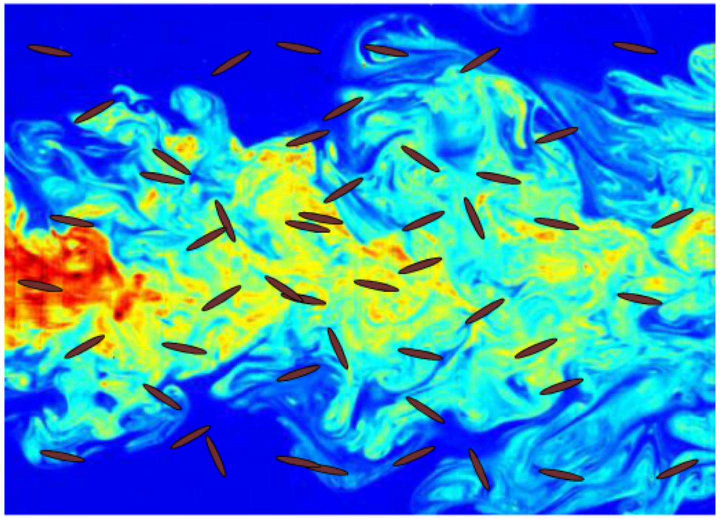
KTH Mechanics

Fluid inertia correction of Jeffery torques for a single spheroid in linear shear flow at low particle Reynolds number

Tomas Rosén and Fredrik Lundell

Big thanks to Dr. Arne Nordmark

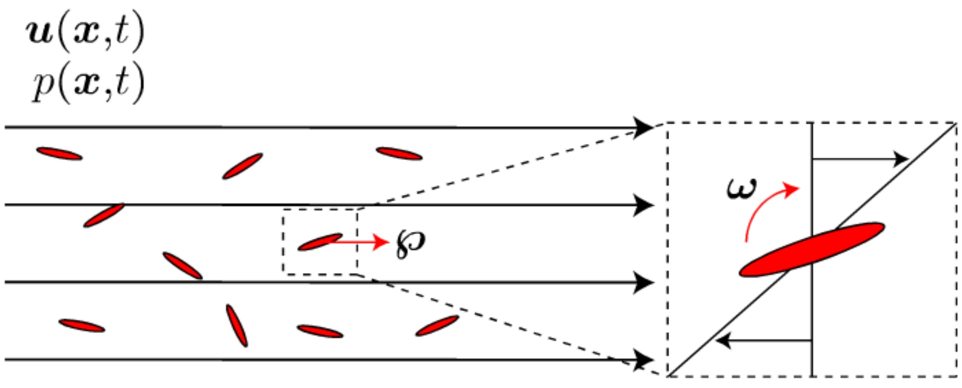
Motivation



Ref: Wikipedia - Turbulence

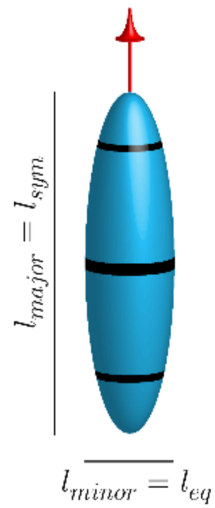
Ref: Motif

Governing equations and definitions

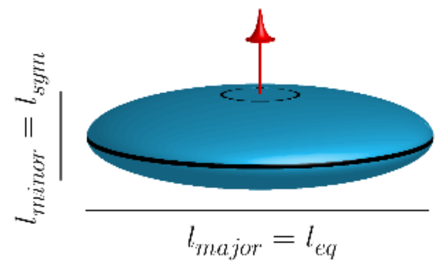


G = shear rate

Prolate spheroid



Oblate spheroid



Fluid motion:

$$\nabla \cdot \boldsymbol{u} = 0$$

$$Re_p \cdot \left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} \right) = -\nabla p + \nabla^2 \boldsymbol{u}$$

Reynolds number:

$$Re_p = \frac{G l_{major}^2}{\nu}$$

Stokes number:

$$St = \frac{\rho_p}{\rho_f} \cdot Re_p$$

Particle motion:

$$St \cdot \frac{\partial \wp}{\partial t} = \boldsymbol{F}$$

$$St \cdot \left(\boldsymbol{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega}) \right) = \boldsymbol{T}$$

Aspect ratio:

$$r_p = \frac{l_{sym}}{l_{eq}}$$

Alt. aspect ratio:

$$r_p^* = \frac{l_{major}}{l_{minor}}$$

Jeffery's orbits

G. B. Jeffery, Proc. R. Soc. London, Ser A 102 (1922)

Assumptions: Particle is massless and infinitely small (compared to fluid length scales)

$$\nabla \cdot \boldsymbol{u} = 0$$

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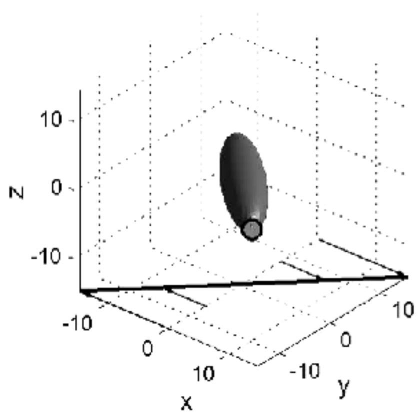
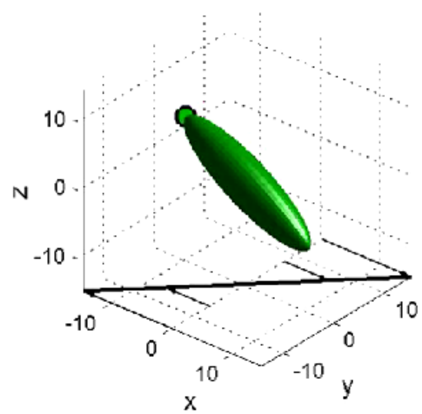
$$St \cdot \left(\boldsymbol{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega}) \right) = \boldsymbol{T}$$

Analytical solution for torque

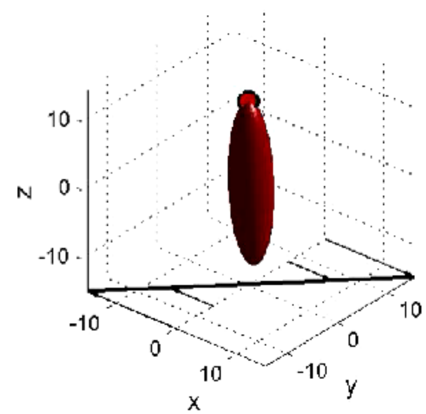
Analytical solution for particle angular velocity



Tumbling



Kayaking

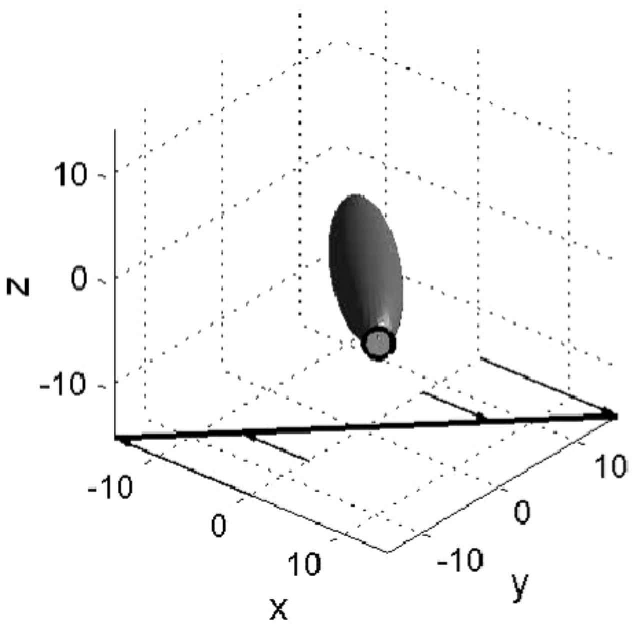


Log-rolling

$$\cdot \left(\ldots \nabla)\boldsymbol{u} \right) = -\nabla p + \nabla^2 \boldsymbol{u}$$

alytical solution for torque

Analytical solution for pa angular velocity



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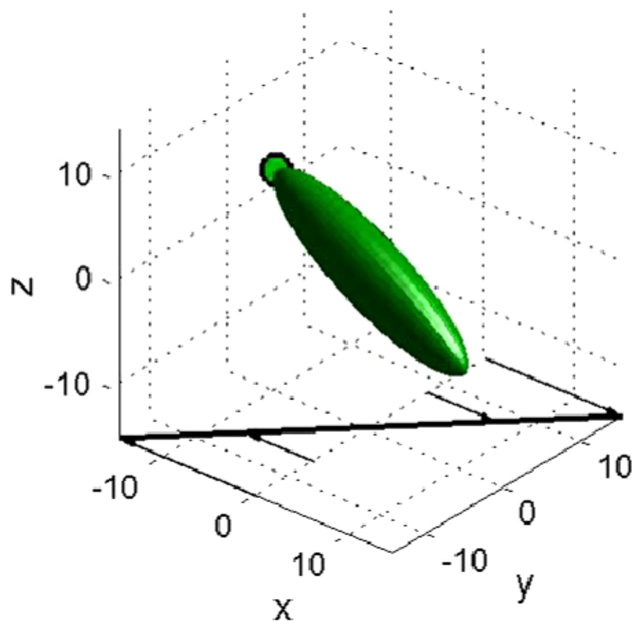

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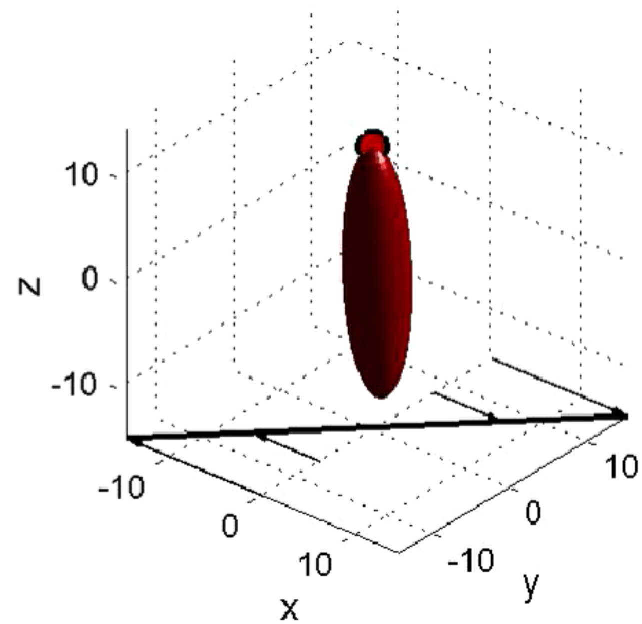

Log-rolling

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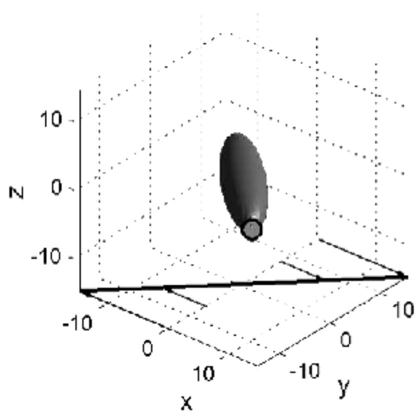
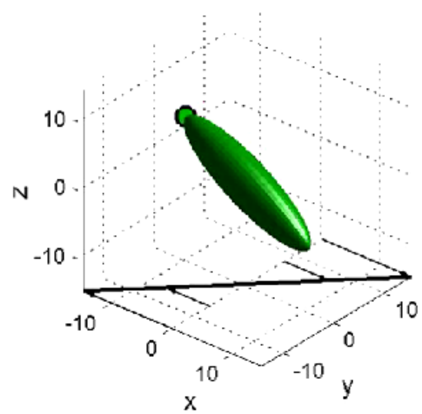


Tumbling



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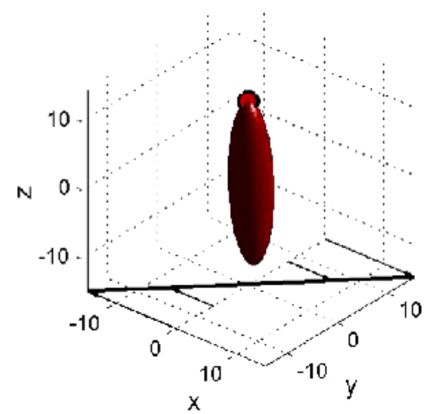

Log-rolling

Lundell & Carlsson approach

F. Lundell and A. Carlsson, Phys. Rev. E 81 (2010)

Assumptions: Particle is small compared to fluid length scales, but particle inertia relevant

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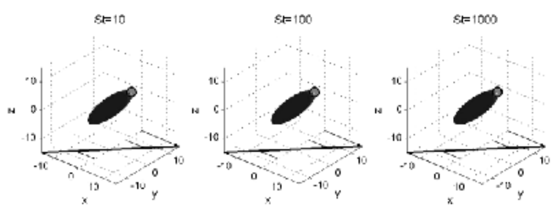
Analytical solution for torque

Analytical solution for particle angular acceleration

Orbit drift



Particle inertia leads to a drift towards Tumbling

Validity of assumption

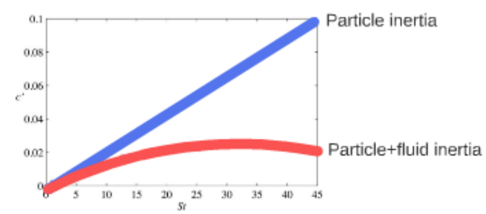
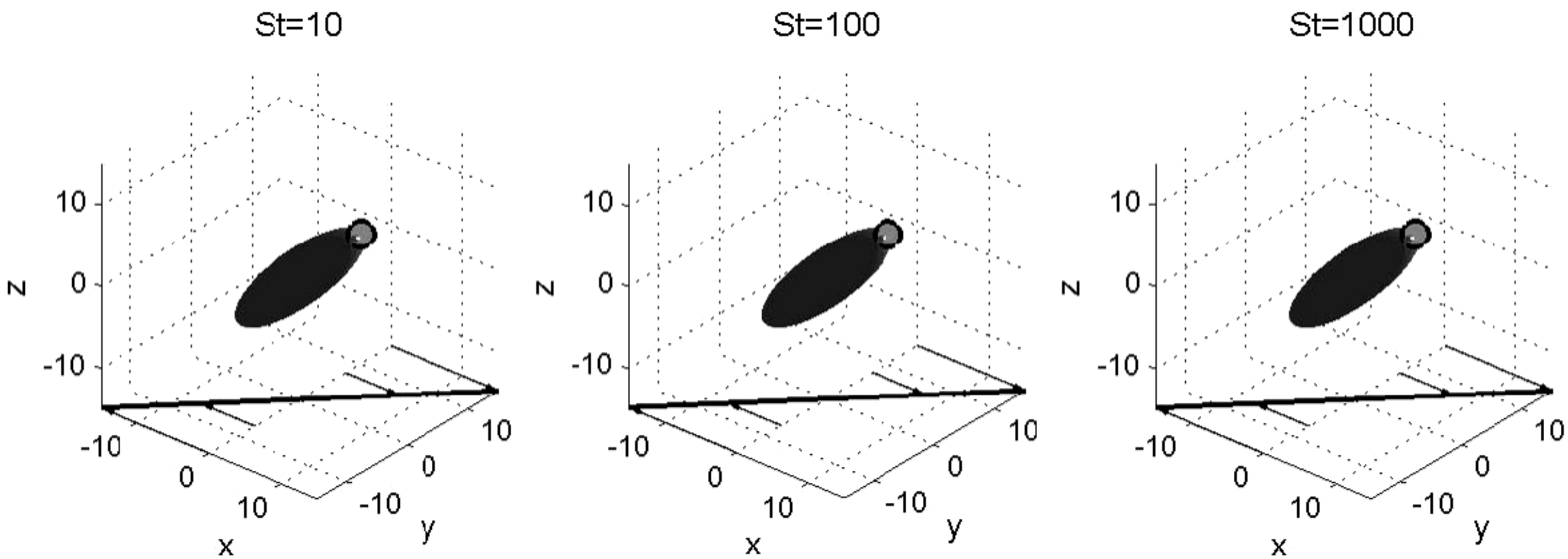


FIG. 8. (Color online) Orbit drift parameter c' as a function of St for the present case without fluid inertia (blue, *) and from simulations including fluid and particle inertia with κ=1.001 by Yu *et al.* [24] (red, ○). k_b=k_c=0.5.

Particle inertia seems to be the only reason for orbit drift at low particle Reynolds numbers

Orbit drift



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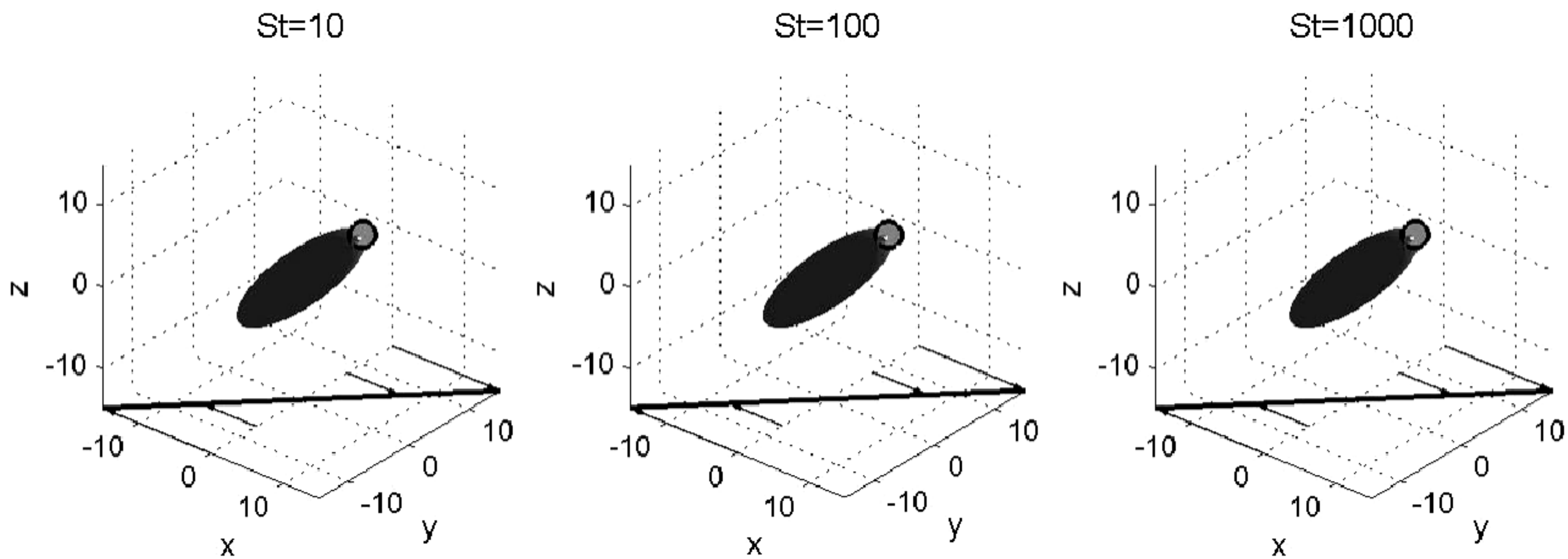
Orbit drift



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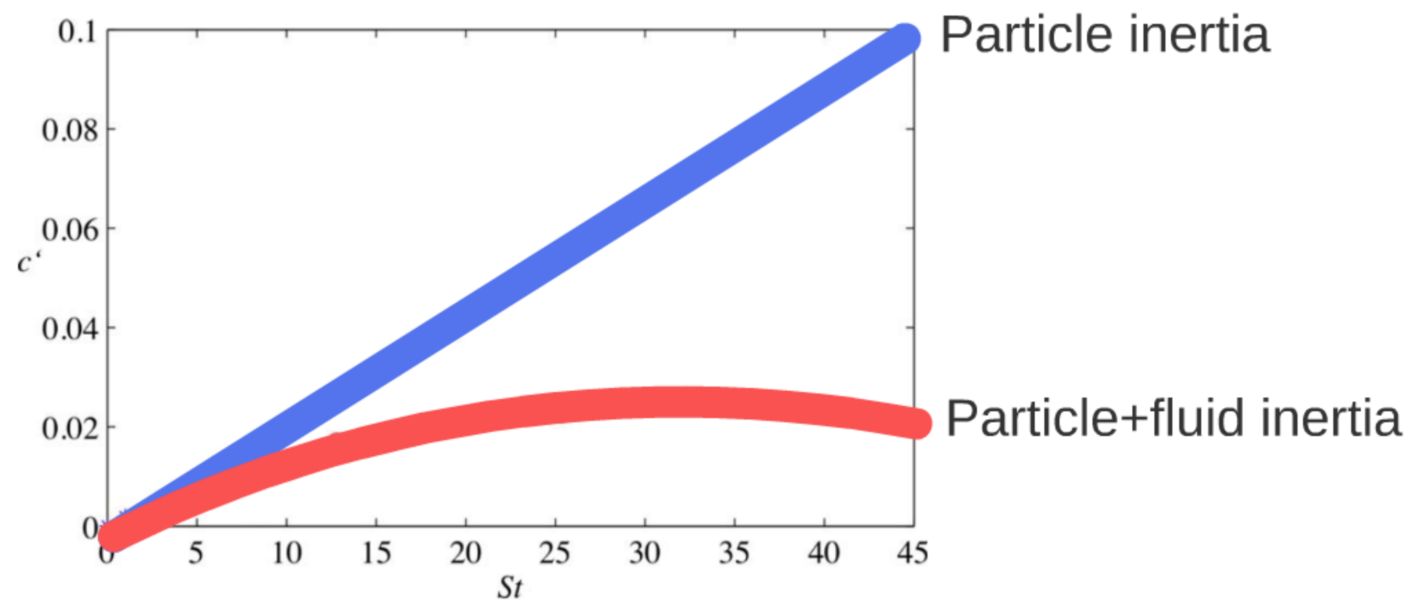

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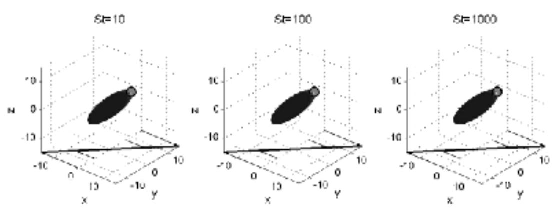
Analytical solution for torque

Analytical solution for particle angular acceleration

Orbit drift



Particle inertia leads to a drift towards Tumbling

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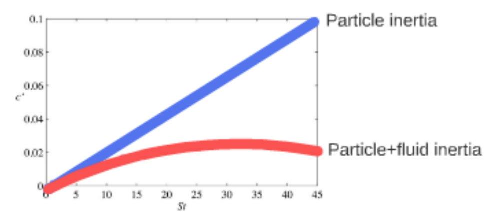

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Particle inertia seems to be the only reason for orbit drift at low particle Reynolds numbers

Rosén et al. approach

T. Rosén et al., J. Fluid Mech. 738 (2014)

Simulations of neutrally buoyant prolate spheroids using the lattice Boltzmann method with external boundary force (LB-EBF) [1]

$$\nabla \cdot \boldsymbol{u} = 0$$

$$Re_p \cdot \left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} \right) = -\nabla p + \nabla^2 \boldsymbol{u}$$

$$St \cdot \left(\boldsymbol{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega}) \right) = \boldsymbol{T}$$

Primary effect of fluid inertia = Orbit drift to Log-rolling

Low Reynolds number = Orbit drift to Tumbling

↓

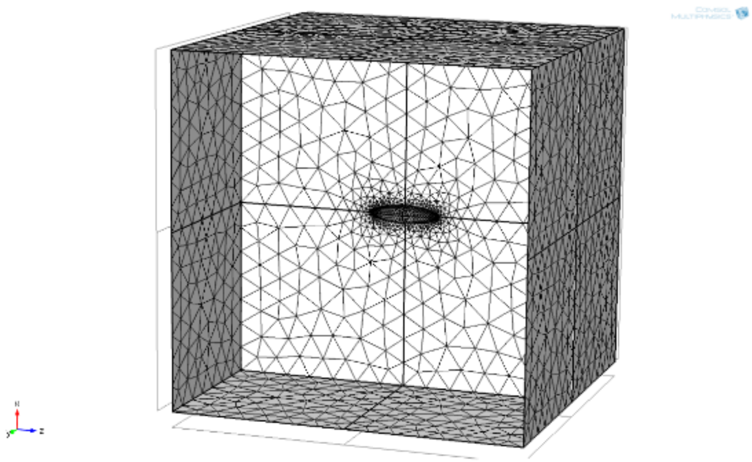
Low Reynolds number = Particle inertia dominates

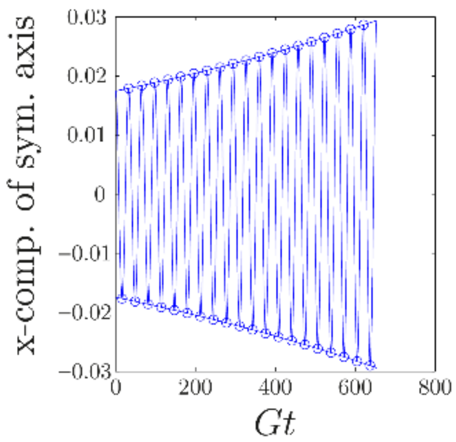
Very computationally expensive method

[1] J. Wu and C. K. Aidun, Int. J. Numer. Meth. Fluids 62 (2010)

Present method

Stability analysis of Log-rolling spheroid in Comsol Multiphysics [1]

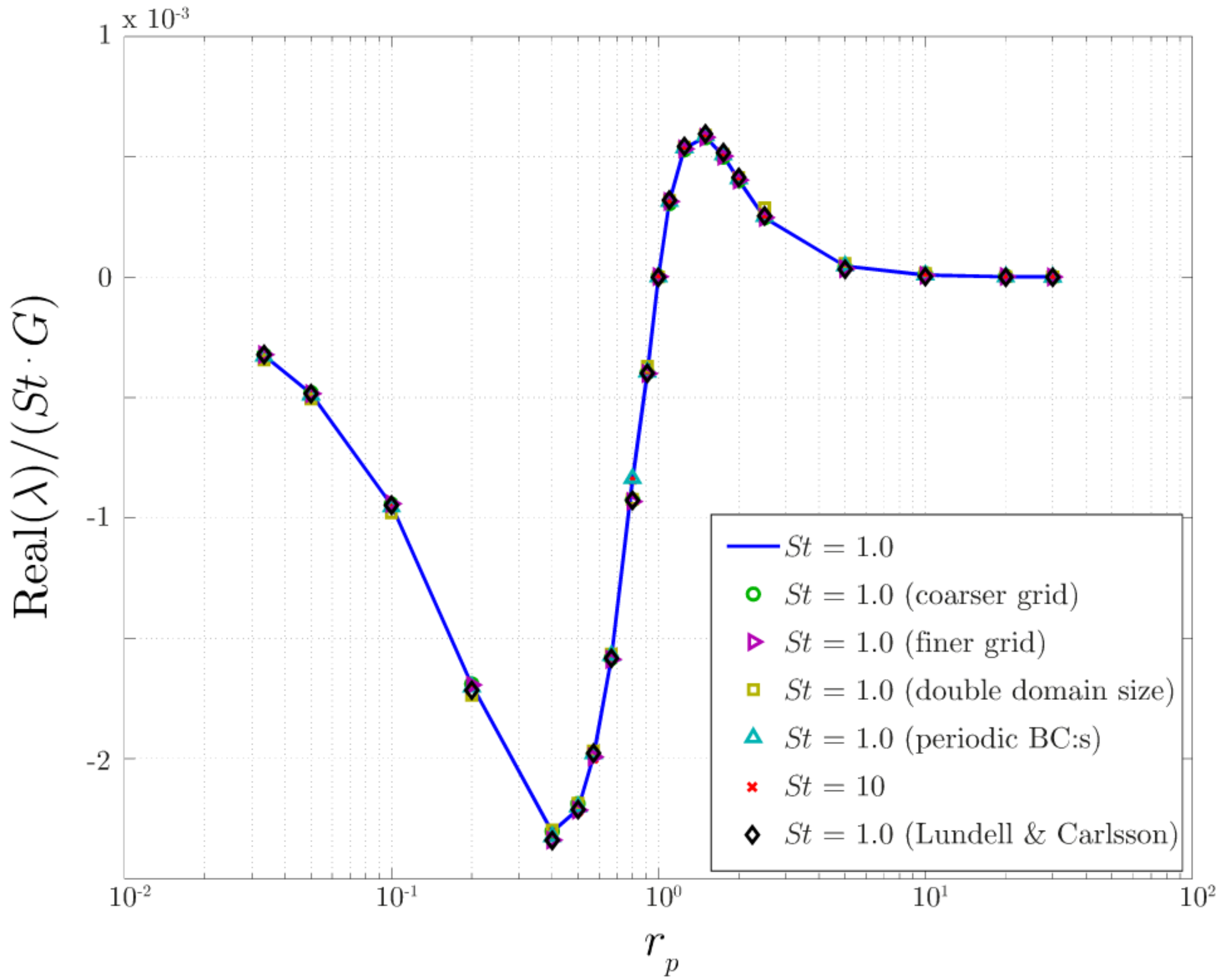




- Fixed but deformable grid
- Velocity boundary condition on particle surface
- Coupled motion of particle and fluid
- Step 1: Solve the stationary flow problem
- Step 2: Use the Eigenvalue solver to analyze stability of the solution
- Real part of Eigenvalue directly related to Orbit drift

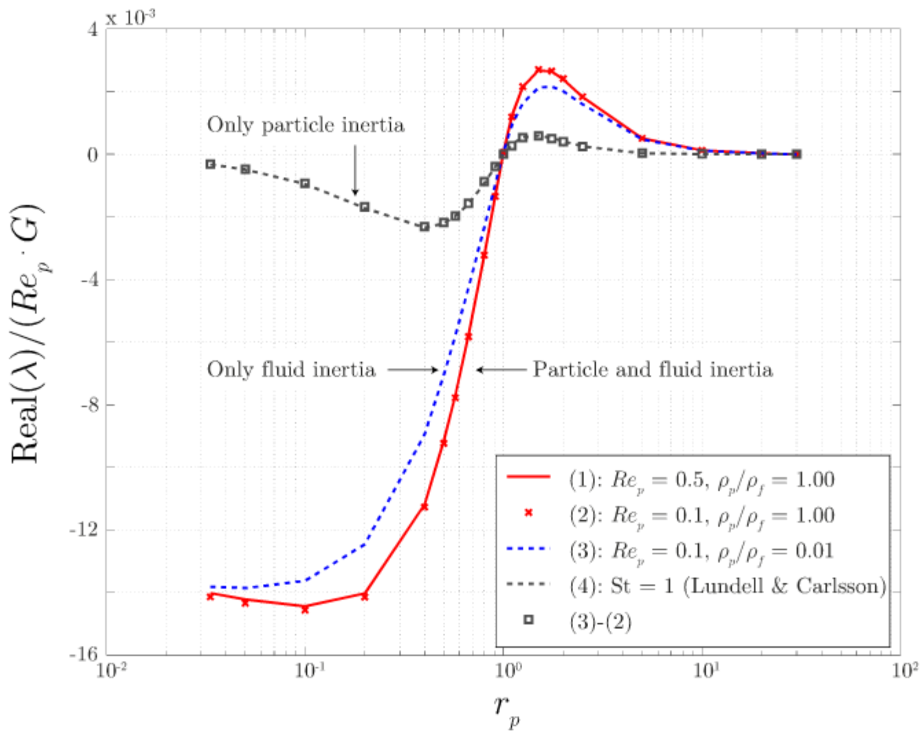
[1] Model built by Dr. Arne Nordmark

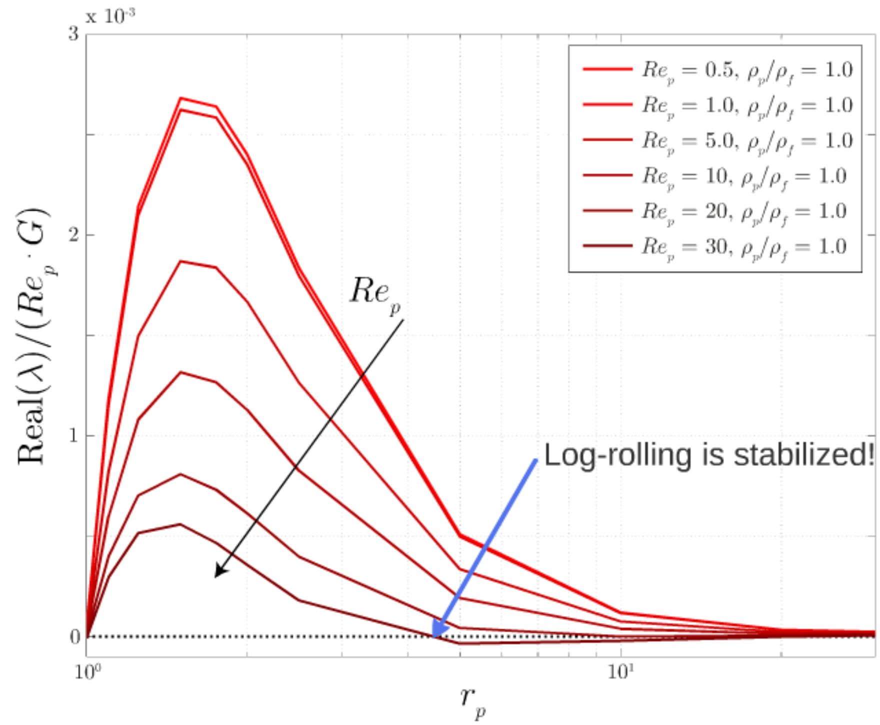
Validation: Effect of particle inertia



Model matches exactly

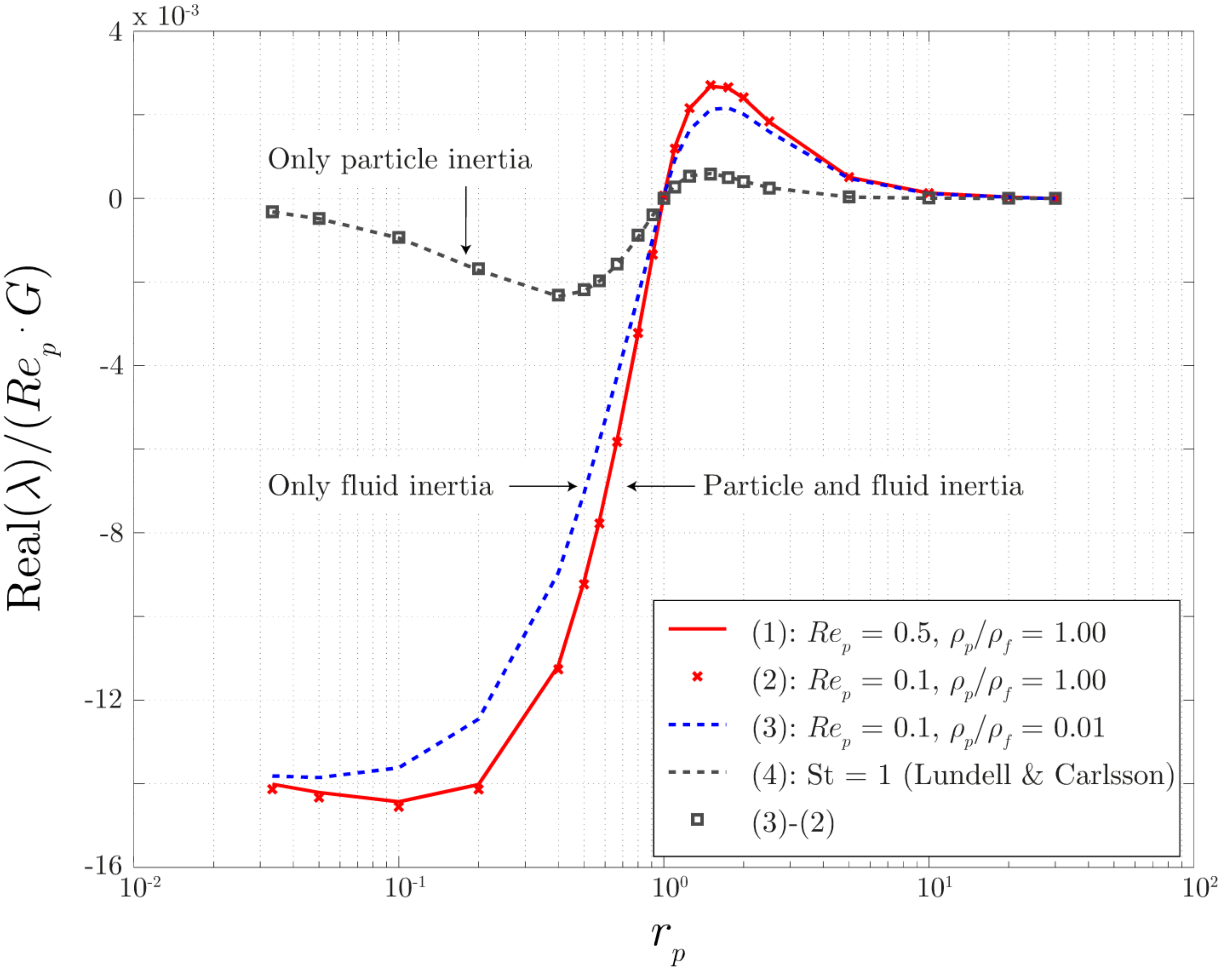
Effect of fluid inertia





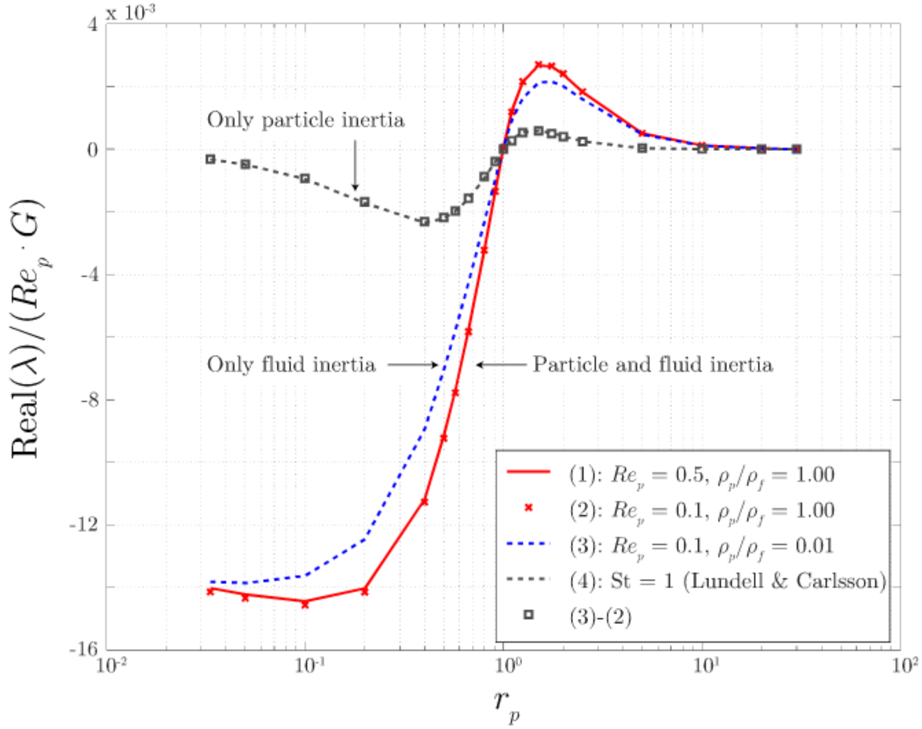
- ***Neutrally buoyant particles:***
  - ***Fluid inertia dominates over particle inertia***

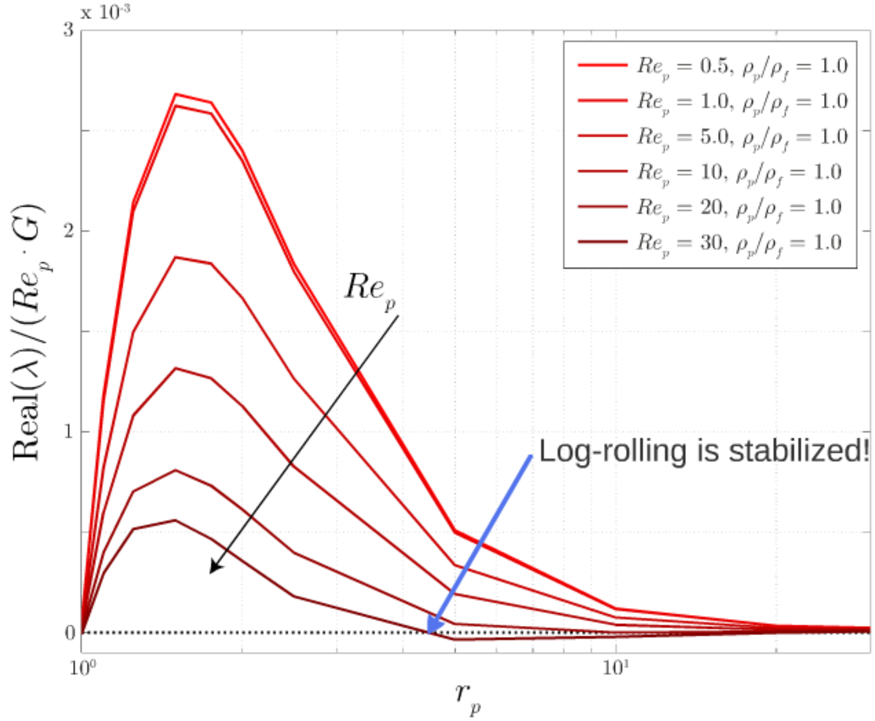
- ***Low particle Reynolds number:***
  - ***Fluid inertia and particle inertia both act in the same direction***



x 10-3
4
0
-4
-8
-12
-16
10-2
10-1
100
101
102
Real(λ)/(Re_p · G)
r_p
Only particle inertia
Only fluid inertia
Particle and fluid inertia
(1): Re_p = 0.5, ρ_p/ρ_f = 1.00
(2): Re_p = 0.1, ρ_p/ρ_f = 1.00
(3): Re_p = 0.1, ρ_p/ρ_f = 0.01
(4): St = 1 (Lundell & Carlsson)
(3)-(2)

Effect of fluid inertia





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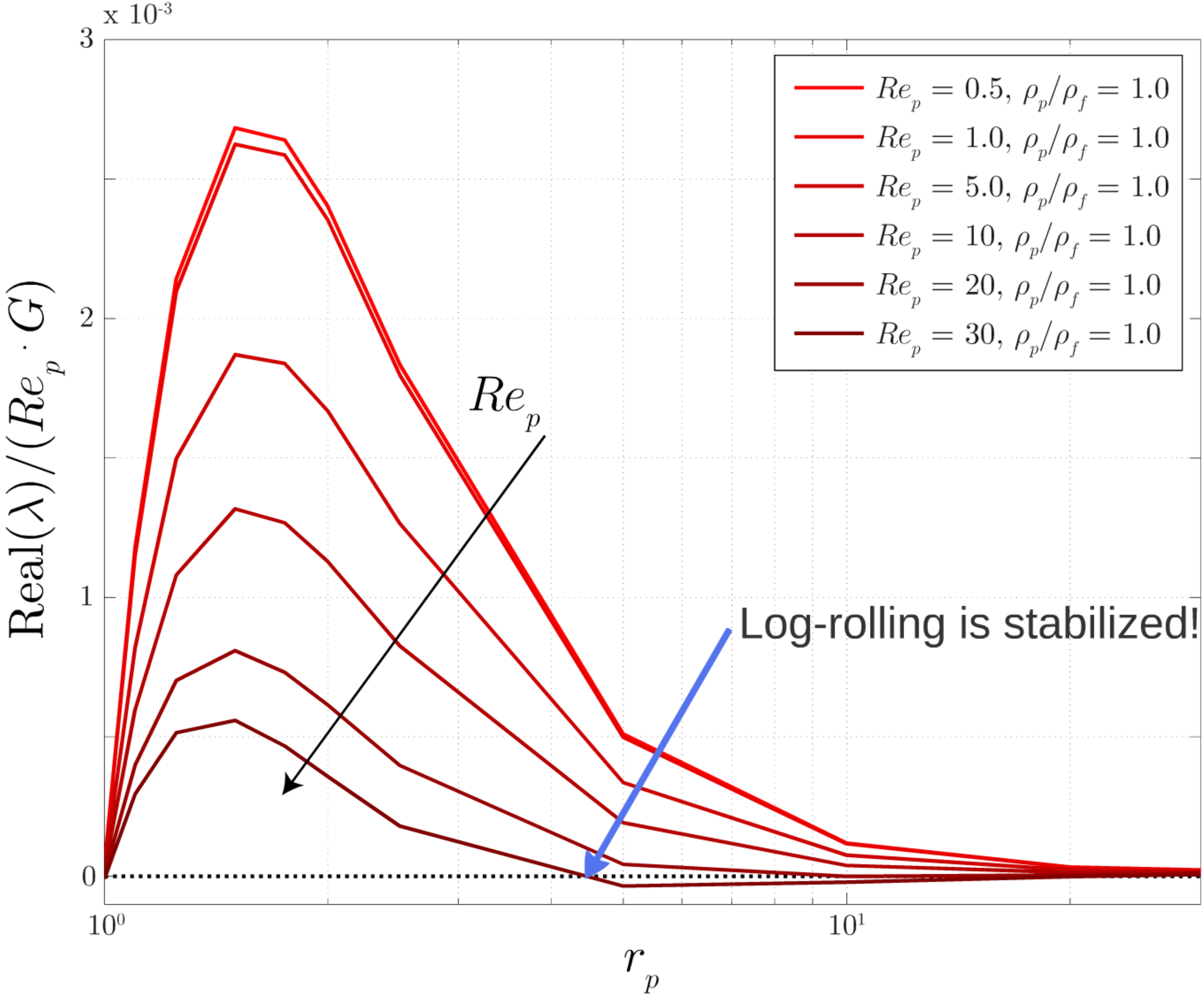
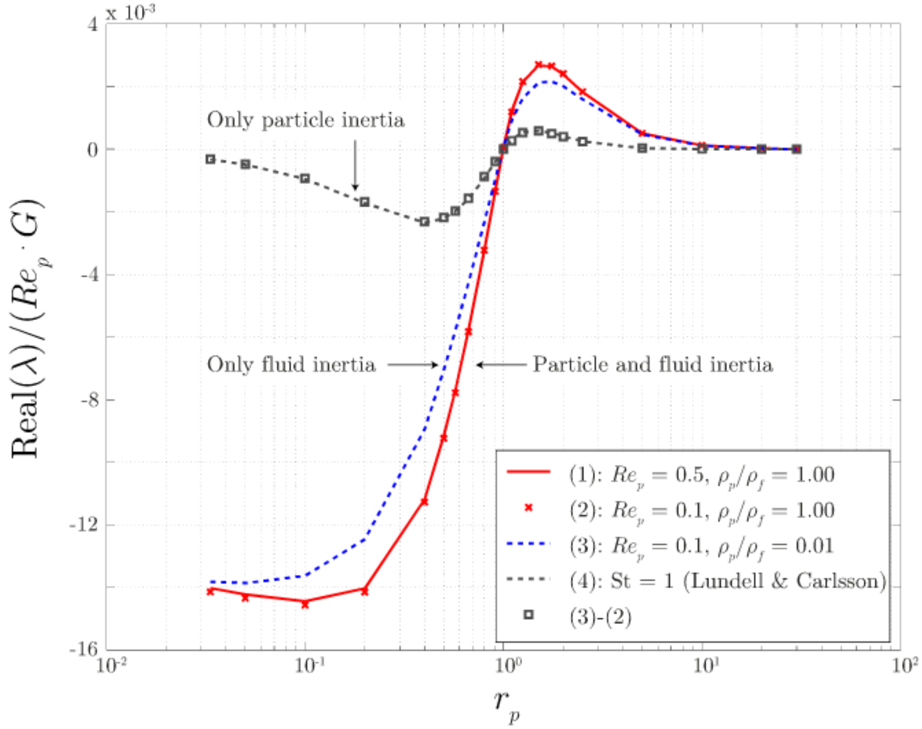
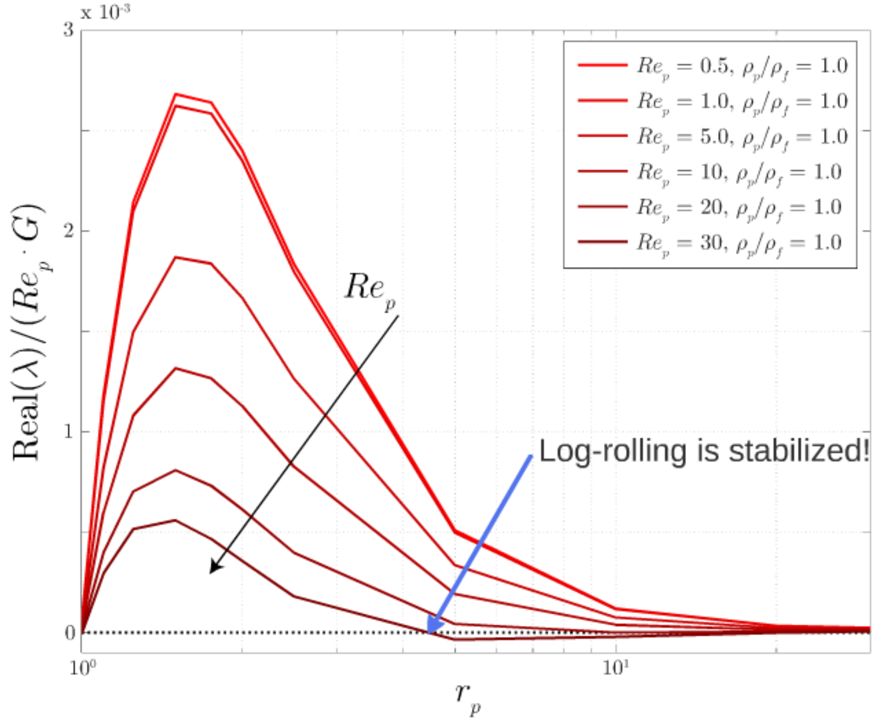

x 10-3
3
2
1
0
Real(λ)/(Re_p · G)
Re_p = 0.5, ρ_p/ρ_f = 1.0
Re_p = 1.0, ρ_p/ρ_f = 1.0
Re_p = 5.0, ρ_p/ρ_f = 1.0
Re_p = 10, ρ_p/ρ_f = 1.0
Re_p = 20, ρ_p/ρ_f = 1.0
Re_p = 30, ρ_p/ρ_f = 1.0
Re_p
Log-rolling is stabilized!
10^0
10^1
r_p

Effect of fluid inertia





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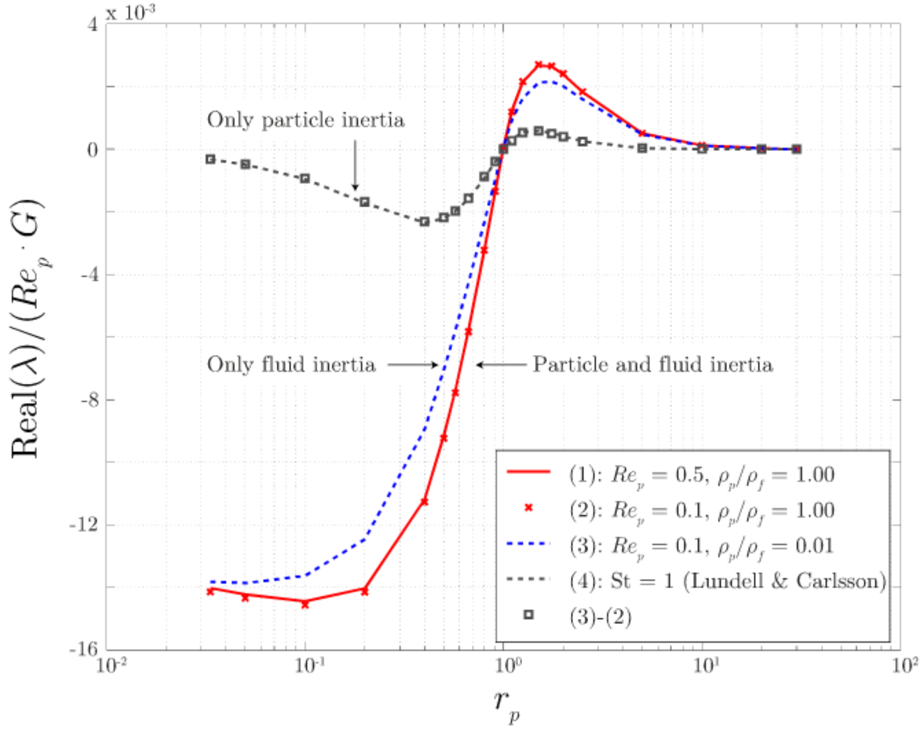
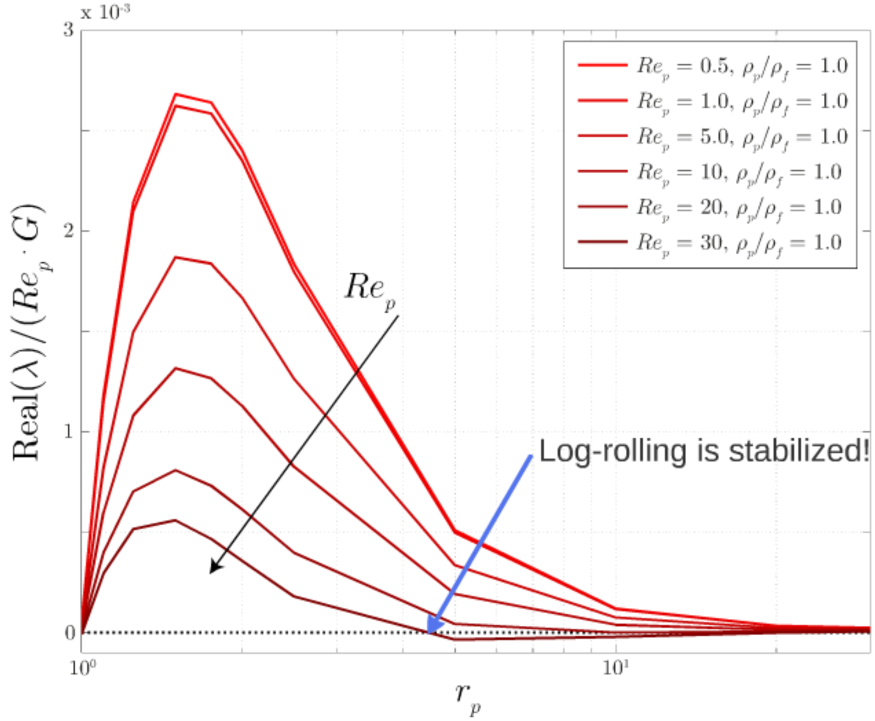
Effect of fluid inertia





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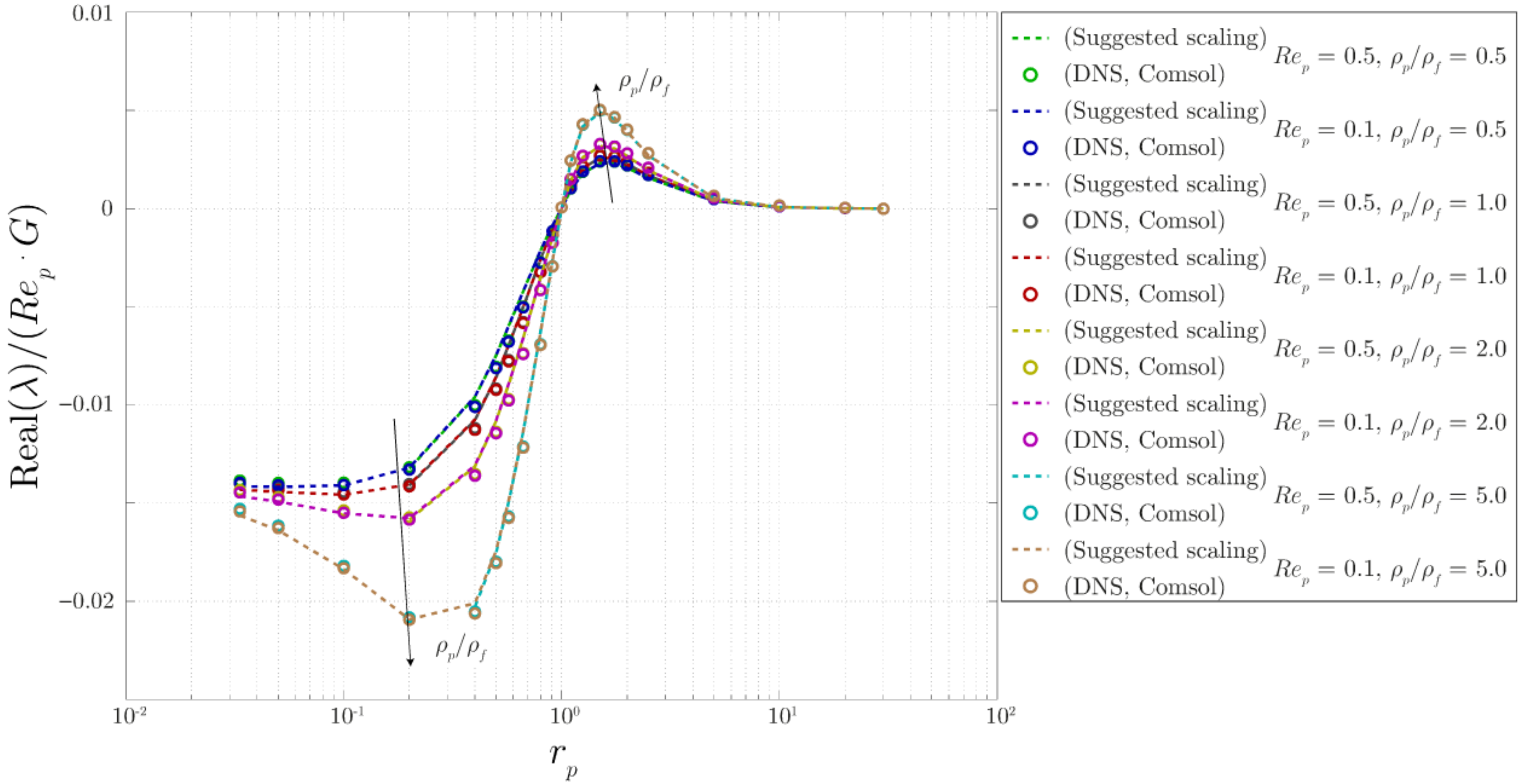
Scaling of Stokes number

$$St^* = \frac{\rho_p}{\rho_f} \cdot Re_p + K \cdot r_p^* \cdot Re_p$$

K = 9/4 (prolate)
K = 23/16 (oblate)



Meaning of scaling

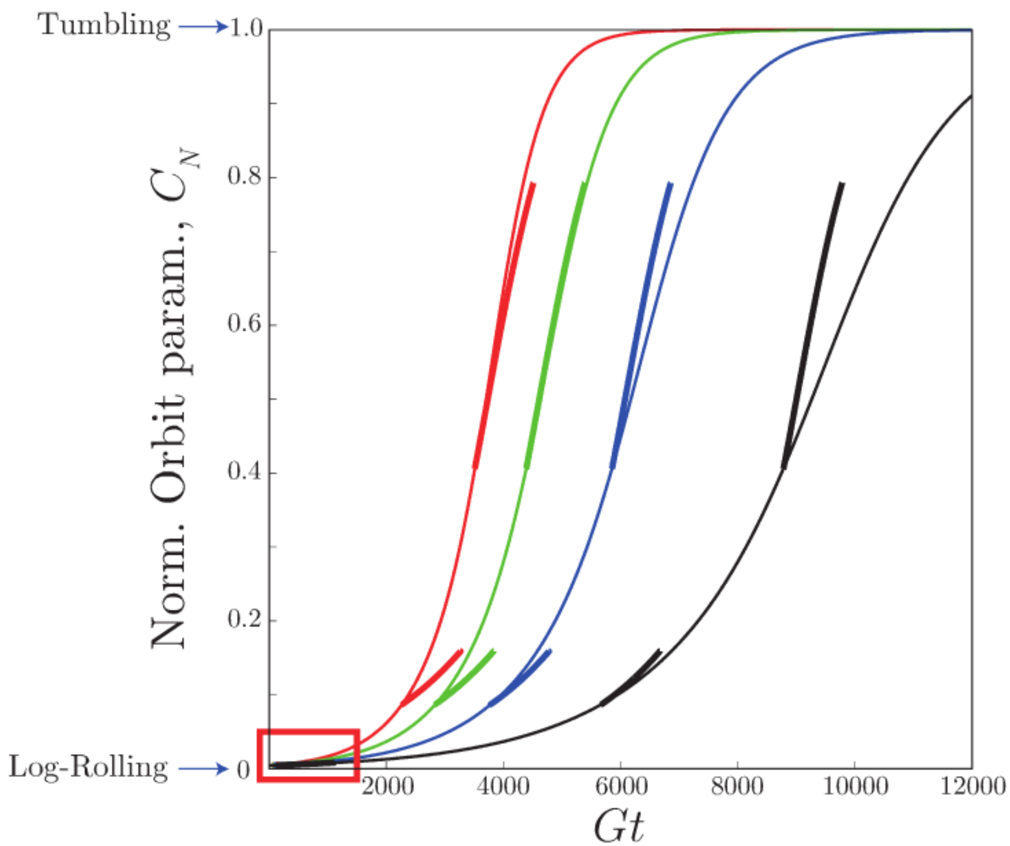
Added mass:

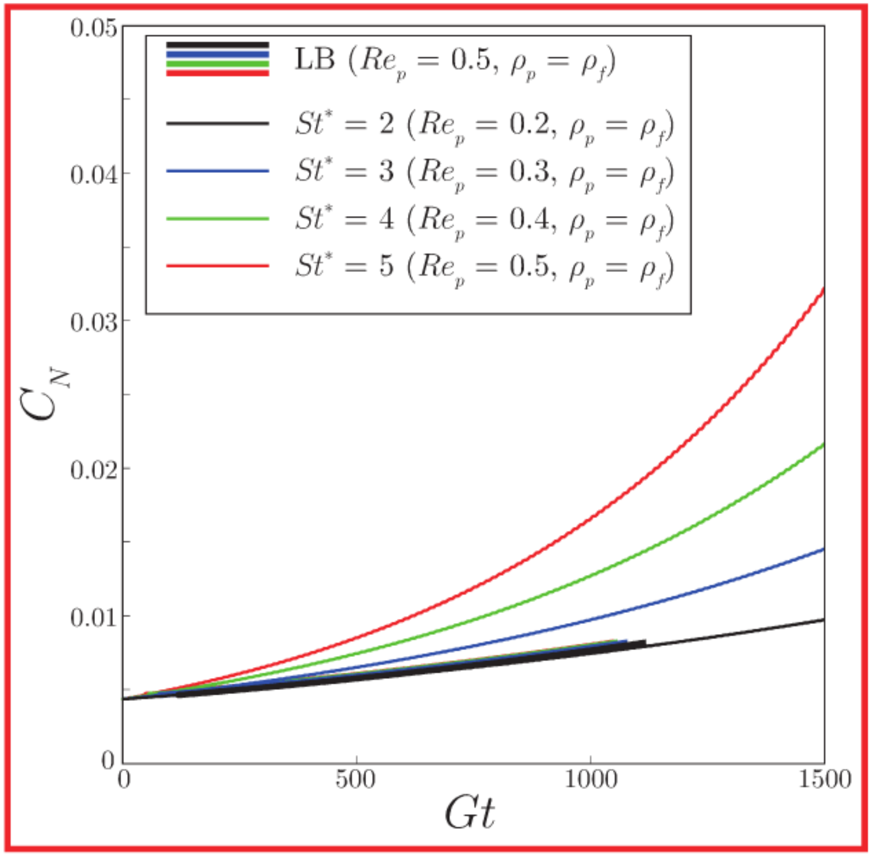
$$\underbrace{Re_p \cdot (\frac{\rho_p}{\rho_f} + K r_p^*)}_{St^*} \cdot [\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega})] = \boldsymbol{T}_{Jeffery}$$

Correction to Jeffery torques:

$$St \cdot [\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega})] = \boldsymbol{T}_{Jeffery} - \underbrace{K \cdot Re_p \cdot r_p^* \cdot [\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega})]}_{\boldsymbol{T}_{corr}}$$

Issues: Generality



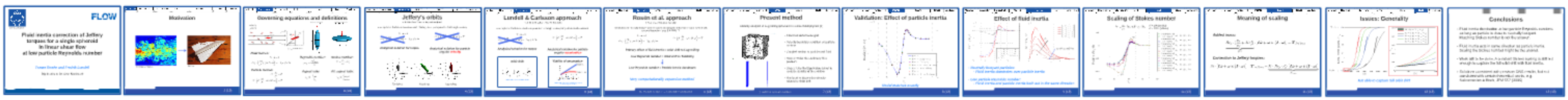


Not able to capture full orbit drift

Conclusions

- Fluid inertia domination at low particle Reynolds numbers as long as particle is close to neutrally buoyant. Matching Stokes number is not the answer.
- Fluid inertia acts in same direction as particle inertia. Scaling the Stokes number might be the answer.
- Work still to be done. A constant Stokes scaling is still not enough to capture the full orbit drift with fluid inertia.
- Solutions consistent with previous DNS results, but not consistent with certain theoretical works, e.g. Subramanian & Koch, JFM 557 (2006)

Thank you!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