8th MC-WG Meeting of COST Action FP1005
October 22-24, 2014, Université de Caen, France

Rheological properties of dilute suspensions of flexible and curved fibers

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Motivation

Characterization of suspension structure through rheology

Important for different industrial processes

- Paper making
- Composite processing
- Dry-forming of pulp mats

Process optimization and improving product quality

Backround

Fiber contribution to deviatoric stress

Rigid, straight fibers

- Batchelor’s theory – known fiber orientation
- Semi-dilute suspensions; hydrodynamic interactions

Flexible fibers, fibers with irregular equilibrium shapes

- No theoretical studies
- Few numerical studies – based on Batchelor’s theory

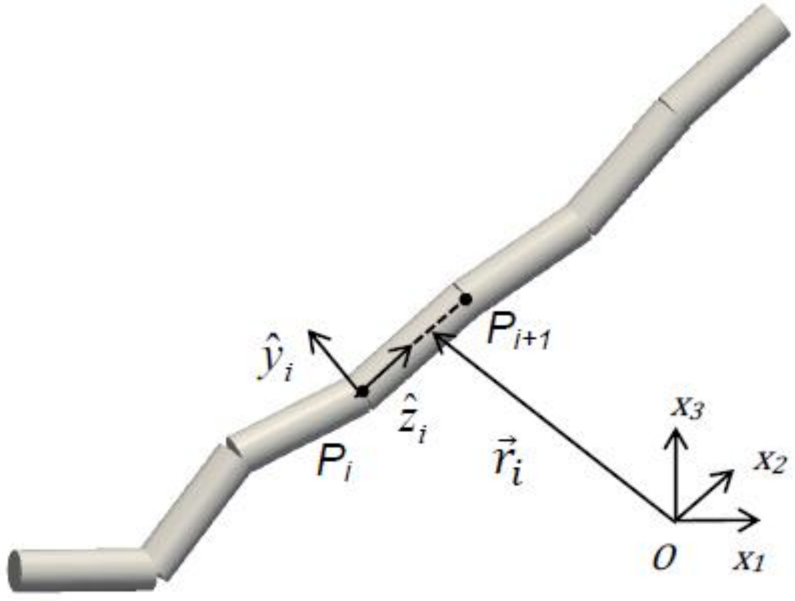
Present work

- Effects of fiber flexibility and fiber curvature on suspension rheological properties
  - Viscosity
  - Normal stress differences
- Dilute sheared suspensions in a Newtonian fluid
- Numerical study using particle-level fiber model

Fiber geometry

Chain of rigid cylindrical segments

Geometrical properties defined for **each segment**



- diameter
- length
- position vector
- unit direction vectors
- equilibrium shape

Fiber equations of motion

Direct application of Euler's laws for **each fiber segment**

- Linear momentum equation

$$m_i \ddot{\boldsymbol{r}}_i = \boldsymbol{F}_i^h + \boldsymbol{X}_{i+1} - \boldsymbol{X}_i$$

h: hydrodynamic

X: connectivity force

- Angular momentum equation

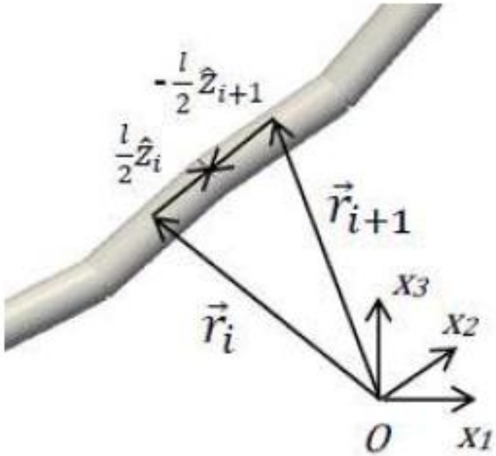
$$\frac{\partial(\boldsymbol{I}_i \cdot \boldsymbol{\omega}_i)}{\partial t} = \boldsymbol{T}_i^h + \boldsymbol{Y}_{i+1} - \boldsymbol{Y}_i + \frac{l_i}{2}\hat{z}_i \times \boldsymbol{X}_{i+1} + \left(\frac{-l_i}{2}\hat{z}_i\right) \times (-\boldsymbol{X}_i)$$

Y: sum of bending and twisting torques

Fiber equations of motion (cont.)

- Connectivity constraint - end points of adjacent segments coincide

$$r_i + \frac{l_i}{2}\hat{z}_i = r_{i+1} - \frac{l_{i+1}}{2}\hat{z}_{i+1}$$



- Connectivity equation - time derivative of connectivity constraint

$$\dot{r}_{i+1} - \dot{r}_i = \frac{l_i}{2}\omega_i \times \hat{z}_i + \frac{l_{i+1}}{2}\omega_{i+1} \times \hat{z}_{i+1}$$

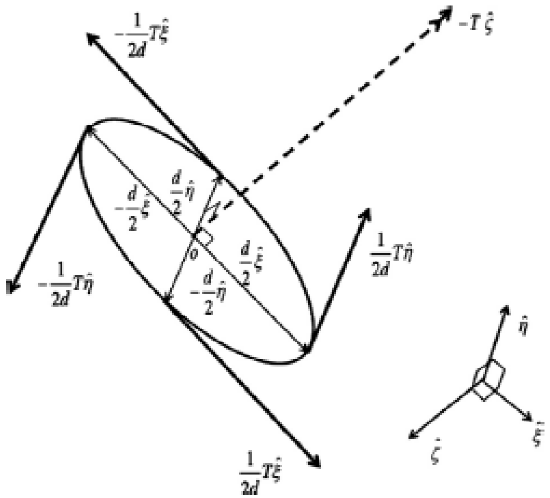
Solve velocities and angular velocities

Evolve segment positions and orientations in time

Direct method for deviatoric stress computation

- Dipole strength of single fiber
- Hydrodynamic forces and torques as localized tractions

$$\mathbf{s}' = \sum_{i=1}^{N} \left[\mathbf{F}_i^h \mathbf{r}_i - \frac{1}{2} \epsilon \cdot \mathbf{T}_i^h - \frac{1}{3} \left(\mathbf{r}_i \cdot \mathbf{F}_i^h \right) \right]$$

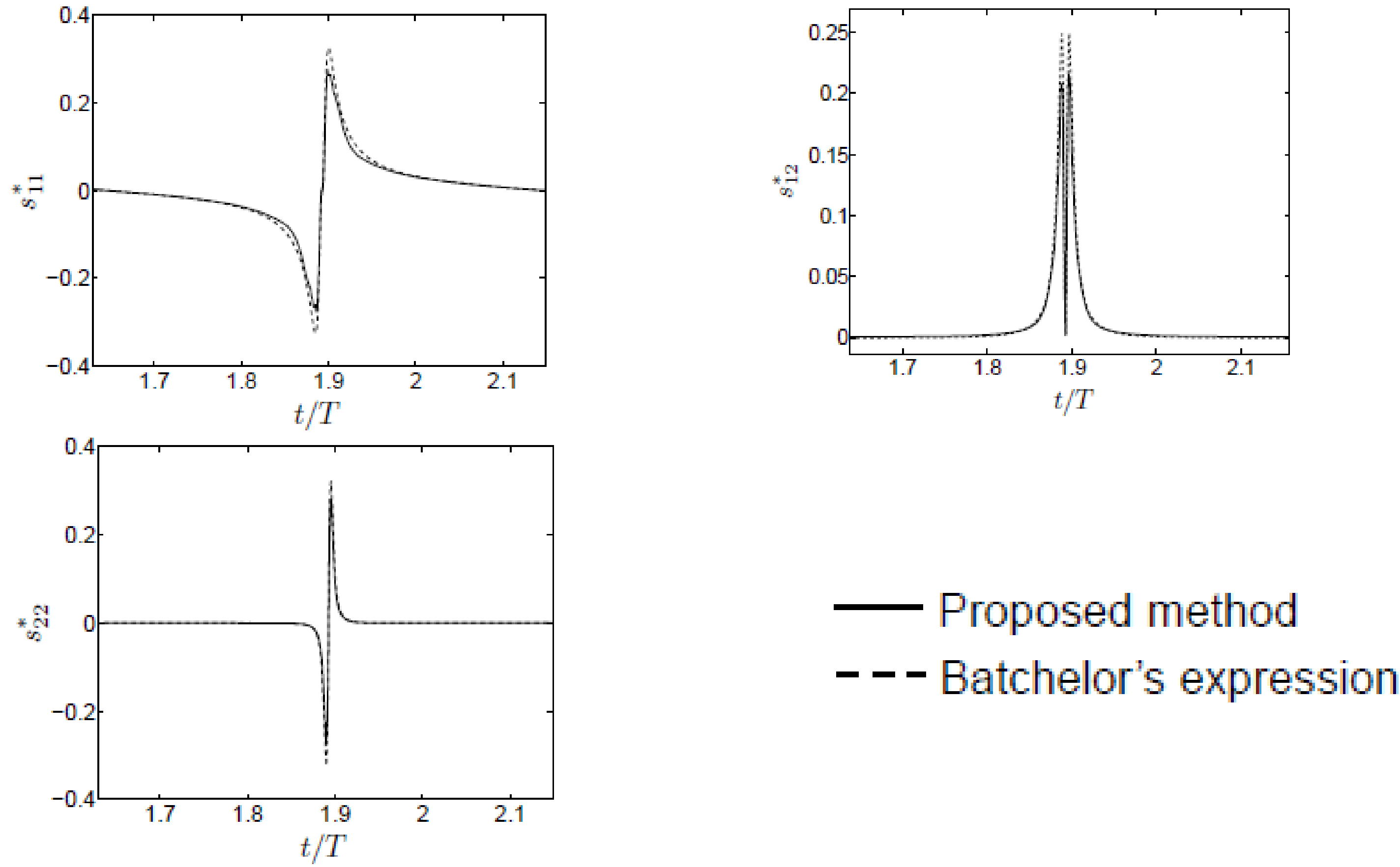


- Flexible fibers
- Irregularly shaped fibers

Hydrodynamic torque exerted by segment on fluid represented by four point forces

Dipole strength validation

Isolated, straight, rigid fiber – agreement with Batchelor's theory



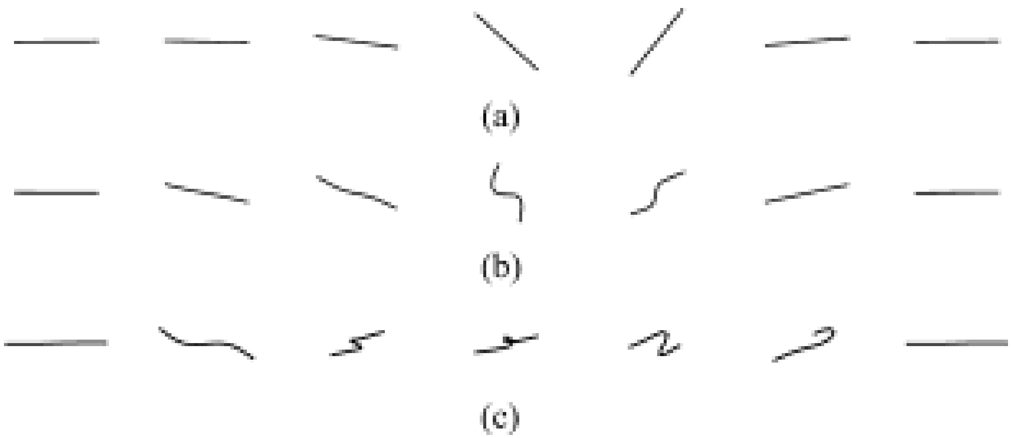
Numerical experiments

- Ergodicity assumption (ensemble average from multiplae fibers equal to time average from single fiber)
- Fibers initially in flow-gradient plane

- A parametric study of orbital drift for fibers initially oriented at an angle to the flow–gradient plane
- Time-scale within which in-plane fiber dynamics is stable

Fiber shape development

Initially straight fibers deform into S-shapes

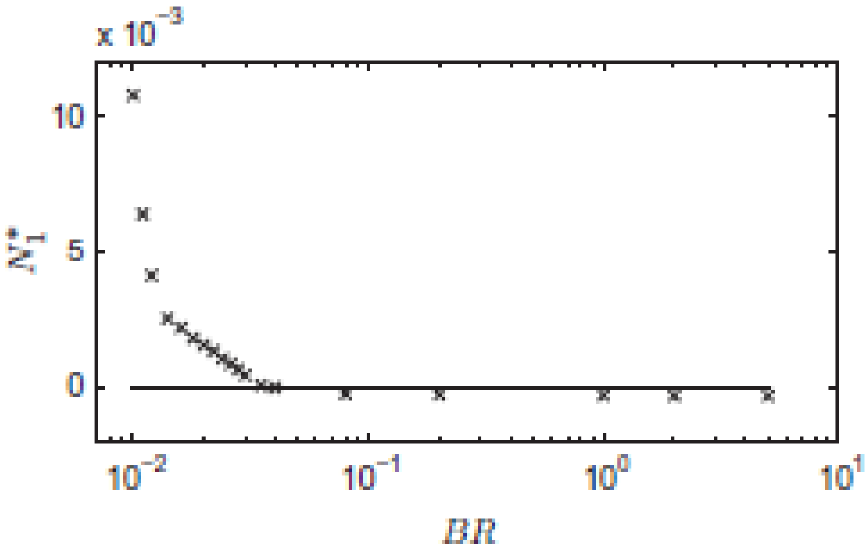


Time–series of images from simulation for different bending ratio:
a) *BR*=1, b) *BR*=0.02, c) *BR*=0.01.

Qualitative agreement with previous numerical study (Schmid *et al*. ,*J. Rheol*., 2000)

Effects of fiber flexibility

Relation between fiber bending ratio *BR* and first normal stress difference N_1^*



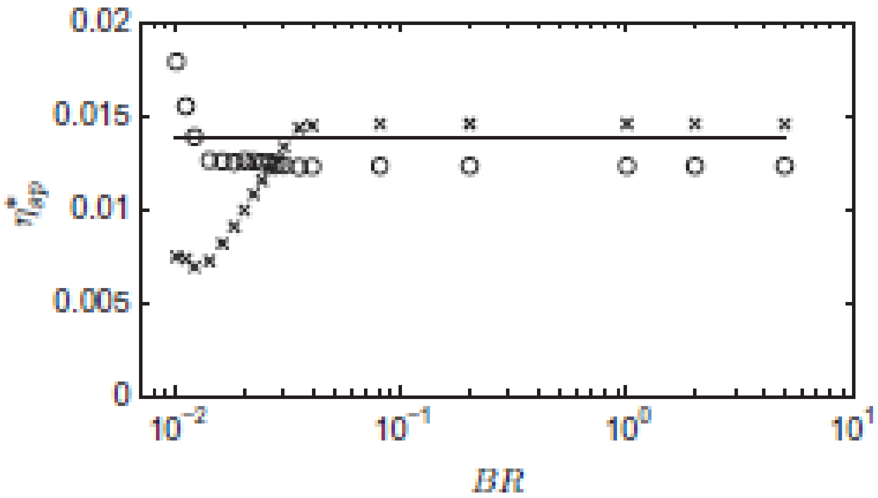
— Theoretical estimate for rigid fiber

× Simulation results

- Increases with fiber flexibility (lower *BR* values)

Effects of fiber flexibility

Relation between fiber bending ratio *BR* and specific viscosity (SV) η^*_{sp}

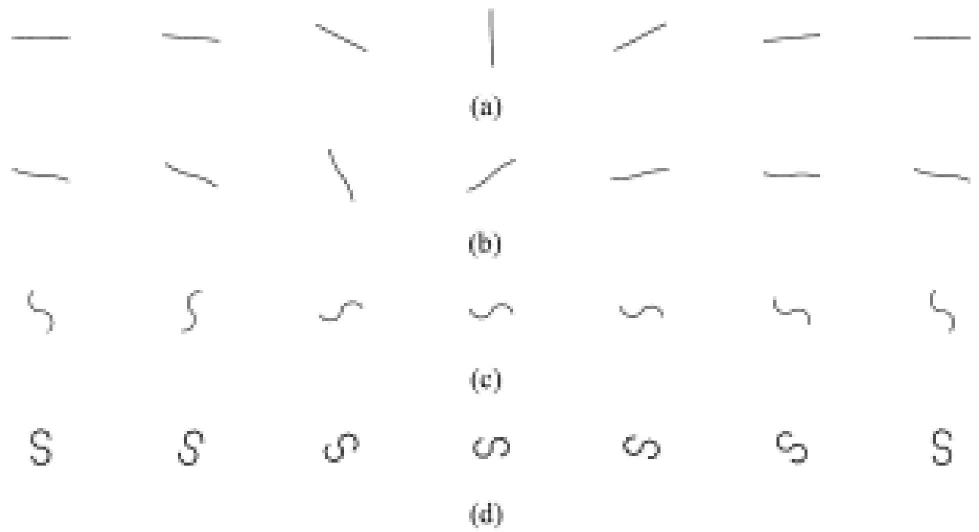


○ Batchelor's expression
× Proposed method

- Batchelor's theory: SV increases with fiber flexibility
  - Observed by other studies (Joung *et al.,J. Non-Newtonian Fluid Mech., 2001)*
- Proposed method: SV decreases with fiber flexibility
  - Reason: reduced viscous dissipation

Curved fibers

- Two co-planar arcsc that together create S-shape; *BR*=5

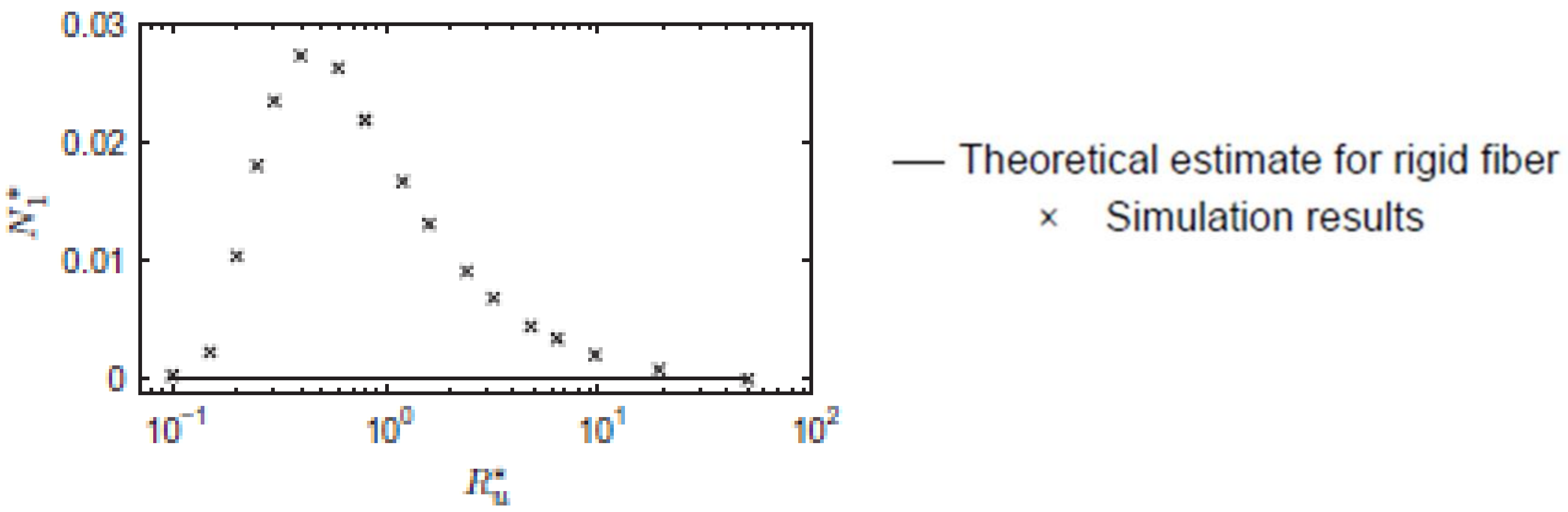


Time–series of images from simulation for different dimensionless radius of curvature:

a) *R*u*=9.6, b) *R*u*=1.2, c) *R*u*=0.2, d) *R*u*= 0.1.

Effects of fiber curvature

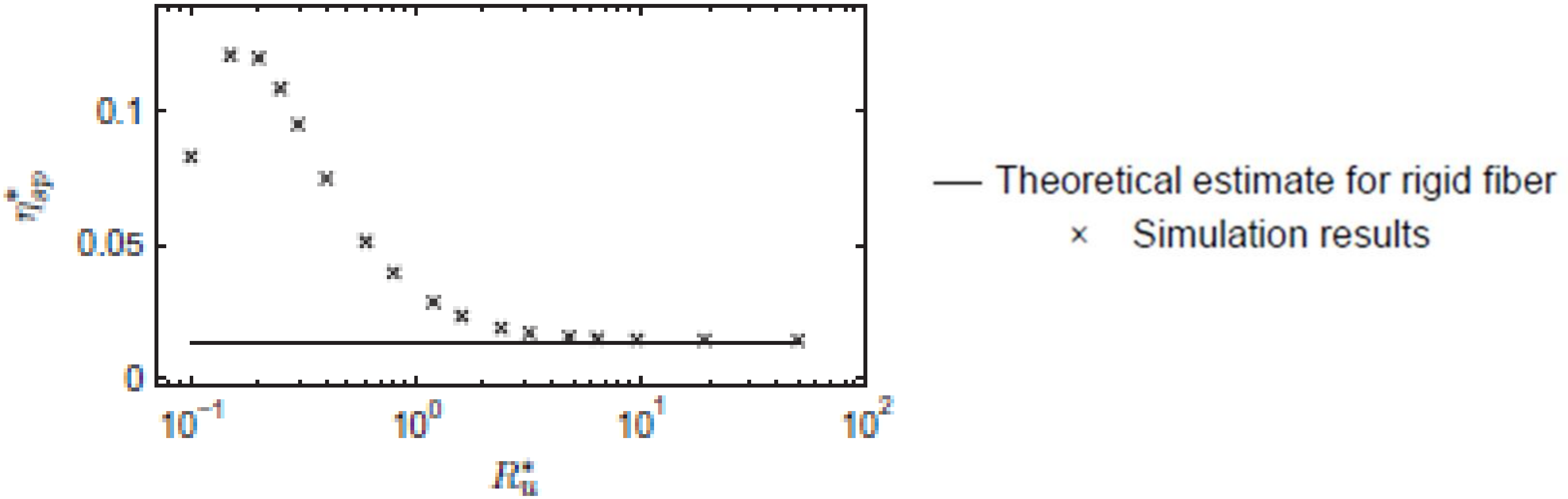
Relation between fiber dimenisonless fiber curvature (DFC) R_u^* and first normal stress difference N_1^*



- Increases when DFC decreases, reaches maximum, vanishes when symmetry is restored

Effects of fiber curvature

Relation between dimensionless fiber curvature R_u^* (DFC) and specific viscosity (SV) η_{sp}^*



- Increases as DFC decreases, reaches maximum, decreases as fiber shape becomes coiled

Conclusions

- Rheology of flexible fiber suspensions
  - Viscosity decreases with fiber flexibility
  - Important to properly account for fiber deformability
  - Experimentalíst – characterize fiber morphology
- Rheology of curved fiber suspensions
  - Viscosity significantly increases with fiber curvature, reaches maximum and decreases when fiber becomes coiled
  - First normal stress difference vanishes when symmetry is restored

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Thank you!

Project was financially supported by Bo Rydin Foundation and SCA Hygiene Products AB