Multiphase Flow Laboratory
University of Udine

COST Training Schoool (Action FP1005)

Martin Luther Universität, Halle-Wittenberg, May 27-29, 2015

Project supported by COST - European Cooperation in Science and Technology

Lecture 3: Examples of Application of the Point-Fiber Approach

Cristian Marchioli

Dept. Electrical, Management and Mechanical Engineering, University of Udine (Italy)

MARTIN-LUTHER-UNIVERSITÄT HALLE-WITTENBERG

UNIVERSITÀ DEGLI STUDI DI UDINE

Multiphase Flow Laboratory
University of Udine

Project supported by COST - European Cooperation in Science and Technology

Outline of Lecture 3

1.1 Fiber-laden Dilute Turbulent Channel Flow
- 1.1.1 Flow solver

1.2 Anisotropy Effects
- 1.2.1 Wall-dependence of scales
- 1.2.2 Local Stokes number

1.3 Phenomenology of Near-Wall Fiber Turbulence Interaction
- 1.3.1 Quasi-streamwise vortices
- 1.3.2 Long/short accumulation regions

1.4 Results
- 1.4.1 Fiber segregation
- 1.4.2 Fiber deposition
- 1.4.3 Fiber orientation
- 1.4.4 Slip velocity
- 1.4.5 Angular velocity

Recall: Dilute suspension of rigid fibers in wall turbulence

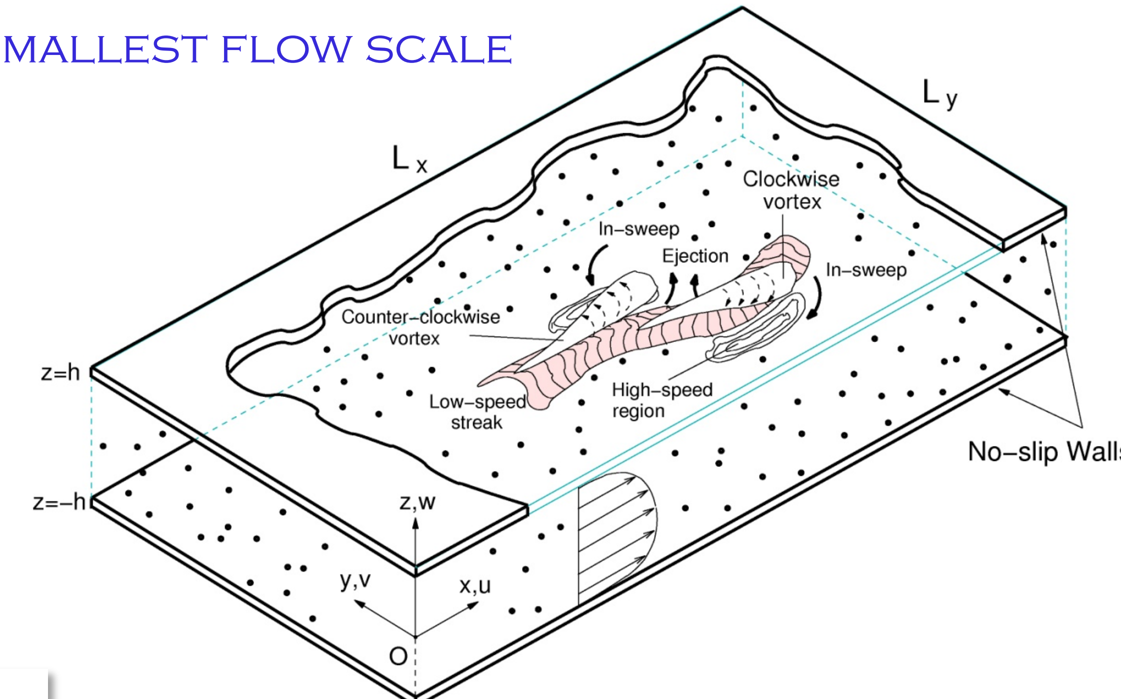
Multiphase Flow Laboratory
University of Udine

Project supported by COST - European Cooperation in Science and Technology

Fibers are modelled as non-deformable prolate ellipsoids evolving in 3D time-dependent fully-turbulent flow (e.g. Marchioli et al, 2010)

Assume:

- Fibers smaller than the smallest flow scale
  - > pointwise
  - > Stokes regime
- Dilute flow
  - > no turbulence modulation
  - > no collisions



First: work with non-dimensional equations!

$$a^+ = \frac{a}{\delta_\nu} = \frac{a u_\tau}{\nu} \quad , \quad \mathbf{x}^+ = \frac{\mathbf{x}}{\delta_\nu} = \frac{\mathbf{x} u_\tau}{\nu}$$

$$t^+ = \frac{t u_\tau^2}{\nu} \quad v_i^+ = \frac{v_i}{u_\tau} \quad , \quad \omega_{i'}^+ = \frac{\omega_{i'} \nu}{u_\tau^2}$$

with $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ the shear velocity

Recall: Complete system of non-dimensional equations

Project supported by COST - European Cooperation in Science and Technology

$$
\begin{cases}
\underline{Kinematics:} \\
\dfrac{d\mathbf{x}^{+}_{(G)}}{dt} = \mathbf{v}^{+} \\
\dfrac{de_0}{dt^{+}} = \dfrac{1}{2}(-e_1\omega^{+}_{x'} - e_2\omega^{+}_{y'} - e_3\omega^{+}_{z'}) \\
\dfrac{de_1}{dt^{+}} = \dfrac{1}{2}(e_0\omega^{+}_{x'} - e_3\omega^{+}_{y'} + e_2\omega^{+}_{z'}) \\
\dfrac{de_2}{dt^{+}} = \dfrac{1}{2}(e_3\omega^{+}_{x'} + e_0\omega^{+}_{y'} - e_1\omega^{+}_{z'}) \\
\dfrac{de_3}{dt^{+}} = \dfrac{1}{2}(-e_2\omega^{+}_{x'} + e_1\omega^{+}_{y'} + e_0\omega^{+}_{z'}) \\
\underline{Dynamics:} \\
\dfrac{d\mathbf{v}^{+}}{dt^{+}} = (\dfrac{S-1}{S})\mathbf{g}^{+} + \dfrac{3}{4\lambda S a^{+2}} \bar{\bar{\mathbf{K}}}_{(e_0,e_1,e_2,e_3)} \cdot (\mathbf{u}^{+} - \mathbf{v}^{+}) \\
\dfrac{d\omega^{+}_{x'}}{dt^{+}} = \omega^{+}_{y'}\omega^{+}_{z'}\left(1 - \dfrac{2}{1+\lambda^2}\right) + \dfrac{20\left[(1-\lambda^2)f' + (1+\lambda^2)(\xi' - \omega^{+}_{x'})\right]}{(\alpha_0 + \lambda^2\gamma_0)(1+\lambda^2)Sa^{+2}} \\
\dfrac{d\omega^{+}_{y'}}{dt^{+}} = \omega^{+}_{x'}\omega^{+}_{z'}\left(\dfrac{2}{1+\lambda^2} - 1\right) + \dfrac{20\left[(\lambda^2-1)g' + (\lambda^2+1)(\eta' - \omega^{+}_{y'})\right]}{(\alpha_0 + \lambda^2\gamma_0)(1+\lambda^2)Sa^{+2}} \\
\dfrac{d\omega^{+}_{z'}}{dt^{+}} = \dfrac{20}{(2\alpha_0)Sa^{+2}}(\chi' - \omega^{+}_{z'})
\end{cases}
$$

RK4

Mixed Explicit -implicit

A note on time integration of equations

Project supported by COST - European Cooperation in Science and Technology

- The system of ODEs is stiff (requires very small time steps)
- Using an explicit method for the dynamics (e.g. RK4) would be computationally very expensive
- Alternative solution: use a mixed explicit-implicit scheme:

$$
\begin{aligned}
\frac{dv_x^+}{dt^+} &= \left(\frac{S-1}{S}\right)g_x^+ + \frac{3}{4\lambda S {a^+}^2}\left[k_{11}(u_x^+ - v_x^+) + \right. \\
&+ \left. k_{12}(u_y^+ - v_y^+) + k_{13}(u_z^+ - v_z^+)\right]
\end{aligned}
$$

A note on time integration of equations

Project supported by COST - European Cooperation in Science and Technology

The system of ODEs is stiff (requires very small time steps)

Using an explicit method for the dynamics (e.g. RK4) would be computationally very expensive

Alternative solution: use a mixed explicit-implicit scheme:

$$\frac{dv_x^+}{dt^+} = \left(\frac{S-1}{S}\right)g_x^+ + \frac{3}{4\lambda S a^{+2}}\left[k_{11}(u_x^+ - v_x^+) + k_{12}(u_y^+ - v_y^+) + k_{13}(u_z^+ - v_z^+)\right]$$

$$\frac{v_x^{n+1} - v_x^n}{\Delta t} = \boxed{\left(\frac{S-1}{S}\right)}g_x^+ + \boxed{\frac{3}{4\lambda S a^{+2}}}\left[k_{11}(u_x^n - v_x^{n+1}) + k_{12}(u_y^n - v_y^n) + k_{13}(u_z^n - v_z^n)\right].$$

$$C_{grav,x} = \left(\frac{S-1}{S}\right)g_x^+$$

$$C_{drag} = \frac{3}{4\lambda S a^{+2}}$$

A note on time integration of equations

Project supported by COST - European Cooperation in Science and Technology

The system of ODEs is stiff (requires very small time steps)

Using an explicit method for the dynamics (e.g. RK4) would be computationally very expensive

Alternative solution: use a mixed explicit-implicit scheme:

$$\frac{dv_x^+}{dt^+} = (\frac{S-1}{S})g_x^+ + \frac{3}{4\lambda S a^{+2}}\left[k_{11}(u_x^+ - v_x^+) + k_{12}(u_y^+ - v_y^+) + k_{13}(u_z^+ - v_z^+)\right]$$

$$\frac{v_x^{n+1} - v_x^n}{\Delta t} = (\frac{S-1}{S})g_x^+ + \frac{3}{4\lambda S a^{+2}}\left[k_{11}(u_x^n - v_x^{n+1}) + k_{12}(u_y^n - v_y^n) + k_{13}(u_z^n - v_z^n)\right]$$

$$C_{grav,x} = \left(\frac{S-1}{S}\right) g_x^+$$

$$C_{drag} = \frac{3}{4\lambda S a^{+2}}$$

$$v_x^{n+1} = \frac{v_x^n + \Delta t\left\{C_{grav,x} + C_{drag}\left[k_{11}u_x^n + k_{12}(u_y^n - v_y^n) + k_{13}(u_z^n - v_z^n)\right]\right\}}{1 + \Delta t \cdot C_{drag} \cdot k_{11}}$$

Multiphase Flow Laboratory
University of Udine

Recall: Dilute suspension of Rigid fibers in wall turbulence

Project supported by COST - European Cooperation in Science and Technology

Simulation parameters:

- Fluid

Re_τ	Fluid	$\rho_F\ [kg/m^3]$	$\nu\ [m^2/s]$	$h\ [cm]$	$u_\tau\ [m/s]$	$\overline{u_x}\ [m/s]$
150	Air	1.3	$1.57 \cdot 10^{-5}$	2.0	0.11775	1.77
150	Water	1000	$1.00 \cdot 10^{-6}$	0.5	0.03000	0.45

- Particles

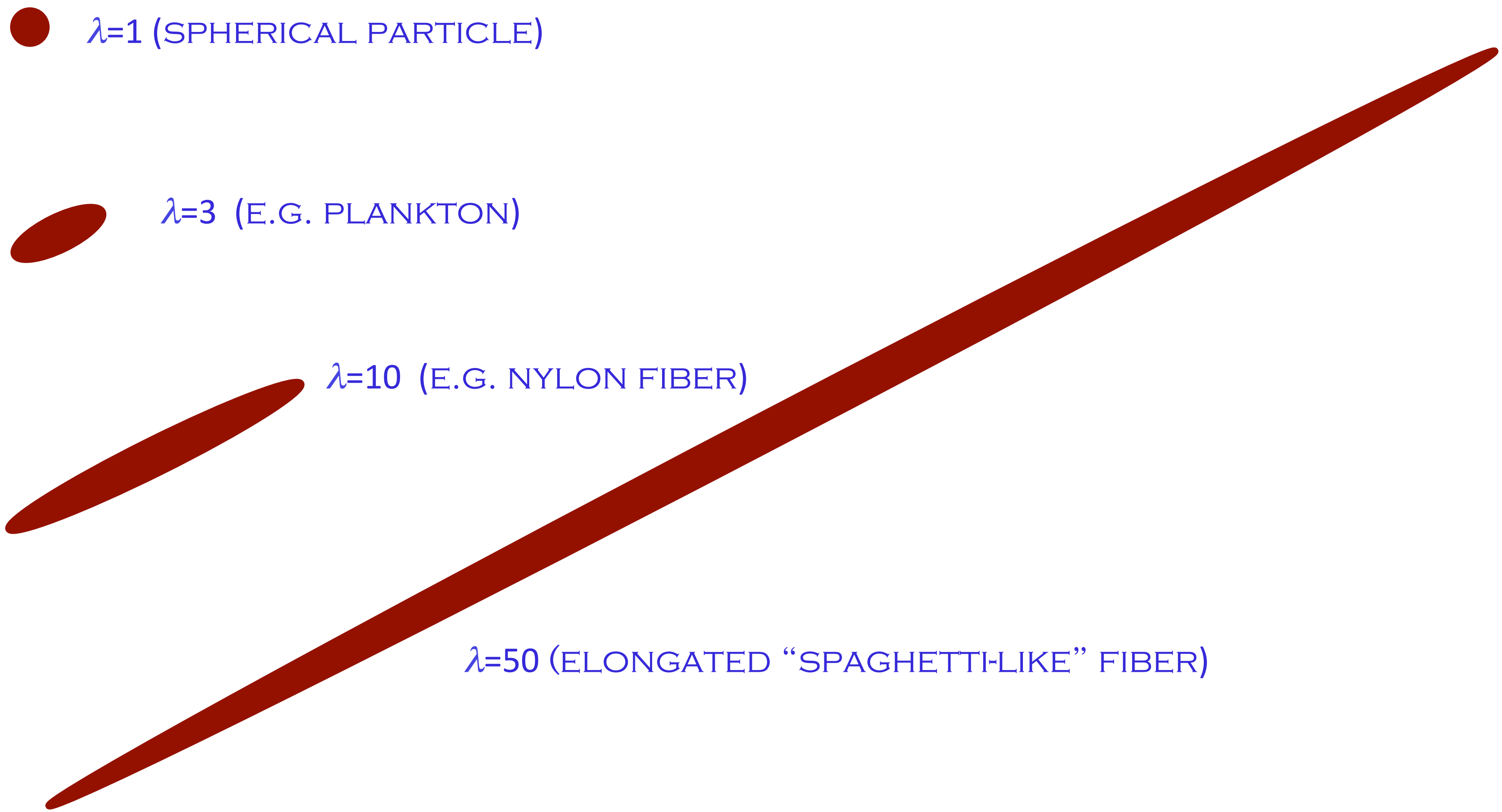
Set	τ_p^+	λ	S	$2b^+$	(μm)	(kg/m^3)
F1-1	1	1.001	34.72	0.72	96.07	45.14
F1-3	1	3	18.57	2.16	287.93	24.14
F1-10	1	10	11.54	7.20	960.09	15.01
F1-50	1	50	7.54	36.00	4800.01	9.80
F5-1	5	1.001	173.60	0.72	96.07	225.68
F5-3	5	3	92.90	2.16	287.93	120.77
F30-3	30	3	557.10	2.16	287.93	724.23
F30-10	30	10	346.30	7.20	960.09	450.19
F30-50	30	50	226.15	36.00	4800.01	294.00
F100-1	100	1.001	3472.33	0.72	96.07	4514.03
F100-3	100	3	1857.00	2.16	287.93	2414.10
F100-10	100	10	1154.33	7.20	960.09	1500.63
F100-50	100	50	753.83	36.00	4800.01	979.98

$$m_P(\lambda = 50) > m_P(\lambda = 10) > m_P(\lambda = 3) > m_P(\lambda = 1.001)$$

Multiphase Flow Laboratory
University of Udine

"Cartoon" of fiber elongation

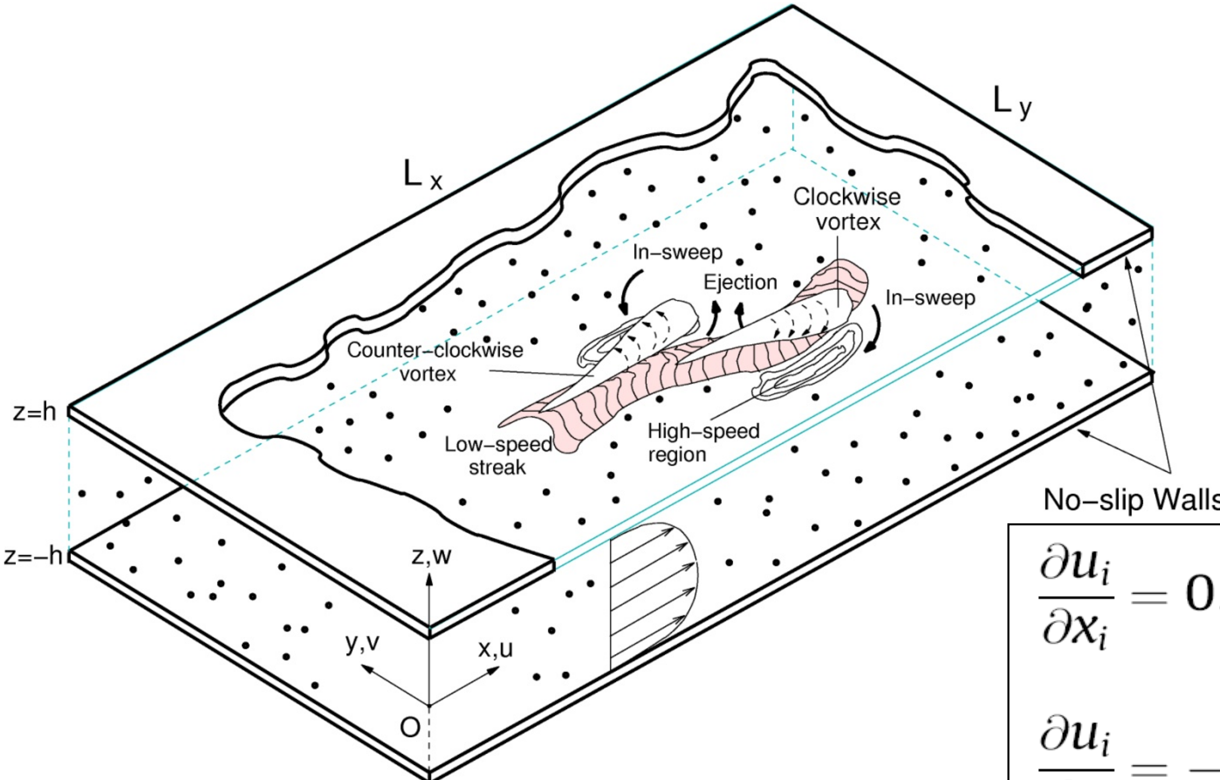
Project supported by COST - European Cooperation in Science and Technology



Multiphase Flow Laboratory
University of Udine

Modelling Approach: Flow Solver

Project supported by COST - European Cooperation in Science and Technology



Point-particle DNS – Flow domain and flow field calculation

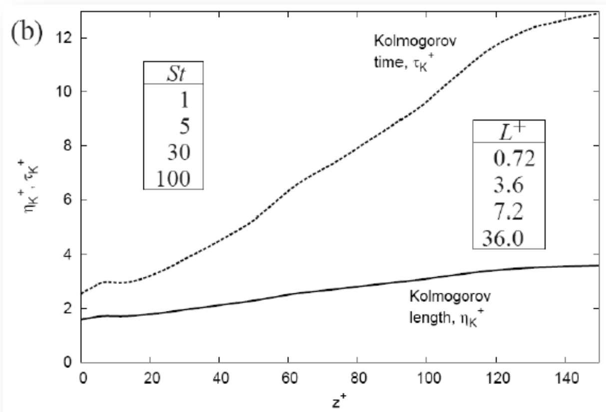
$$\frac{\partial u_i}{\partial x_i} = 0,$$

$$\frac{\partial u_i}{\partial t} = -u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{Re_\tau} \frac{\partial^2 u_i}{\partial x_j^2} - \frac{\partial p}{\partial x_i} + \delta_{1,i}$$

- Time-dependent 3D turbulent gas flow
- Channel size: $L_X \times L_Y \times L_Z = 4\pi h \times 2\pi h \times 2h$
- Pseudo-spectral DNS: Fourier modes (1D FFT) in the homogeneous directions (x and y), Chebychev coefficients in the wall-normal direction (z)
- Time integration: Adams-Bashforth (convective terms), Crank-Nicolson (viscous terms)

Observation 1: Anisotropy effects on fiber dynamics

Project supported by COST - European Cooperation in Science and Technology



The dissipative scales τ_K, $\tau\eta$ change with wall distance

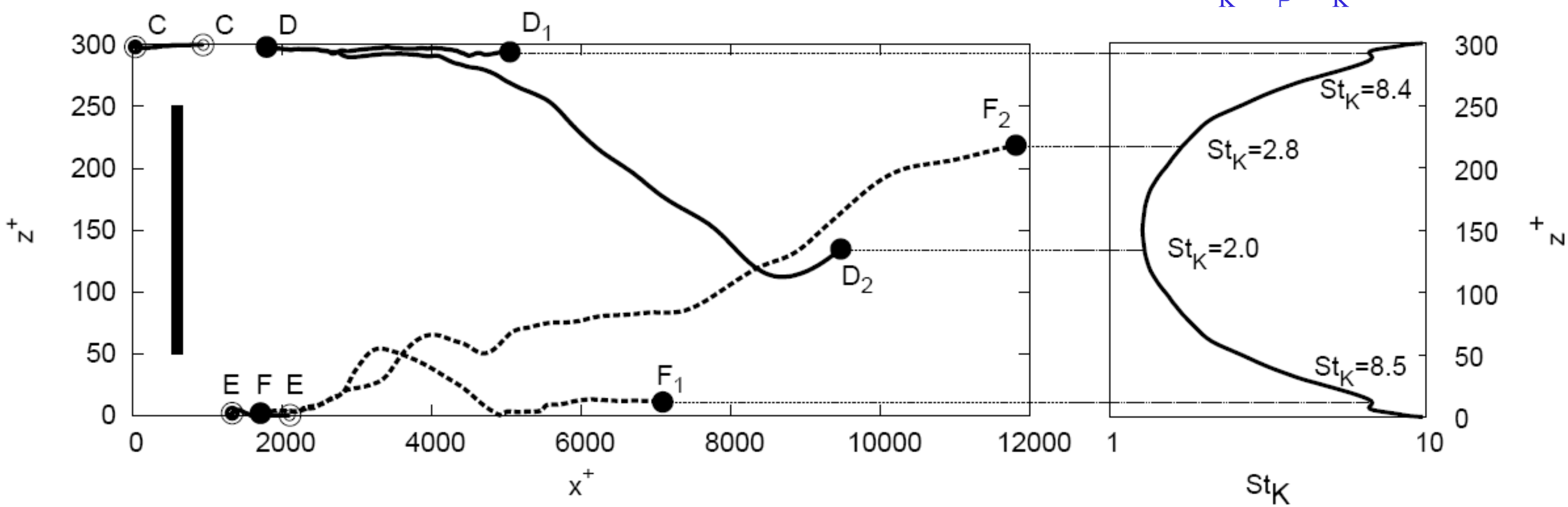
Dissipative Kolmogorov time/length scale

$\tau_K^+=\tau_K / \tau_F$

$\eta_K^+=\eta_K / \eta_F$

Renormalized Stokes number

$St_K=\tau_P/\tau_K$



Hence particle inertia changes with wall distance!!

Multiphase Flow Laboratory
University of Udine

Observation 2: Turbulent particle deposition onto a wall

Project supported by COST - European Cooperation in Science and Technology

Qualitative explanation of instantaneous wall transfer mechanism

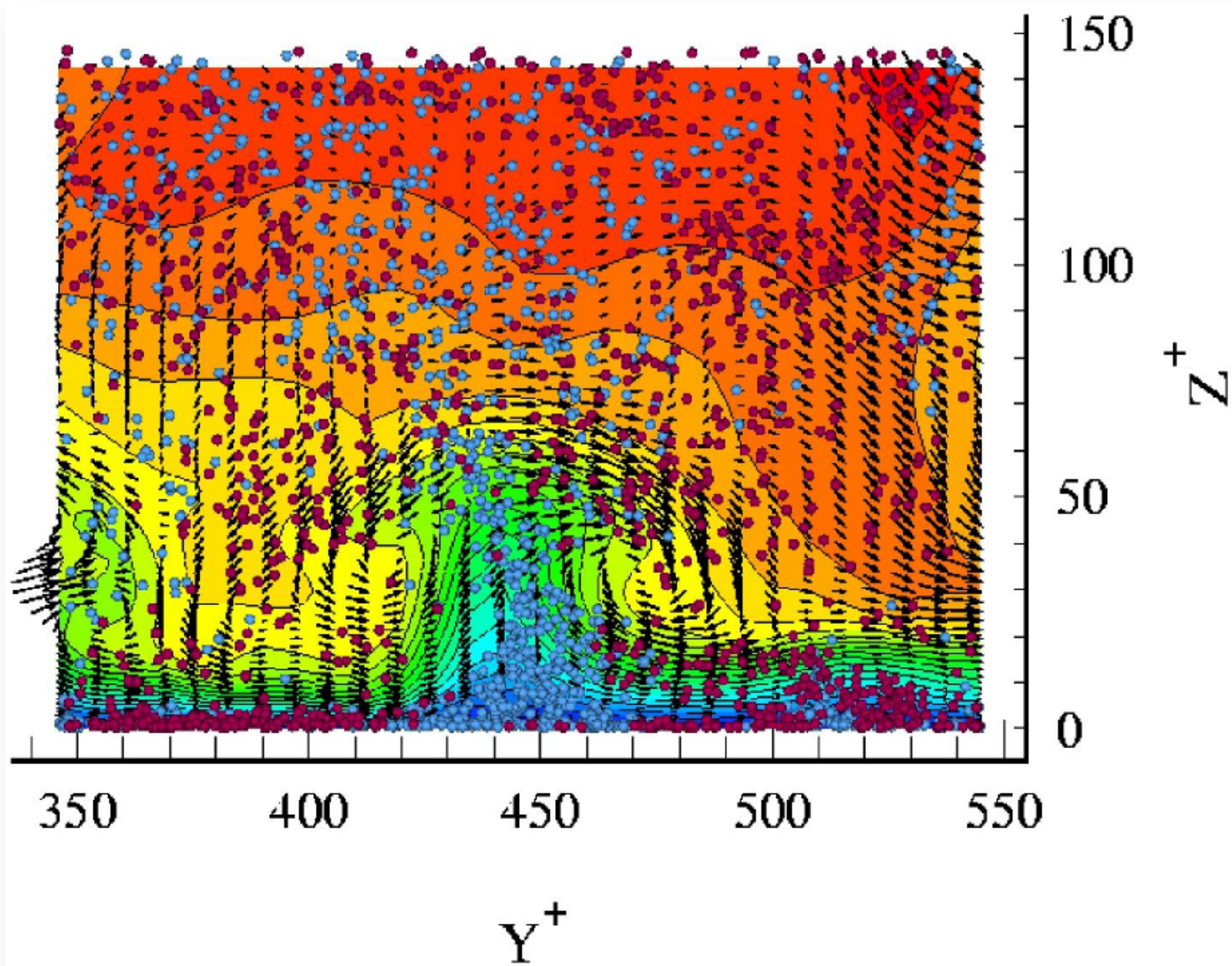
Deposition and entrainment are controlled by turbulence structures localized in time and space

Red: high stream-wise velocity

Blue: low stream-wise velocity

Purple particles: to the wall

Light blue particles: off the wall

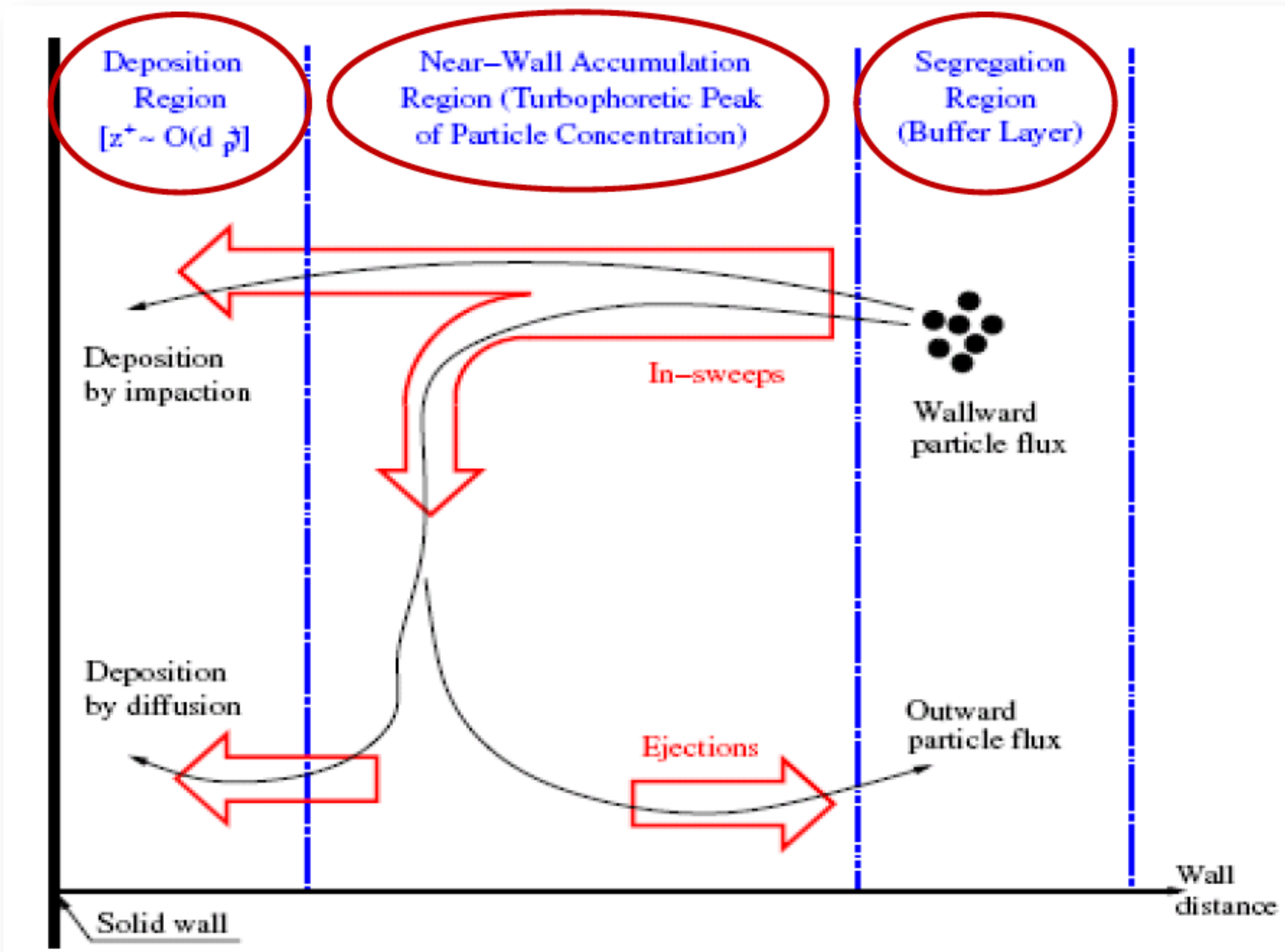


Multiphase Flow Laboratory
University of Udine

Turbulent particle deposition onto a wall: physics from DNS

Project supported by COST - European Cooperation in Science and Technology

We can picture particle wall deposition as a multi-step process. Particles are **first** accumulated in the buffer region and **then** deposited



From microscale phenomena to macroscale effects

Project supported by COST - European Cooperation in Science and Technology

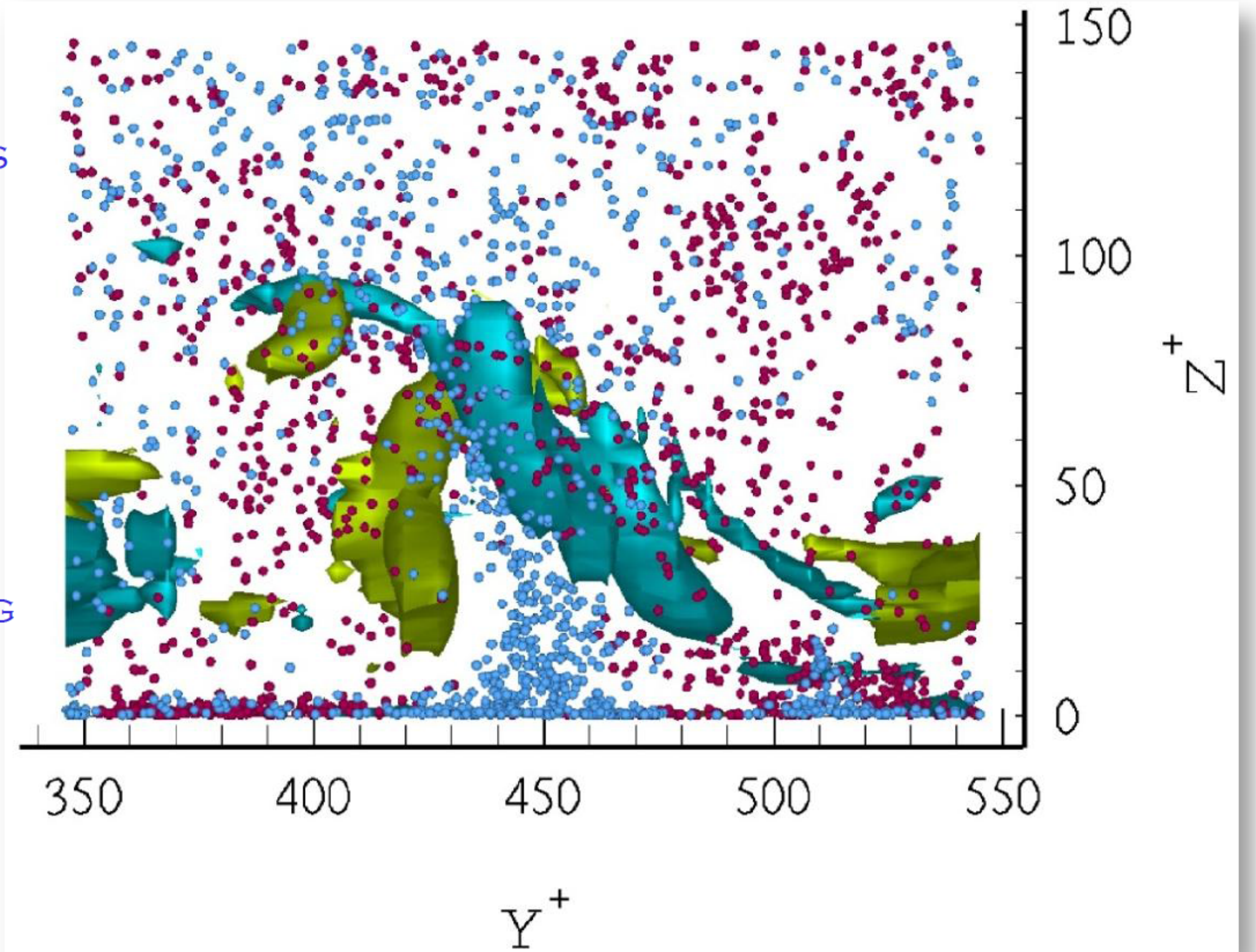
Physical mechanisms leading to deposition ore resuspension are determined by particle interaction with near-wall structures

Green:
Counter-CLKWS rotating quasi-STRMWS vortex (ω_x)

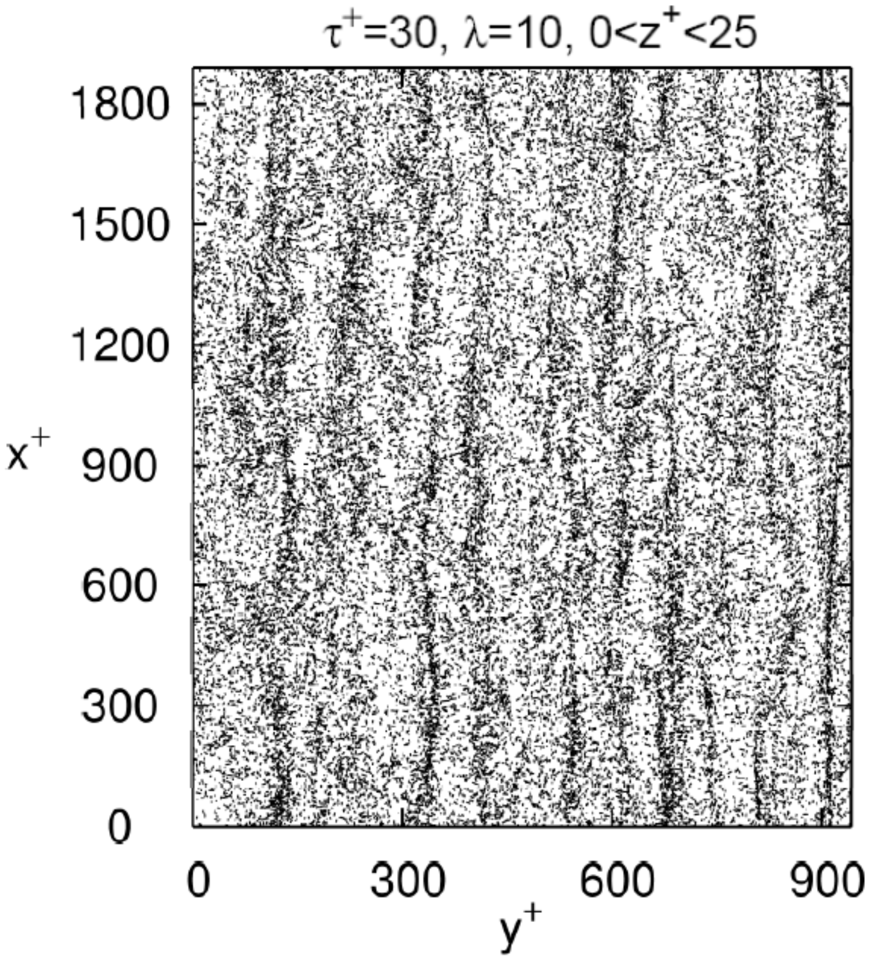
Blue:
CLKWS rotating quasi-STRMWS vortex (ω_x)

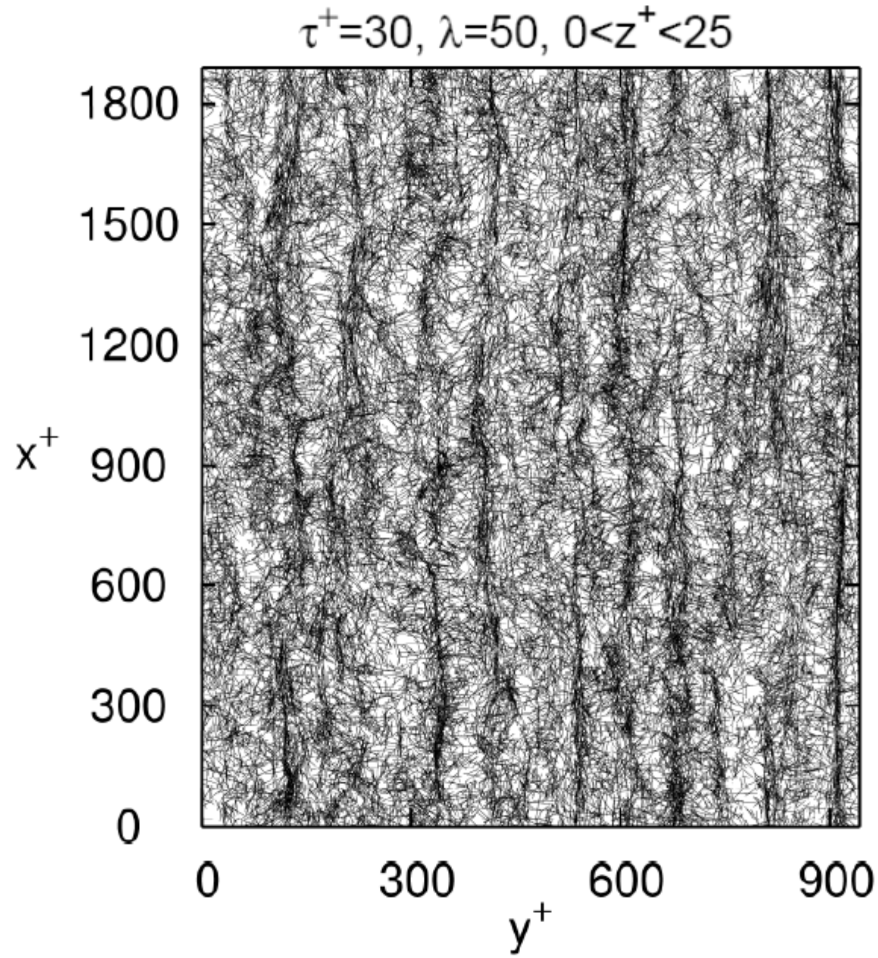
Red:
Particles approaching the wall

Pale blue:
Particles leaving the wall



Recall: Fibers in the near-wall region



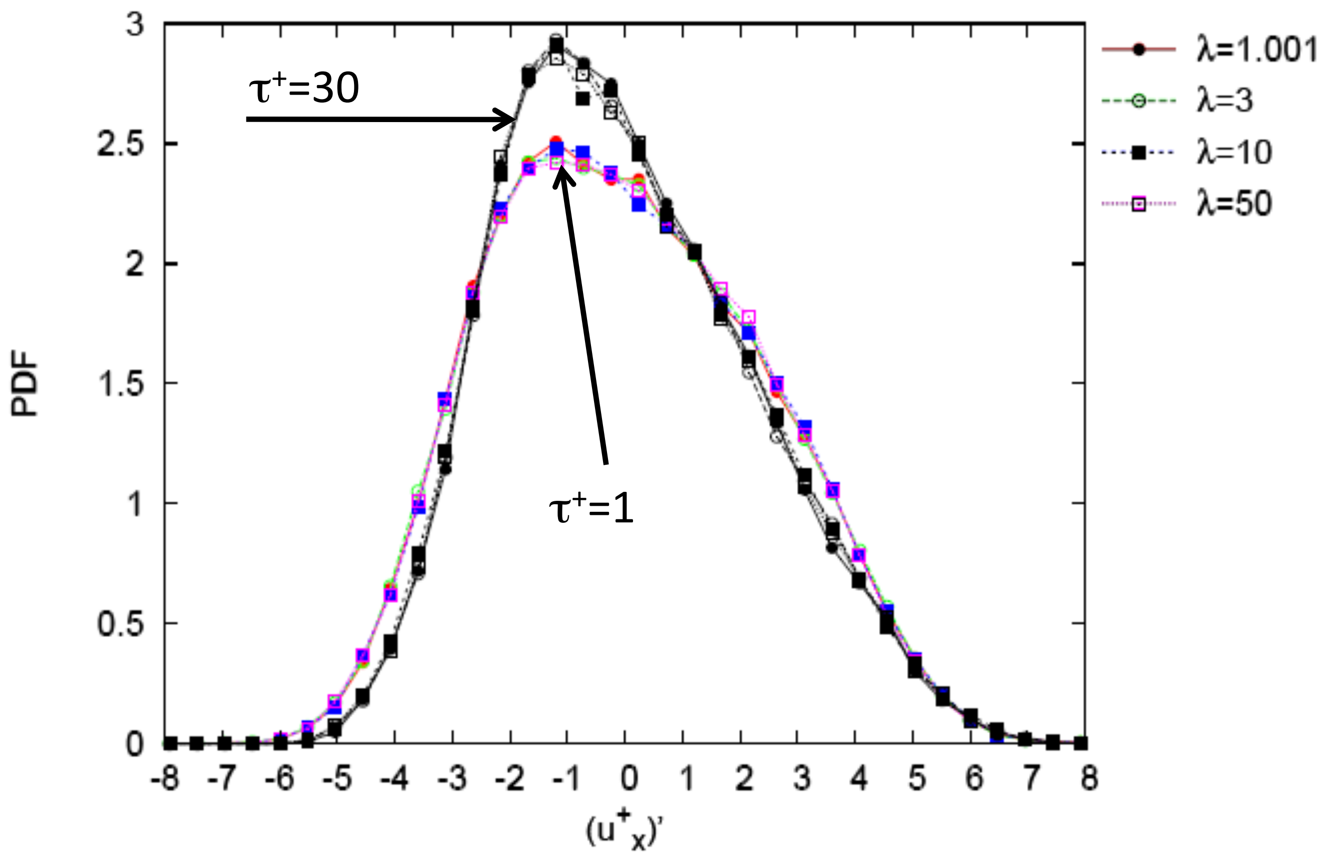


Fibers segregate into streaks which superpose to the fluid low-speed velocity streaks

Multiphase Flow Laboratory
University of Udine

Recall: Fibers in the near-wall region

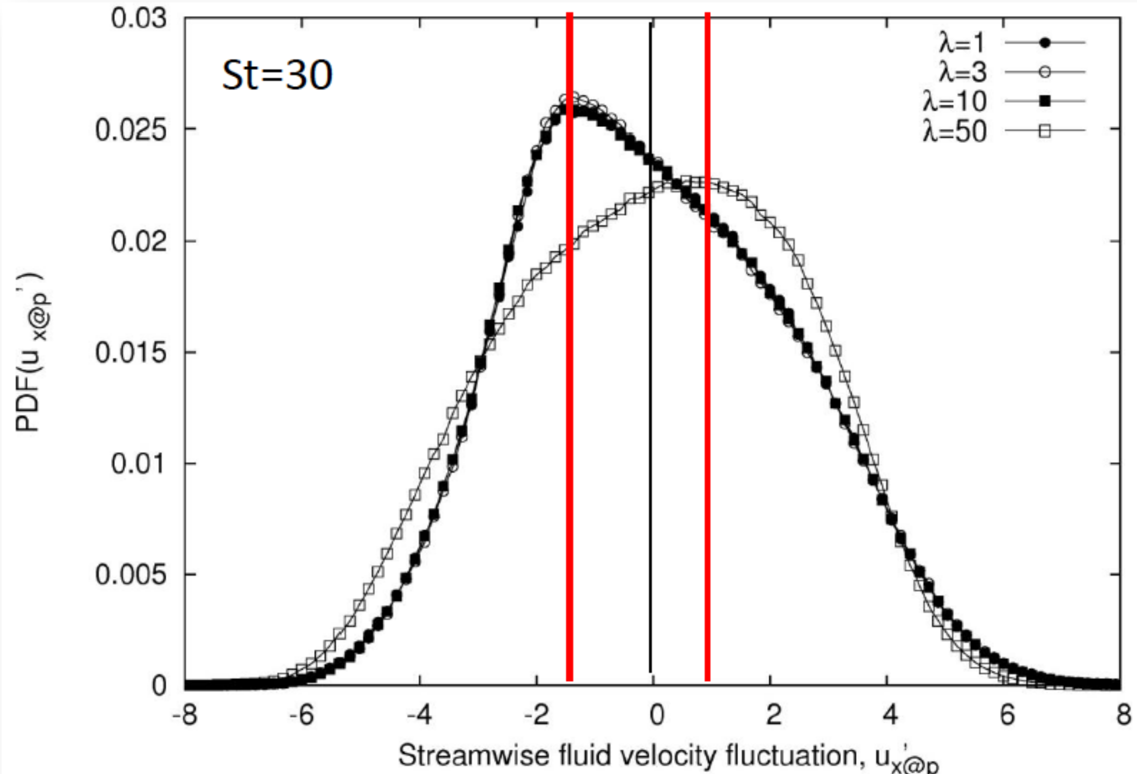
Project supported by COST - European Cooperation in Science and Technology



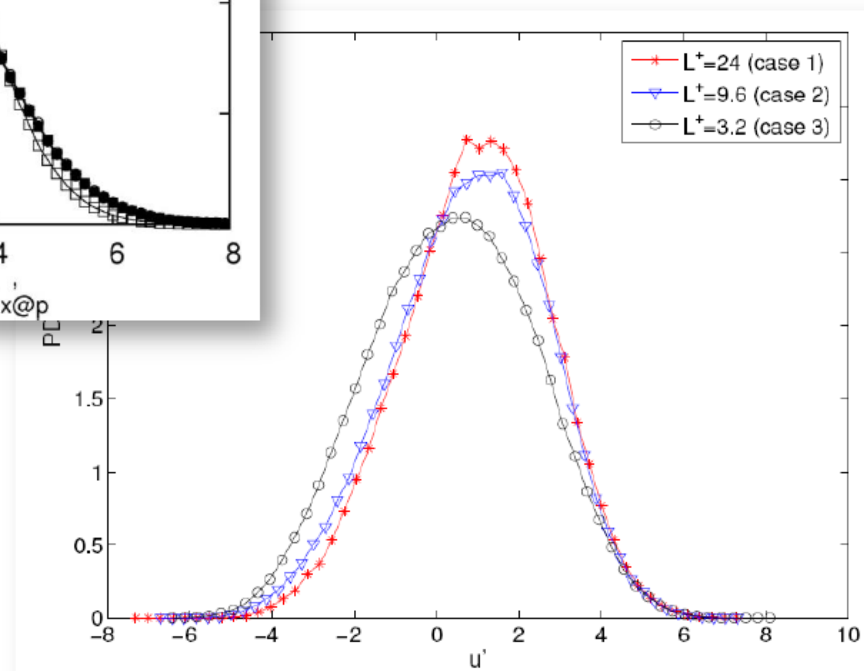
Fibers segregate into streaks which superpose to the fluid's low-speed velocity streaks

But the degree of segregation does not depend on λ!

Digression: How "strong" is the particle-wall interaction model?



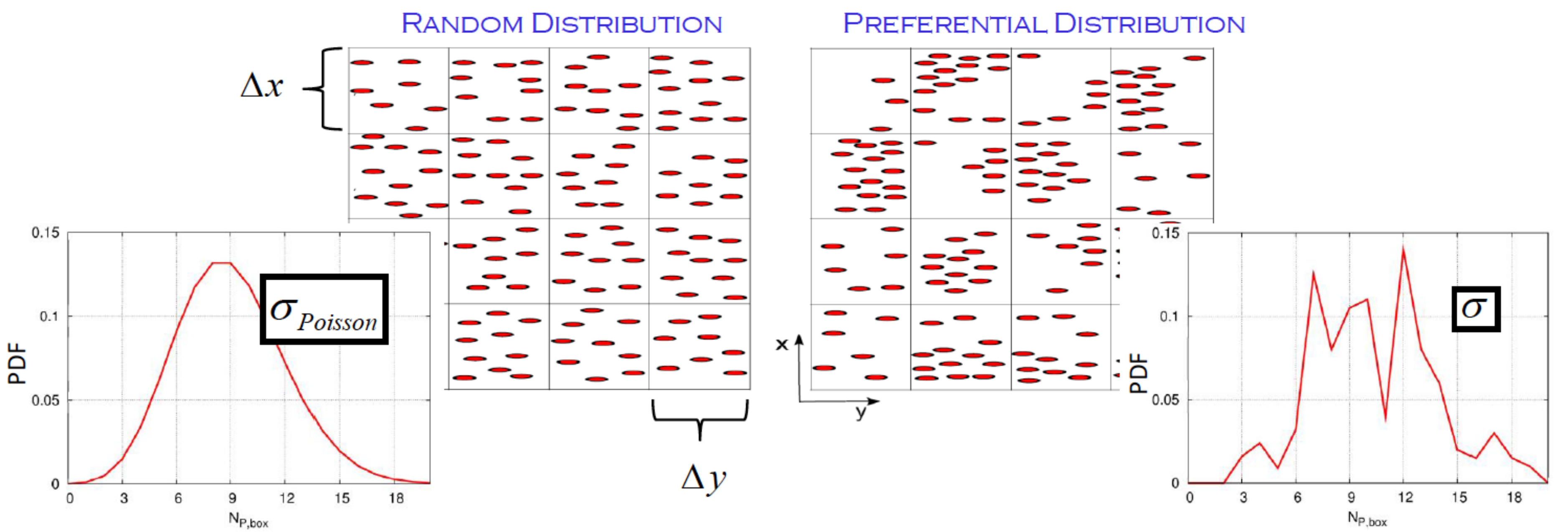
Let us change the "rule" for detecting particle-wall impact...



The PDF for the longer fibers indicates segregation into HSS! This has been observed also in finite-size simulations (Do-Quang et al., Phys Fluids, 2013)

Results: Quantification of local fiber segregation

- Turbulence segregates fibers
- Higher-inertia fibers appear more segregated
- How to quantify segregation? As deviation from a random distribution *



σ: standard deviation

m: mean number of particles per cell

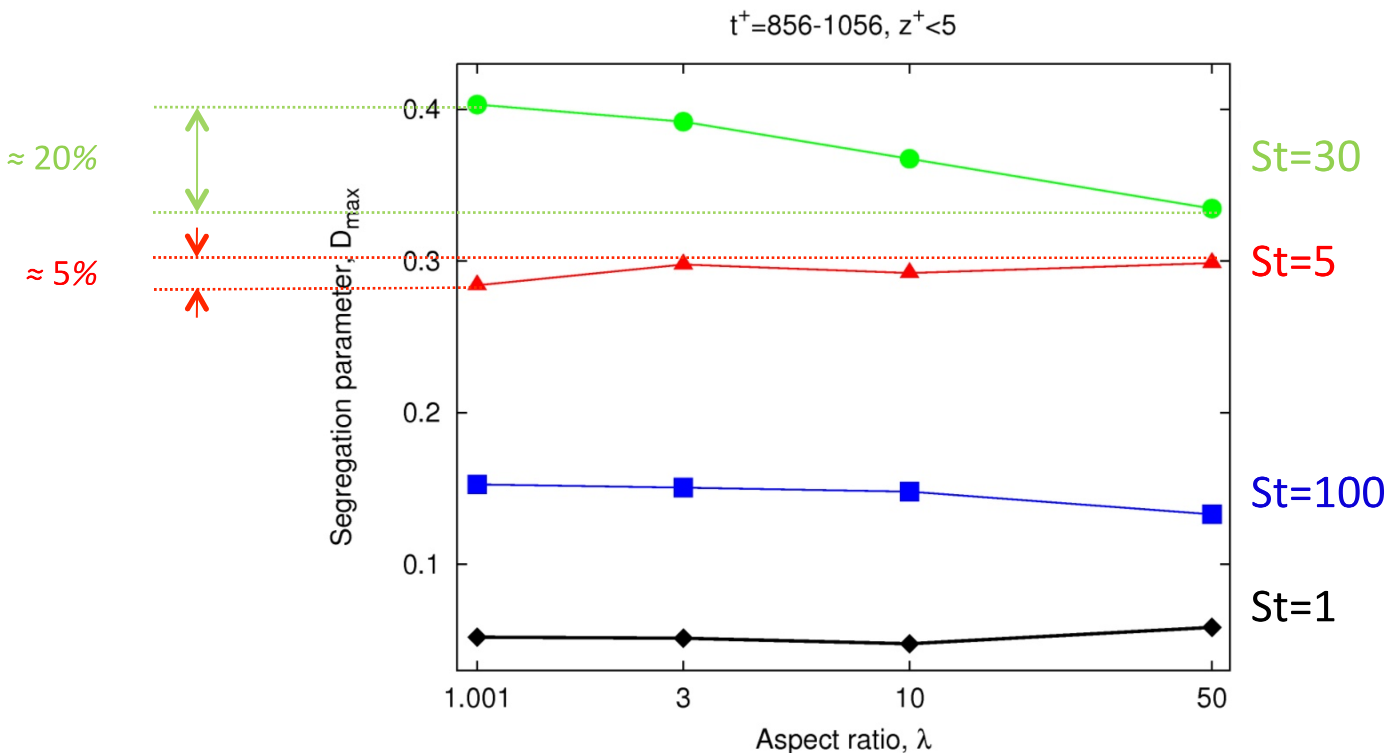
$$\Rightarrow \boxed{D = \frac{\sigma - \sigma_{Poisson}}{m} = D_{(\Delta x, \Delta y)}} \Rightarrow D_{MAX}$$

Multiphase Flow Laboratory
University of Udine

Results: Quantification of local fiber segregation

Project supported by COST - European Cooperation in Science and Technology

The values of D_{MAX} are averaged in time and computed for $z^+ < 5$ (viscous sublayer)



The degree of segregation in the near-wall region depends also on λ (not only on *St*).

The influence of λ on segregation changes sensibly for different *St* (different particle inertia).

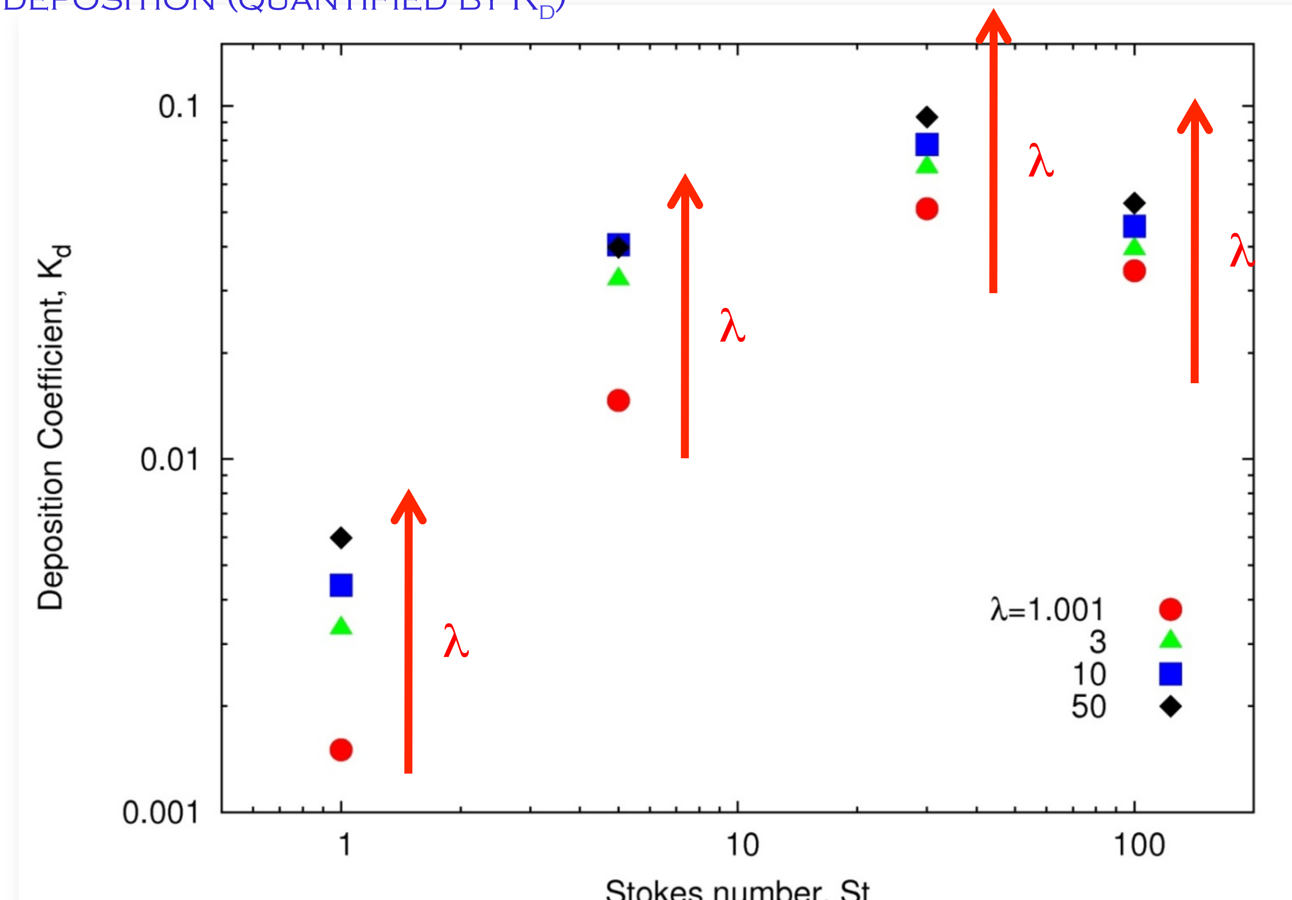
Results: Quantification of fiber deposition rate

Longer fibers tend to segregate less

From Eulerian-Lagrangian studies of spherical particle dispersion in TBL, we know that segregation controls deposition

Let's look at fiber deposition (quantified by K_D)

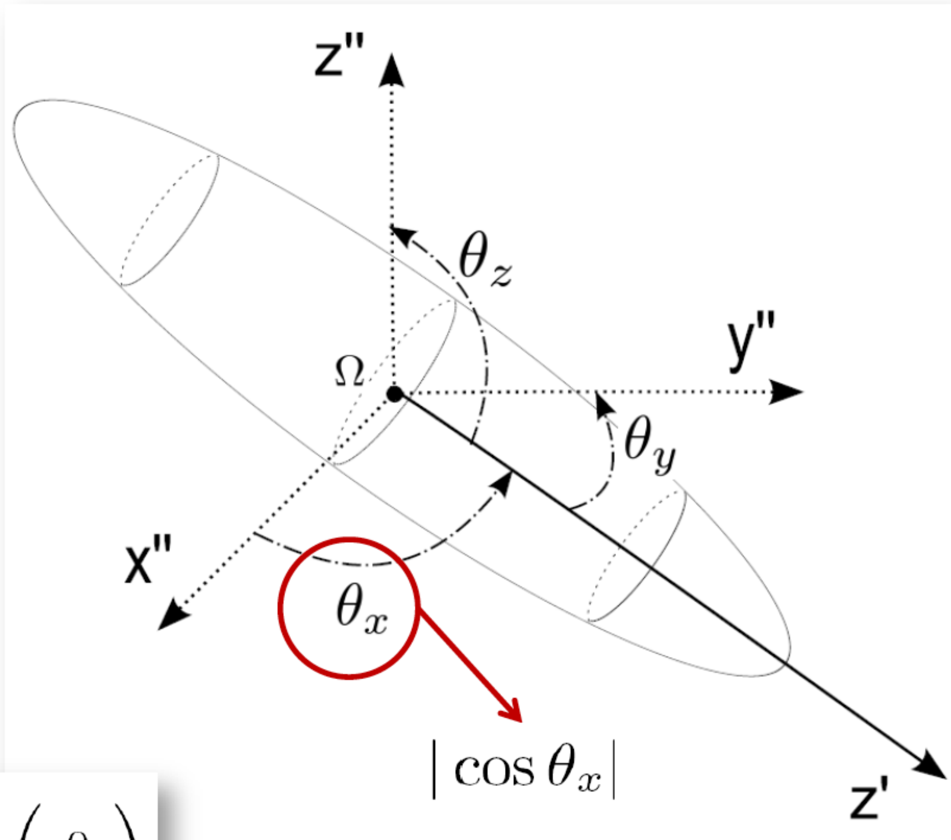
$$K_d = \frac{J \quad [kg/m^2 s]}{C \quad [kg/m^3]}$$



Results: Fiber orientation statistics

Project supported by COST - European Cooperation in Science and Technology

Use direction cosines to compute orientation



$$\begin{pmatrix} \cos\theta_x \\ \cos\theta_y \\ \cos\theta_z \end{pmatrix} = R_{Eul}^{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\cos\theta_x = R_{31},\ \cos\theta_y = R_{32},\ \cos\theta_z = R_{33}$$

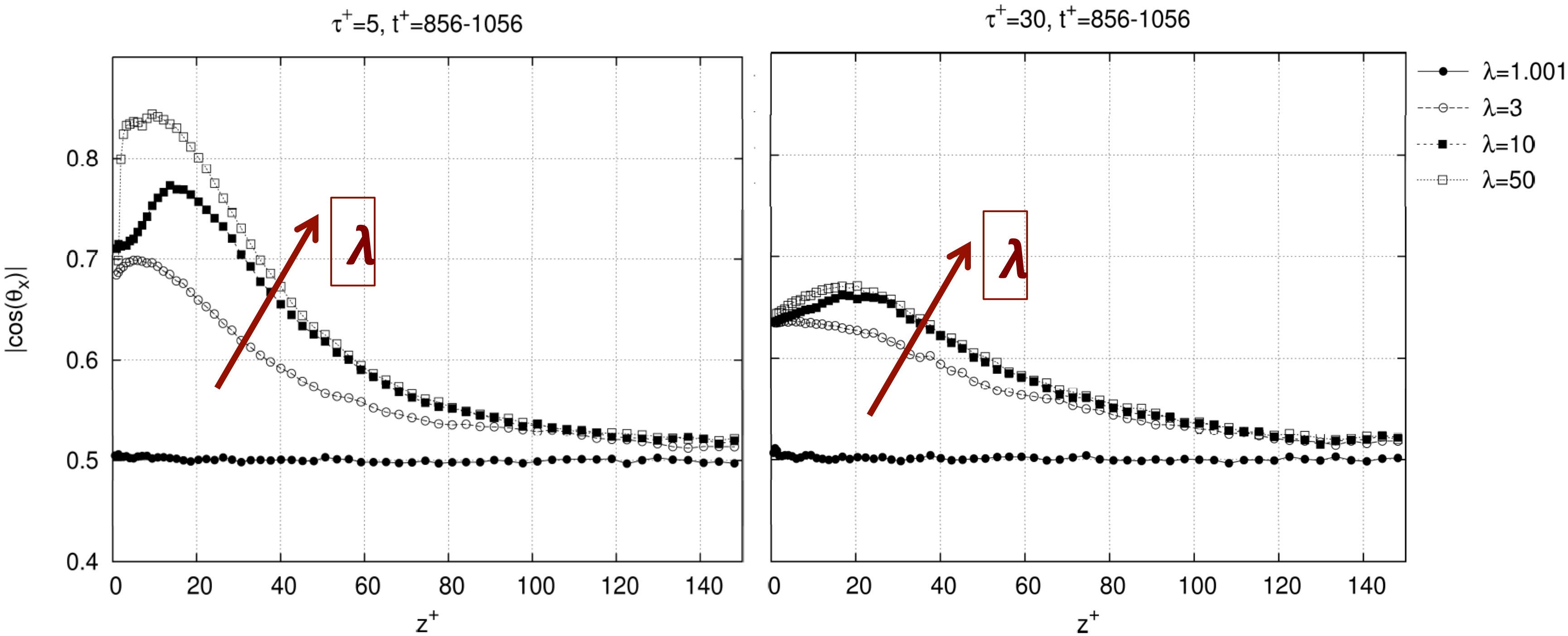
Multiphase Flow Laboratory

University of Udine

Results: Fiber orientation statistics

Project supported by COST - European Cooperation in Science and Technology

Fiber orientation with respect to the co-moving frame (given by the ensemble-averaged value of the direction cosines).



Orientation **strongly depends on *λ*** and on the Stokes number as well.

Fibers preferentially align in the streamwise (mean flow) direction. That's the most probable orientation!

Fiber orientation becomes isotropic in the center of the channel.

(see e.g. Marchioli et al., Phys. Fluids, 2010; *Mortensen, Phys. Fluids, 2008).*

Results: Fiber orientation alignment frequency

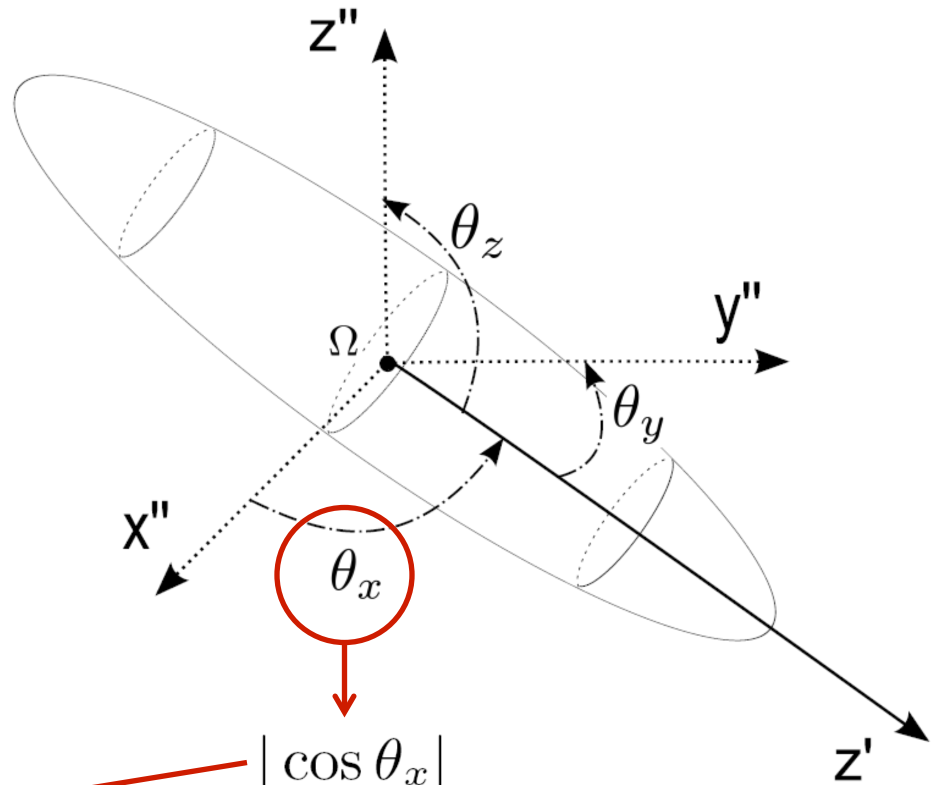
Project supported by COST - European Cooperation in Science and Technology

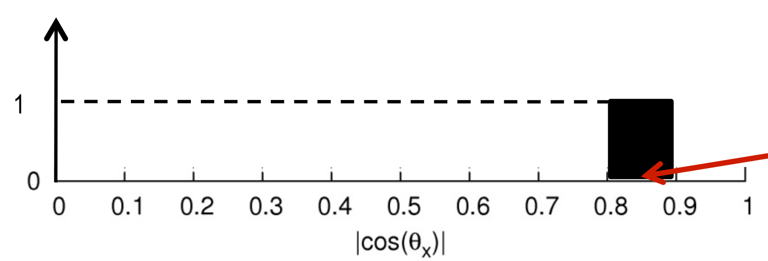
How long do fibers in the near-wall region remain aligned with the mean flow?

Is this alignment a "stable condition"?

Calculate Alignment Frequency:

- Divide the interval [0,1] into *N* orientation classes (here *N*= 10).
- At each time step:
  - Compute $|\cos(\theta_i)|$ for each fiber
  - Identify the orientation class sampled
  - Increment the time step counter for that class
- Compute percent values

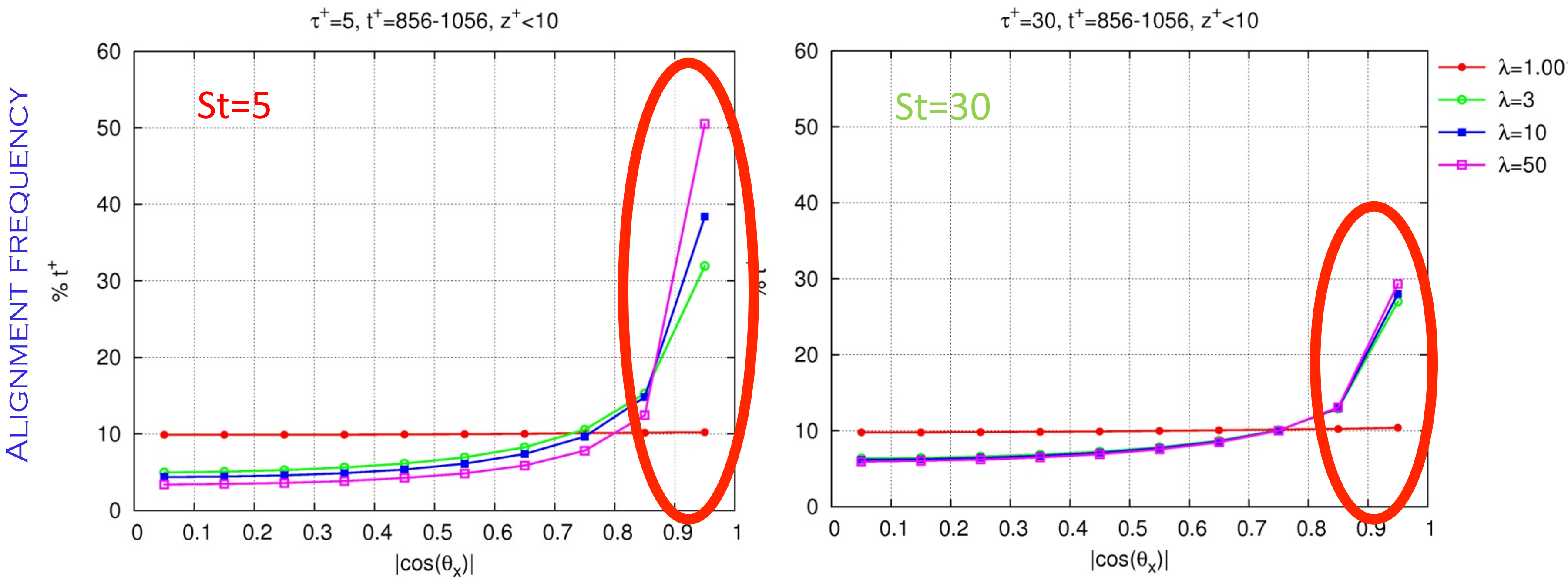




Multiphase Flow Laboratory
University of Udine

Results: Fiber orientation alignment frequency

Project supported by COST - European Cooperation in Science and Technology



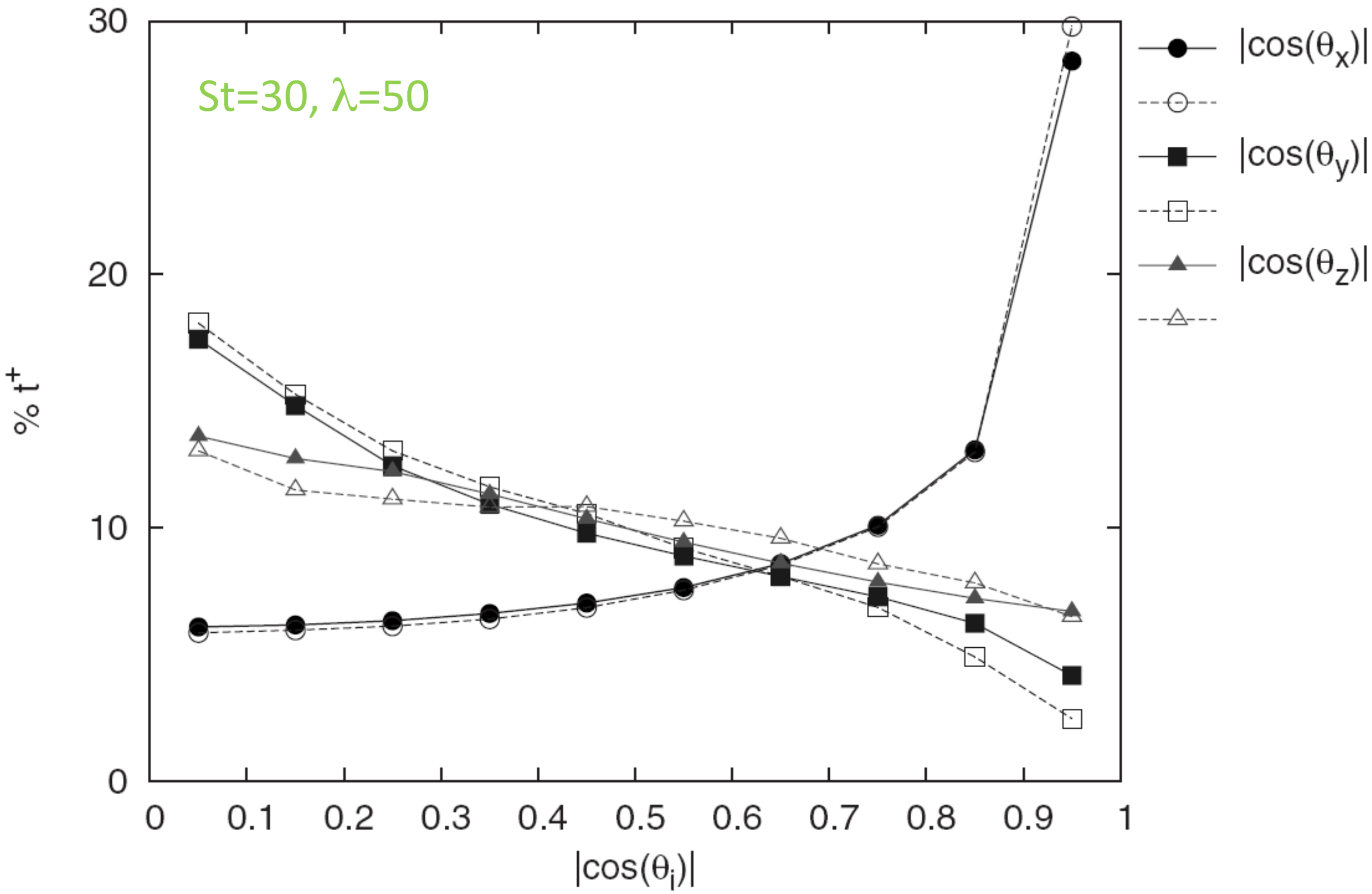
Fibers are aligned with the mean flow for just 50% of the time in the most favourable case (St=5, λ=50).

Much less in the other cases.

Results: Fiber orientation
alignment frequency

Project supported by COST - European Cooperation in Science and Technology

Alignment frequency

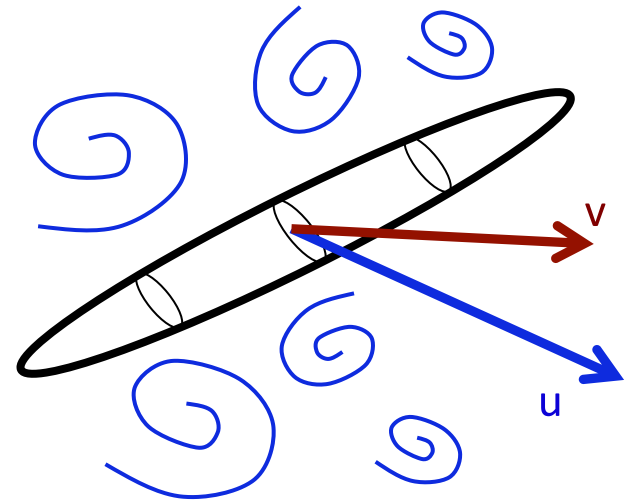


Alignment frequency statistics do not change if computed accounting only for fibers segregated into near-wall streaks

Results: Characterization of relative ("slip") velocity

Slip velocity is a crucial variable in:

1. Euler-Lagrange simulations:
   - One-way coupling: determines the drag experienced by fibers
   - Two-way coupling: determines reaction force from fibers on fluid
2. Two-fluid modeling of particle-laden flows
   - Modeling SGS fiber dynamics in LES flow fields
   - Crossing trajectory effects on time decorrelation tensor of u



u: fluid velocity "seen"

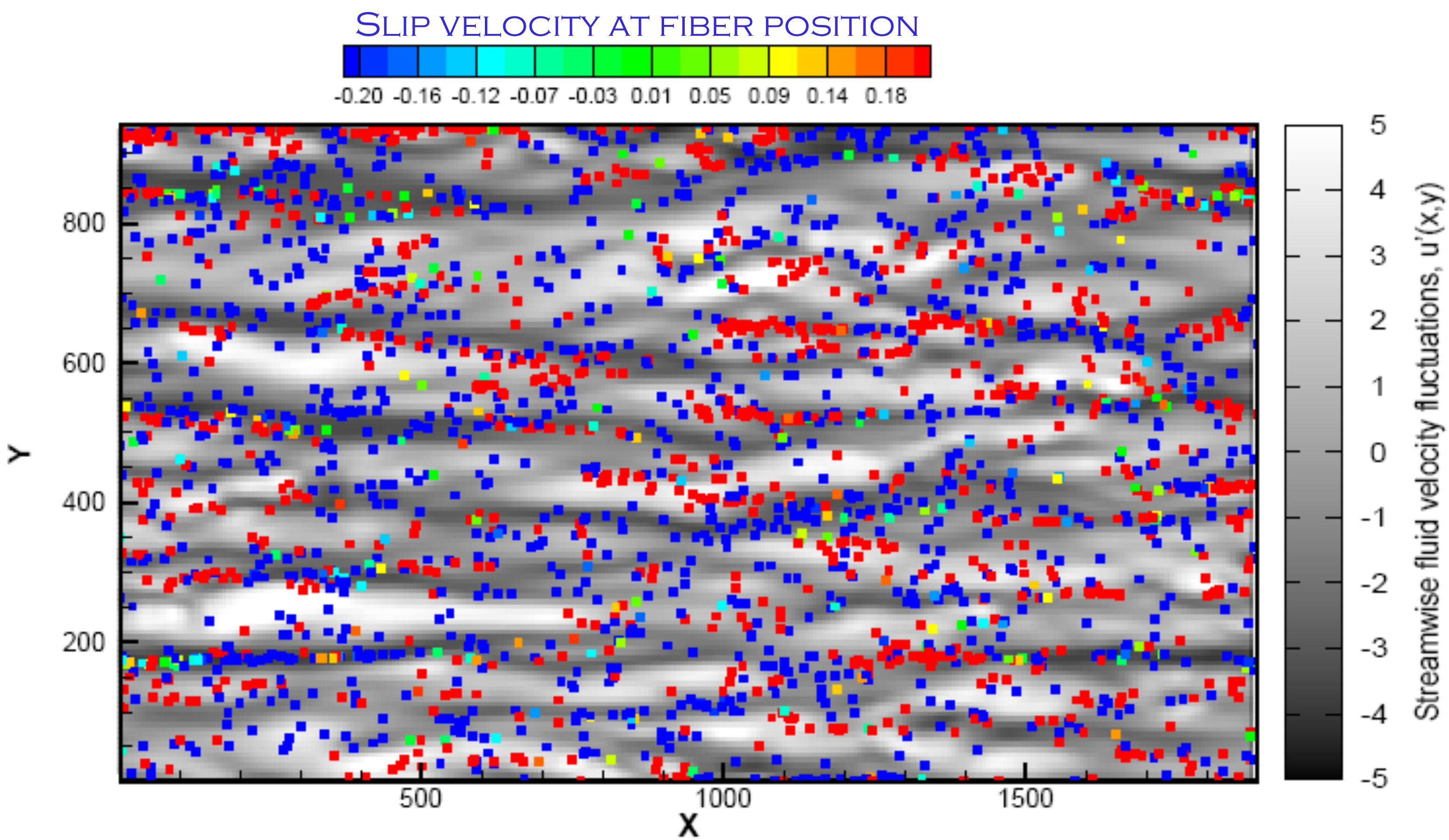
v: fiber velocity

Δu=u-v: slip velocity

⇨ statistical characterization of Δu at varying fiber inertia and elongation

Results: Characterization of relative ("slip") velocity

Project supported by COST - European Cooperation in Science and Technology



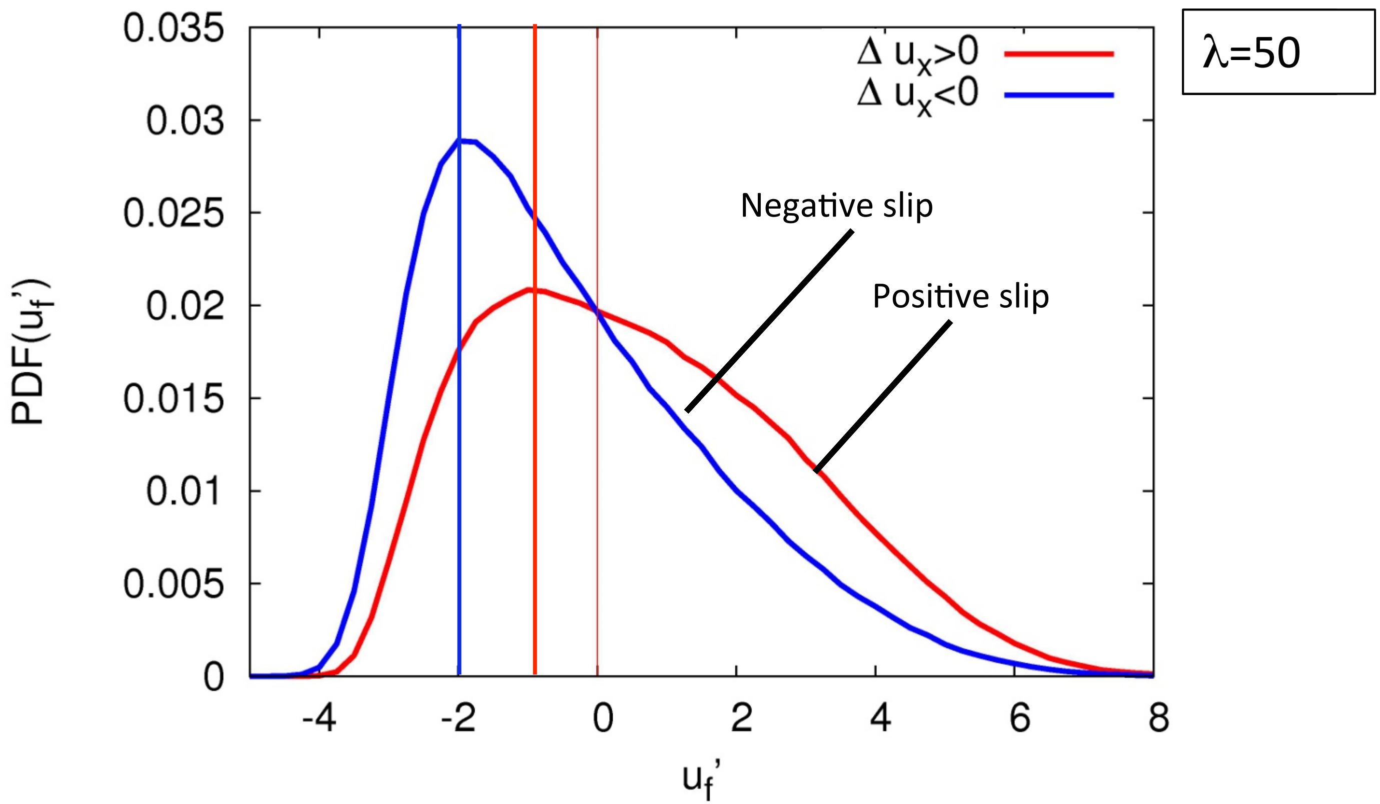
Sample snapshot for St=30, λ=50 fibers

LSS seem to correlate better with negative slip velocity...

Use slip velocity to analyze fiber accumulation in LSS

Project supported by COST - European Cooperation in Science and Technology

Effect of fiber elongation on conditioned PDF(u_f') – St=30

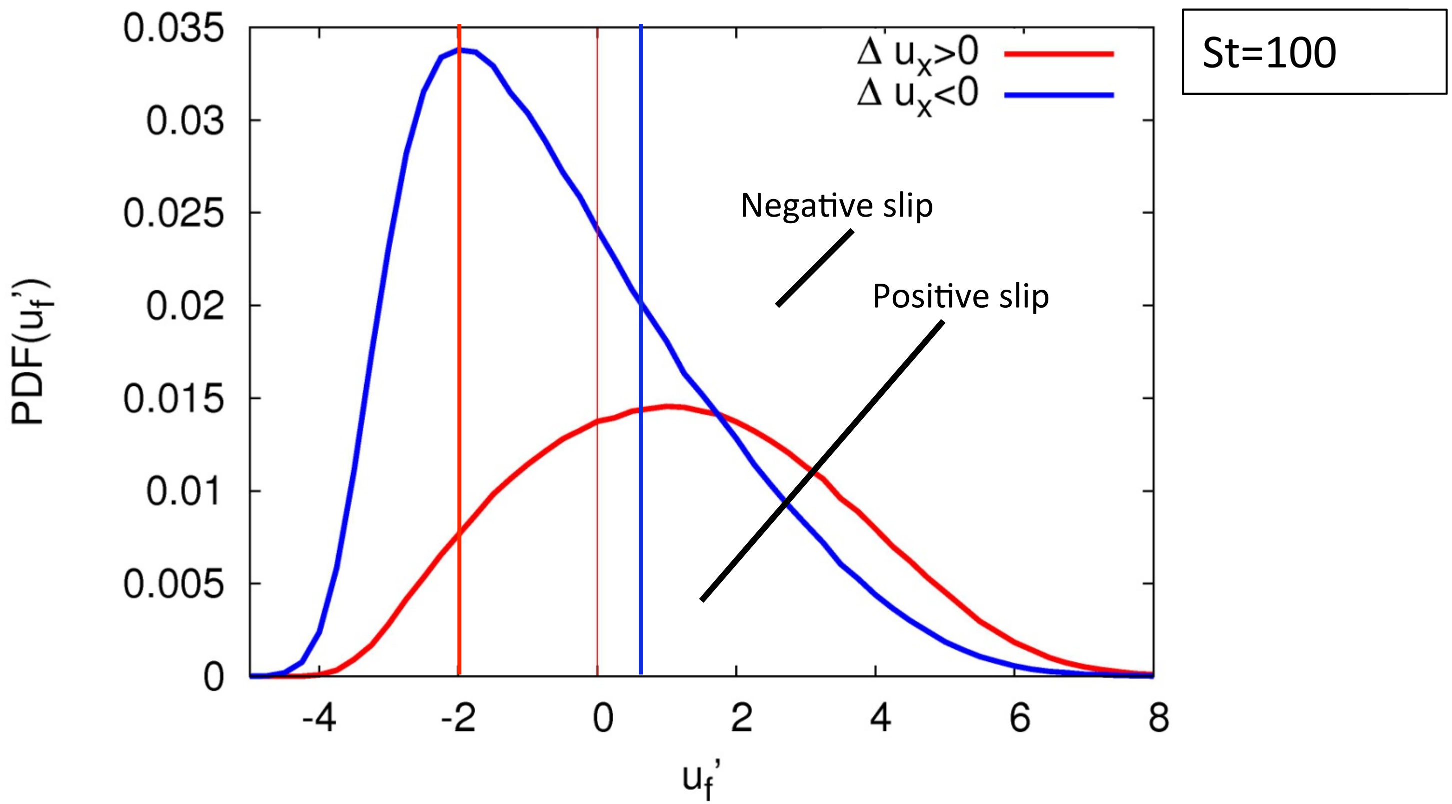


The influence of λ is not dramatic: only a change in the peak values is observed (no PDF shape change)

Use slip velocity to analyze fiber accumulation in LSS

Project supported by COST - European Cooperation in Science and Technology

Effect of fiber inertia on conditioned PDF(u_f') – λ=10

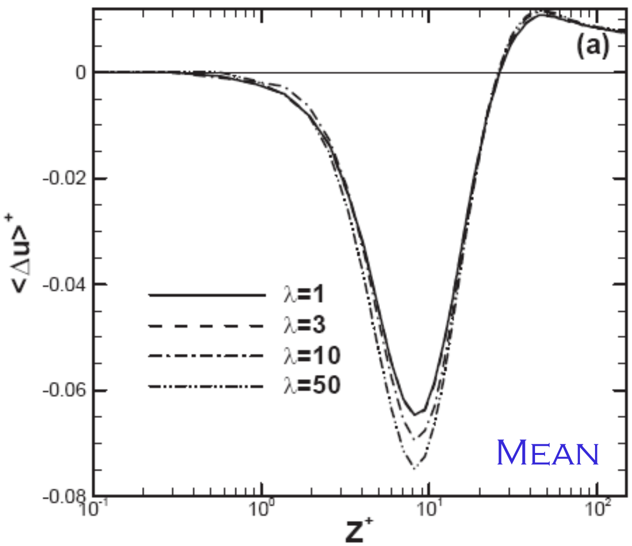


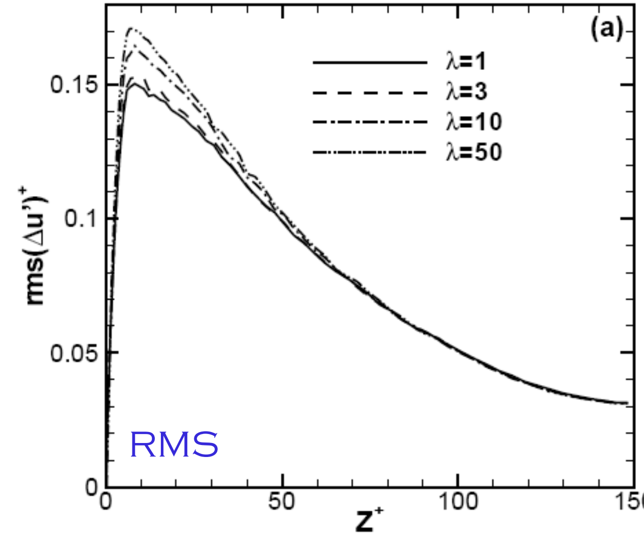
Significant PDF shape change with curve "inversion" between St=5 and St=30

Results - Streamwise Slip Vel. Mean and RMS values

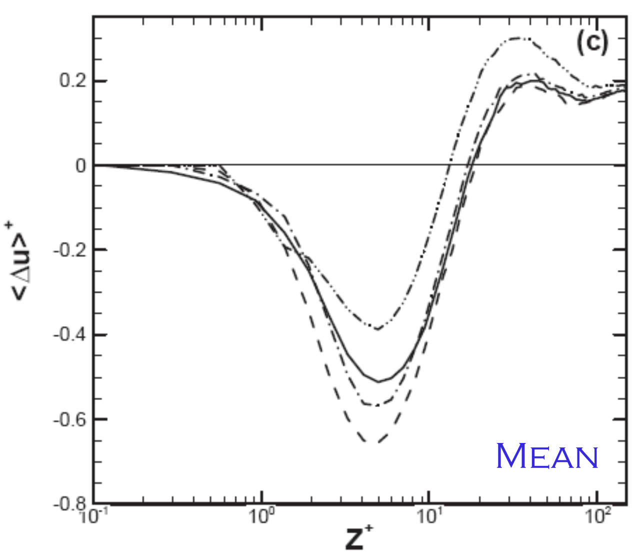
Project supported by COST - European Cooperation in Science and Technology

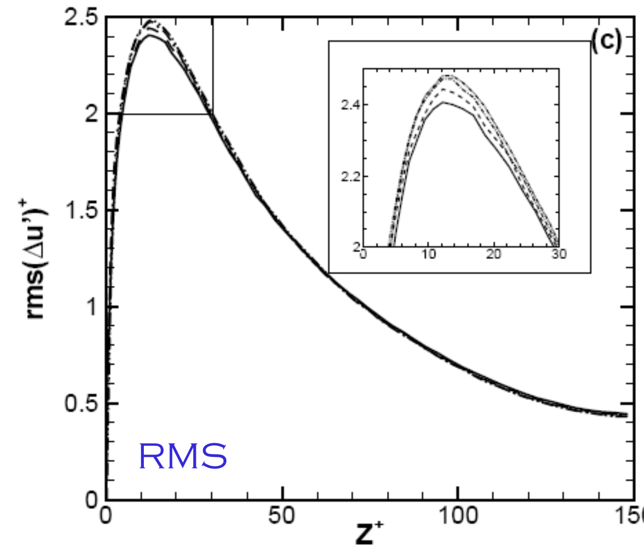
St=1





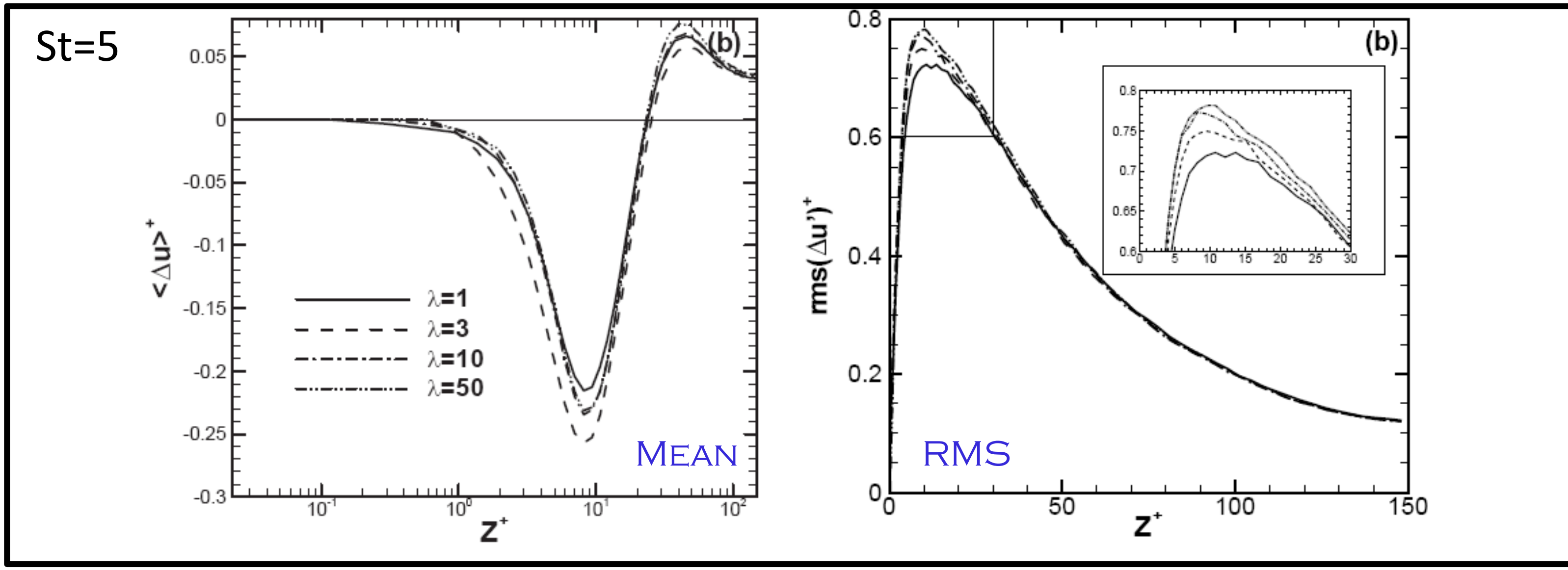
St=30

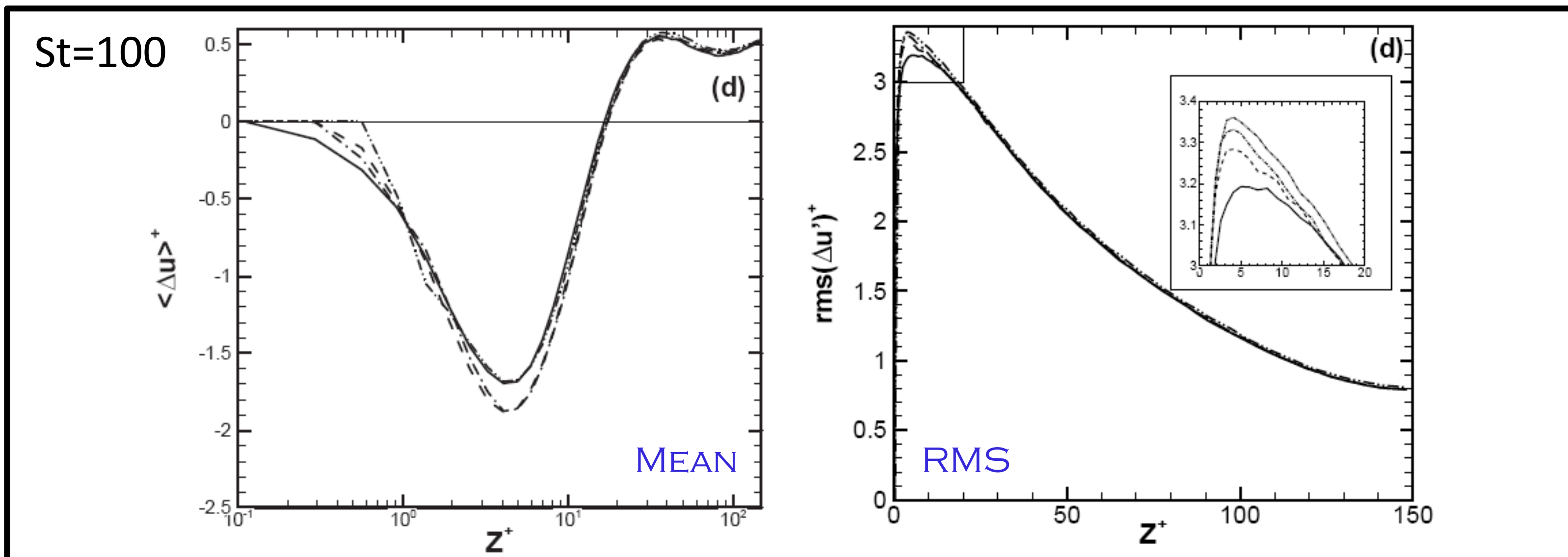




Results - Streamwise Slip Vel. Mean and RMS values

Project supported by COST - European Cooperation in Science and Technology



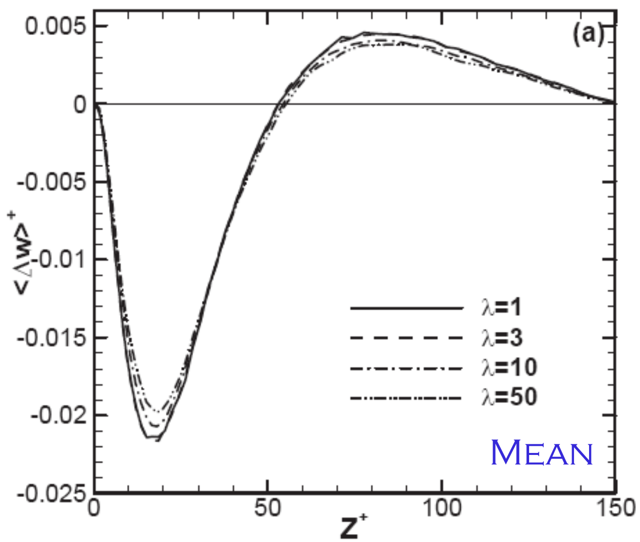


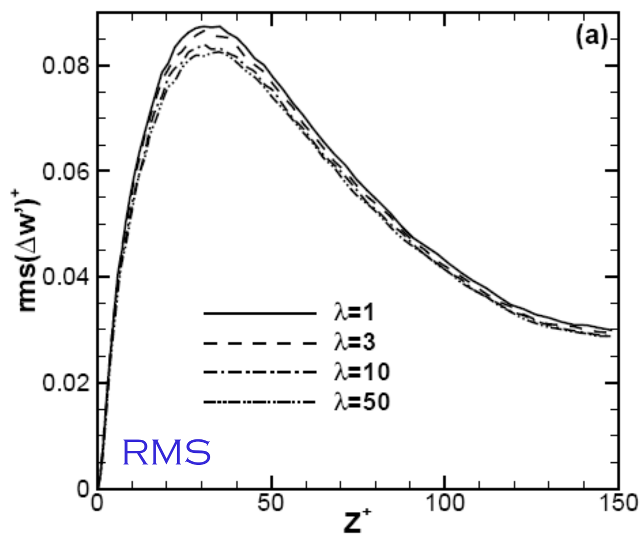
Multiphase Flow Laboratory
University of Udine

Results - Wall-normal Slip Vel. Mean and RMS values

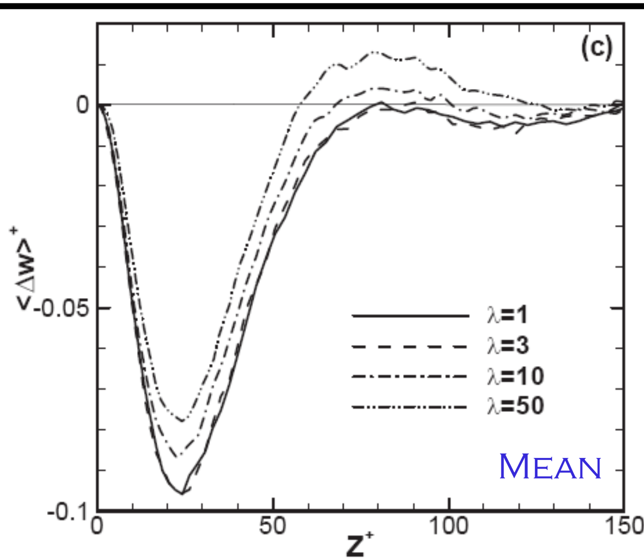
Project supported by COST - European Cooperation in Science and Technology

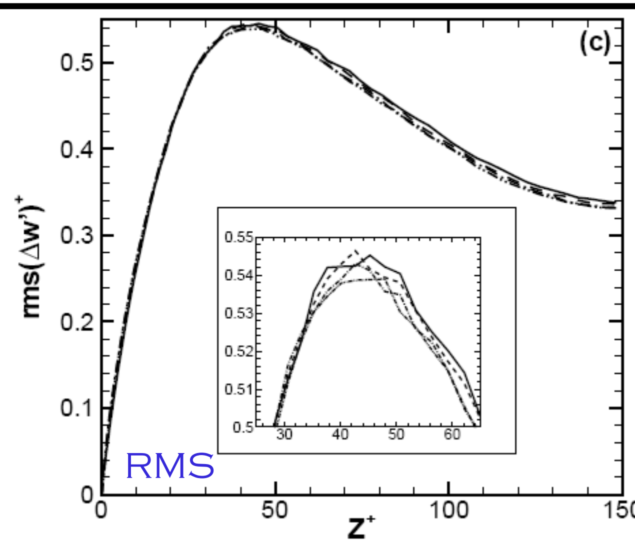
St=1





St=30





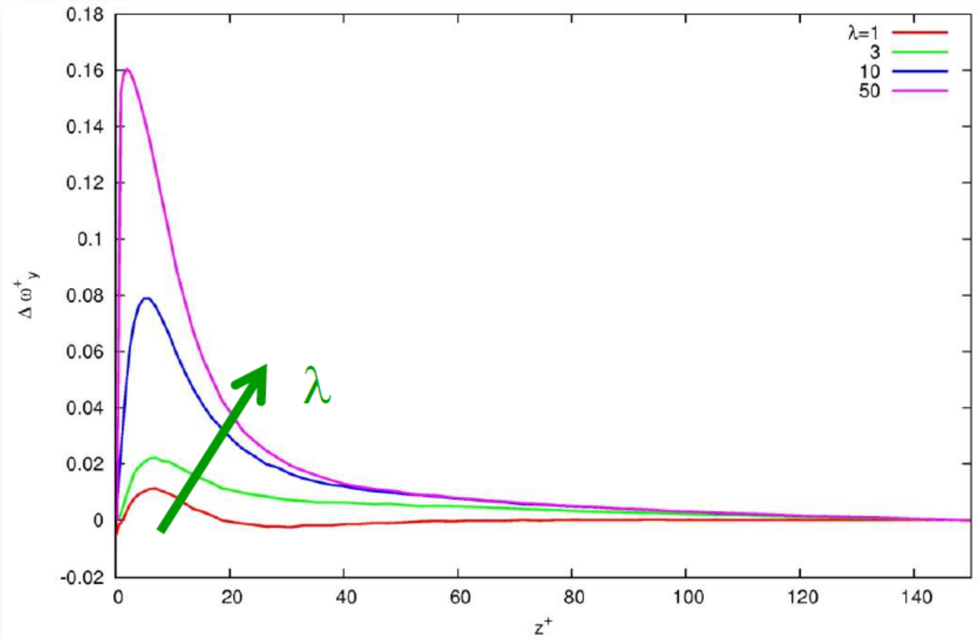
Multiphase Flow Laboratory
University of Udine

Results - Slip spin statistics

Project supported by COST - European Cooperation in Science and Technology

Evaluate slip spin statistics...

Relative spin between fluid and fibers determines fiber rotational dynamics...



Mean slip spin for the St=30 fibers in the Re=150 flow (spanwise component)

$\underline{Dynamics:}$

$$\frac{d\mathbf{v}^+}{dt^+} = \left(\frac{S-1}{S}\right)\mathbf{g}^+ + \frac{3}{4\lambda Sa^{+2}}\bar{\bar{\mathbf{K}}}_{(e_0,e_1,e_2,e_3)} \cdot (\mathbf{u}^+ - \mathbf{v}^+)$$

$$\frac{d\omega^+_{x'}}{dt^+} = \omega^+_{y'}\omega^+_{z'}\left(1 - \frac{2}{1+\lambda^2}\right) + \frac{20\left[(1-\lambda^2)f' + (1+\lambda^2)(\xi' - \omega^+_{x'})\right]}{(\alpha_0 + \lambda^2\gamma_0)(1+\lambda^2)Sa^{+2}}$$

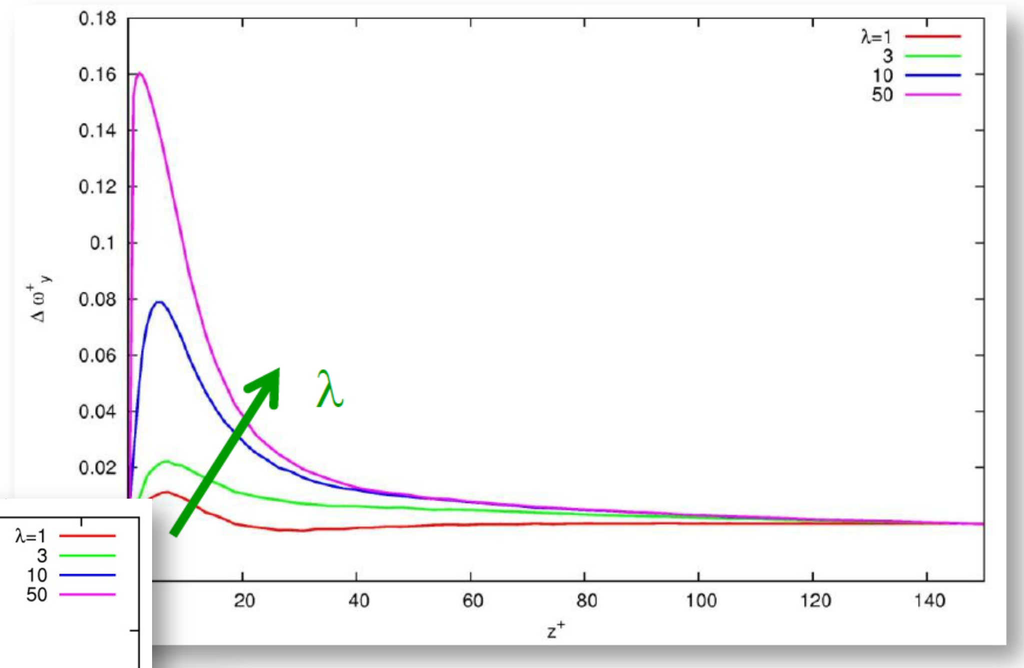
$$\frac{d\omega^+_{y'}}{dt^+} = \omega^+_{x'}\omega^+_{z'}\left(\frac{2}{1+\lambda^2} - 1\right) + \frac{20\left[(\lambda^2-1)g' + (\lambda^2+1)(\eta' - \omega^+_{y'})\right]}{(\alpha_0 + \lambda^2\gamma_0)(1+\lambda^2)Sa^{+2}}$$

$$\frac{d\omega^+_{z'}}{dt^+} = \frac{20}{(2\alpha_0)Sa^{+2}}(\chi' - \omega^+_{z'})$$

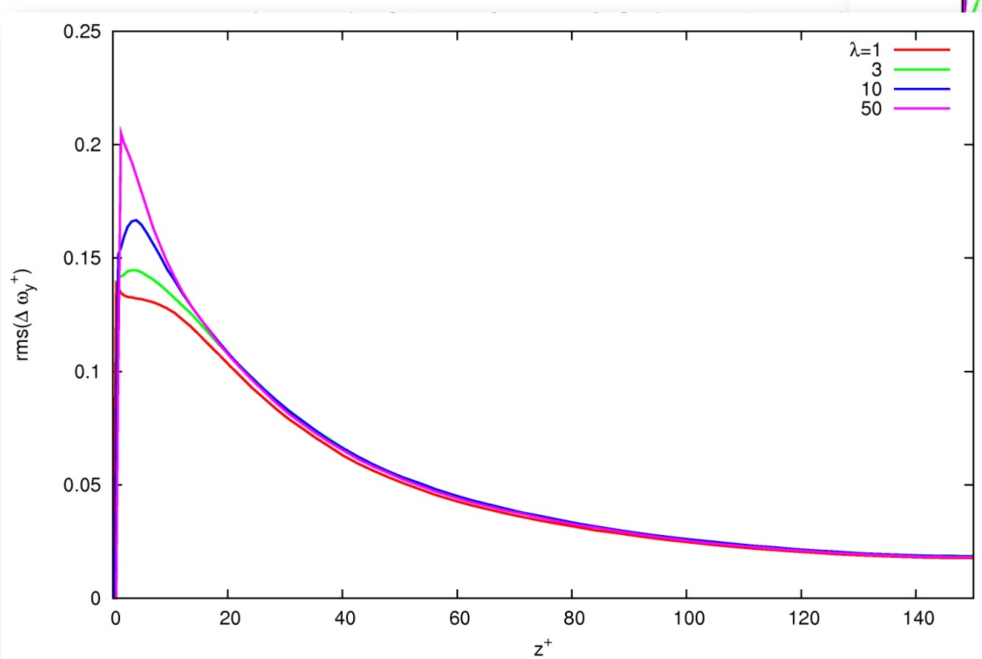
Results - Slip spin statistics

Evaluate slip spin statistics...

Relative spin between fluid and fibers highly influenced by fiber inertia...



Mean slip spin for the St=100 fibers in the Re=150 flow (spanwise component)

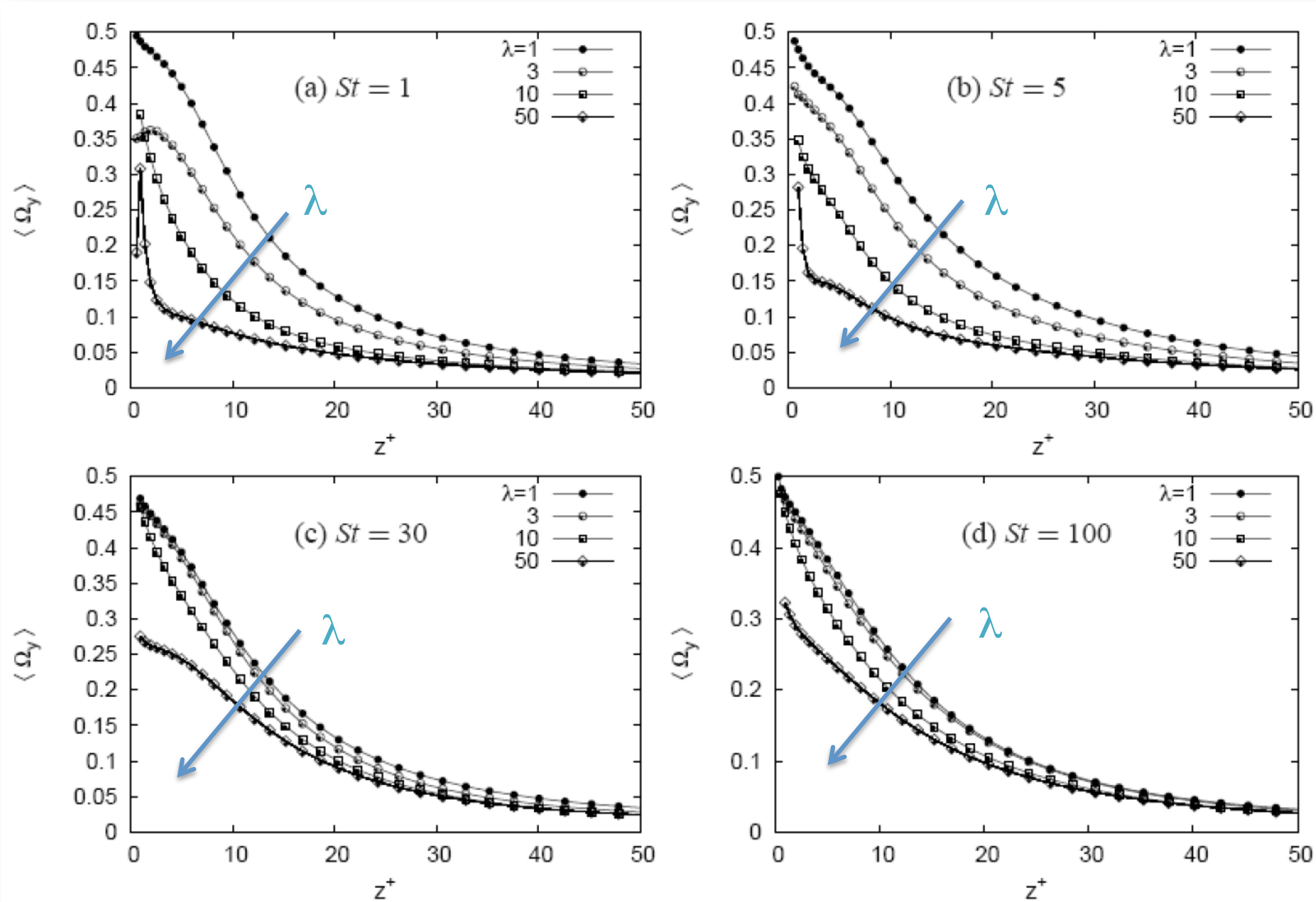


Slip spin rms for the St=100 fibers in the Re=150 flow (spanwise component)

Results – fiber rotation statistics (angular velocity)

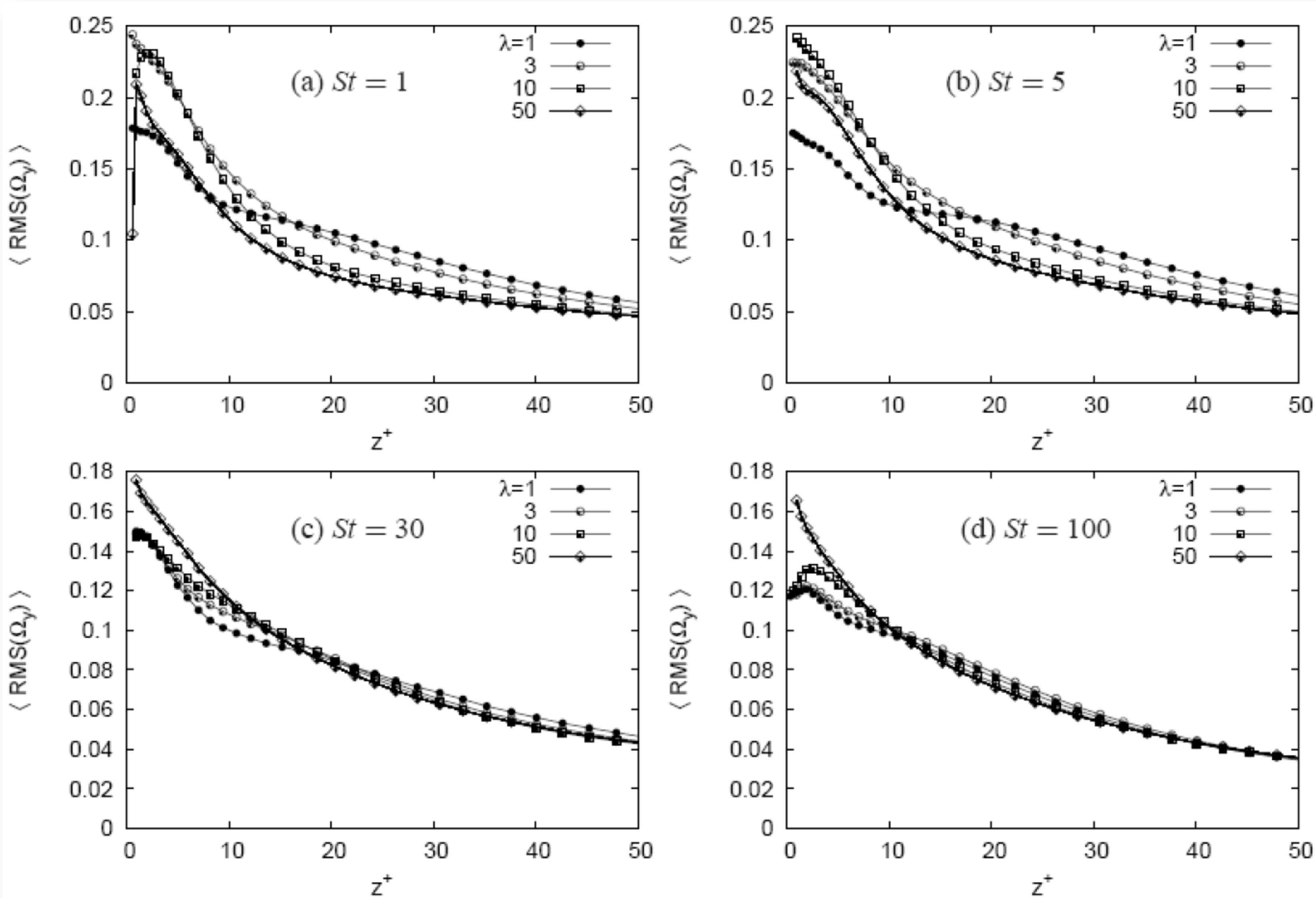
Project supported by COST - European Cooperation in Science and Technology

Mean fiber spanwise angular velocity:



Results – fiber rotation statistics (angular velocity)

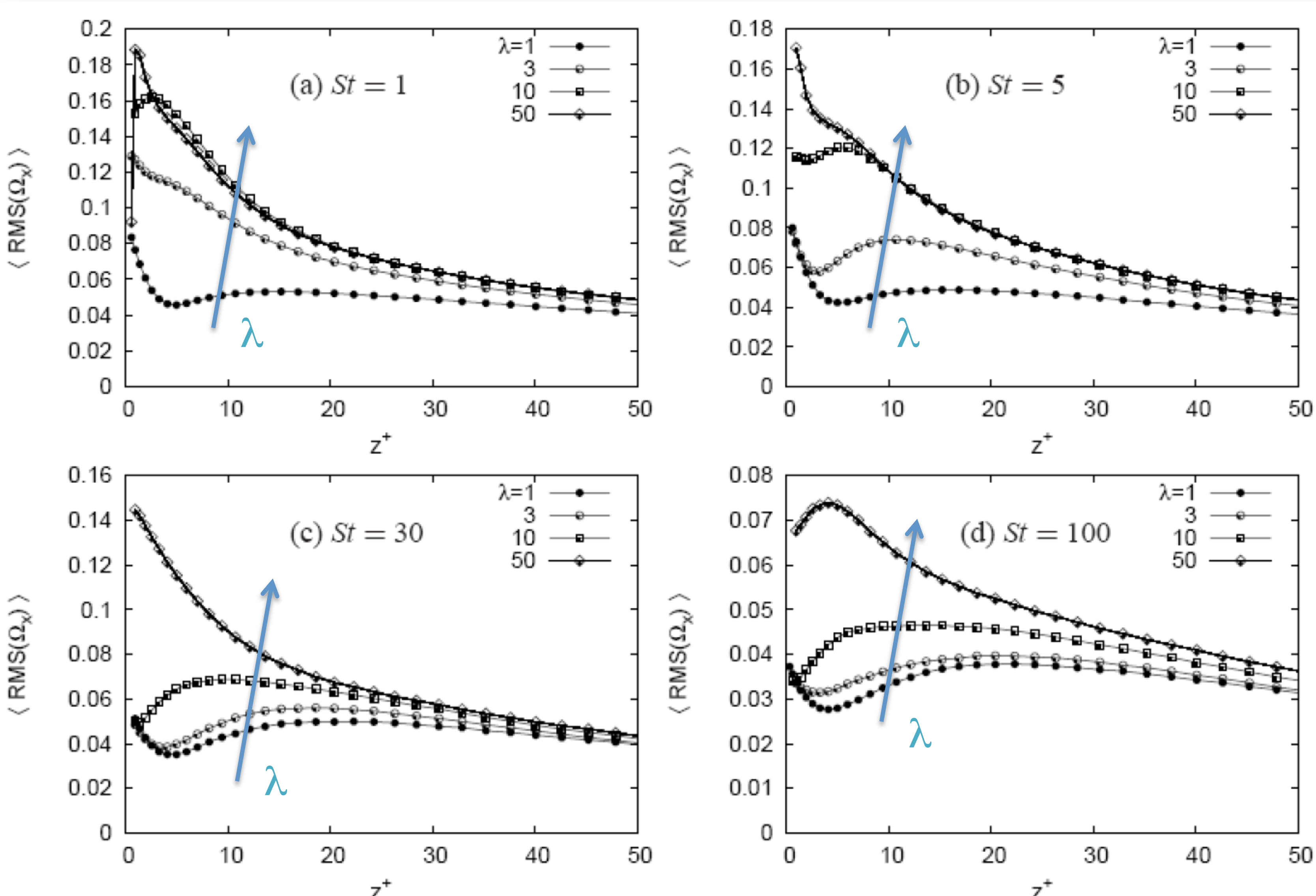
RMS of fiber spanwise angular velocity:



Results – fiber rotation statistics (angular velocity)

Project supported by COST - European Cooperation in Science and Technology

RMS of fiber streamwise angular velocity:

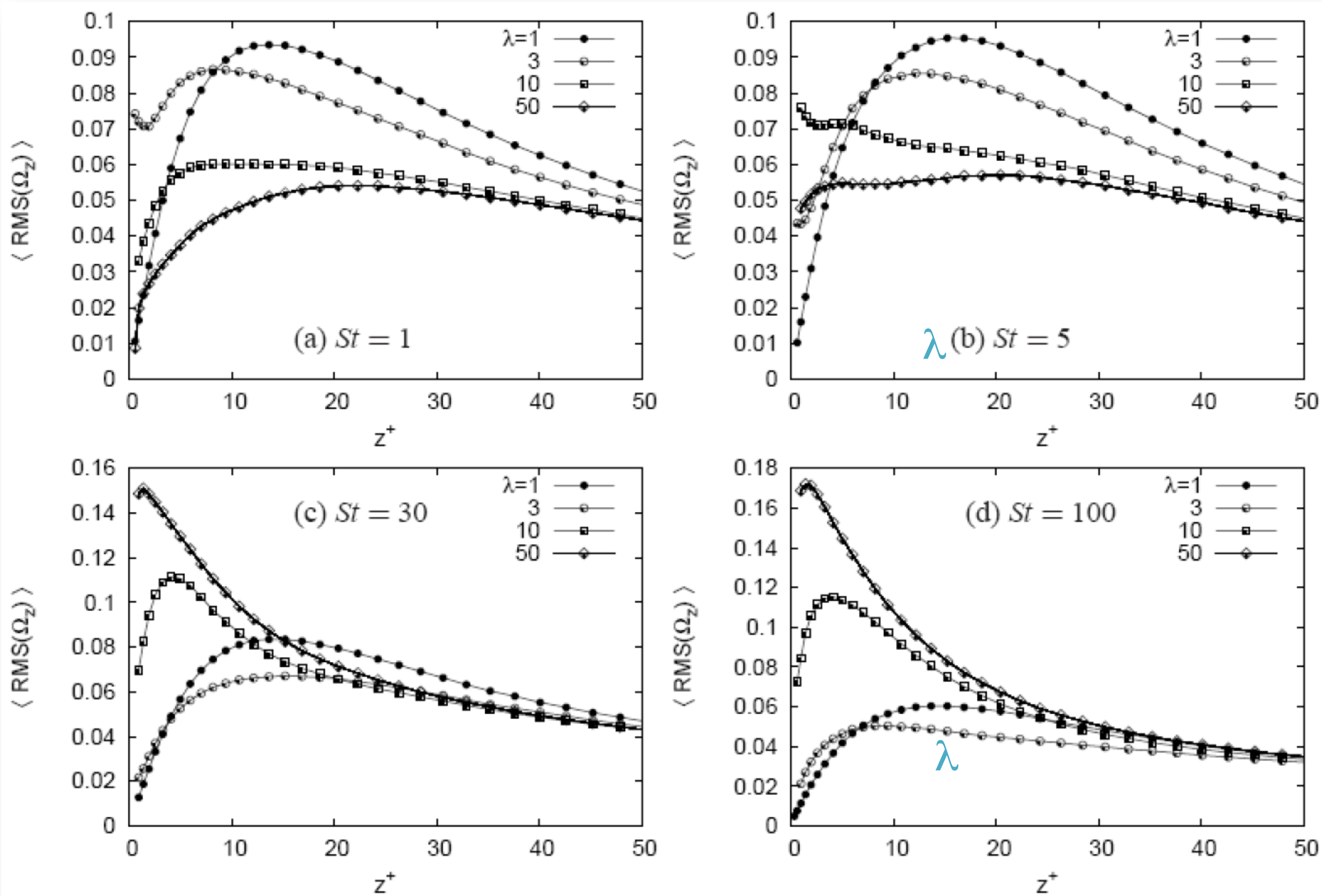


Multiphase Flow Laboratory
University of Udine

Results – fiber rotation statistics (angular velocity)

Project supported by COST - European Cooperation in Science and Technology

RMS of fiber wall-normal angular velocity:



Results – fiber rotation statistics (angular velocity)

To characterize the fiber rotation process, one can look at:

Lagrangian autocorrelation coefficients of the fiber's angular velocity (normalized by the variance of the Gaussian Distribution)

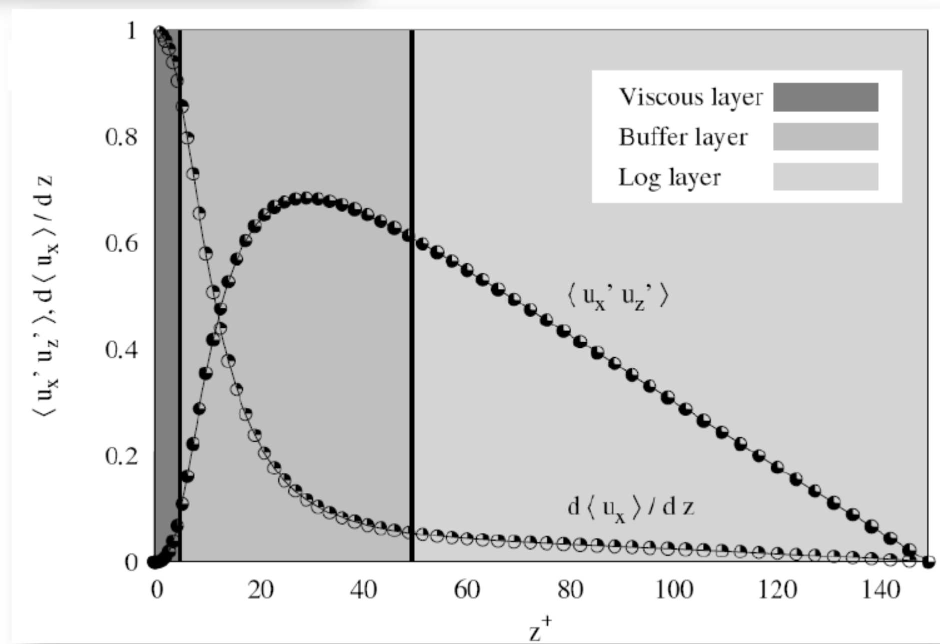
$$R_{\Omega_i,\Omega_i}(\tau) = \frac{\langle \Omega_i(\mathbf{x}_p(t_0),t_0)\Omega_i(\mathbf{x}_p(t_0+\tau),t_0+\tau)\rangle}{\langle \Omega_i(\mathbf{x}_p(t_0),t_0)^2\rangle^{1/2}\langle \Omega_i(\mathbf{x}_p(t_0+\tau),t_0+\tau)^2\rangle^{1/2}}$$

Lagrangian integral timescale

$$T_L^{ii} = \int_0^\infty R_{\Omega_i,\Omega_i}(\tau)\mathrm{d}\tau$$

Turbulent rotational diffusivity

$$\Gamma_\Omega(\tau) = \int_0^\tau R_{\Omega_i,\Omega_i}(t)\mathrm{d}t$$



Multiphase Flow Laboratory

University of Udine

Results – fiber rotation statistics (angular velocity)

Project supported by COST - European Cooperation in Science and Technology

Question: can we model fiber rotation in a simple way?

For instance, is there any region of the channel flow where we can model rotation within the theory of diffusion?

If yes, then rotation could be described as a **Ornstein-Uhlenbeck** process, which is completely characterized by:

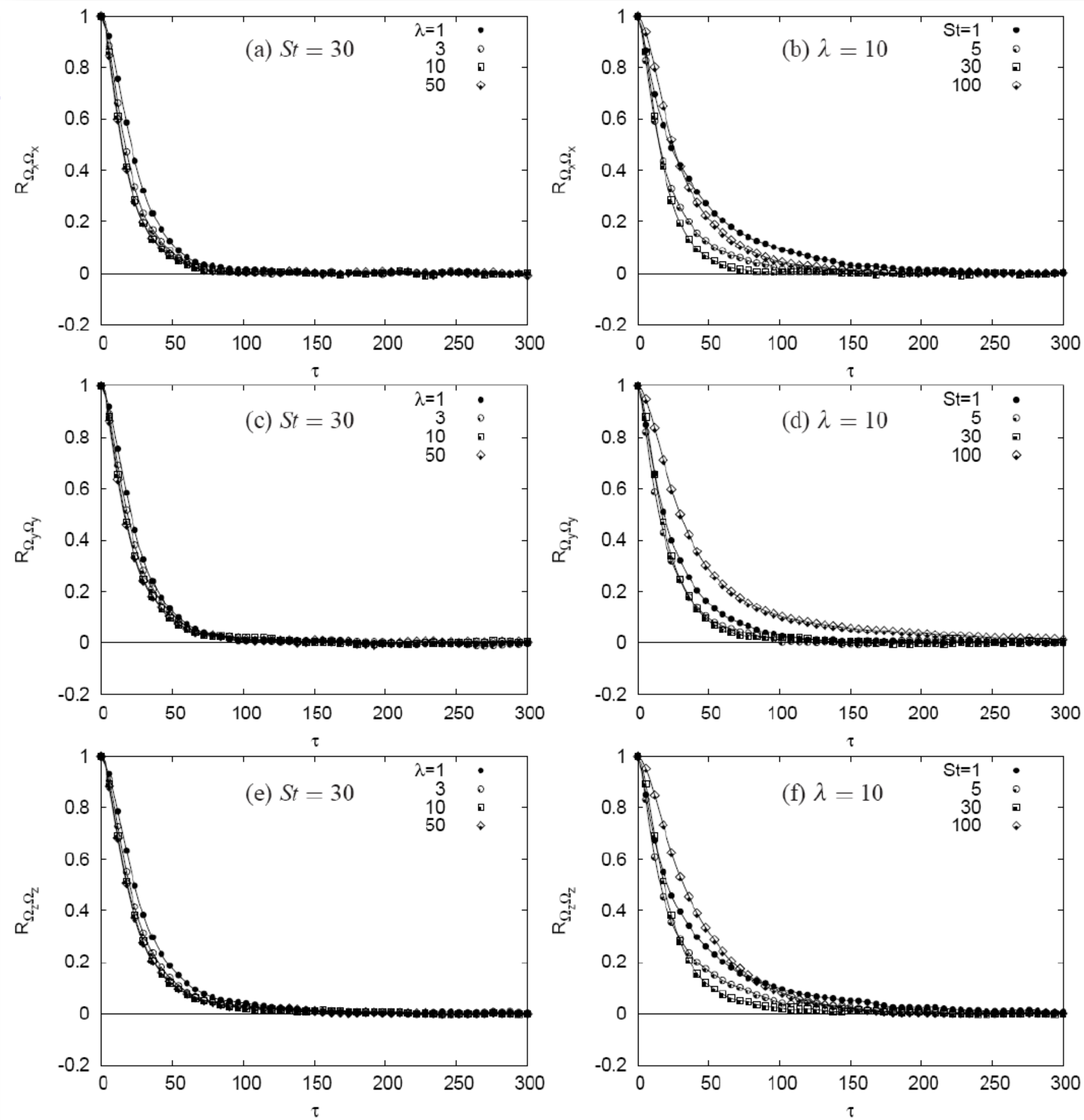
- statistically-stationary Gaussian distribution
- autocorrelation which takes the specific form of a negative exponential:

$$R_{\Omega_i,\Omega_i}(\tau) = e^{-\frac{\tau}{T_L^{ii}}} \longrightarrow \Gamma_{\Omega_i}(\tau) = T_L^{ii}\left(1 - e^{-\frac{\tau}{T_L^{ii}}}\right)$$

Results – Autocorrelation

Project supported by COST - European Cooperation in Science and Technology

Log layer ($z^+ > 50$)

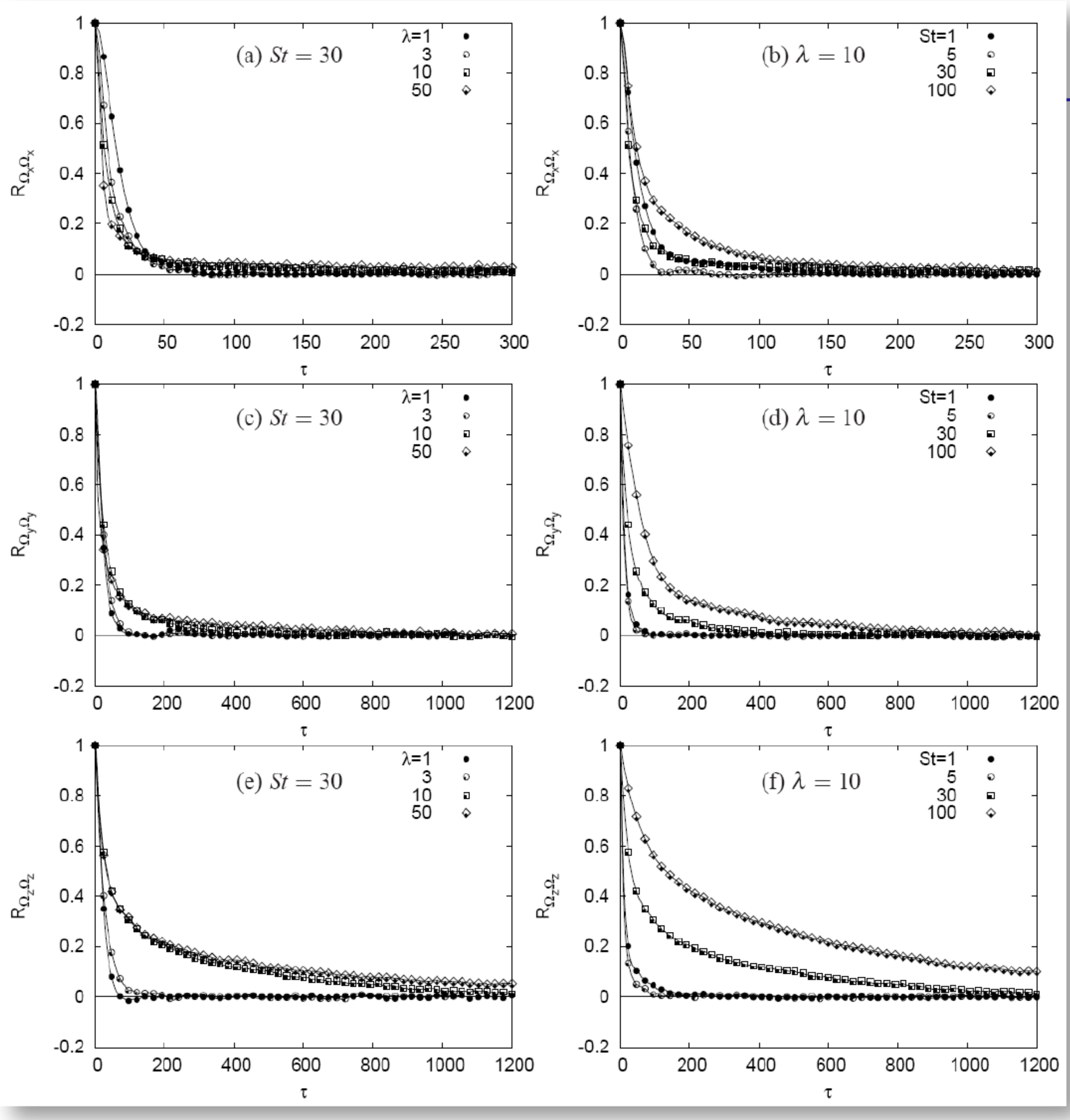


Results – Autocorrelation

Multiphase Flow Laboratory
University of Udine

Project supported by COST - European Cooperation in Science and Technology

Buffer layer
($5 < z^+ < 50$)

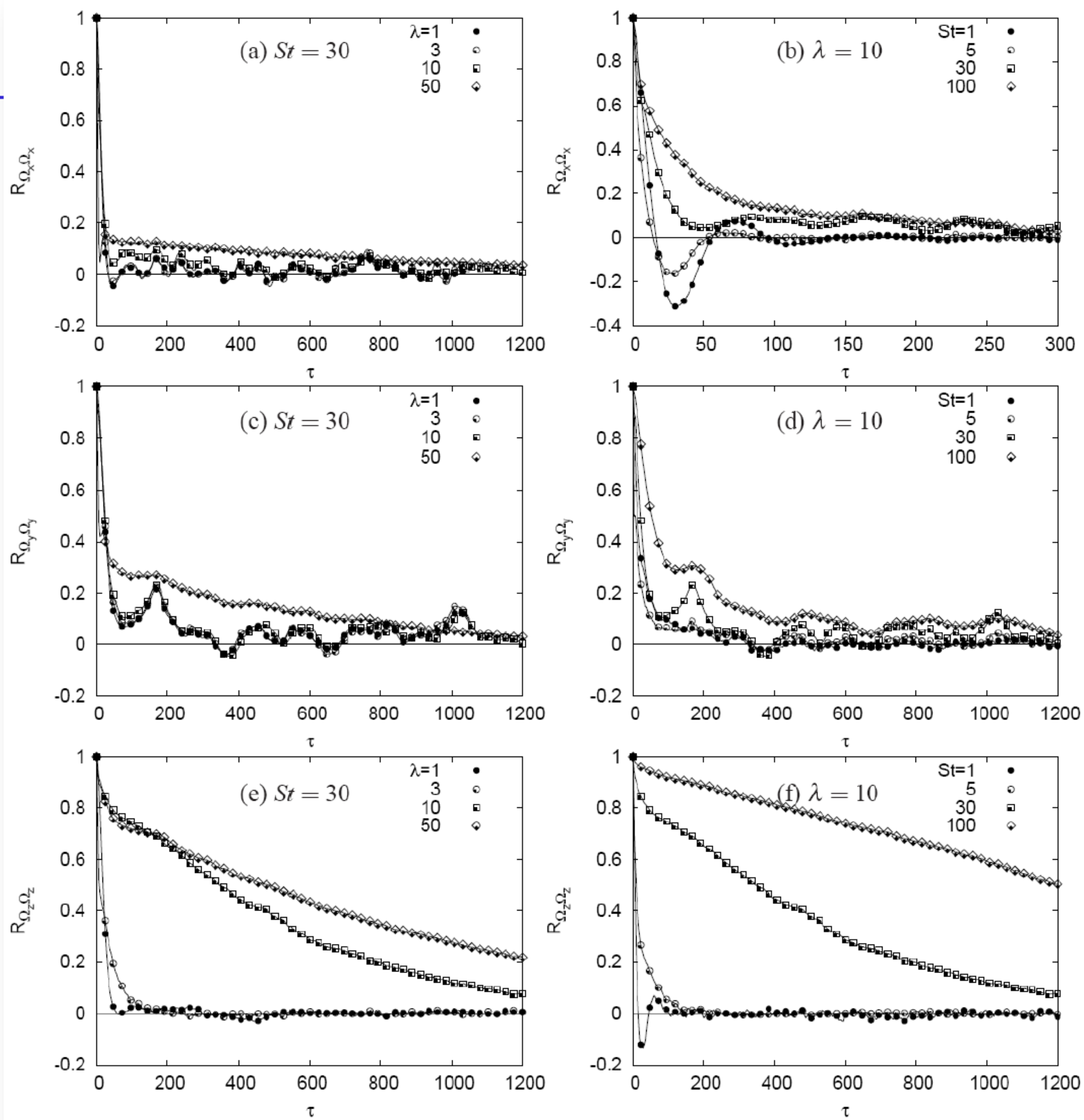


Multiphase Flow Laboratory
University of Udine

Results – Autocorrelation

Project supported by COST - European Cooperation in Science and Technology

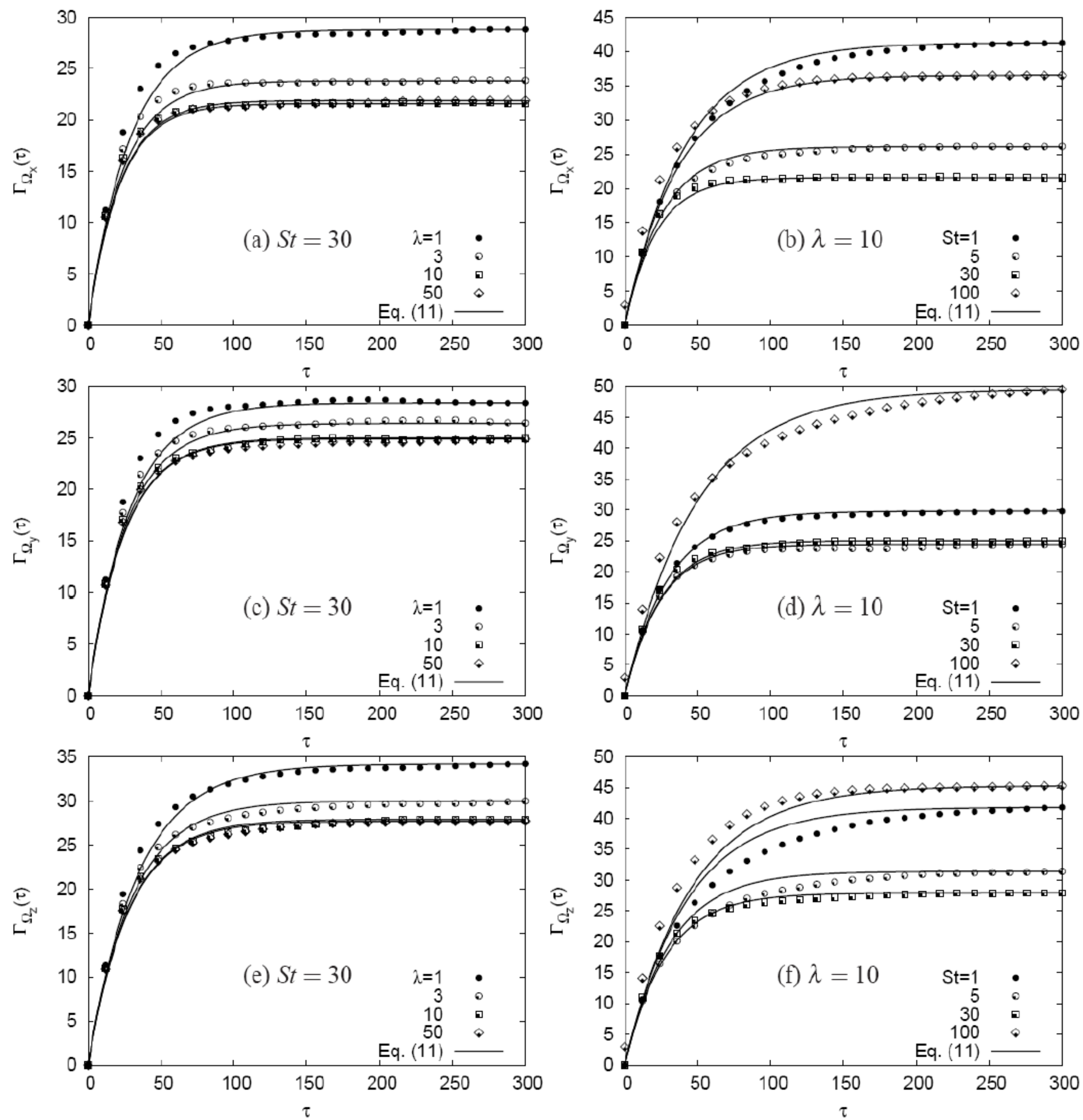
Viscous layer
($z^+ < 5$)



Results – Rotational diffusivity

Project supported by COST - European Cooperation in Science and Technology

Log layer ($z^+ > 50$)

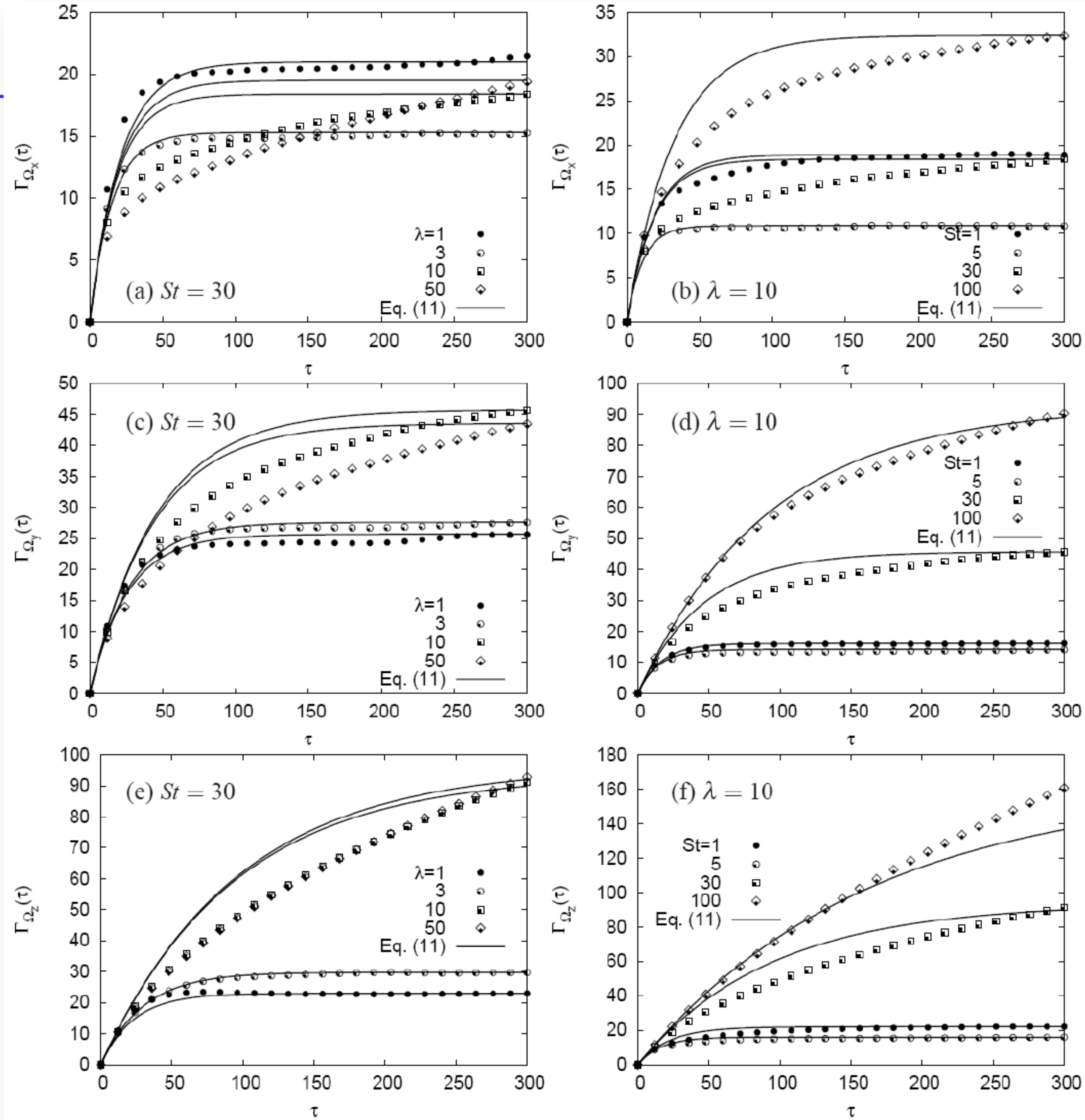


Multiphase Flow Laboratory
University of Udine

Results – Rotational diffusivity

Project supported by COST - European Cooperation in Science and Technology

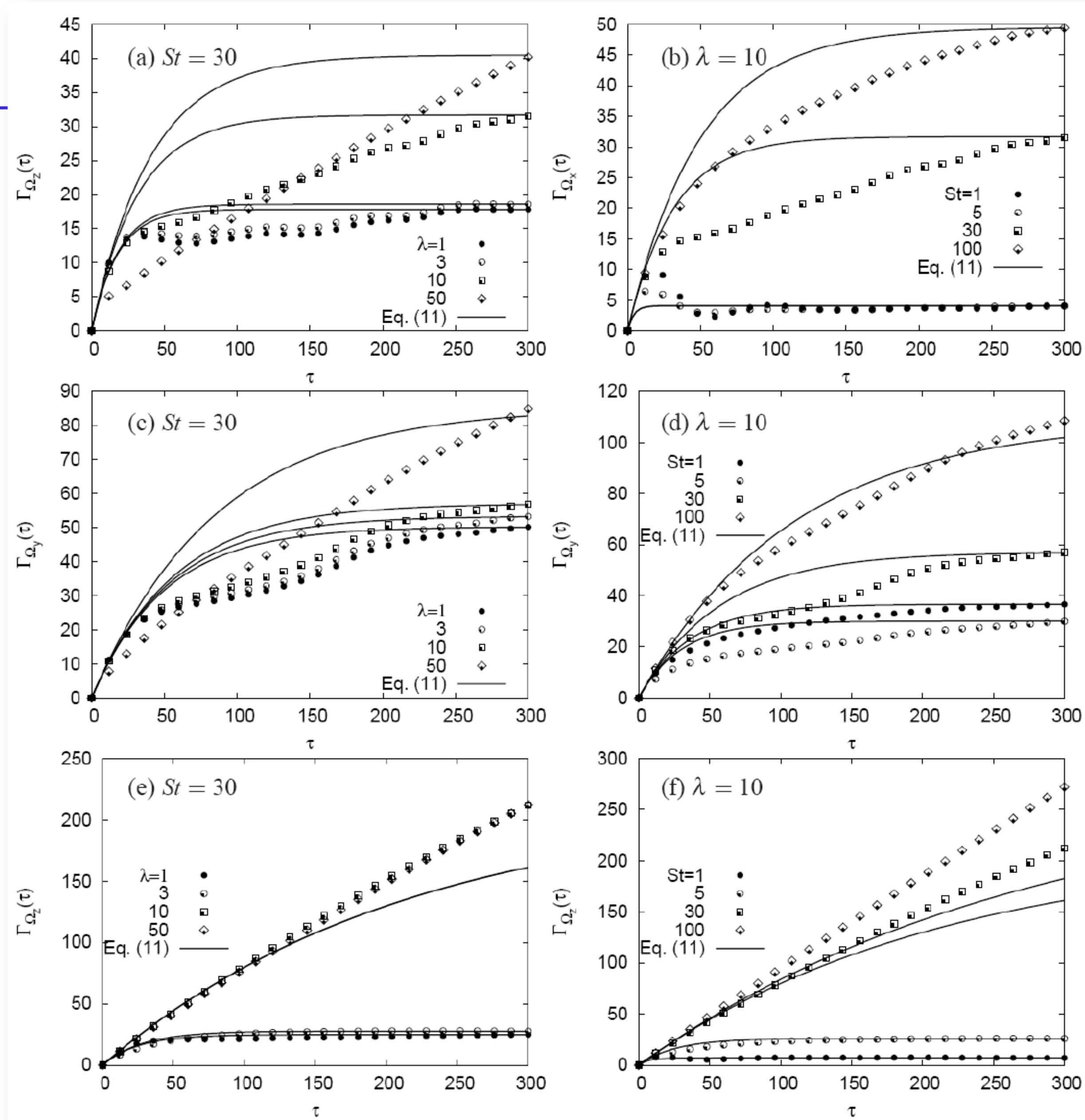
Buffer layer
($5 < z^+ < 50$)



Results – Rotational diffusivity

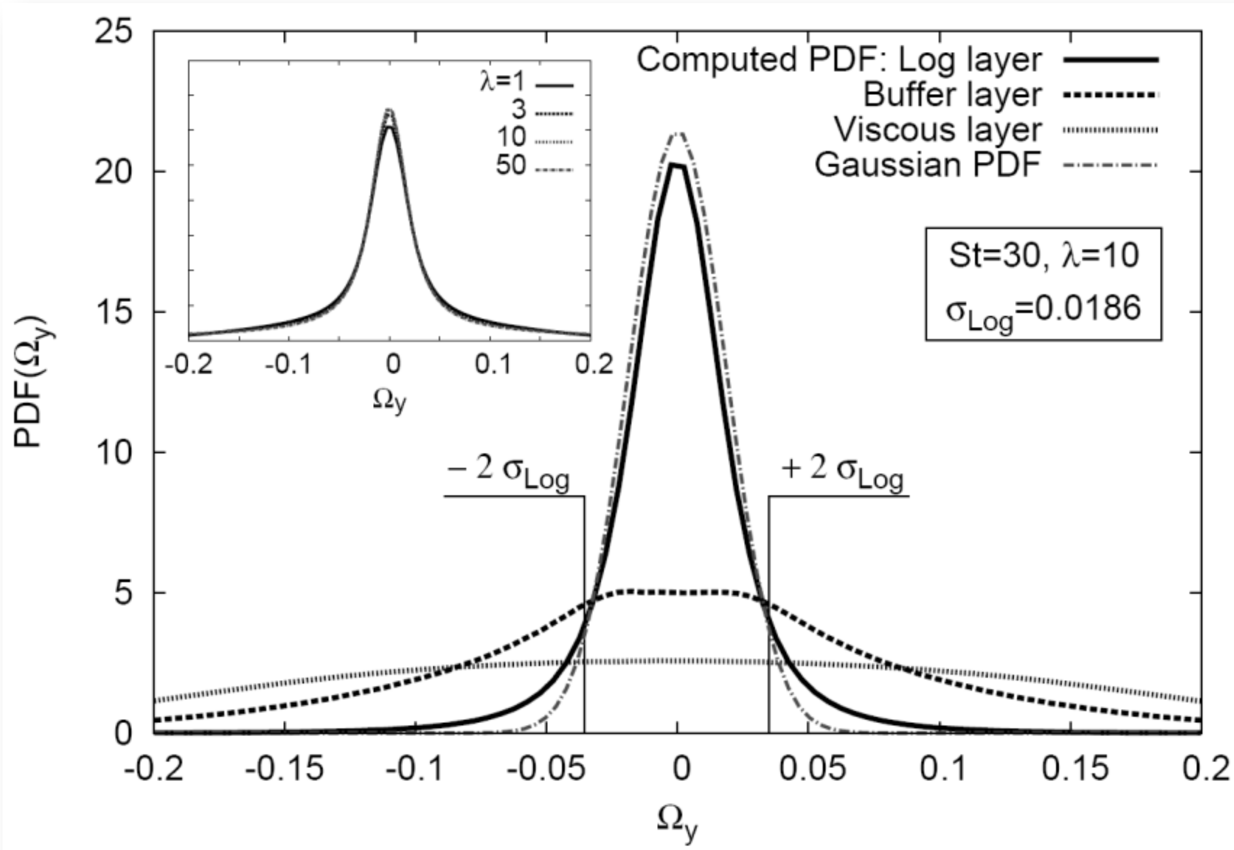
Project supported by COST - European Cooperation in Science and Technology

Viscous layer
($z^+ < 5$)



Results – PDF of Fiber Angular Velocity

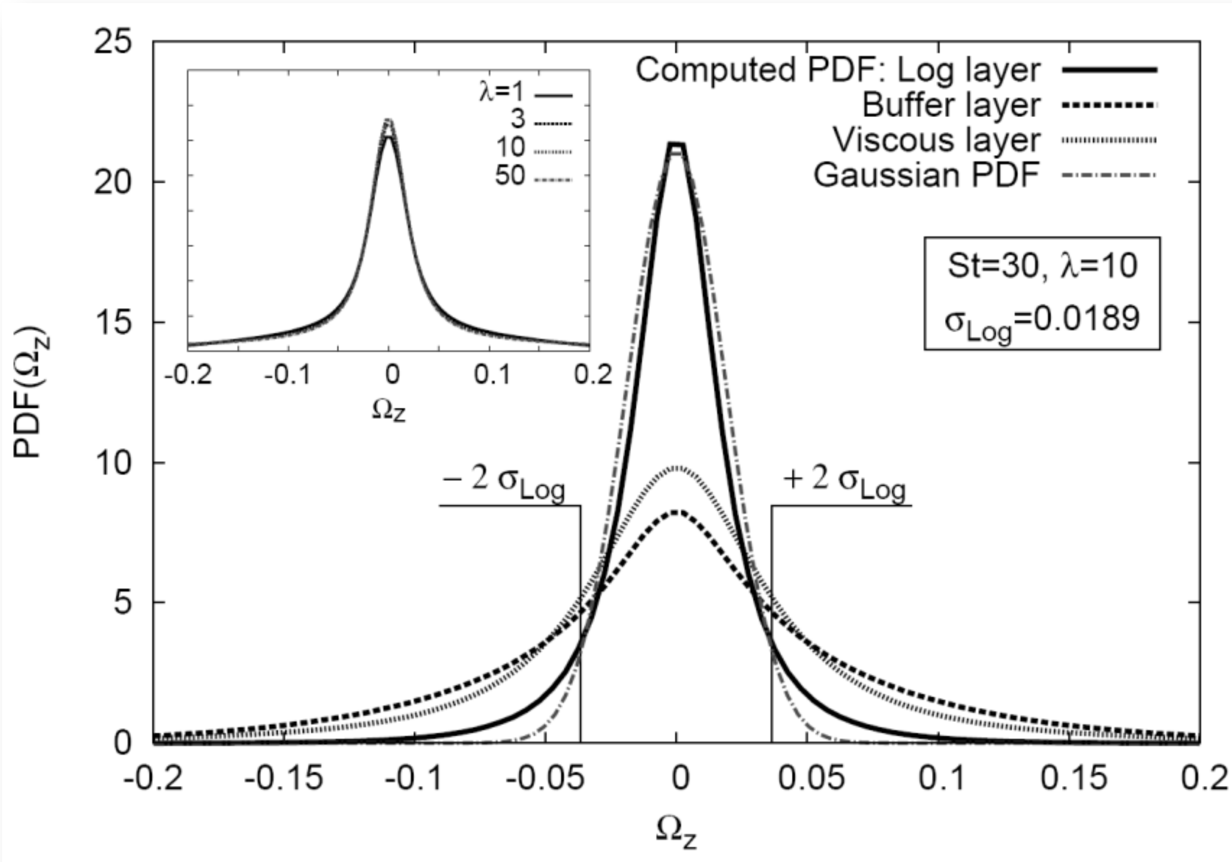
PDF of the fiber angular velocity components in the different sub-regions of the channel (for St=30 and λ=10)



PDFs in the log layer are close to Gaussian except for values of Ω_i larger than $2\sigma_{Log}$

Results – PDF of Fiber Angular Velocity

PDF of the fiber angular velocity components in the different sub-regions of the channel (for St=30 and λ=10)

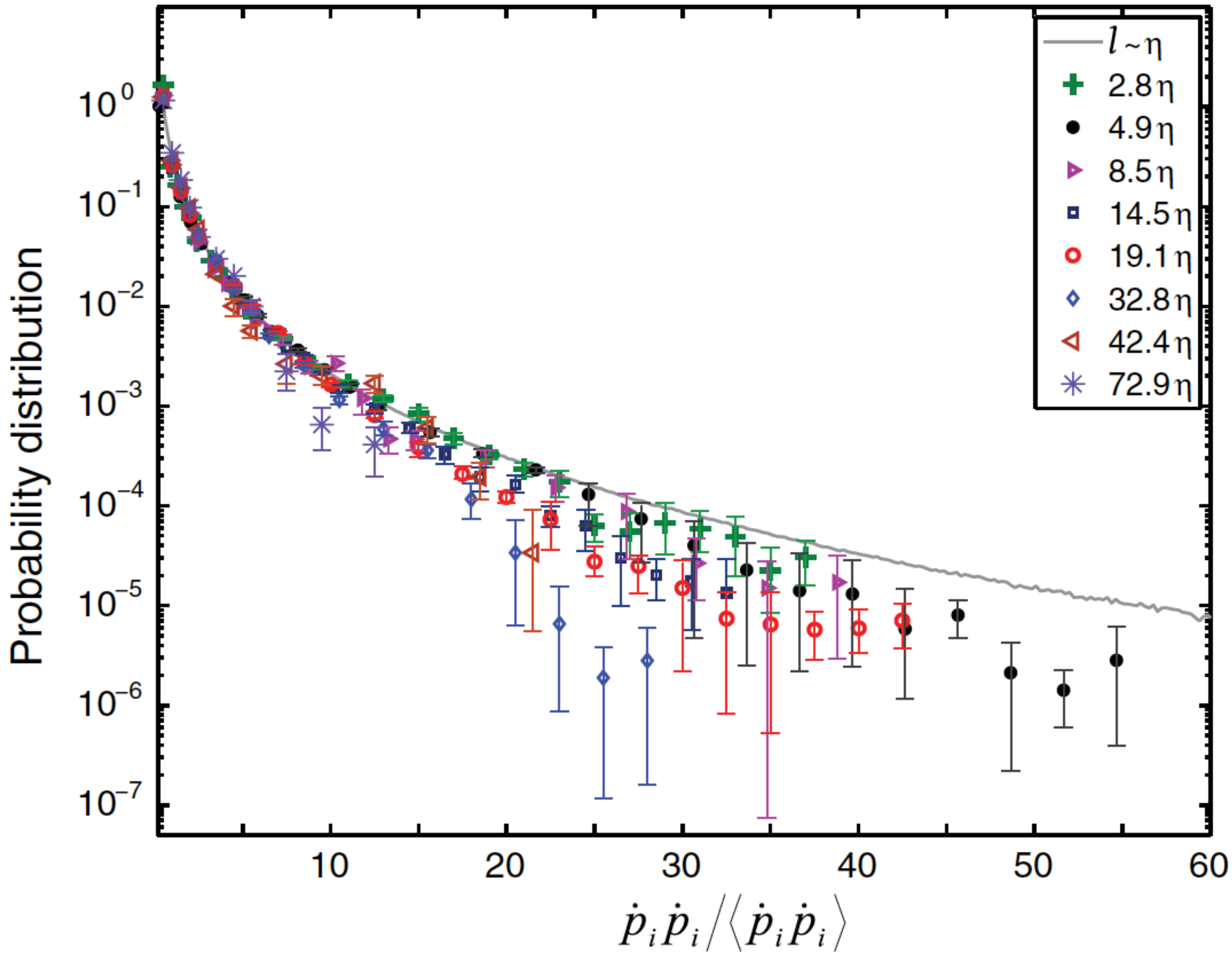


PDFs in the log layer are close to Gaussian except for values of Ω_i larger than $2\sigma_{Log}$

Multiphase Flow Laboratory
University of Udine

Results – PDF of fiber rotation rate

Project supported by COST - European Cooperation in Science and Technology



PDF of fiber rotation rate squared for different fiber lengths (from Parsa & Voth, 2014)

Project supported by COST - European Cooperation in Science and Technology

Lessons Learned

L1. All statistics are strongly affected by inertia (quantitative modification)

L2. Shape (elongation) affects most of the statistics but only from a qualitative point of view

L3. Fiber alignment with mean flow direction is very unstable (especially for large inertia particles)

L4. Segregation into streaks does not affect preferential alignment

L5. RMS of slip velocity exceeds the corresponding mean value by roughly 3 to 5 times: the instantaneous slip velocity may thus frequently change sign. This also holds for the slip spin

L6. Fiber rotation in wall-bounded turbulence can be described within the theory ofdiffusion as a Ornstein-Uhlenbeck process only far away from the walls

Multiphase Flow Laboratory
University of Udine

Project supported by COST - European Cooperation in Science and Technology

References

R1. Do-Quang M, Amberg G, Brethouwer G, Johansson AV, Phys Rev E, vol 89 (2014), pp 013006

R2. Marchioli C, Fantoni M, Soldati A , Phys Fluids, vol 22 (2010), pp 033301

R3. Marchioli C, Soldati A, Acta Mech, vol. 224 (2013) pp 2311-2328

R4. Mortensen PH, Andersson HI, Gillissen JJJ, Boersma BJ, Phys. Fluids, 20 (2008), pp 093302

R5. Soldati A, Marchioli C, Int. J. Multiphase Flow, vol 35 (2009), pp 827-839

R6. Zhang H, Ahmadi G, Fan F-G, McLaughlin JB, Int J Multiphase Flow, vol 27 (2001) pp 971-1009

R7. Zhao F, van Wachem BGM, Acta Mech, vol 224 (2013) pp 2331- 2358

R8. Parsa S, Voth GA, Phys. Rev. Lett, vol 112 (2014), pp 024501

Thank You
For Your Kind Attention!

Any questions?

Final considerations: Modelling Issues in Eulerian-Lagrangian Simulations

Cristian Marchioli

Dept. Electrical, Management and Mechanical Engineering, University of Udine (Italy)

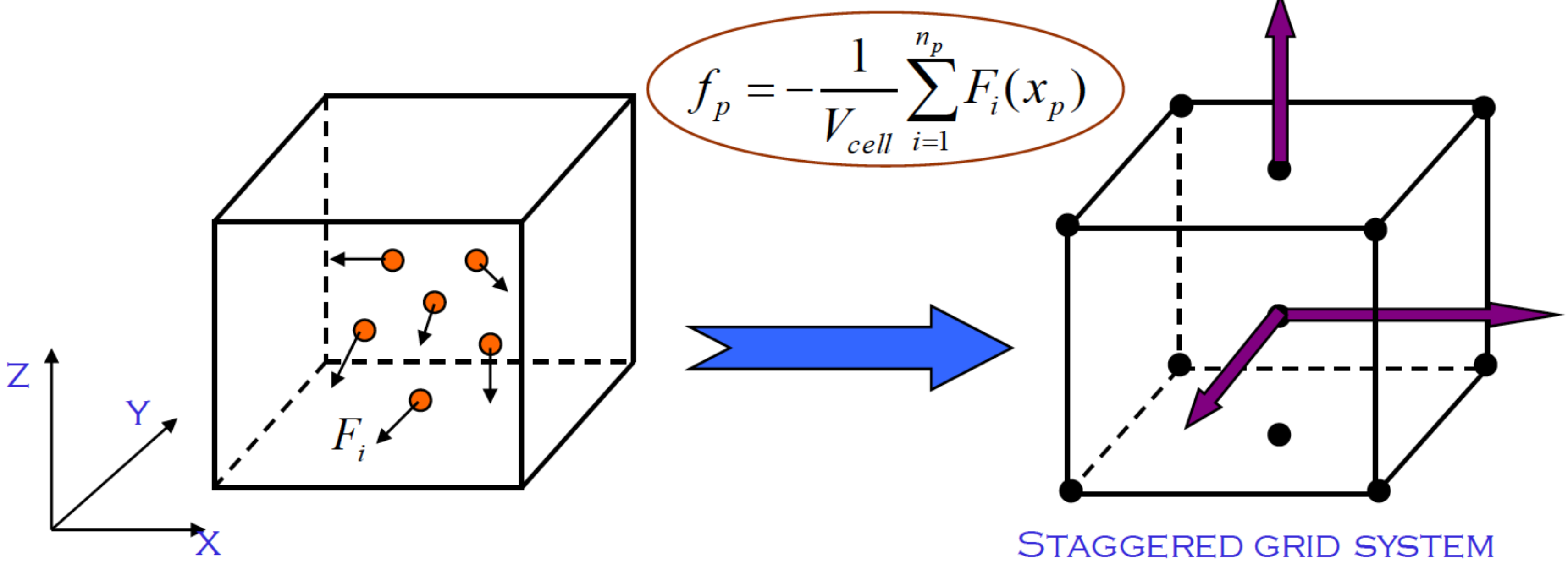
Fiber-Fluid Coupling: Force-coupling

- Drag force is the point-force on the particle

$$m\frac{dv}{dt} = F_i(x_p)$$

- Body force in the momentum equation of fluid:

$$\rho\left[\frac{\partial u}{\partial t} + (u\cdot\nabla)u\right] = -\nabla p + \mu\nabla^2 u + f_p$$



Fiber-Fluid Coupling: Force-coupling

Start from Cauchy's equations of motion, i.e. the principle of linear momentum conservation, expressed in Cartesian tensor notation:

$$\rho \frac{Du_i}{Dt} = \frac{\partial T_{ji}}{\partial x_j} + \rho f_p$$

where T_{ji} is a stress tensor and f_p is a body force. For a Newtonian fluid, the stress tensor is:

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

For details see: Andersson et al, J Fluid Mech, vol 696 (2012), pp 319-329

Fiber-Fluid Coupling: Force-coupling

Multiphase Flow Laboratory
University of Udine

Project supported by COST - European Cooperation in Science and Technology

T_{JI} is a stress tensor in a fluid-particle mixture:

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Micropolar fluids
Eringen (1966)

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \boxed{\mu_r\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) - 2\mu_r \varepsilon_{mij}\omega_m}$$

Fluid angular velocity

$$\Omega_m = \frac{1}{2}\varepsilon_{mji}\frac{\partial u_i}{\partial x_j} \Longrightarrow \boxed{\mu_r\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) = 2\mu_r \Omega_m \varepsilon_{mij}}$$

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \boxed{\underbrace{2\mu_r \varepsilon_{mij}\left(\Omega_m - \omega_m\right)}_{T_{ij}^P}}$$

See Andersson et al, J Fluid Mech (2012)

Multiphase Flow Laboratory
University of Udine

Fiber-Fluid Coupling: Force-coupling

Project supported by COST - European Cooperation in Science and Technology

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \boxed{\underbrace{2\mu_r \varepsilon_{mij}\left(\Omega_m - \omega_m\right)}_{T^P_{ij}}}$$

- T_{ij}^{p} represents the effect of particles on the motion of the fluid-particle mixture

- $\boldsymbol{\omega}_m$ represents the field of micro-rotation, namely the particle angular velocity

The micro-rotation field is obtained from a transport equationfor angular momentum (see Eringen 1966 & Lukaszewicz,1999)

See Andersson et al, J Fluid Mech (2012)

FIBER-FLUID COUPLING: FORCE-COUPLING

Project supported by COST - European Cooperation in Science and Technology

FROM TENSOR ANALYSIS: FOR ANY VECTOR **-N_m** THERE EXISTS AN ANTI-SYMMETRIC TENSOR OF SECOND ORDER THAT CONTAINS THE SAME INFORMATION AS THE VECTOR (SEE E.G. IRGENS, 2008).

THE TORQUE VECTOR **-N_m** CAN THUS BE OBTAINED FROM A *PARTICLE STRESS TENSOR*

$$-N_m = -\frac{1}{2}\varepsilon_{mij} T^P_{ij}\Delta \qquad \textit{WITH} \quad T^P_{ij} = -T^P_{ji}$$

IN PRACTICE, THE TORQUE IS THE SUM OF TORQUES OVER ALL PARTICLES INSIDE THE GRID CELL UNDER CONSIDERATION:

$$T^P_{ij} = \frac{1}{\Delta}\sum_{\ell=1}^{n_p} \varepsilon_{mij} N_m = \frac{1}{\Delta}\sum_{\ell=1}^{n_p} \begin{vmatrix} 0 & +N_3 & -N_2 \\ -N_3 & 0 & +N_1 \\ +N_2 & -N_1 & 0 \end{vmatrix}$$

SEE ANDERSSON ET AL, J FLUID MECH (2012)

Fiber-Fluid Coupling: Force-coupling

Project supported by COST - European Cooperation in Science and Technology

Momentum equation with force- and torque-coupling

$$\rho \frac{Du_i}{Dt} = \frac{\partial T_{ji}}{\partial x_j} + \frac{\partial T_{ji}{}^{p}}{\partial x_j} + \rho f_p$$

where:

$$T_{ij} = T_{ji} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$T_{\mathrm{ji}}^{P} = -T_{\mathrm{ij}}^{P} = -\frac{1}{\Delta}\sum_{\ell=1}^{n_p} \varepsilon_{mij} N_m$$

$$f_p = -\frac{1}{\Delta}\sum_{i=1}^{n_p} F_i(x_p)$$

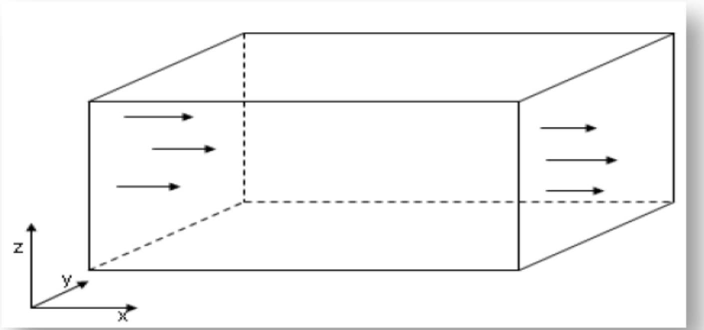
See Andersson et al, J Fluid Mech (2012)

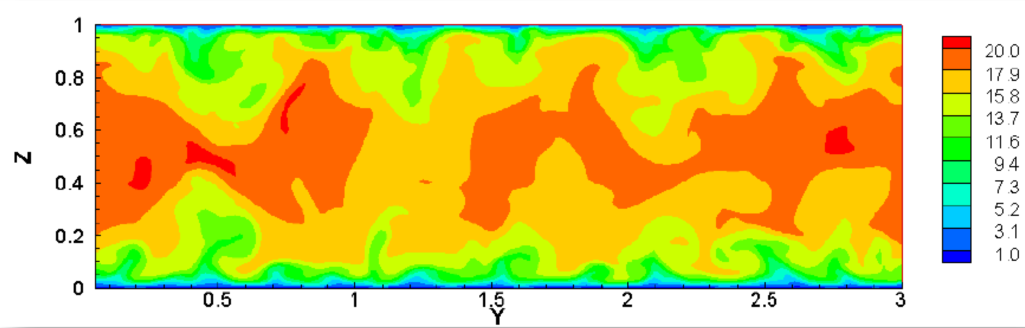
Full force/torque coupling: Some sample results

Multiphase Flow Laboratory
University of Udine
Project supported by COST - European Cooperation in Science and Technology

- Re =360
- Channel size: 6h*3h*h

- Particle number 2.5 million
- Minor axis radius=0.001
- Aspect ratio: 5
- Translational response time: 30

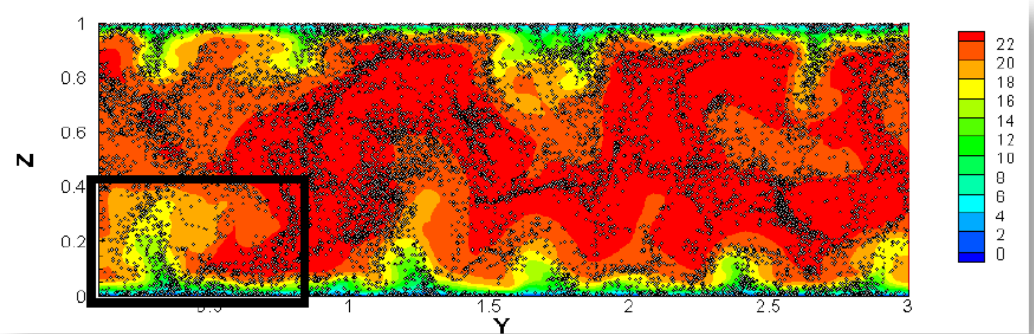




Fluid streamwise velocity in cross-sectional yz plane:

... without fibers

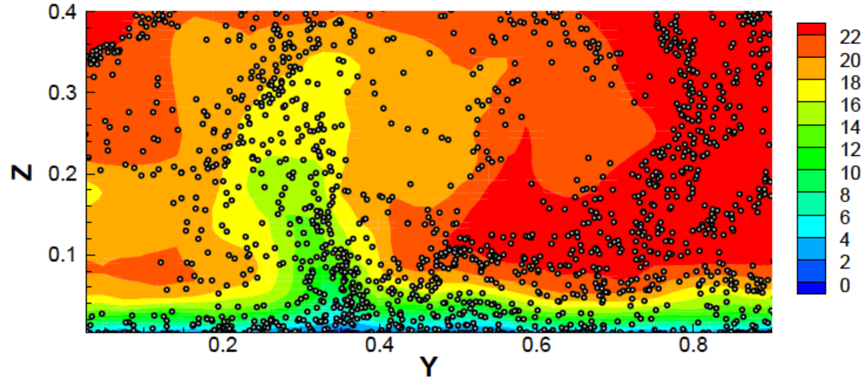
... and with fully-coupled fibers



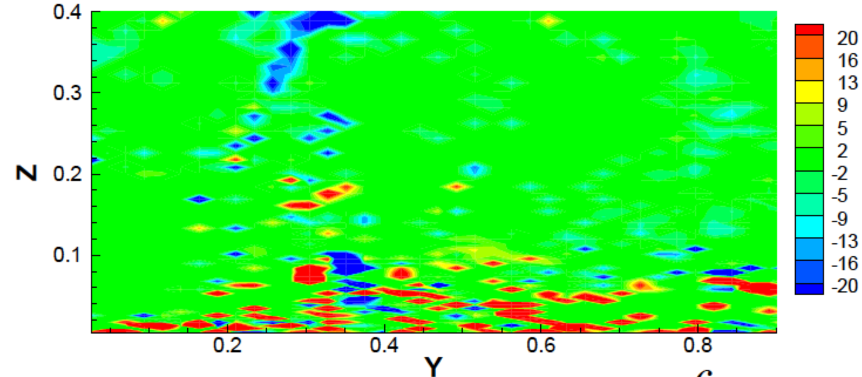
Multiphase Flow Laboratory
University of Udine

Full force/torque coupling: Some sample results

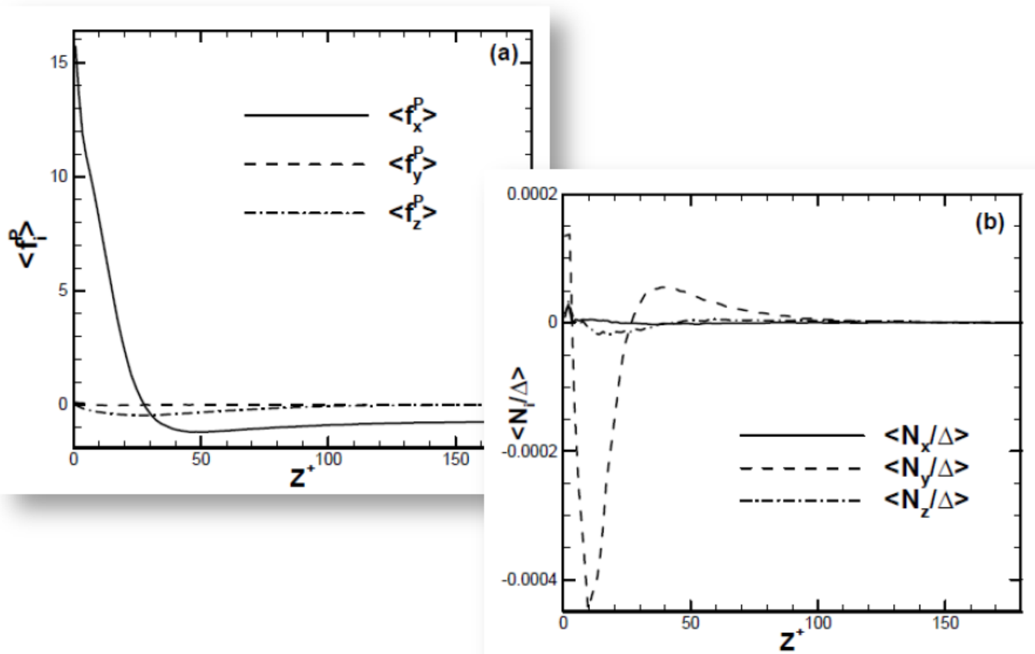
Project supported by COST - European Cooperation in Science and Technology

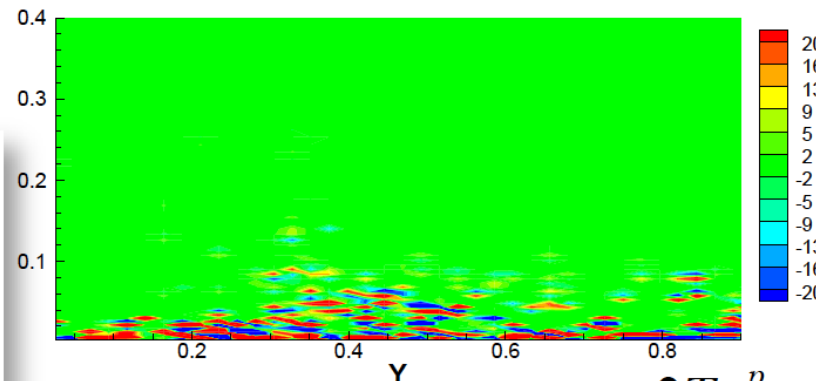


Streamwise velocity contour



Streamwise force on fluid (f_p)



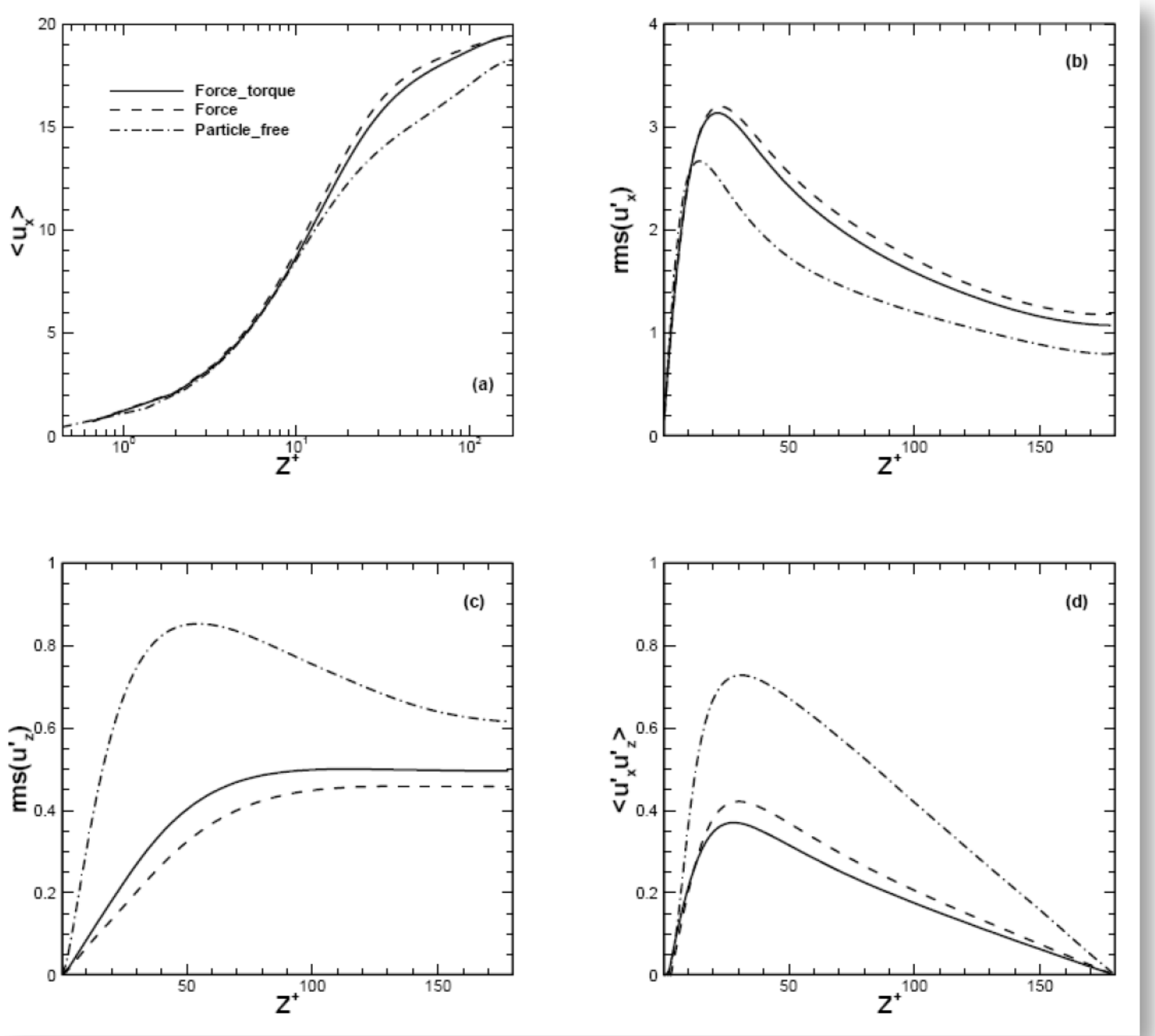


Streamwise torque-force on fluid ($\dfrac{\partial T_{ji}{}^{p}}{\partial x_j}$)

Multiphase Flow Laboratory
University of Udine

Full force/torque coupling: Some sample results

Project supported by COST - European Cooperation in Science and Technology

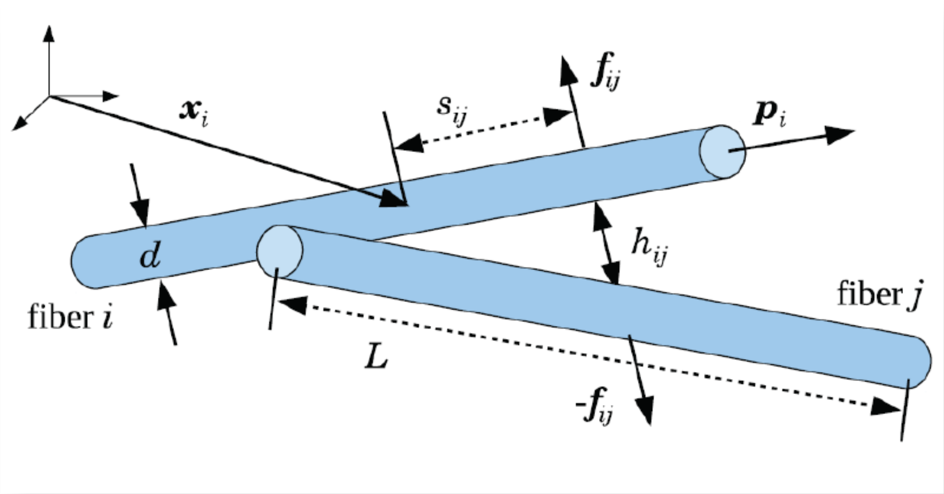


Interactions between Contacting fibers

Multiphase Flow Laboratory
University of Udine

Project supported by COST - European Cooperation in Science and Technology

Collisions between fibers can be modeled rather easily (see Snook et al, Phys Fluids, 2012)



Center-of-mass velocity:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{u}(\boldsymbol{x}_i) + \xi^{-1}\left(\boldsymbol{I} + \boldsymbol{p}_i\boldsymbol{p}_i\right) \cdot \boldsymbol{F}_i$$

Rotational velocity:

$$\dot{\boldsymbol{p}}_i = \boldsymbol{\Omega} \cdot \boldsymbol{p}_i + B\left(\boldsymbol{I} - \boldsymbol{p}_i\boldsymbol{p}_i\right) \cdot \boldsymbol{E} \cdot \boldsymbol{p}_i + \frac{12\xi^{-1}}{L^2}\boldsymbol{\mathcal{T}}_i \times \boldsymbol{p}_i$$

Whenever two fibers i and j reach a minimum separation distance h_{ij} a short-range repulsive force is applied along their common normal:

$$\boldsymbol{f}_{ij} = \begin{cases} 0 & \text{if } h_{ij} > \epsilon, \\ f_0\boldsymbol{n}_{ij} & \text{if } h_{ij} \le \epsilon, \end{cases}$$

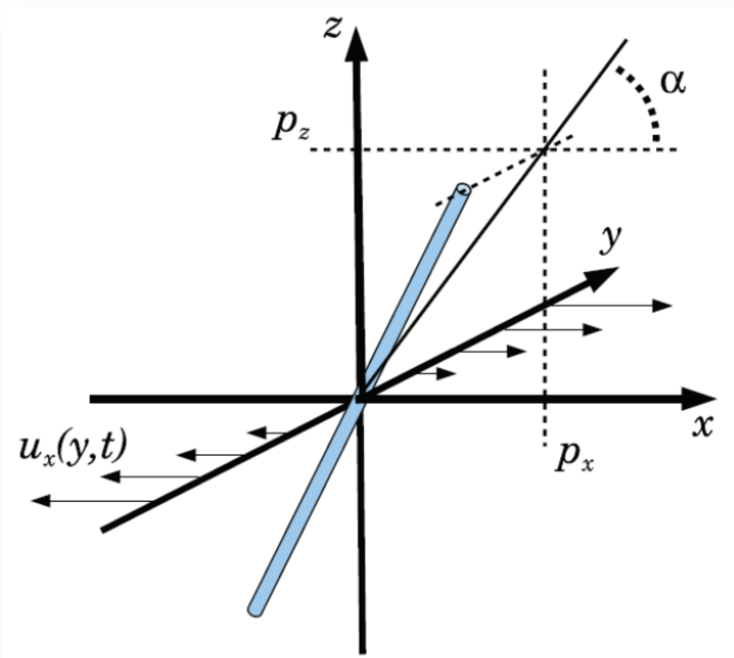
$$\boldsymbol{F}_i = \sum_{j=1}^{N_f} \boldsymbol{f}_{ij}$$

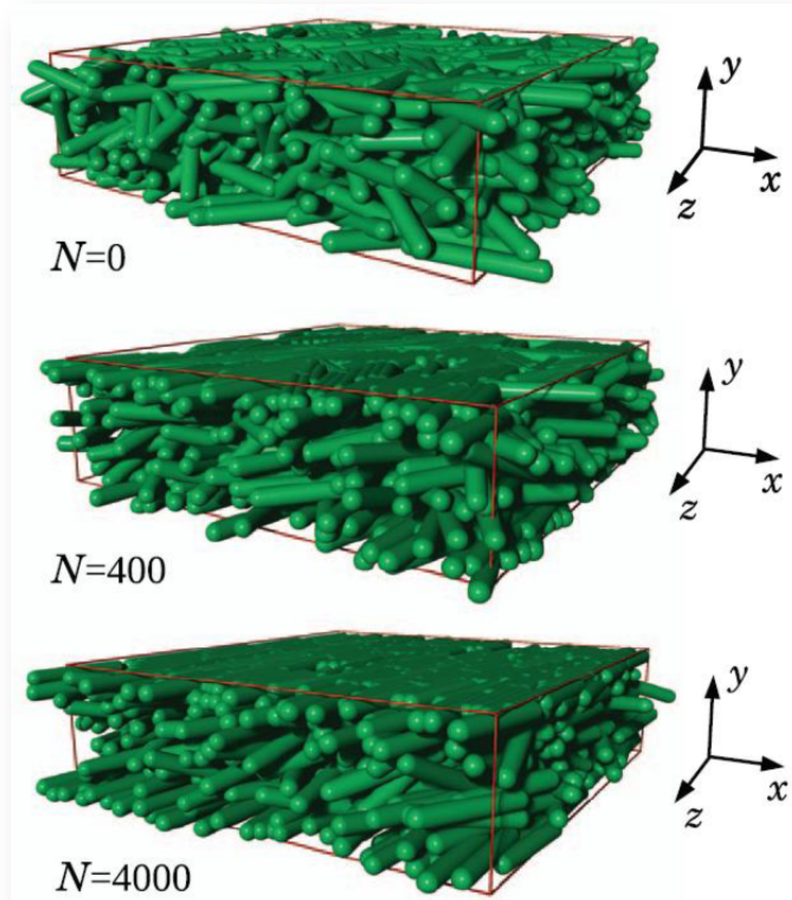
$$\boldsymbol{\mathcal{T}}_i = \sum_{j=1}^{N_f} s_{ij}\boldsymbol{p}_i \times \boldsymbol{f}_{ij}$$

$$\boldsymbol{n}_{ij} = \pm(\boldsymbol{p}_i \times \boldsymbol{p}_j)/|\boldsymbol{p}_i \times \boldsymbol{p}_j|.$$

Interactions between Contacting fibers

The interaction model can accurately predict the orientation distribution of inertialess fibers in oscillatory shear flow In the absence of long-range disturbances and hydrodynamic interactions (Snook et al, Phys Fluids, 2012)



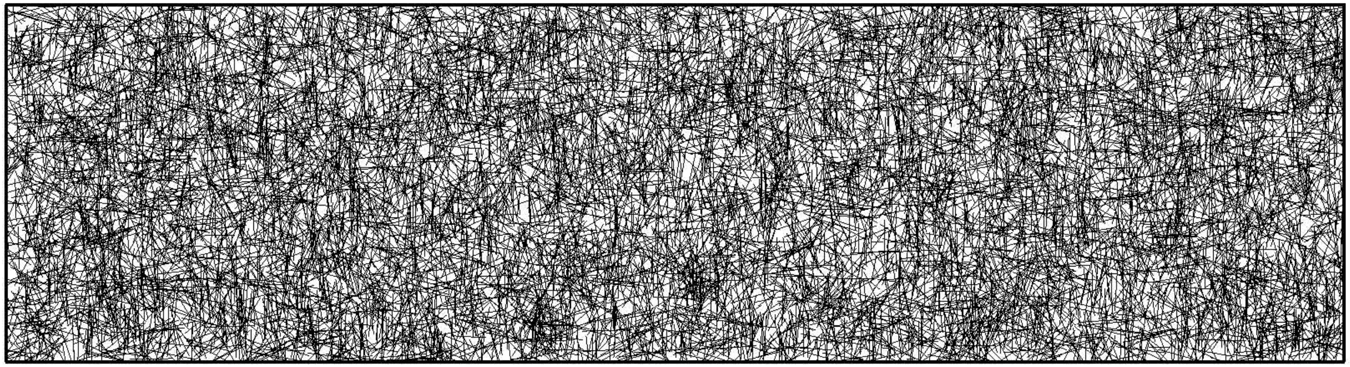


Fibers orient in response to gradients in the flow but disorient in response to fiber-fiber interactions

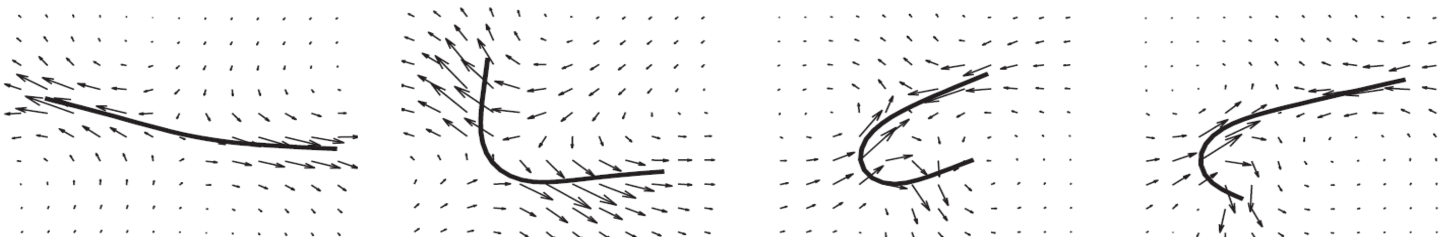
Multiphase Flow Laboratory
University of Udine

Modelling deformability: rigid vs flexible fibers

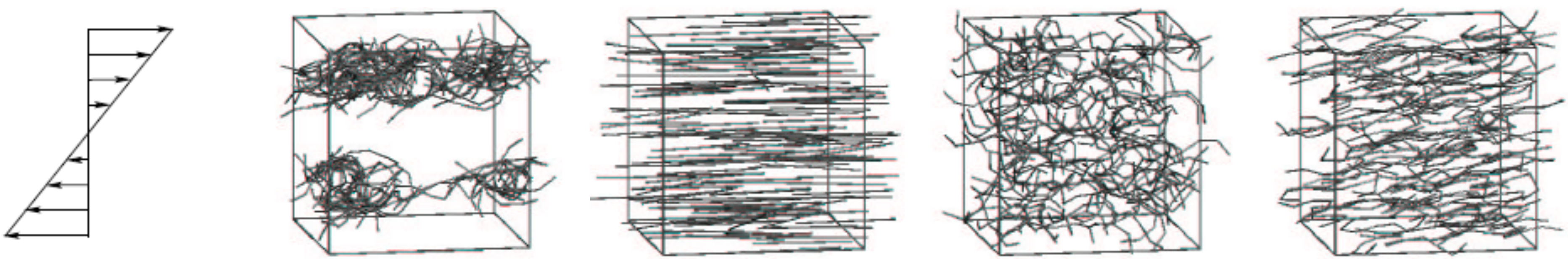
"simple" Rigid fibers in "complex" turbulent flows



"complex" flexible fibers in "simple" shear flow



Isolated flexible fiber in unbounded shear flow (lindstrom and Uesaka, Phys. Fluids, 2007)



flexible fibers (with different bending stiffness) in linear shear flow (l.h. switzer, PhD thesis, 2002)

Multiphase Flow Laboratory
University of Udine

Modelling fiber deformability/flexibility

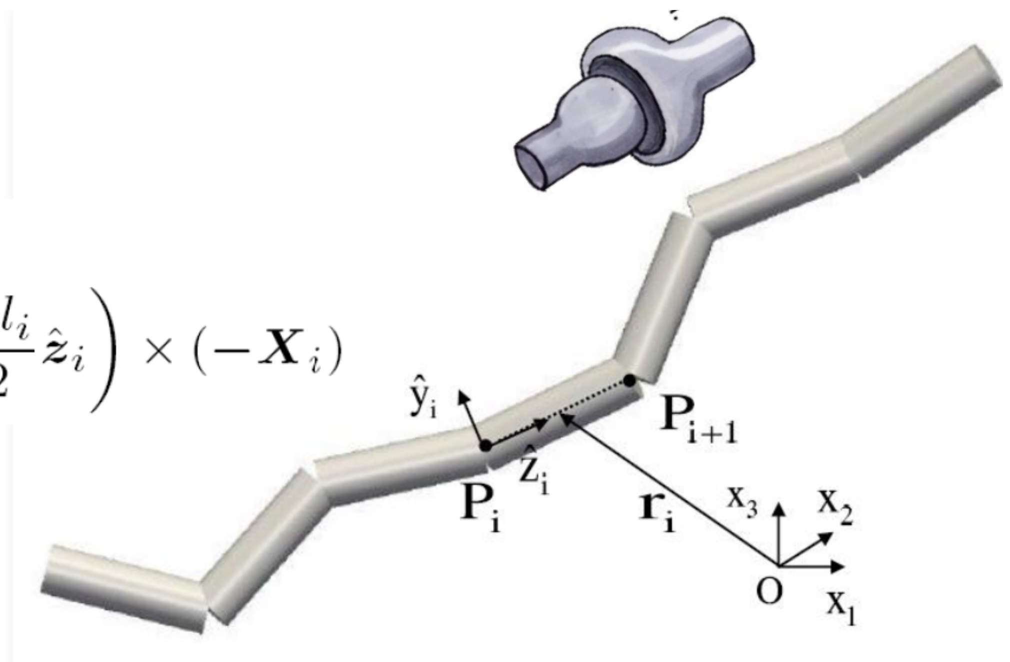
Project supported by COST - European Cooperation in Science and Technology

Bead model: flexible fiber = chain of segments/spheres connected ball-and-socket, or spheroidal, joints (Andric et al., 2013; Slowicka et al., 2013, Derksen, 2010; Lindstrom & Uesaka, 2007; ...)

The model solves for Euler's first and second law for each segment:

$$m_i \ddot{\boldsymbol{r}}_i = \boldsymbol{F}_i^h + \boldsymbol{F}_i^w + \boldsymbol{X}_{i+1} - \boldsymbol{X}_i$$

$$\frac{\partial(\boldsymbol{I}_i \cdot \boldsymbol{\omega}_i)}{\partial t} = \boldsymbol{T}_i^h + \frac{l_i}{2}\hat{\boldsymbol{z}}_i \times \boldsymbol{X}_{i+1} + \left(\frac{-l_i}{2}\hat{\boldsymbol{z}}_i\right) \times (-\boldsymbol{X}_i)$$



and imposes a connectvity constraint between segments (end points of two adjacent segments must coincide):

$$\boldsymbol{r}_i + \frac{l_i}{2}\hat{\boldsymbol{z}}_i = \boldsymbol{r}_{i+1} - \frac{l_{i+1}}{2}\hat{\boldsymbol{z}}_{i+1}$$

$$\dot{\boldsymbol{r}}_i - \dot{\boldsymbol{r}}_{i+1} = \frac{l_i}{2}\boldsymbol{\omega}_i \times \hat{\boldsymbol{z}}_i + \frac{l_{i+1}}{2}\boldsymbol{\omega}_{i+1} \times \hat{\boldsymbol{z}}_{i+1}$$

Connectivity equation

Modelling fiber deformability/flexibility

Multiphase Flow Laboratory
University of Udine

Project supported by COST - European Cooperation in Science and Technology

Hydrodynamic forces and torques acting on each fiber segment (due to the velocity difference Δv between the segment and the surrounding fluid (Andric et al, Acta Mech, 2013)

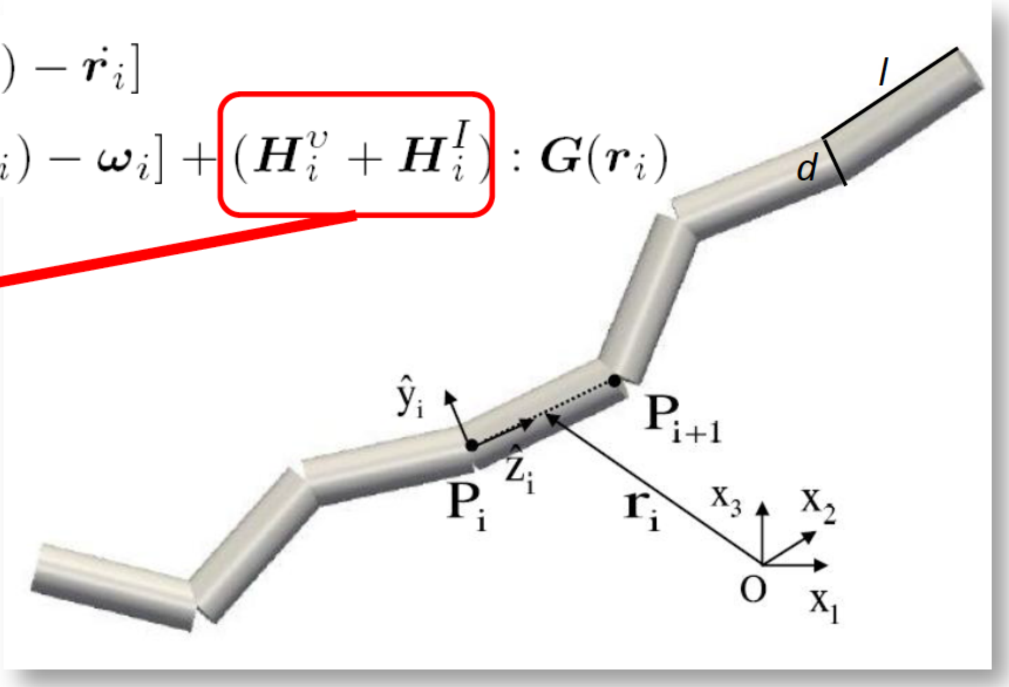
$$\boldsymbol{F}_i^h = \boldsymbol{F}_i^{h,v} + \boldsymbol{F}_i^{h,I} = \boxed{(\boldsymbol{A}_i^v + \boldsymbol{A}_i^I)} \cdot [\boldsymbol{v}(\boldsymbol{r}_i) - \dot{\boldsymbol{r}}_i]$$

$$\boldsymbol{T}_i^h = \boldsymbol{T}_i^{h,v} + \boldsymbol{T}_i^{h,I} = \boxed{(\boldsymbol{C}_i^v + \boldsymbol{C}_i^I)} \cdot [\boldsymbol{\Omega}(\boldsymbol{r}_i) - \boldsymbol{\omega}_i] + \boxed{(\boldsymbol{H}_i^v + \boldsymbol{H}_i^I)} : \boldsymbol{G}(\boldsymbol{r}_i)$$

Viscous

Dynamic

Resistance tensors



The viscous drag force of a fiber segment is approximated with that of a prolate spheroid!

Segment Reynolds number:

$$\mathrm{Re}_s = \rho d \Delta v / \eta$$

$$\Delta v = l\dot{\gamma}$$ Characteristic velocity diff.

$$\dot{\gamma}_i = \sqrt{1/2(\boldsymbol{G}_i : \boldsymbol{G}_i)}$$ Characteristic shear rate

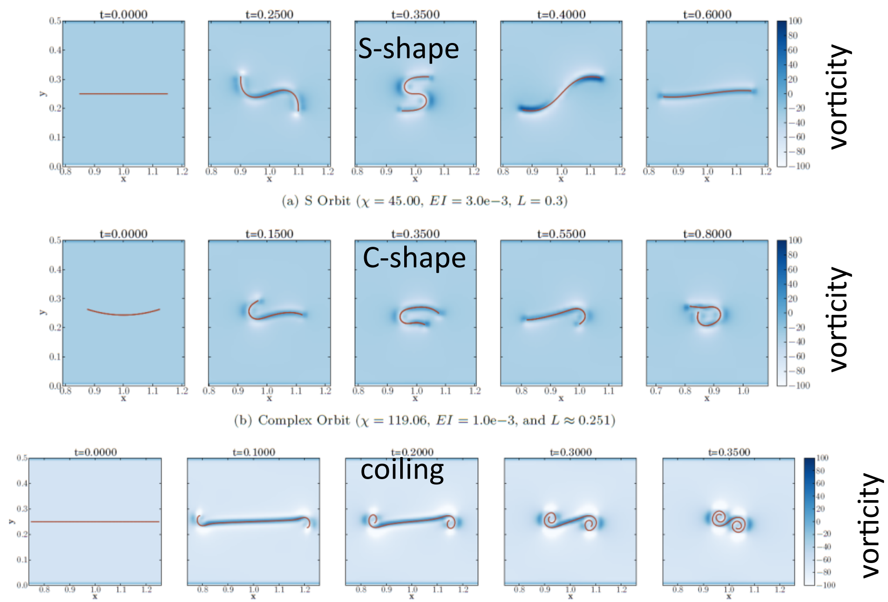
Multiphase Flow Laboratory
University of Udine

Modelling fiber deformability/flexibility

Project supported by COST - European Cooperation in Science and Technology

Realistic models for flexible fibers should incorporate:

- Finite bending stiffness
- Inertia of fiber segments
- Non-creeping fiber-fluid interactions
- Long-/short-range Hydrodynamic fiber-fiber interactions
- Self-interaction
- Two-way coupling



(Courtesy of: Wiens & stockie, 2014)

Models incorporating these features exist (see e.g. Lindstrom & uesaka, phys. Fluids, 2007) but have been used for isolated fibers (qualitative results e.g. regimes of motion), not for **large** systems of **many** fibers in **complex** flows.

Note: <u>wall collision of flexible fibers</u>?!?

Multiphase Flow Laboratory
University of Udine

Modelling fiber deformability/flexibility

Project supported by COST - European Cooperation in Science and Technology

Fibers are keen to bend under the action of local velocity gradients of turbulence if they are long.

Below a certain critical length, they will not deform.

This length can be found assuming that the spatial conformation of a flexible fibers are analog to conformation of flexible polymers in a good solvent, namely a solvent where monomer-monomer, monomer-solvent, and solvent-solvent interactions are very close (Brouzet et al., PRL, 2014).

$$\underbrace{P_\ell \sim \rho \ell^3 \epsilon}_{\text{Power of the turblent field}} \quad = \quad \underbrace{P_b(\ell) = (EI)^2/\alpha \ell^5}_{\text{Power required to bend the fiber}}$$

$$\Downarrow$$

Persistence length $\quad l_p^P \propto \ell_m = (EI)^{1/4}/(\rho\eta\epsilon)^{1/8}$