TECHNISCHE UNIVERSITÄT WIEN
Vienna | Austria

Internal and surface waves in turbulence

Francesco Zonta [a], George Giamagas [a, b], Alessio Roccon [b], Pejman Hadi Sichani [c], Alfredo Soldati [a,b]

[a] TU Wien - Institute of Fluid Mechanics and Heat Transfer
[b] University of Udine - Department of Environmental and Energy Engineering Science
[c] University of Rochester - Department of Mechanical Engineering

1. Stably-stratified turbulence: internal waves

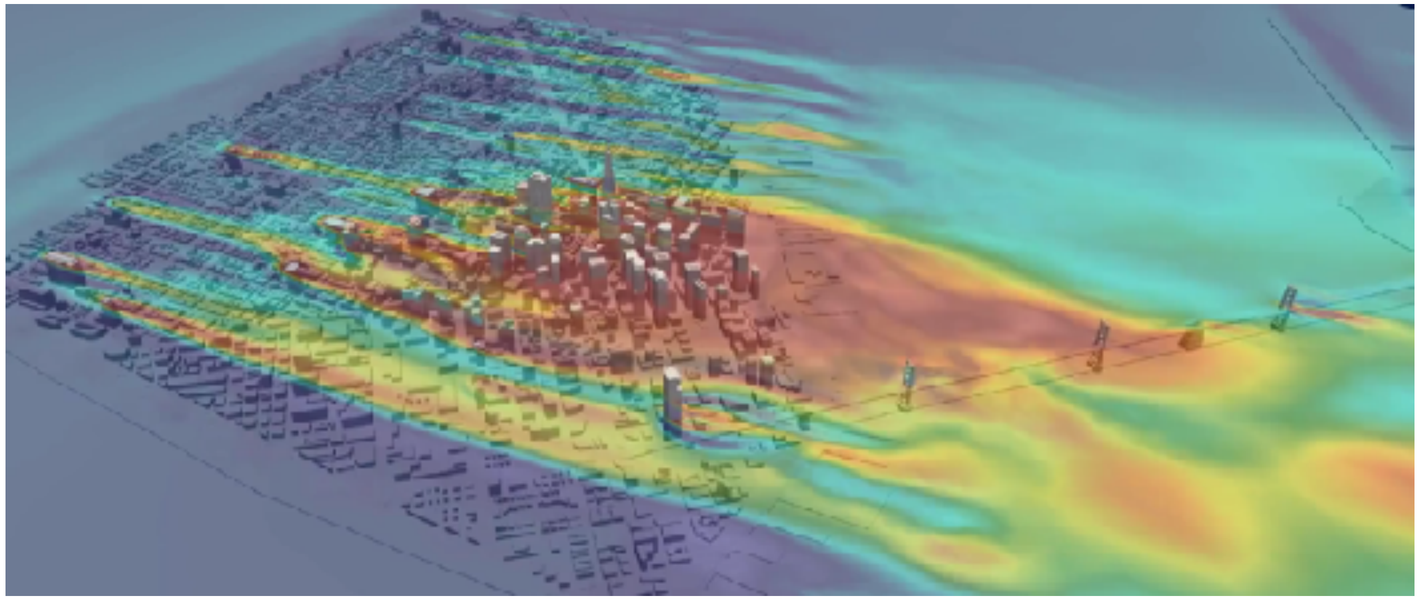
2. Viscosity-stratified turbulence: interfacial waves

Motivation
(Thermally) Stably-stratified wall-bounded turbulence

Algae/phytoplankton surfacing in lakes and oceans (which are usually stably stratified).

Algae blooms
(lake Erie, US)

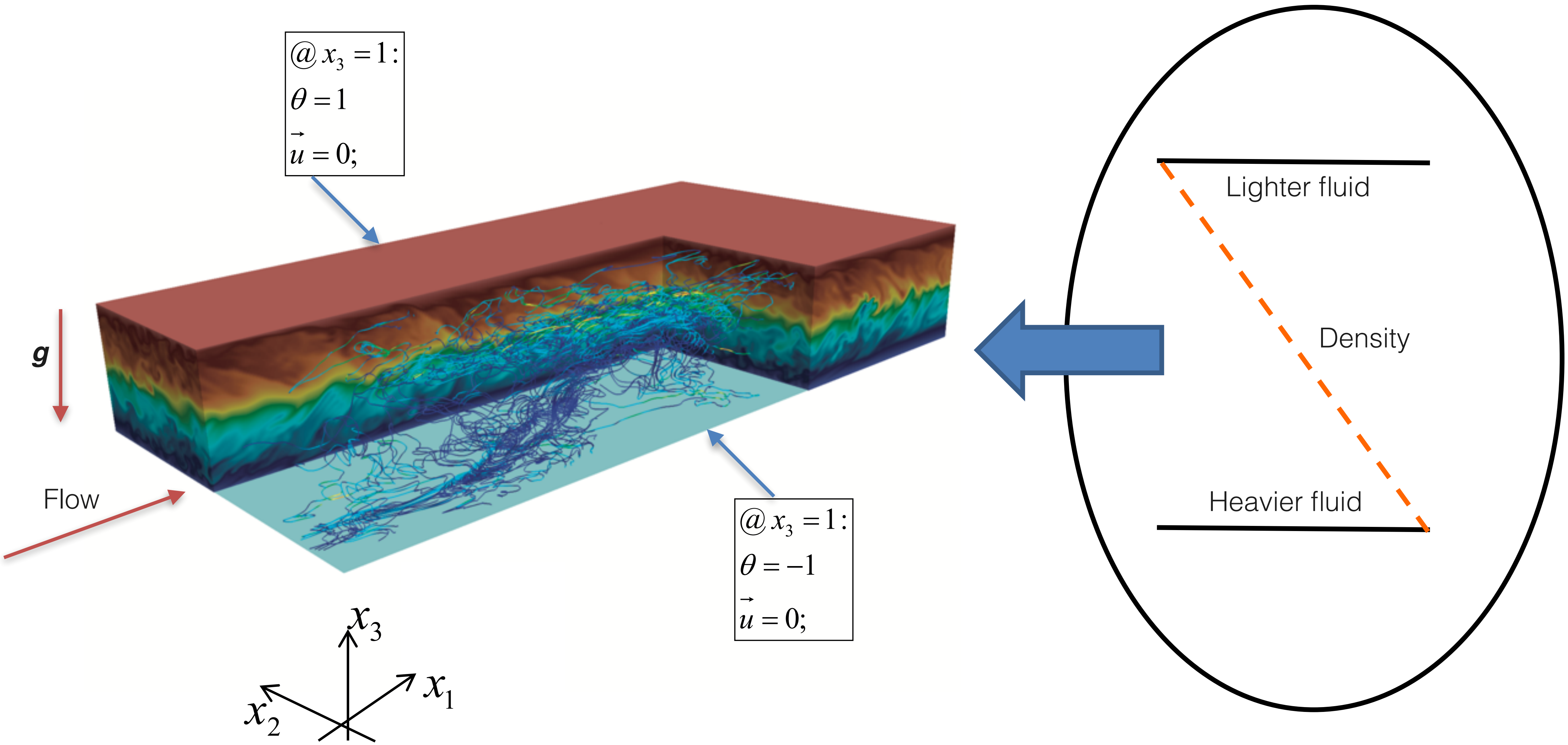
Dispersion of pollutants in winter
(when the atmospheric boundary layer
is often stably-stratified)



Flow configuration: Stably-stratified wall-bounded turbulence

Aim of the work: study the interaction between *turbulence* and *stable stratification* in a wall-bounded flow, at "large" Re;

We take the simplest possible configuration:

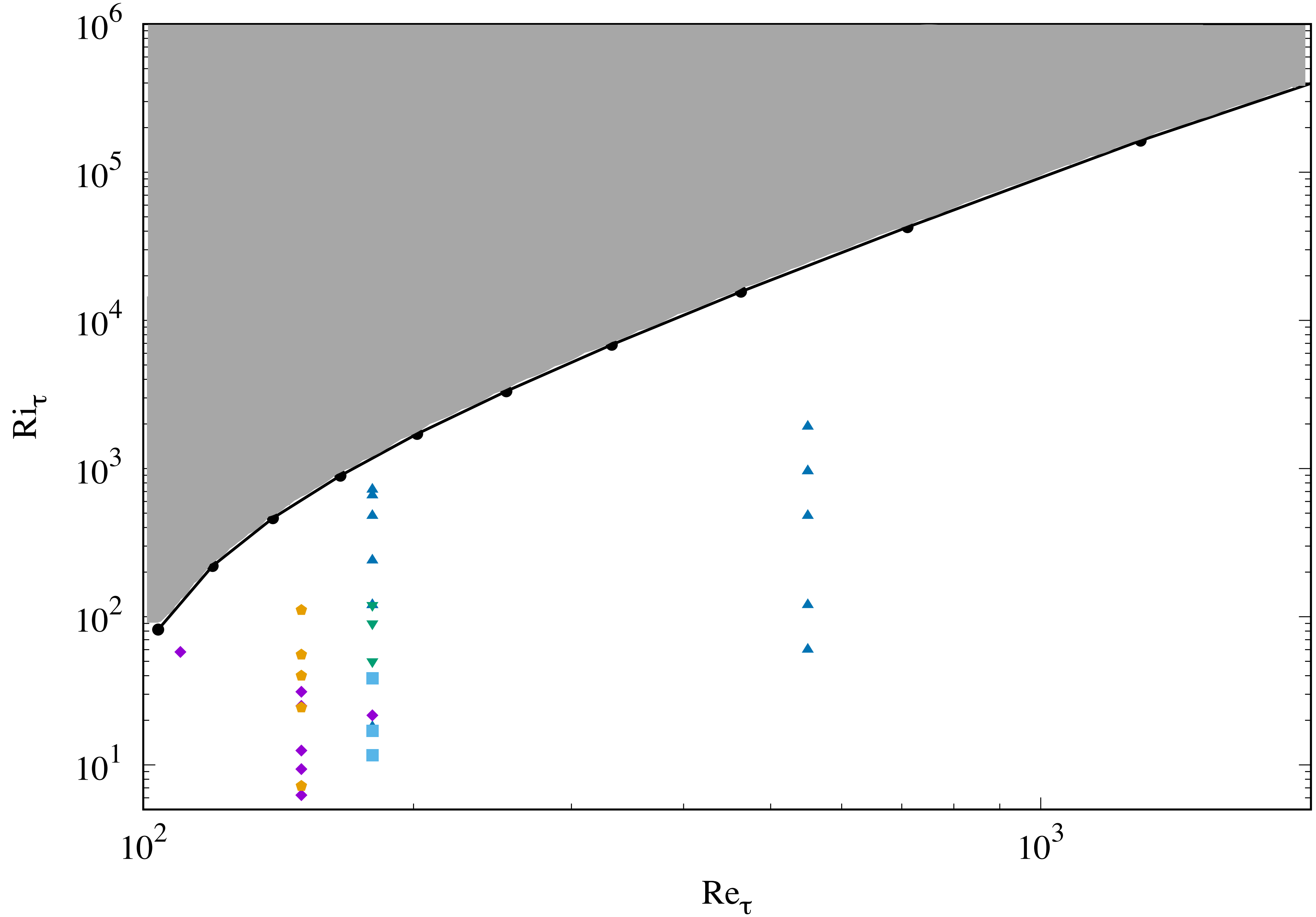


Flow phenomenology: stability map

$$Re_\tau = \frac{u_\tau h}{\nu_0},$$
turbulence importance

$$Ri_\tau = \frac{g\Delta\rho h}{\rho_0 u_\tau^2}$$
stratification importance

$$Pr = \frac{\nu_0}{\kappa_0}$$

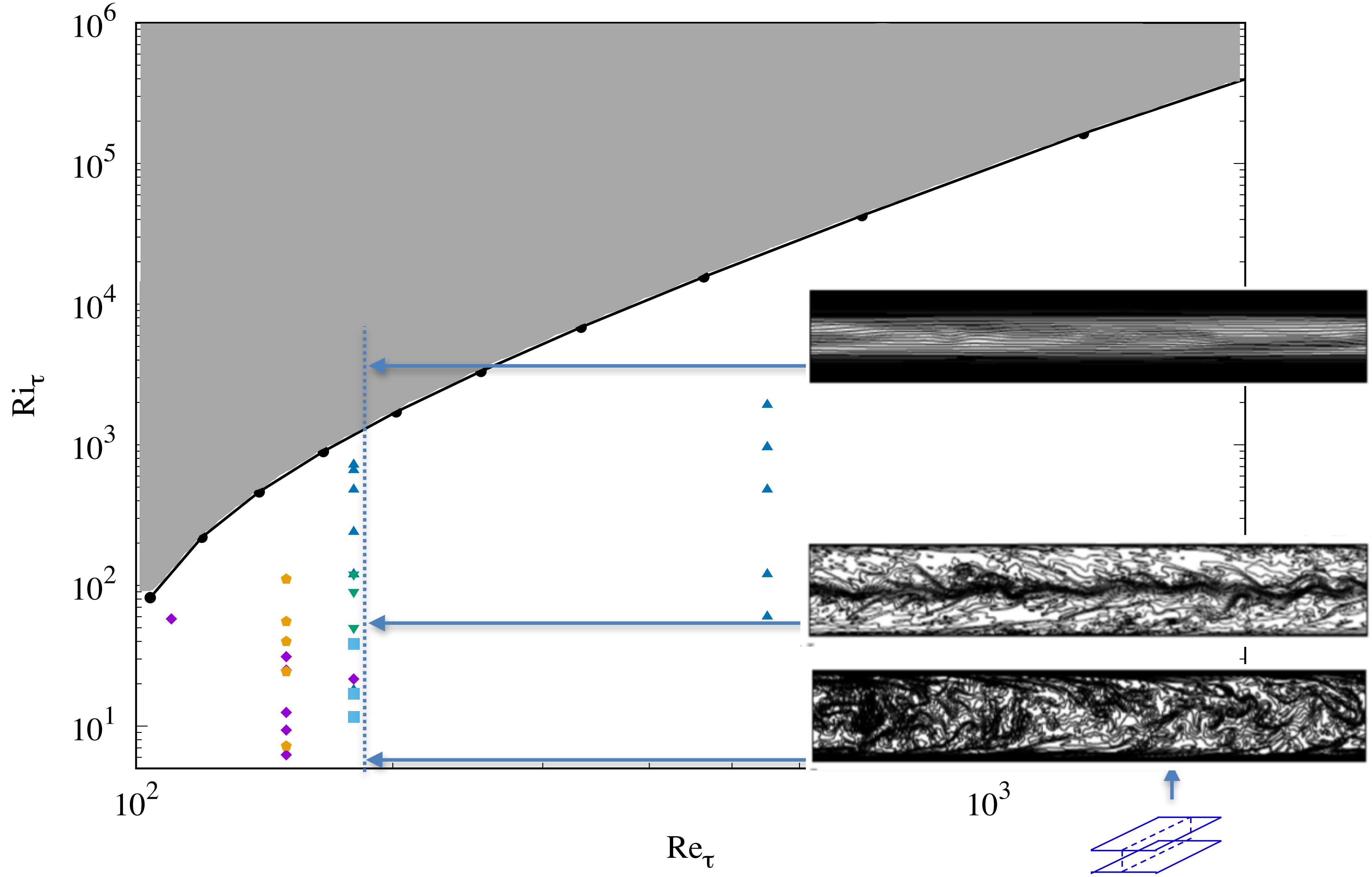


[1] Zonta F and Soldati A, 2018. Applied Mechanics Review, *70, 040801*.

Flow phenomenology: stability map

$$Re_\tau = \frac{u_\tau h}{\nu_0}, \qquad Ri_\tau = \frac{g \Delta\rho h}{\rho_0 u_\tau^2} \qquad Pr = \frac{\nu_0}{\kappa_0}$$

turbulence importance — stratification importance



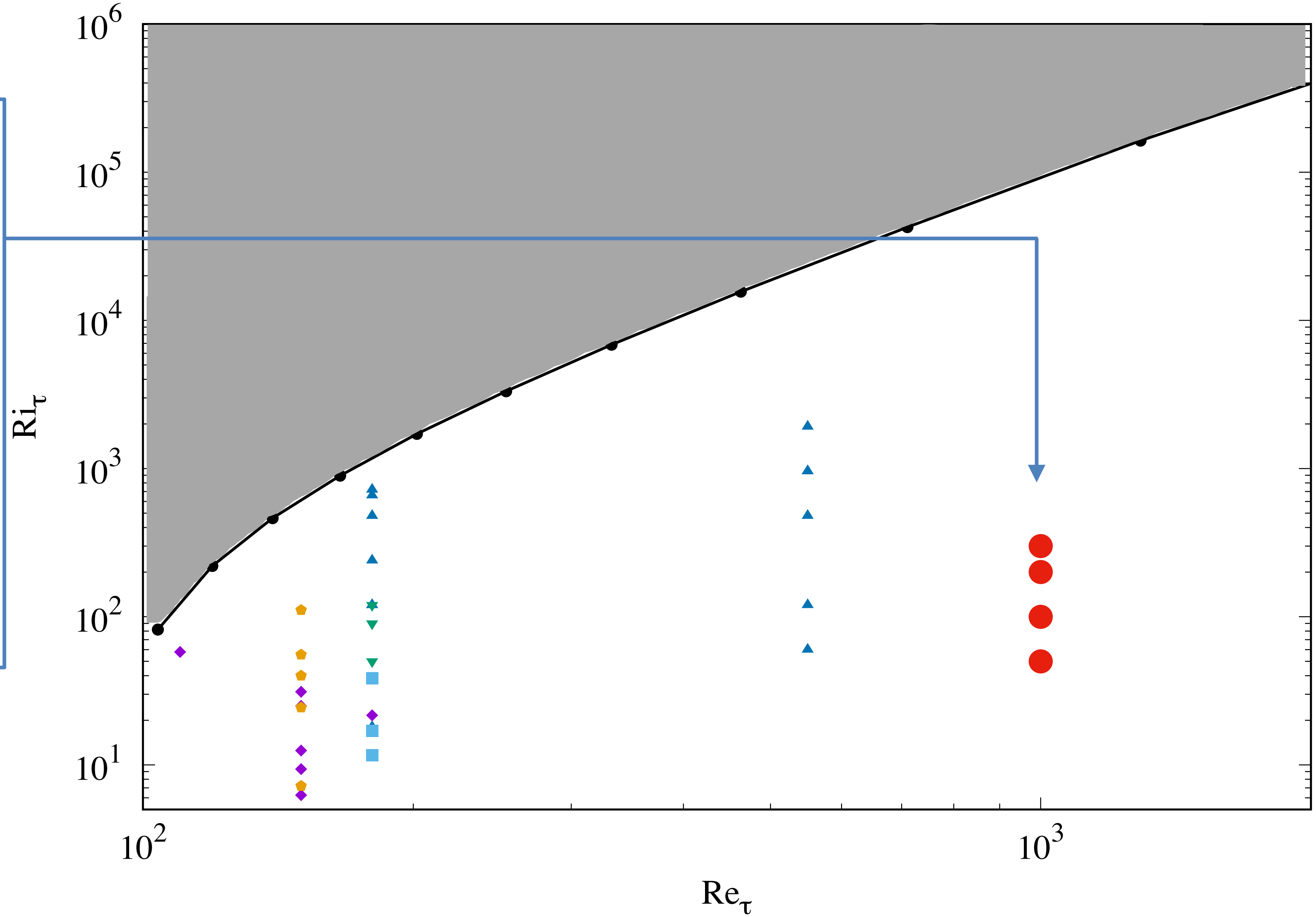
[1] Zonta F and Soldati A, 2018. Applied Mechanics Review, *70, 040801*.

Flow phenomenology: stability map

$$Re_\tau = \frac{u_\tau h}{\nu_0}, \qquad Ri_\tau = \frac{g \Delta\rho h}{\rho_0 u_\tau^2} \qquad Pr = \frac{\nu_0}{\kappa_0}$$

turbulence importance — stratification importance

- **No study in closed Poiseuille channel at Re_t>550**
- We want to move to Re_t=1000, moderate stratification
- DNS data crucial for turbulence modeling (Pr_t, Ri_g, mixing efficiency… Previous talk!)



[1] Zonta F and Soldati A, 2018. Applied Mechanics Review, *70, 040801*.

Numerical approach: OB approximation

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{Du_i}{Dt} = -\frac{\partial p'}{\partial x_i} + \frac{1}{Re_\tau}\frac{\partial^2 u_i}{\partial x_j{}^2} - Ri_\tau \theta \delta_{3,i} + \delta_{1,i}$$

$$\frac{D\theta}{Dt} = \frac{1}{Re_\tau Pr}\frac{\partial^2 \theta}{\partial x_j{}^2}$$

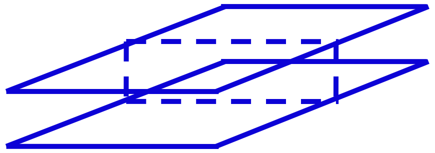
- Direct Numerical Simulation, channel turbulence with air
- Imposed pressure gradient
- Pseudo-Spectral Fourier-Chebyshev method

Simulation	Pr	Re_τ	Ri_τ	Grid
S0	0.71	1000	0	$1024 \times 1024 \times 1025$
S50	0.71	1000	50	$1024 \times 1024 \times 1025$
S100	0.71	1000	100	$1024 \times 1024 \times 1025$
S200	0.71	1000	200	$1024 \times 1024 \times 1025$
S300	0.71	1000	300	$1024 \times 1024 \times 1025$

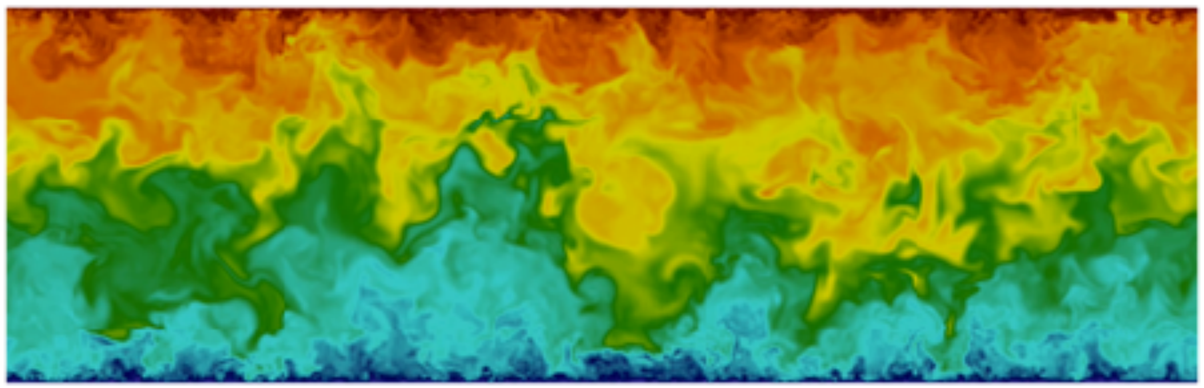
$$Re_\tau = \frac{u_\tau h}{\nu_0} = 1000, \qquad Pr = \frac{\nu_0}{\kappa_0} = 0.71\ , \qquad Ri_\tau = \frac{g\beta_0 \Delta\theta h}{u_\tau^2}\ .$$

Temperature field

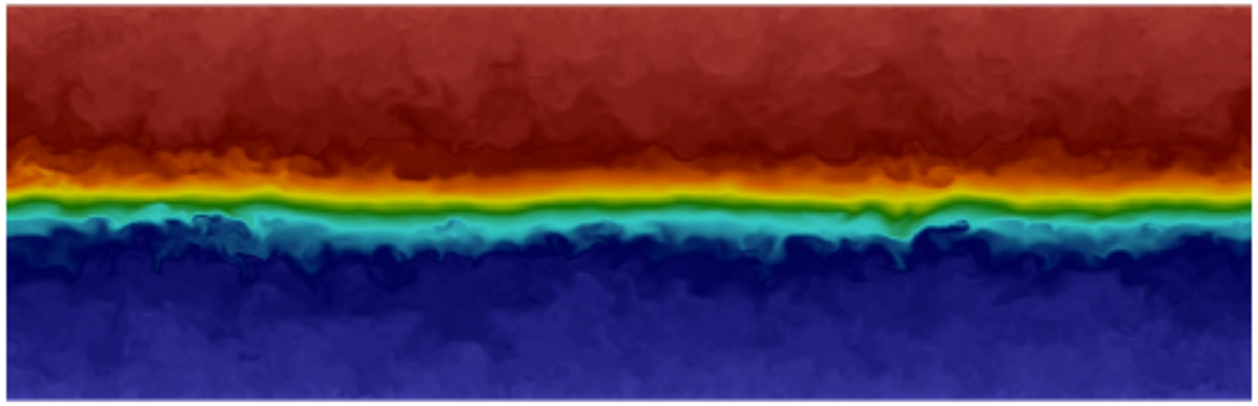
Visualization of temperature on a cross section



$Ri_\tau = 0$



$Ri_\tau = 200$



Neutrally buoyant

- Higher mixing
- Mixing is driven by turbulence structures

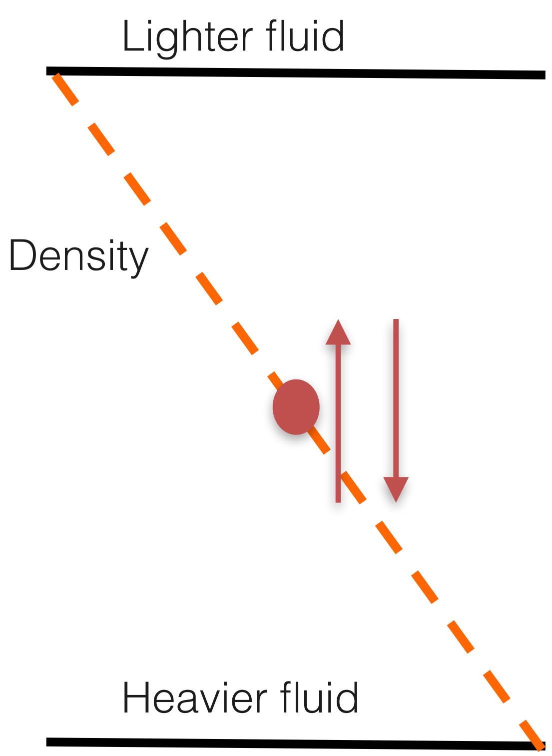
Stably stratified

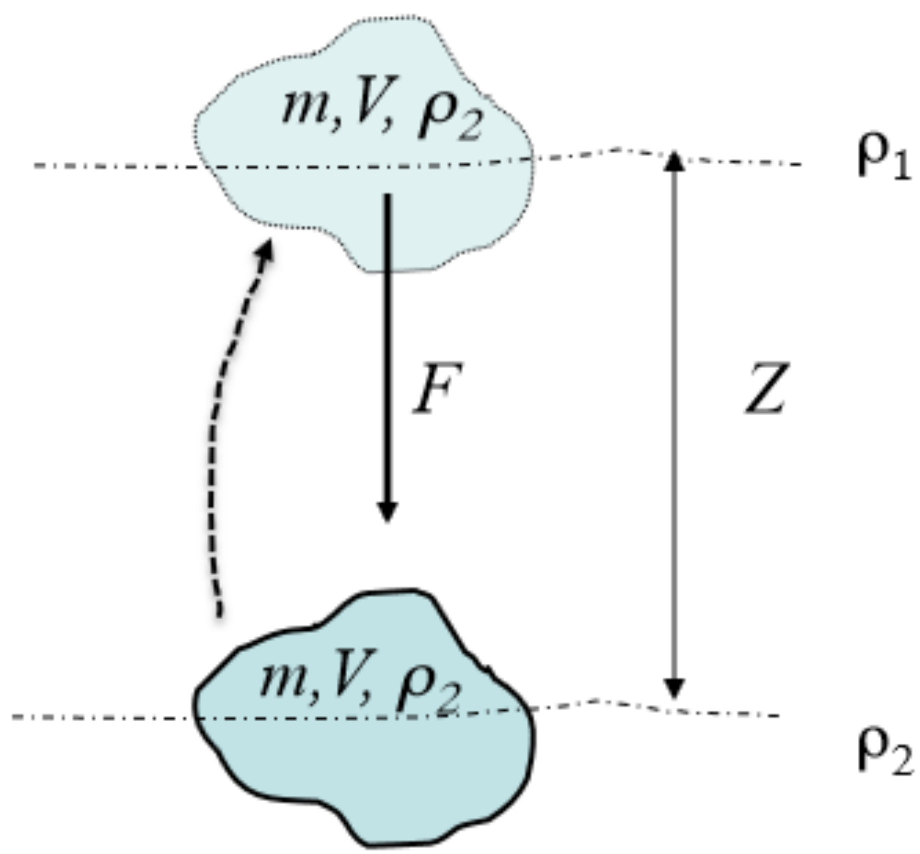
- Lower mixing
- Internal waves act like barriers

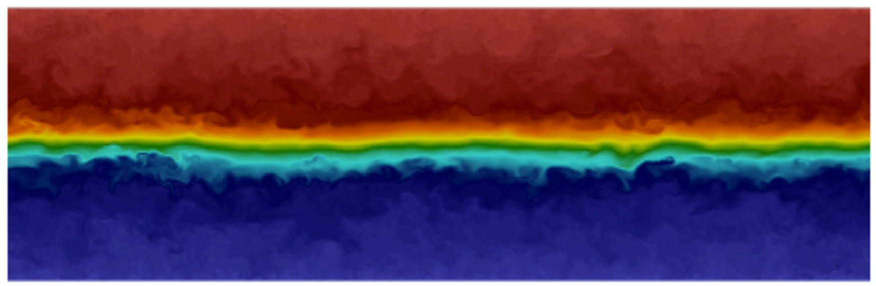
Internal Gravity Waves - IGW

The origin of Internal Waves:

Force balance on a fluid particle

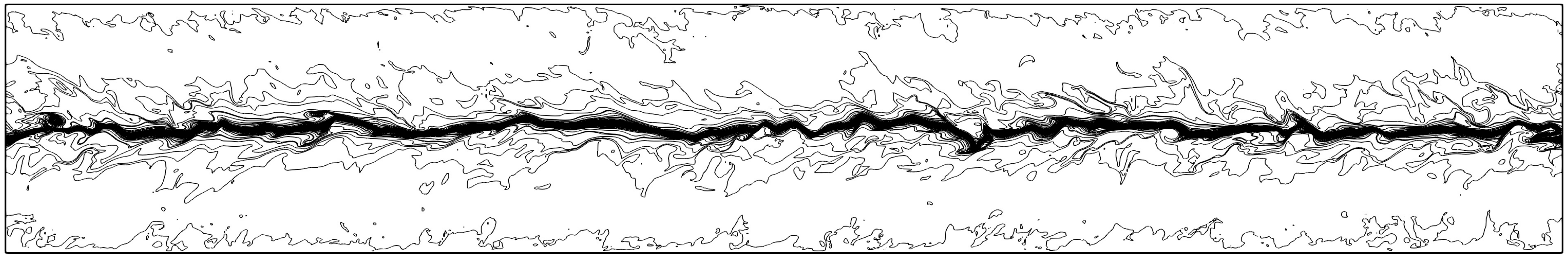


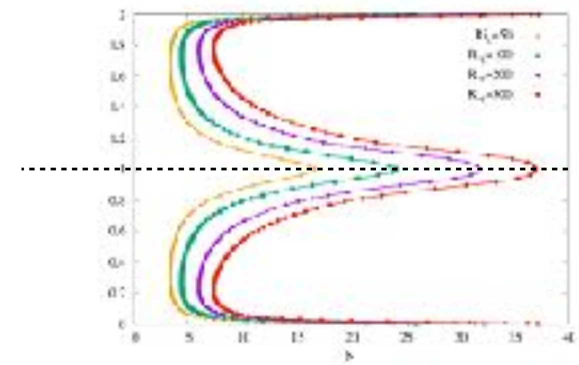




$$
\begin{cases}
\dfrac{\partial^2 Z}{\partial t^2} = \dfrac{g}{\rho}\dfrac{\partial \rho}{\partial z} Z = N^2 Z \\
N = \left(\dfrac{g}{\rho}\dfrac{\partial \rho}{\partial z}\right)^{1/2}
\end{cases}
$$

$$
\left[N = \left(-g\beta \frac{\partial \langle \theta \rangle}{\partial z} \right)^{1/2} \right]
$$

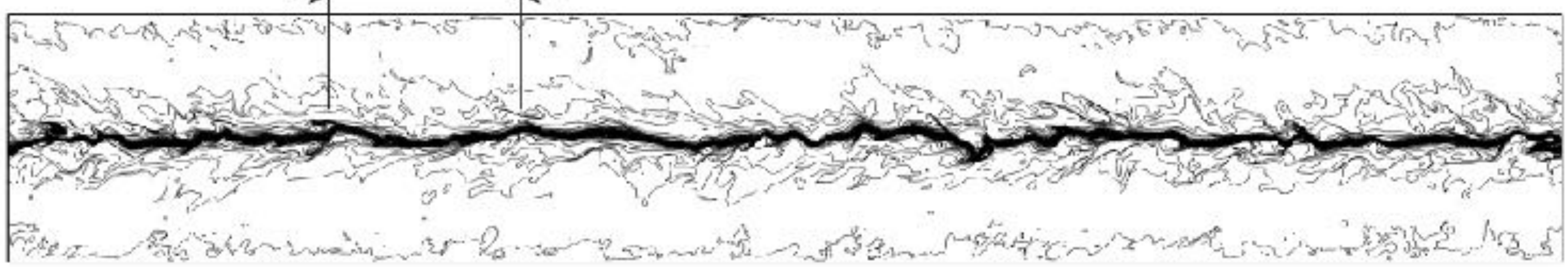


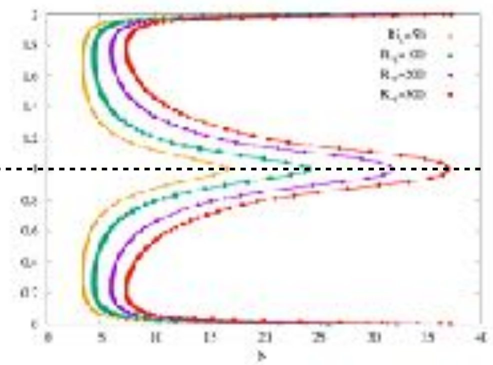


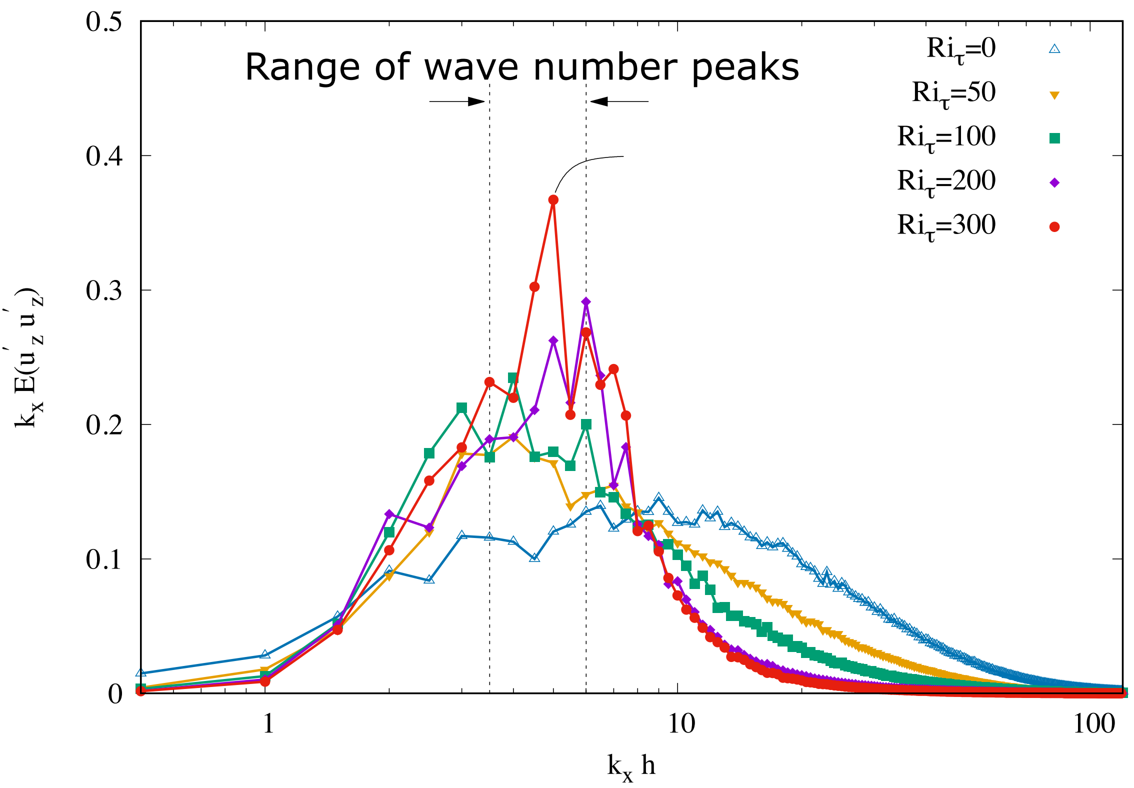
$$N = \sqrt{-g\beta \frac{\partial \langle \theta \rangle}{\partial z}}$$

$$N^* = N u_\tau^* / h^*$$

Internal Gravity Waves - IGW







Energy spectral density of the wall-normal velocity fluctuations

$$N = \sqrt{-g\beta \frac{\partial \langle \theta \rangle}{\partial z}}$$

$$\langle u_{max} \rangle \longrightarrow \langle u_{wave} \rangle$$

$$u^*_{wave} = u_{wave} \times u^*_\tau$$

$$N^* = N u^*_\tau / h^*$$

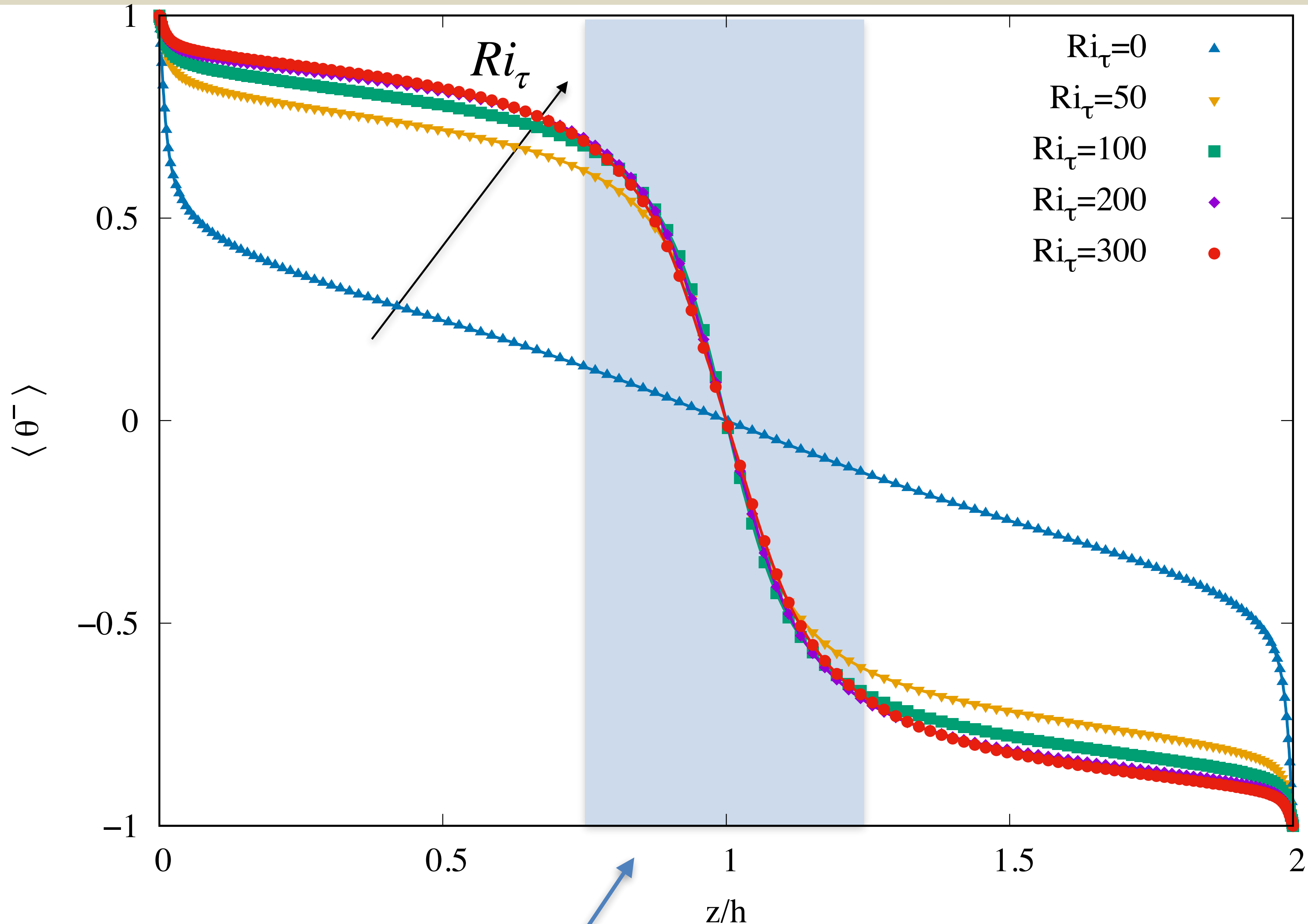
$$f^*_{wave} = u^*_{wave} \times k^*_x / 2\pi$$

$$N^* \sim f^*_{wave}$$

This links the Brunt-Väisälä frequency with the peak of the spectral energy

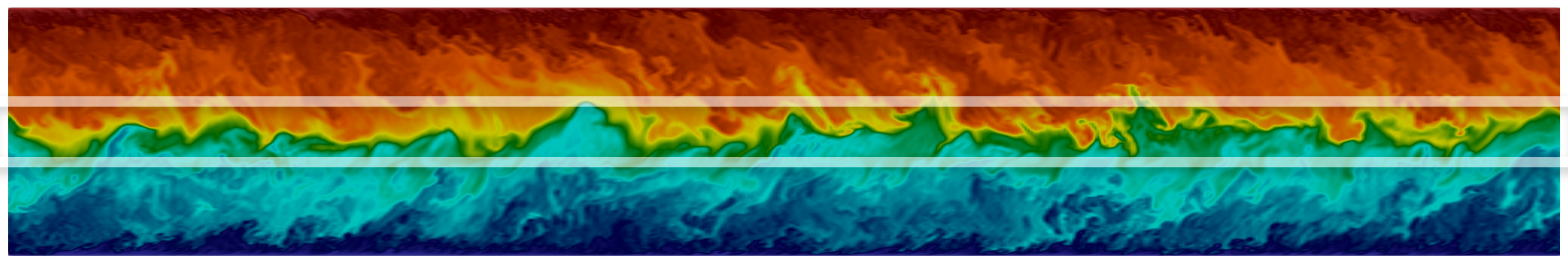
[2] Zonta F, et al 2022. Journal of Fluid Mechanics, *945, A3*.

Mean Temperature



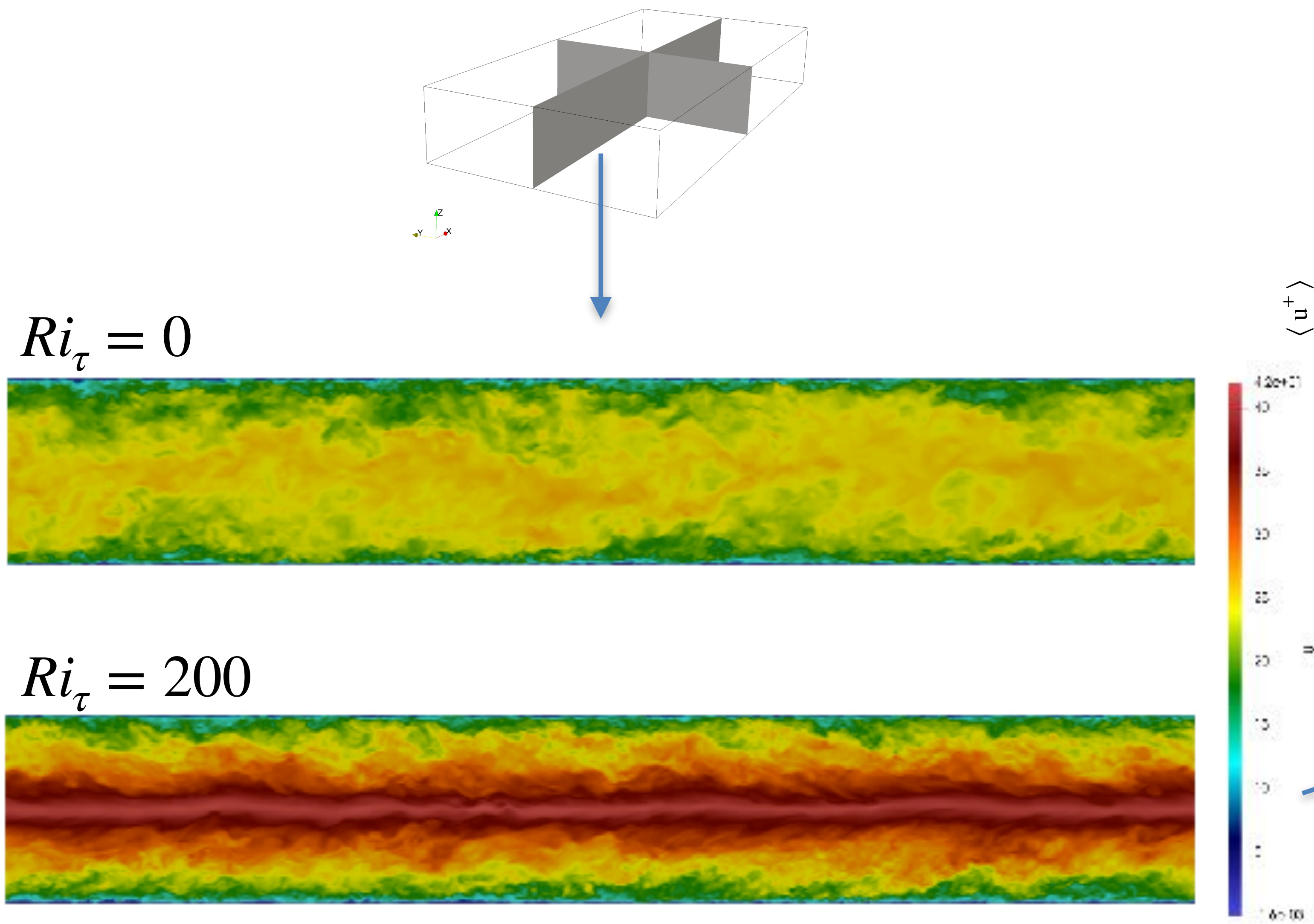
Main observations:

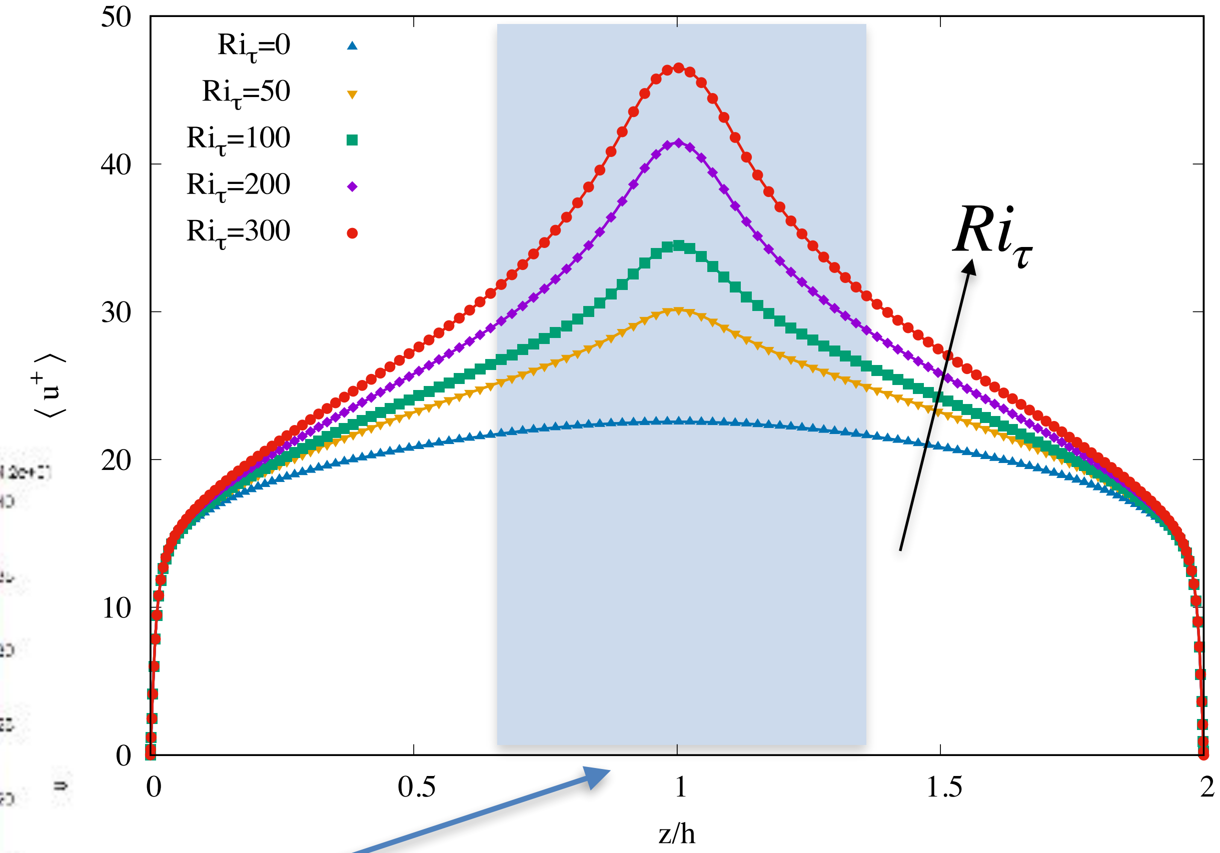
- IGWs (thermocline) behave like a thick interface
- Fluid parcels must climb a potential barrier
- Wall normal transport of heat is reduced



Velocity field

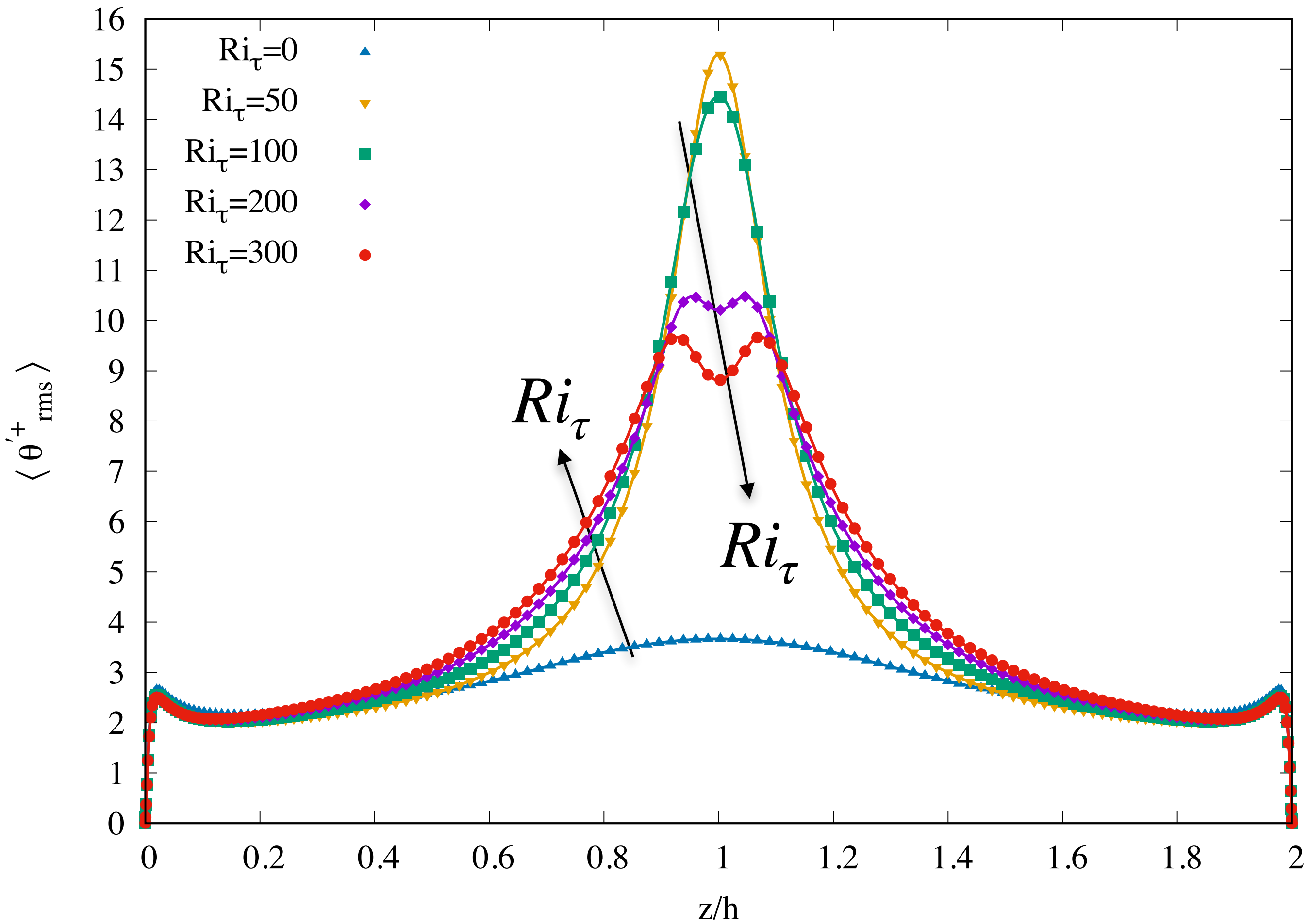
The temperature distribution has a remarkable effect on the velocity field





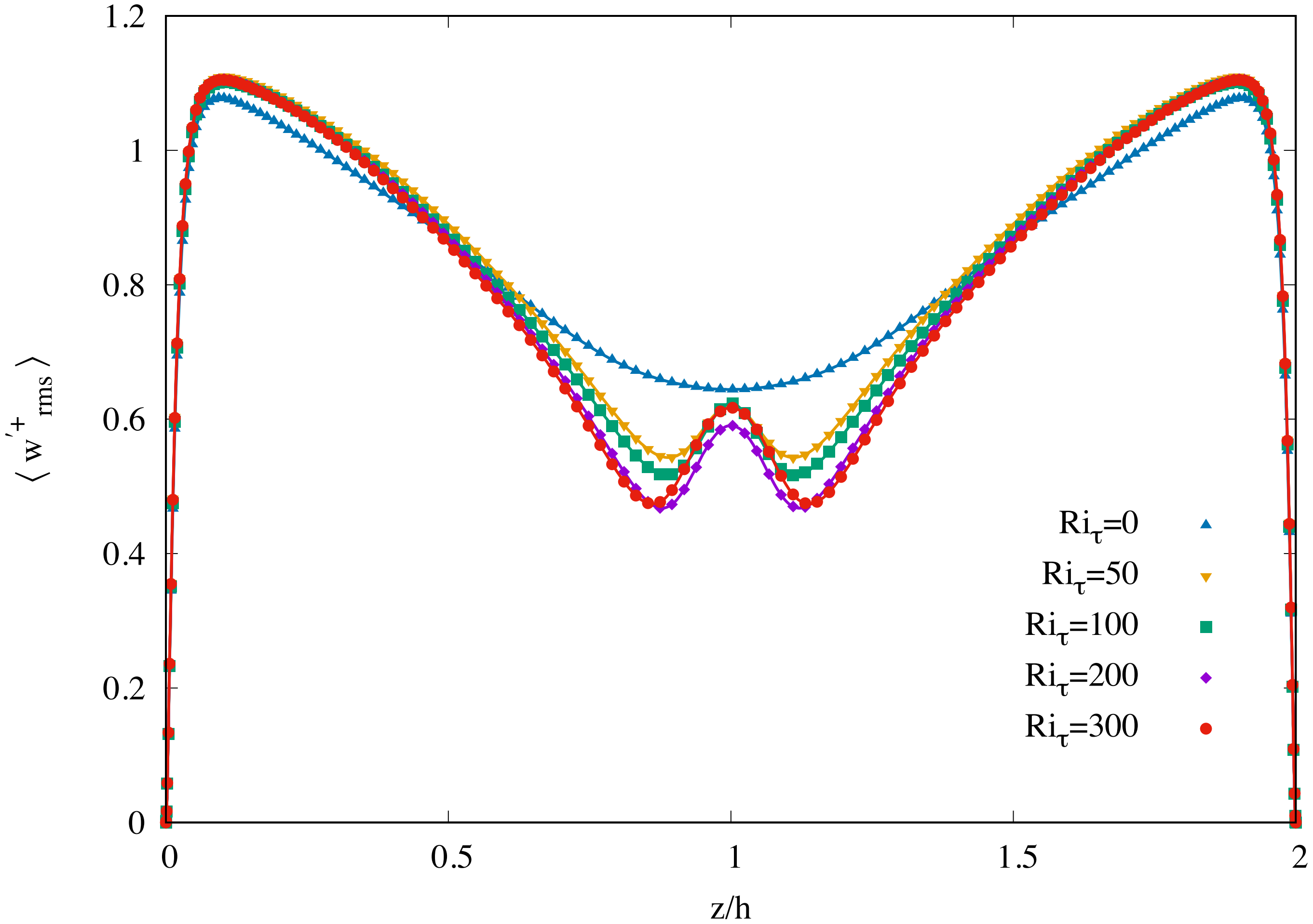
- Wall-normal transport of momentum is reduced (reduced friction factor)
- Stratified flow is ''faster'' and less homogeneous (layering)

Temperature and velocity Fluctuations

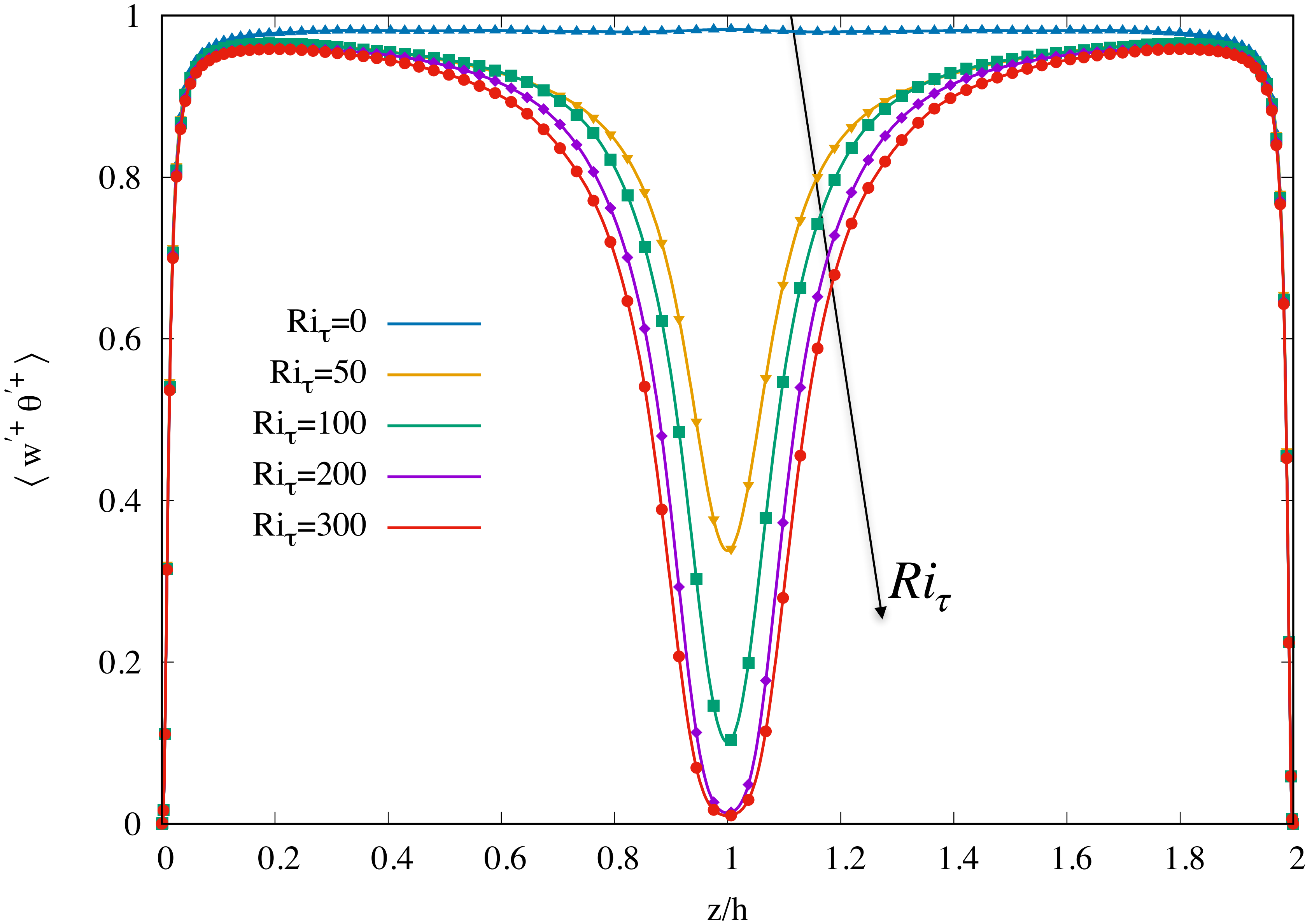


Non-monotonic behavior

Large fluctuations at the channel center



Buoyancy flux I

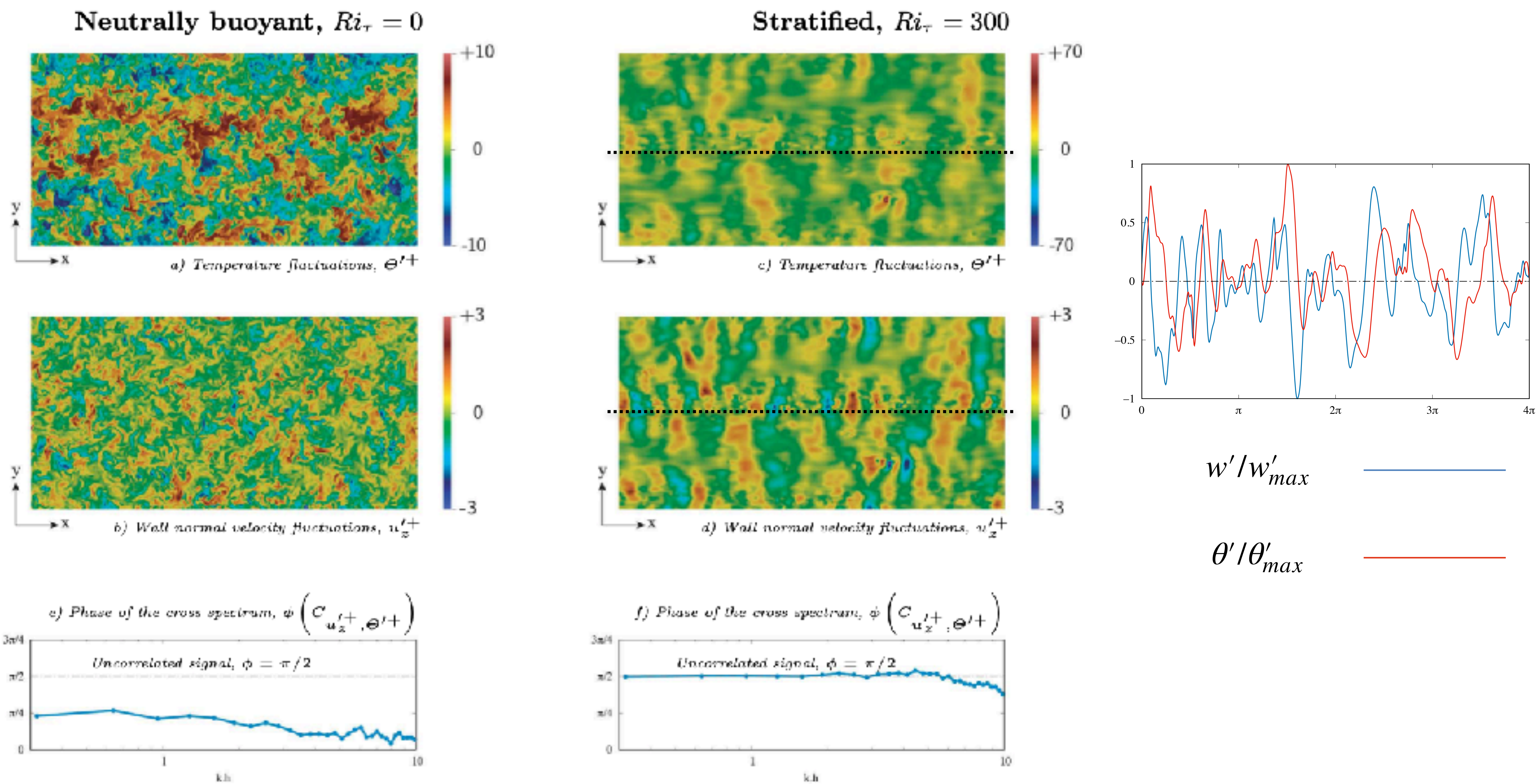


Main observation:

- Although fluctuations are large at the channel center, buoyancy flux is small…why?

Buoyancy flux II

Temperature and vertical velocity fluctuations in a horizontal plane parallel to the wall and located at the channel center, and corresponding cross spectrum



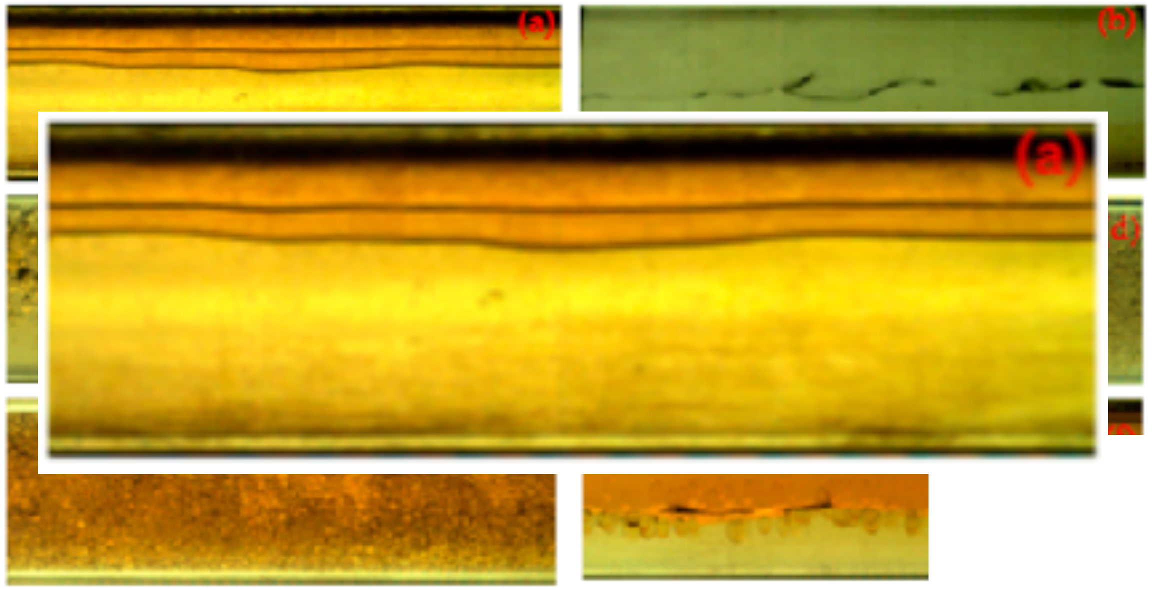
[2] Zonta F, et al 2022. Journal of Fluid Mechanics, *945, A3*.

TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

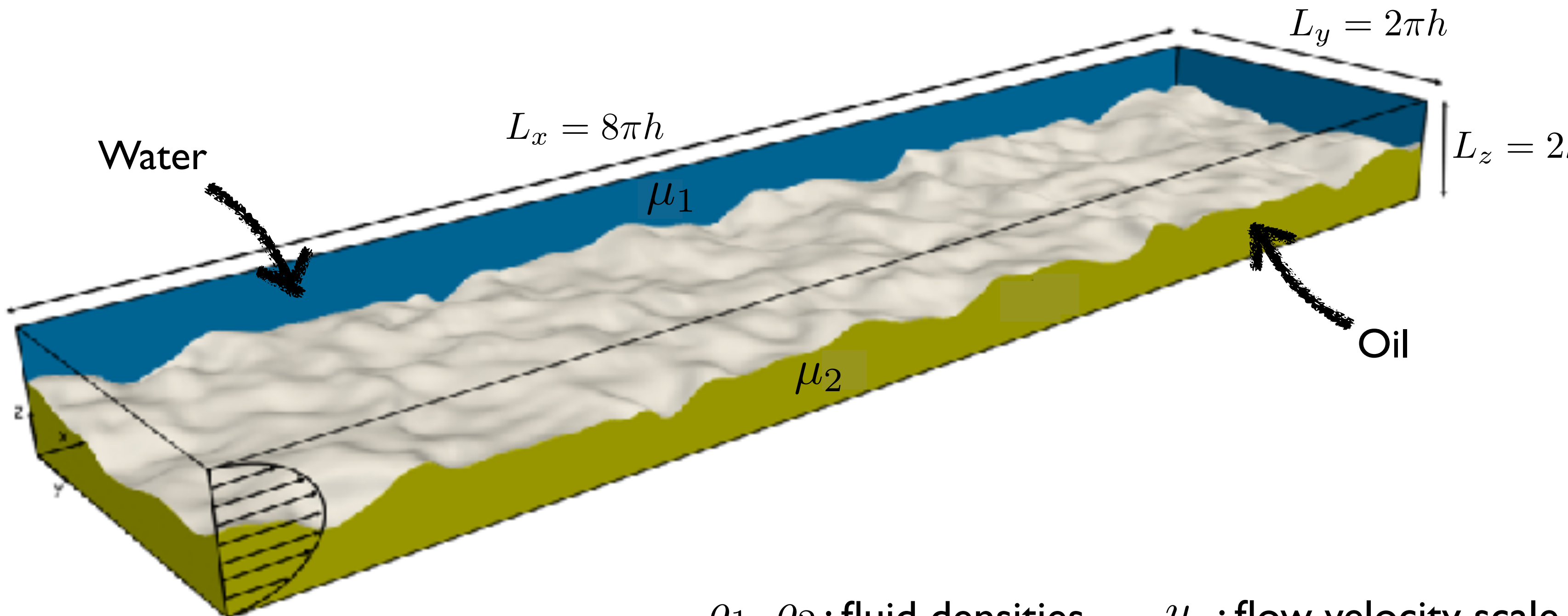
1. Stably-stratified turbulence: internal waves

2. Viscosity-stratified turbulence: interfacial waves

Wall-bounded oil-water flow



Phenomenology of oil-water flow in pipes and channels [1]



Reynolds number: $Re_\tau = \rho_2 u_\tau h / \mu_2$

Weber number: $We = \rho_2 u_\tau^2 h / \sigma$

Viscosity ratio: $\mu_r = \mu_1 / \mu_2$

ρ_1, ρ_2: fluid densities

μ_1, μ_2: fluid viscosities

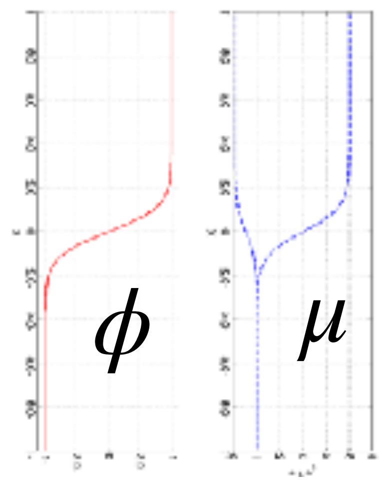
u_τ: flow velocity scale

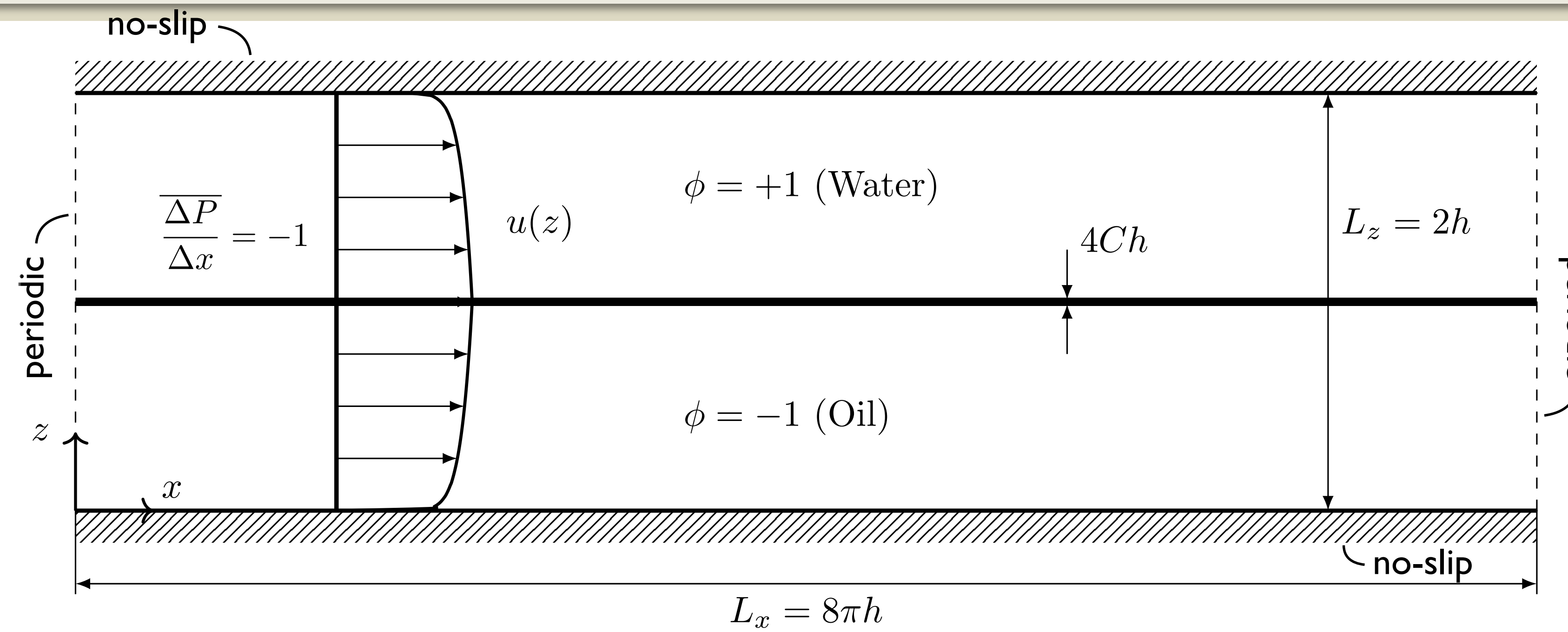
σ : surface tension

Assumption: Matched densities $\longrightarrow$ $\rho_1 = \rho_2$

[1] Li, L.; Kong, L.; Xie, B.; Fang, X.; Kong, W.; Liu, X.; Wang, Y.; Zhao, F. The Influence on Response of a Combined Capacitance Sensor in Horizontal Oil–Water Two-Phase Flow. Appl. Sci. 2019, 9, 346.

Direct Numerical Simulation (DNS) - Phase Field Method (PFM)





Flow

- Continuity: $\nabla \cdot \mathbf{u} = 0$

- Navier-Stokes equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re_\tau} \nabla \cdot [\mu(\phi, \mu_r)(\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \frac{3}{\sqrt{8}} \frac{Ch}{We} \nabla \cdot (|\nabla \phi|^2 \mathbf{I} - \nabla \phi \otimes \nabla \phi)$$

Interface

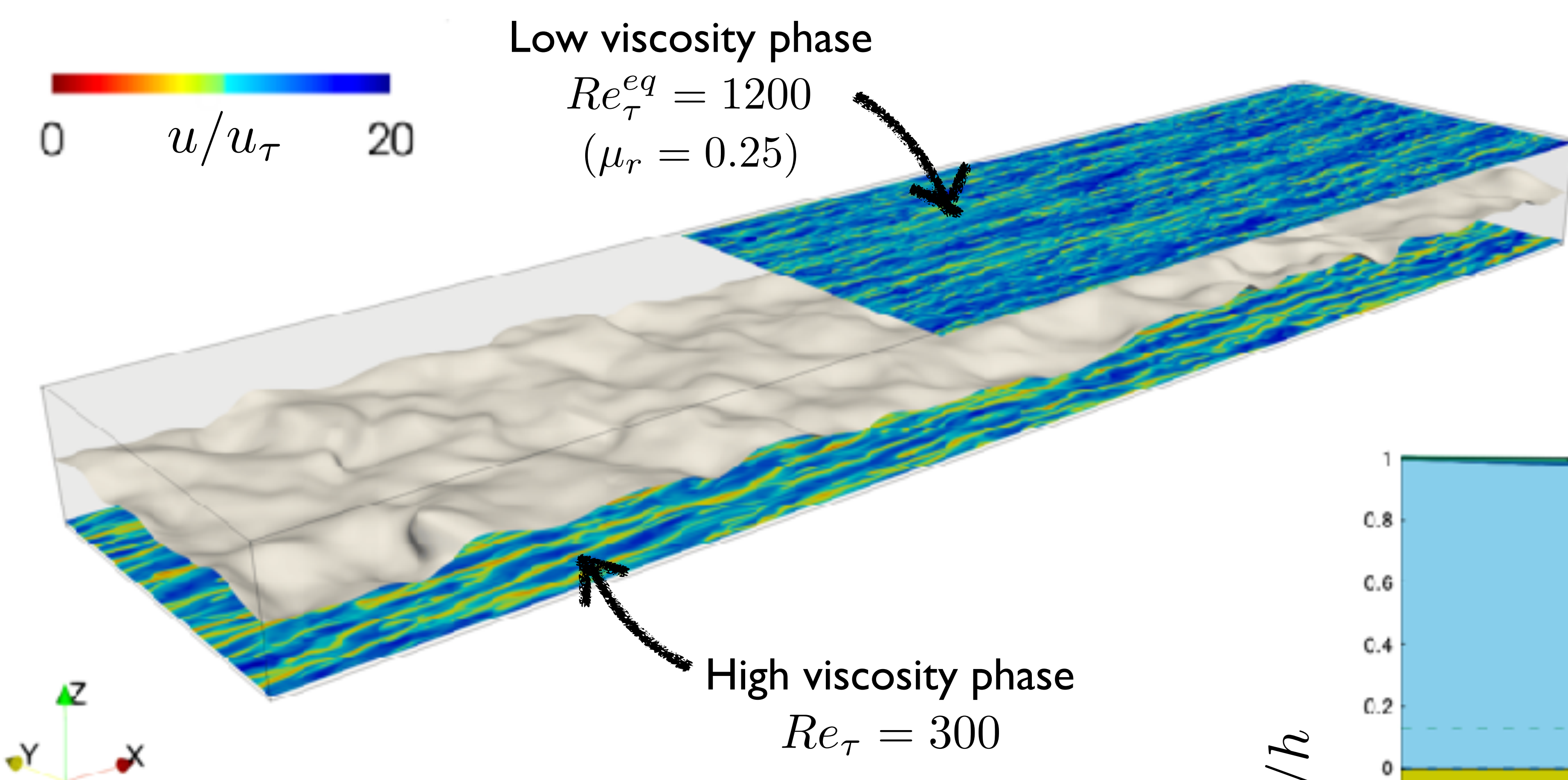
- Advection-diffusion equation:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = \frac{1}{Pe_\phi} \nabla^2 (\phi^3 - \phi - Ch^2 \nabla^2 \phi)$$

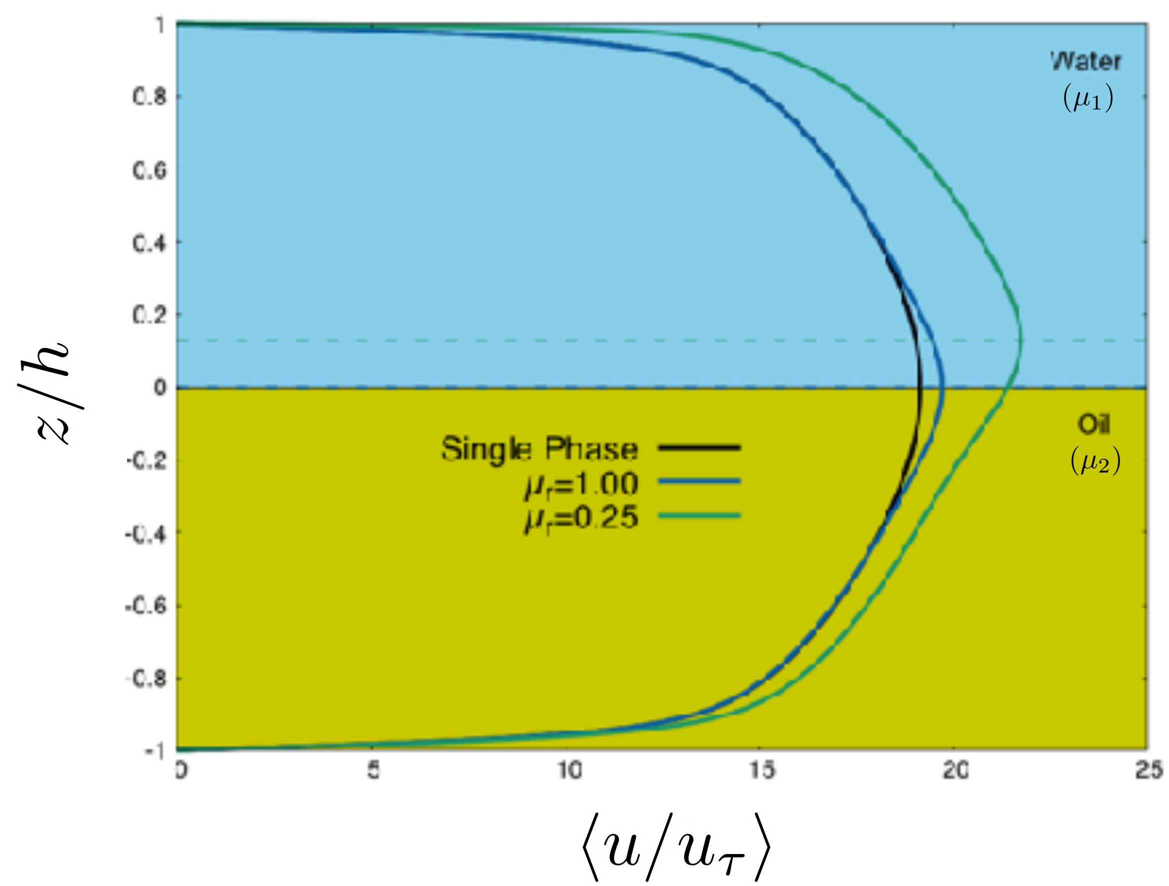
Cahn number: $Ch = \frac{\xi}{h}$

Péclet number: $Pe_\phi = \frac{u_\tau h}{\beta m_\phi}$

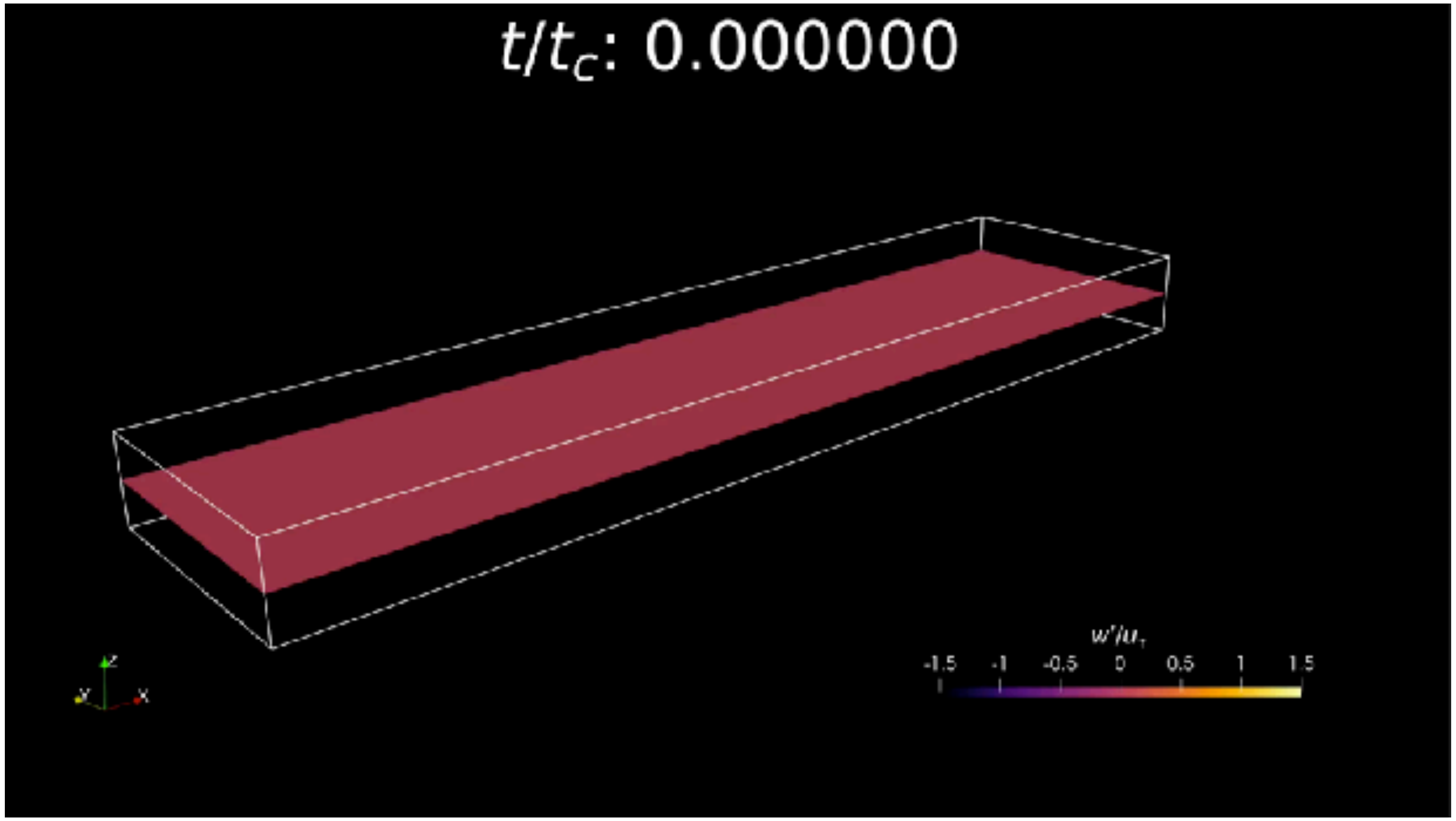
Plan of experiments

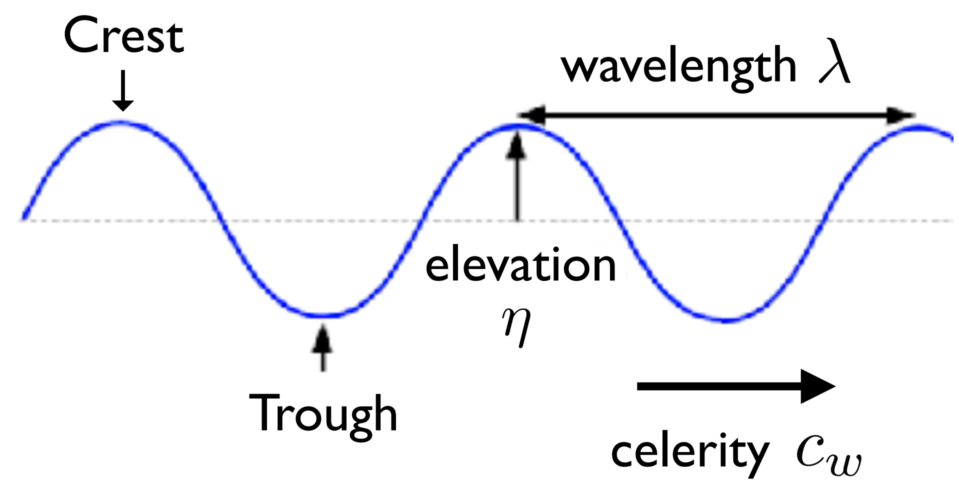


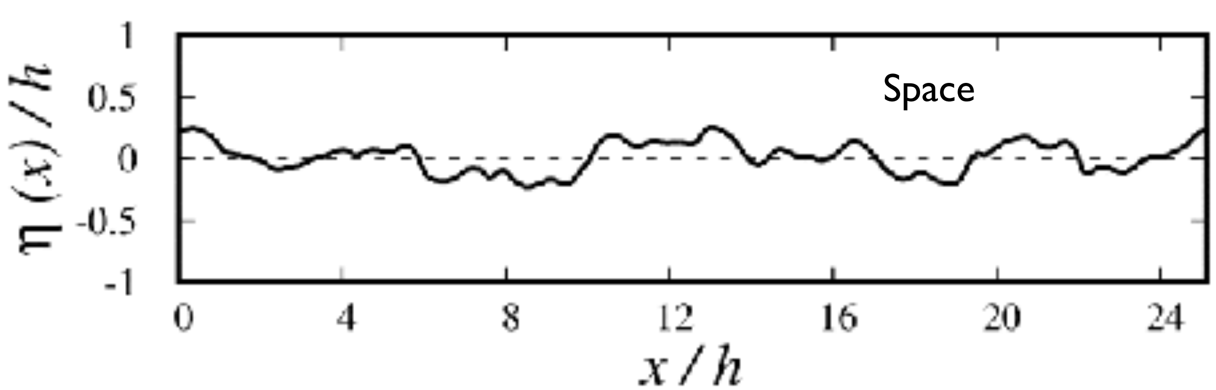
Simulation	Re_τ	We	N_x	N_z	N_y
Single Phase	300	-	1024	513	256
$\mu_r = 1.00$	300	1.0	1024	513	256
$\mu_r = 0.25$	300	1.0	2048	513	1024

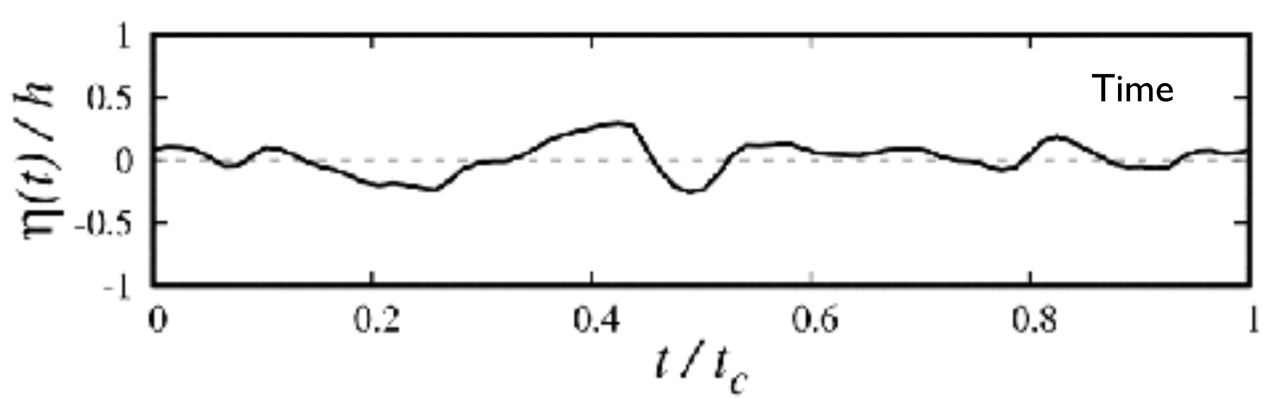


Capillary wave growth

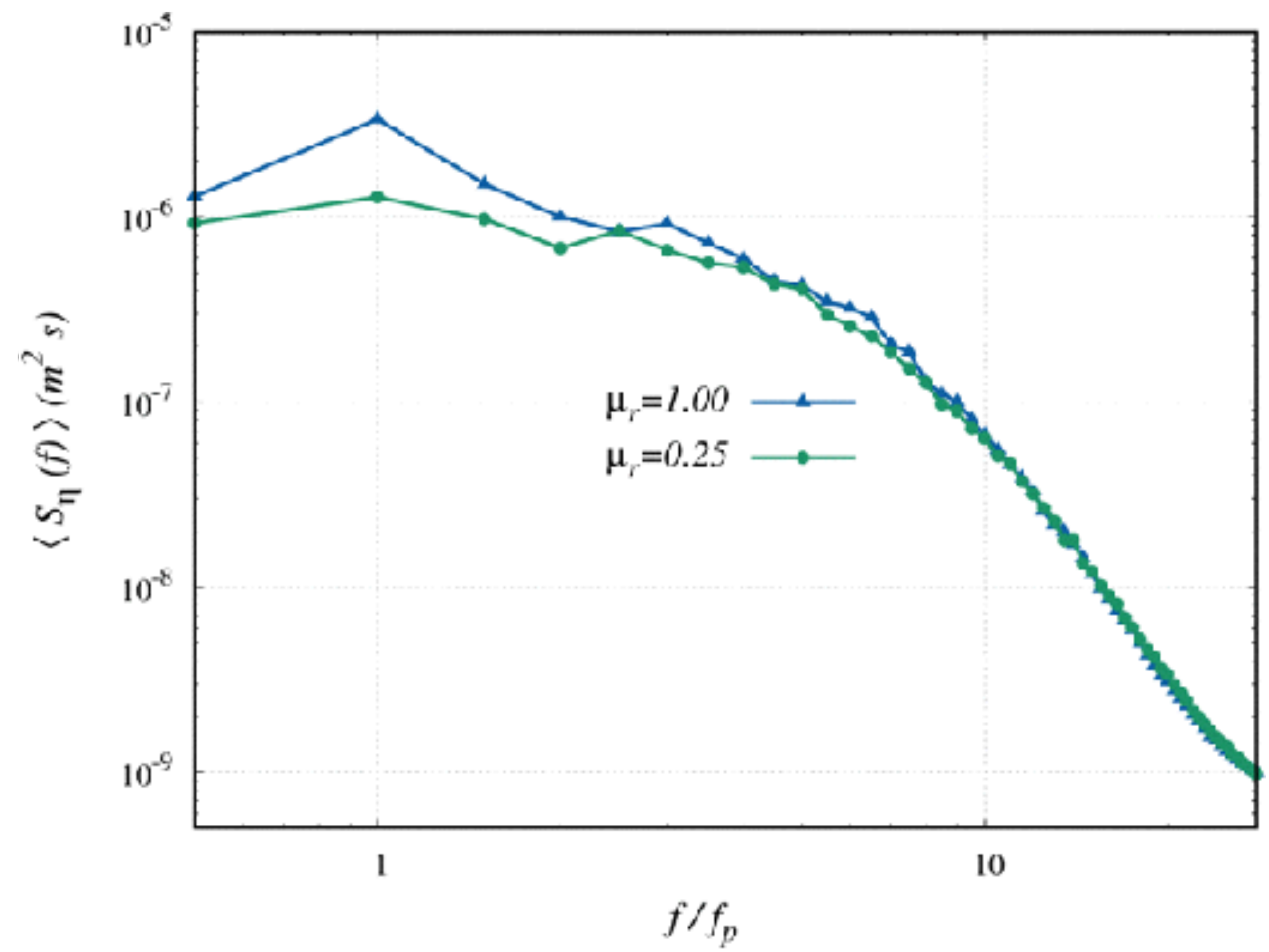


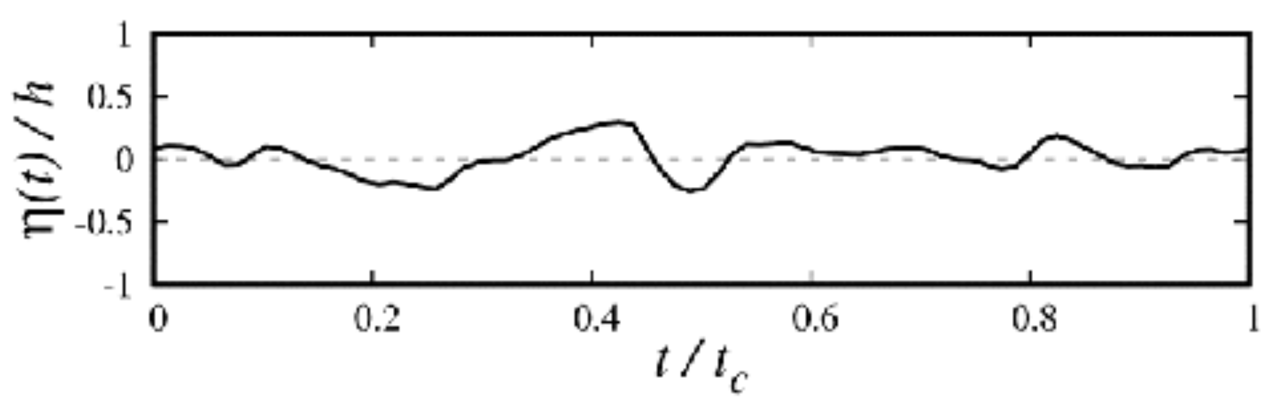






Frequency power spectra of wave elevation - Wave turbulence theory (WTT)





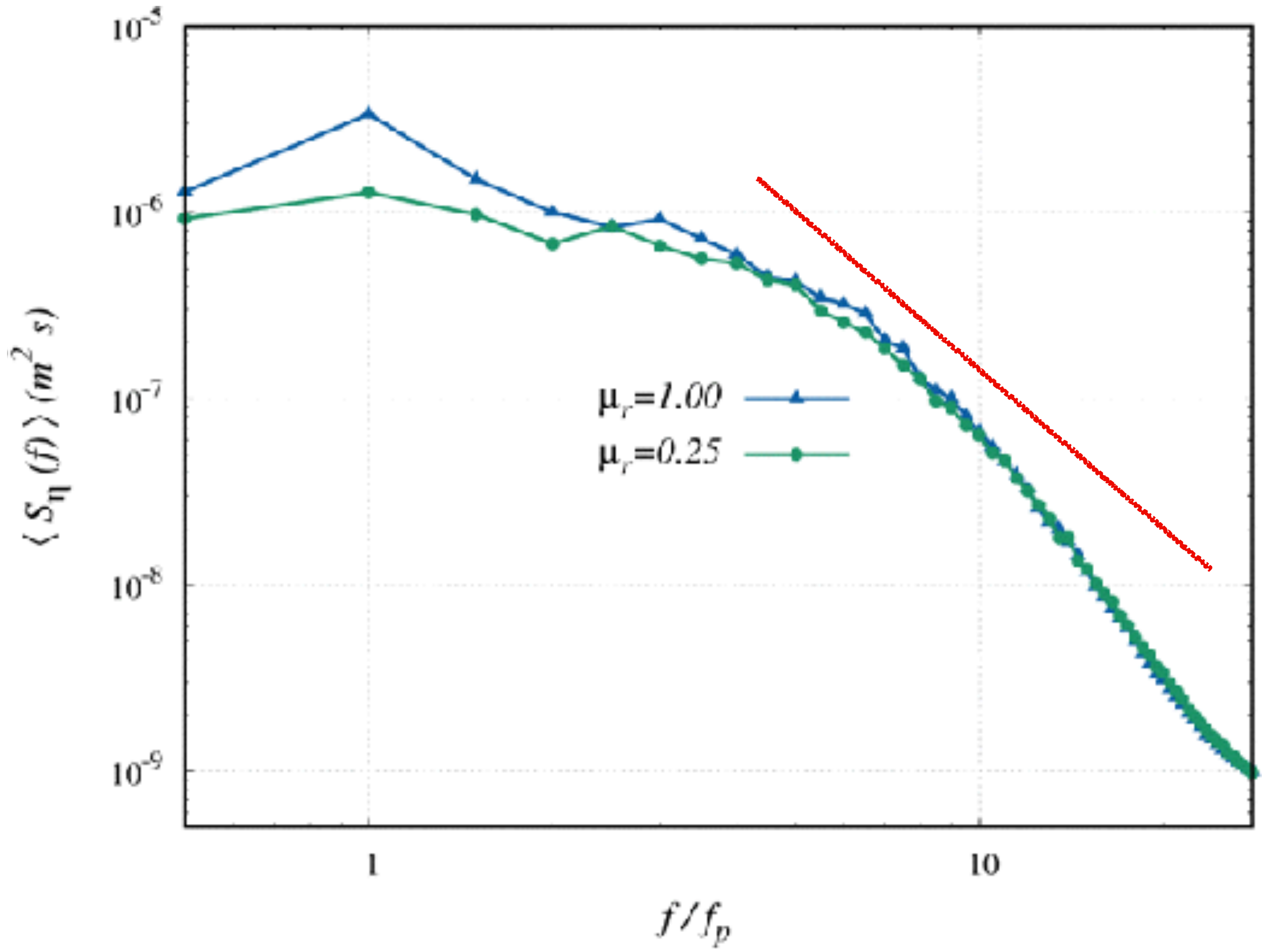
$^* f = c_w / \lambda$

[1] Zakharov, V. E. & Filonenko, N. N. 1967 Weak turbulence of capillary waves. Journal of Applied Mechanics and Technical Physics 8 (5), 37–40.

[2] Balkovsky, E., Falkovich, G., Lebedev, V. & Shapiro, I. Ya. 1995 Large-scale properties of wave turbulence. Phys. Rev. E 52, 4537–4540.

Frequency power spectra of wave elevation - Wave turbulence theory (WTT)

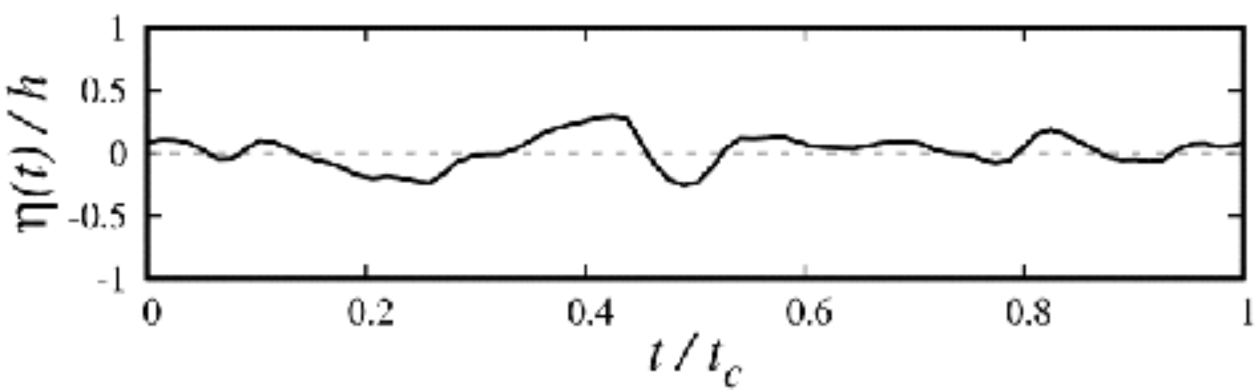
Wave Turbulence Theory predictions:



High-frequency waves [1]:

$$S_\eta(f) = f^{-8/3}$$

"Waves not at equilibrium - energy transfer among wave scales"



Hp of Wave Turbulence Theory:
Weak nonlinearities;
Negligible dissipation;
Inertial regime, far from injection and dissipation

$^*f = c_w/\lambda$

[1] Zakharov, V. E. & Filonenko, N. N. 1967 Weak turbulence of capillary waves. Journal of Applied Mechanics and Technical Physics 8 (5), 37–40.
[2] Balkovsky, E., Falkovich, G., Lebedev, V. & Shapiro, I. Ya. 1995 Large-scale properties of wave turbulence. Phys. Rev. E 52, 4537–4540.

Frequency power spectra of wave elevation - Wave turbulence theory (WTT)

Wave Turbulence Theory predictions:

Low-frequency waves [2]:

$$S_\eta(f) = f^{-1}$$

"Waves at statistical equilibrium"

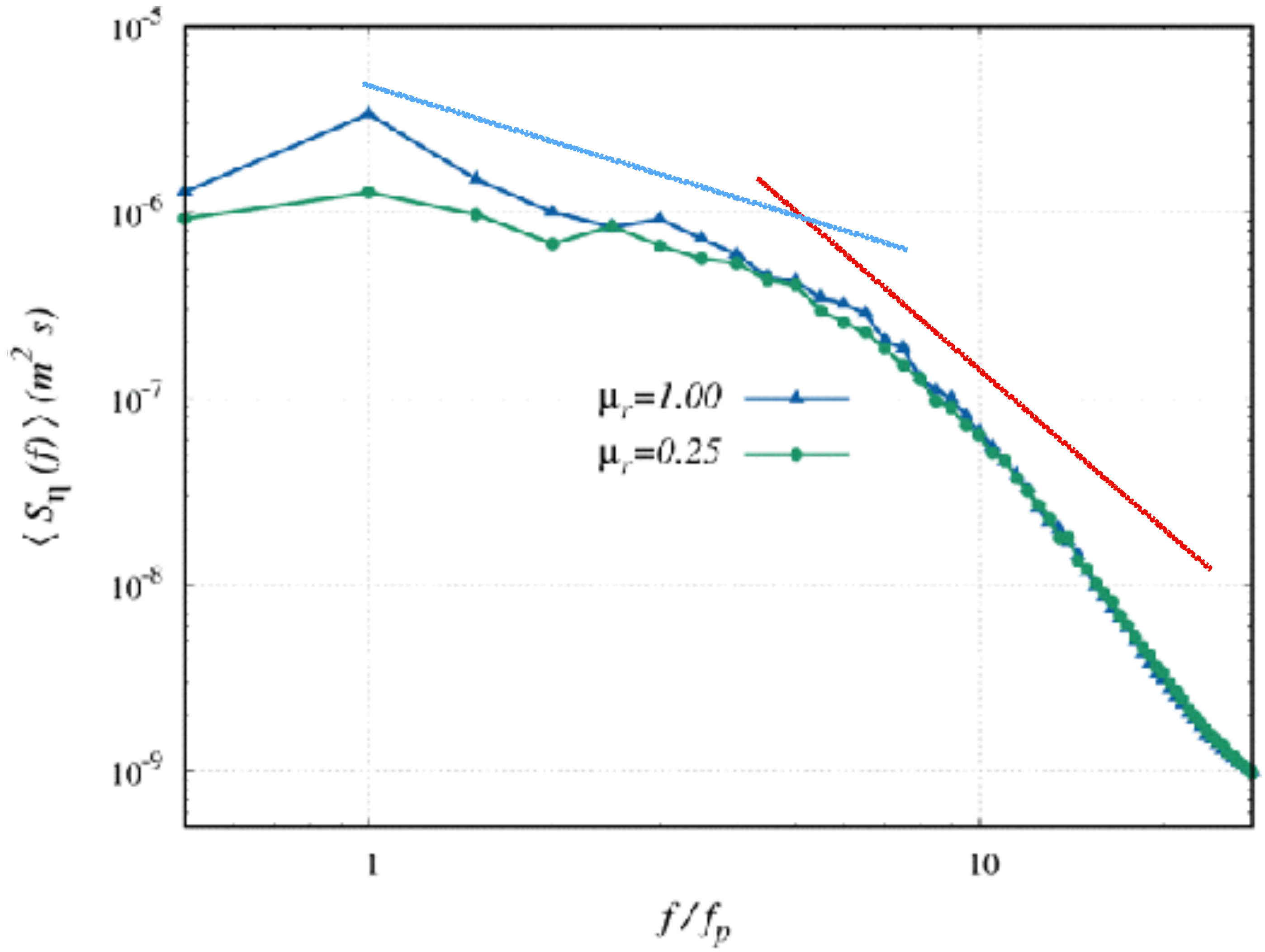
High-frequency waves [1]:

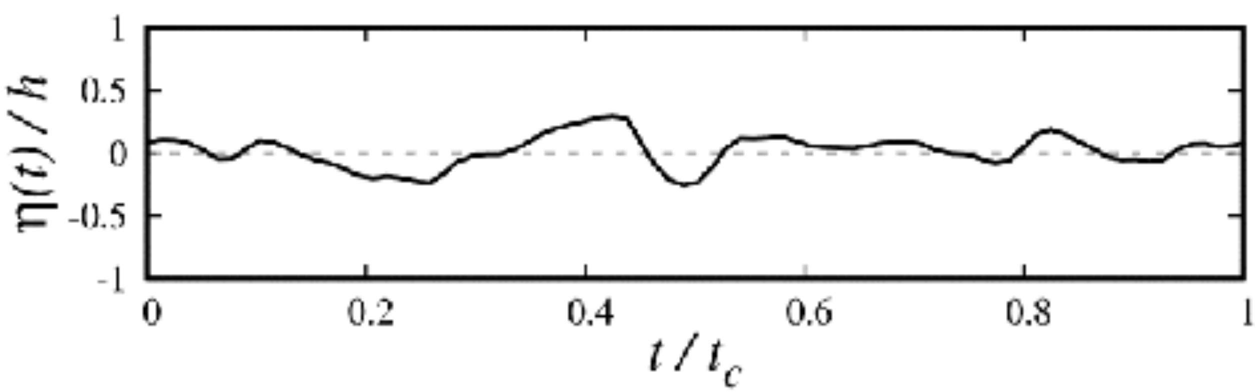
$$S_\eta(f) = f^{-8/3}$$

"Waves not at equilibrium - energy transfer among wave scales"

Hp of Wave Turbulence Theory:
Weak nonlinearities;
Negligible dissipation;
Inertial regime, far from injection and dissipation

$^*f = c_w/\lambda$





[1] Zakharov, V. E. & Filonenko, N. N. 1967 Weak turbulence of capillary waves. Journal of Applied Mechanics and Technical Physics 8 (5), 37–40.
[2] Balkovsky, E., Falkovich, G., Lebedev, V. & Shapiro, I. Ya. 1995 Large-scale properties of wave turbulence. Phys. Rev. E 52, 4537–4540.

Wavenumber power spectra of wave elevation - Wave turbulence theory (WTT)

Wave Turbulence Theory predictions:

Large-scale waves [2]:

$$S_\eta(k) = k^{-1}$$

"Waves at statistical equilibrium"

First part (Super- Hinze scale waves [3]):

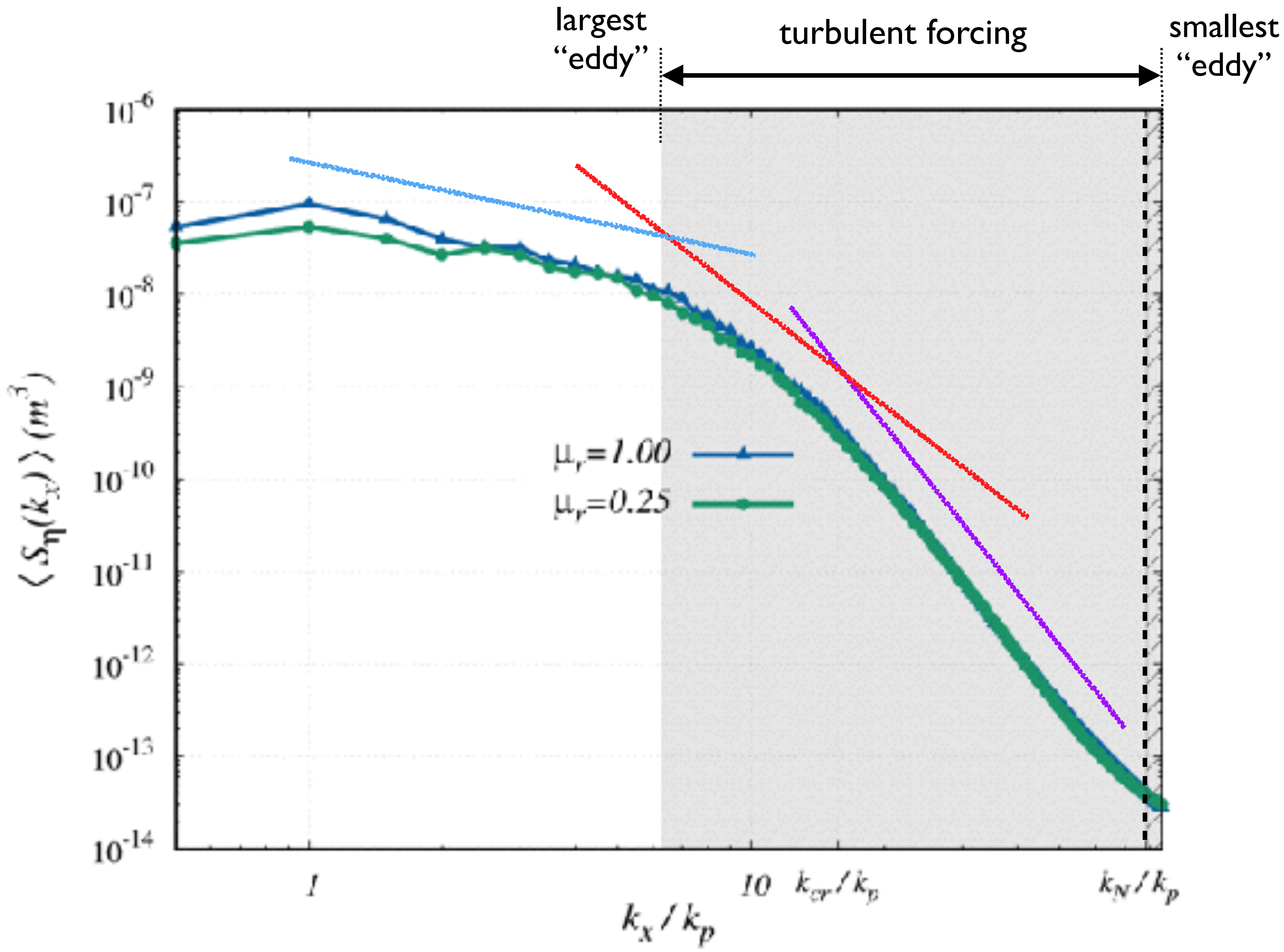
$$S_\eta(k) = k^{-4}$$

"Waves not at equilibrium - energy transfer among wave scales"

Second part (Sub- Hinze scale waves [4]):

$$S_\eta(k) = k^{-6}$$

"Strong damping due to surface tension"



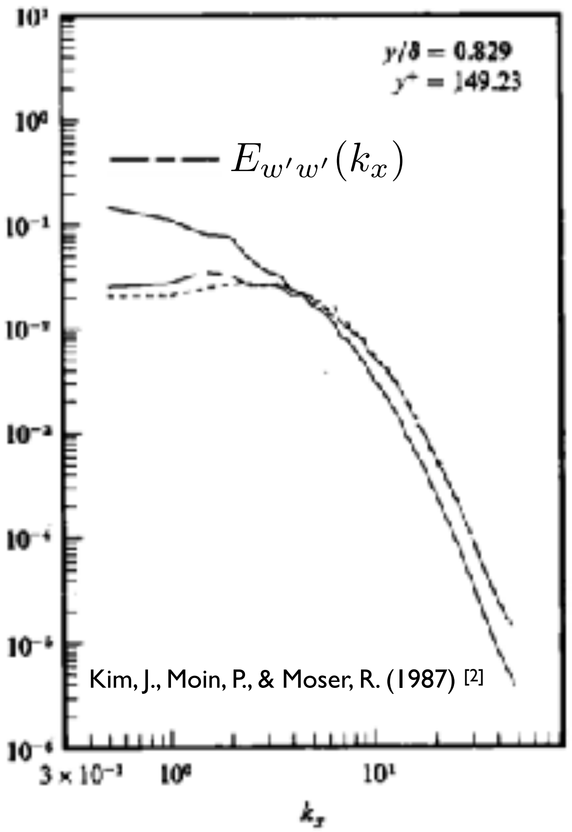
--- Numerical resolution cut-off (shortest wave)

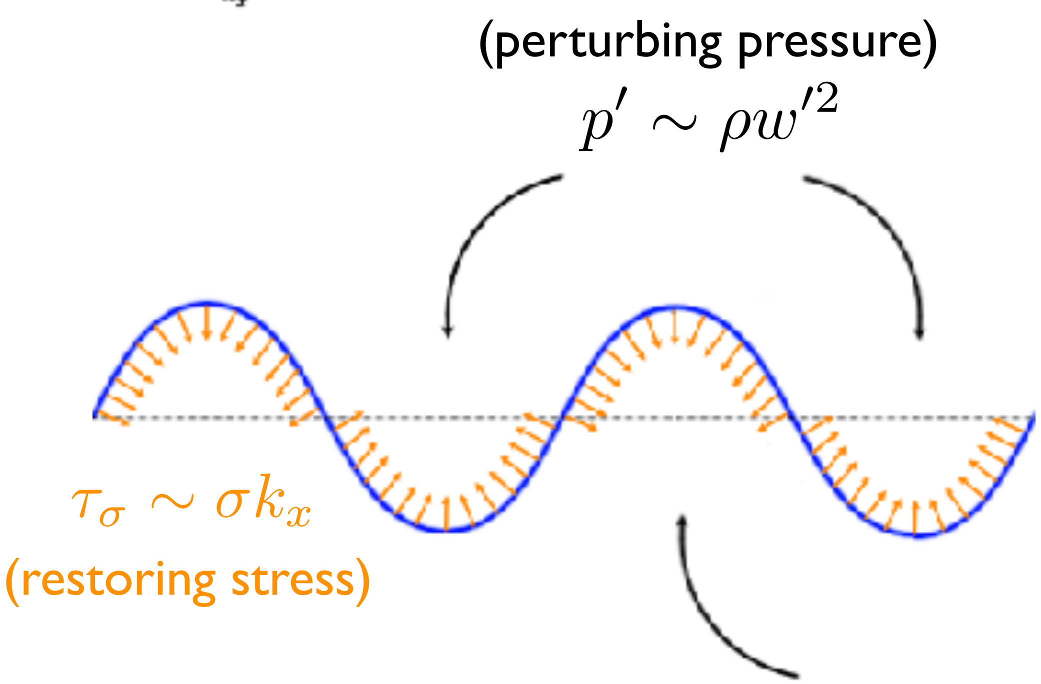
* $k = 2\pi/\lambda$

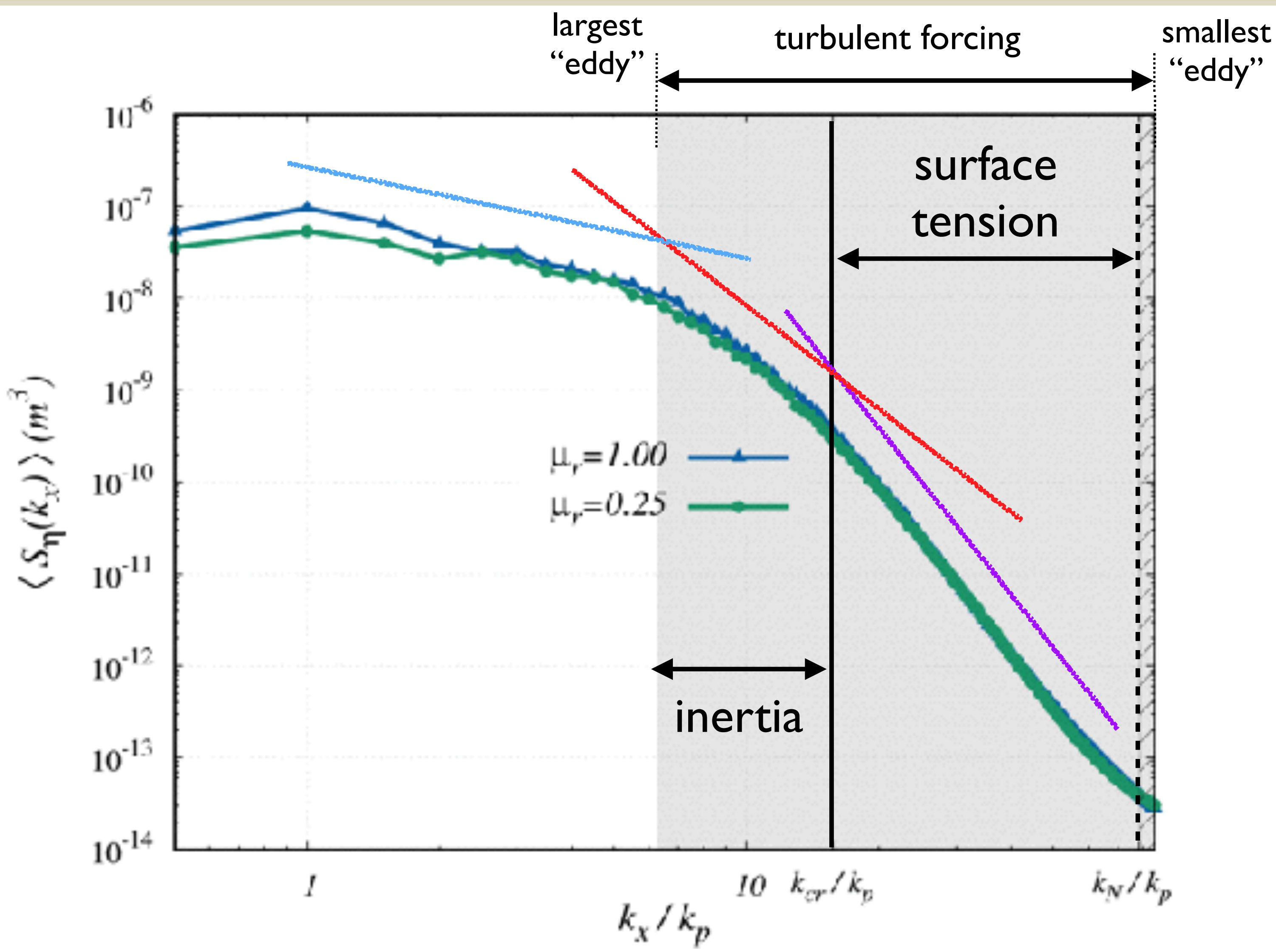
[1] Hinze, J. O. 1955 Fundamentals of the hydrodynamic mechanism of splitting in dispersion processes. AIChE Journal 1 (3), 289–295.
[2] Balkovsky, E., Falkovich, G., Lebedev, V. & Shapiro, I. Ya. 1995 Large-scale properties of wave turbulence. Phys. Rev. E 52, 4537–4540.
[3] Zakharov, V. E. & Filonenko, N. N. 1967 Weak turbulence of capillary waves. Journal of Applied Mechanics and Technical Physics 8 (5), 37–40.
[4] Savelsberg, R. & Van De Water, W. 2008 Turbulence of a free surface. *Phys. Rev. Lett.* 100, 034501

TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

Wavenumber power spectra of wave elevation - Wave turbulence theory (WTT)







Kolmogorov- Hinze critical length scale [1]

$$k_{cr} = 2\pi / (We^{-3/5} Re_\tau^{-2/5} |\epsilon_c|^{-2/5})$$

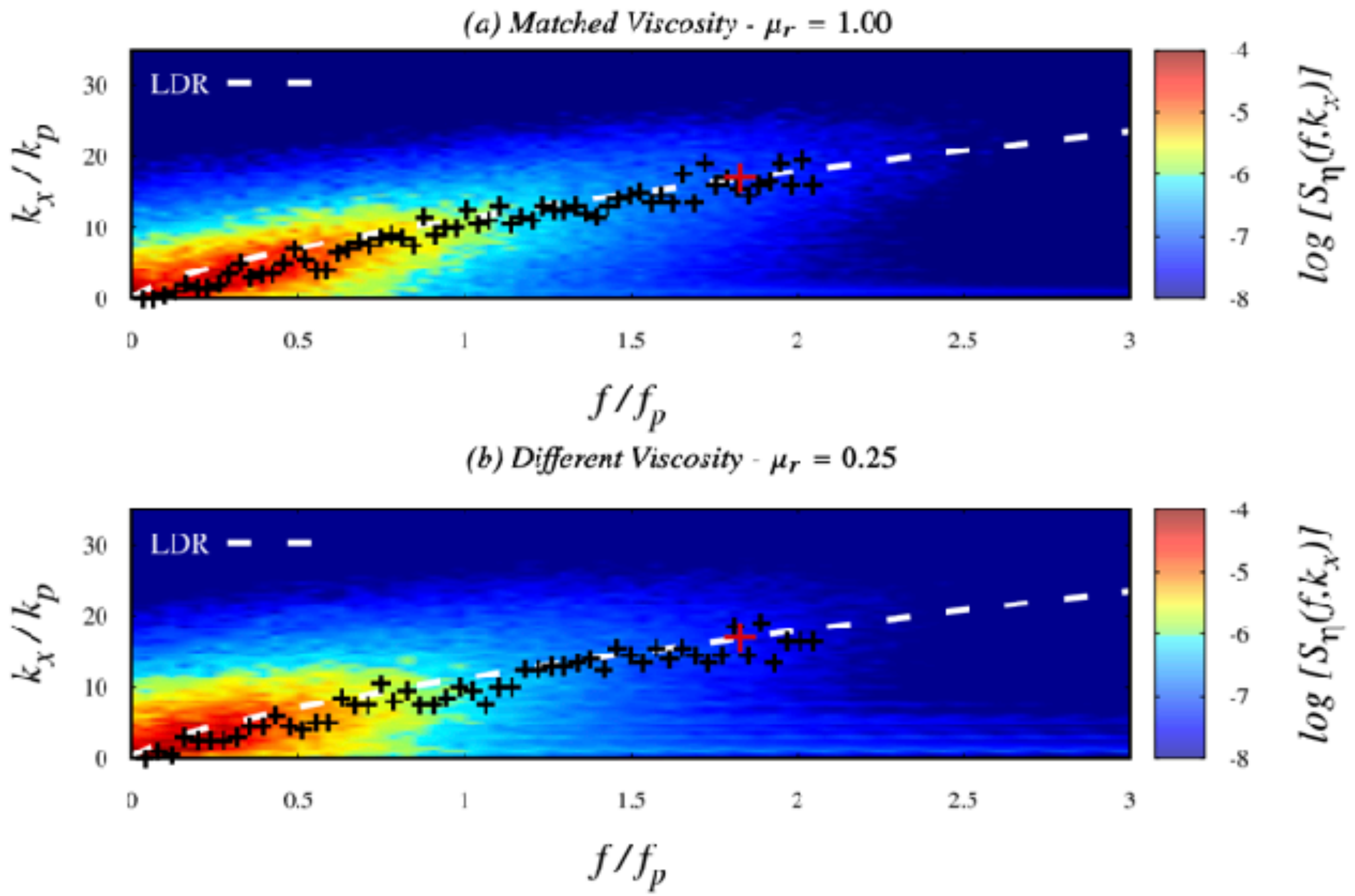
[2] Kim, J., Moin, P., & Moser, R. (1987). Turbulence statistics in fully developed channel flow at low Reynolds number. Journal of Fluid Mechanics, 177, 133-166.

Spatiotemporal power spectra of wave elevation - Wave dispersion

Wave celerity (or frequency) is predicted by the Dispersion Relation (DR) [1]

$$\omega^2 = (\sigma/\rho)k^3 \tanh(kh)$$ (wave celerity: $c_w = \omega/k$)

Frequency - wavenumber space →



$^*\omega = 2\pi f$

[1] Lamb H. 1932. Hydrodynamics. Springer-Verlag, Berlin

[2] Giamagas G, et al 2023. Journal of Fluid Mechanics, *In press*.

TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

Conclusions & Prospectives

DNS of wall bounded stratified turbulence at $\text{Re}_\tau = 1000$ have been performed for different stratification levels, $\text{Ri}_\tau = 0 - 300$

Compared to forced convection, **stratification reduces mixing and wall normal transport of momentum/heat**

The stratified flow becomes ''less homegenous''; **Internal waves are generated** at the channel center

Although velocity and temperature fluctuations may be large at the channel center, **the buoyancy flux is monotonically decreasing**

DNS of viscosity stratified oil-water channel turbulence has been performed

Analysis of interfacial waves shows: **energy equipartition** at scales larger than forcing

Kolmogorov-Hinze scale marks the threshold between growing waves and waves damped by surface tension

Propagation of waves in agreement with **linear dispersion relation**

HLRS

Perspectives:

Thermally-stratified turbulence: Tracking particles/species in thermally stratified turbulence

CINECA

Viscosity stratified turbulence: Adding surfactants; Wave breaking

VIENNA SCIENTIFIC CLUSTER

Thank you !