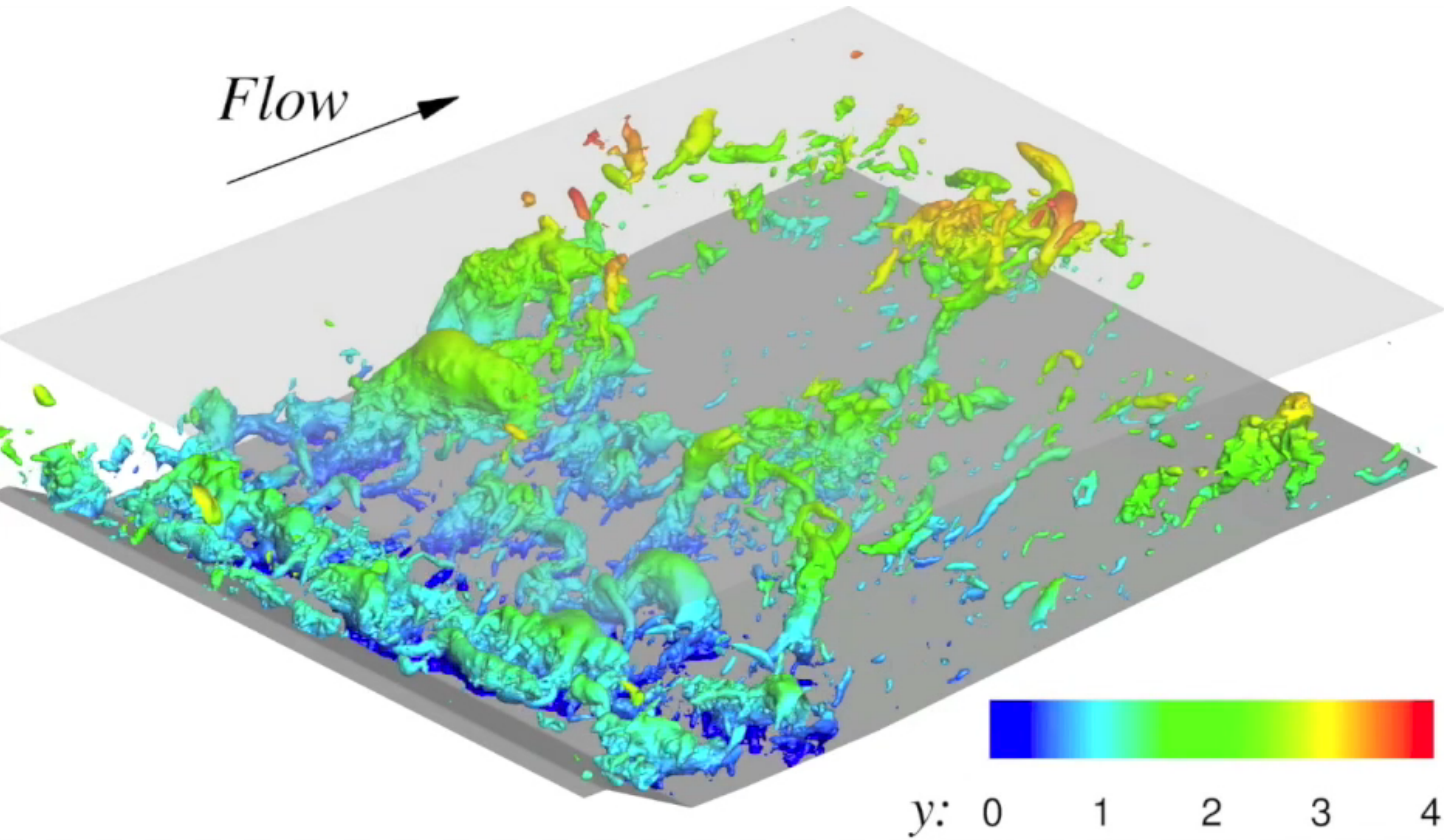
Outline

- Motivation
- Governing equations for LES
- Boundary conditions
- Subfilter-scale modelling
- Validation of an LES
- **Applications**
  - *Flow over dunes*
  - *Wall jets*
  - *Rough-wall boundary layers*
  - *Impinging jets*
- Hybrid RANS/LES methods
- Challenges
- Conclusions

Applications
Flow over River Dunes (*M. Omidyeganeh*)



Dunes: Motivation

- Interaction of a flow field with a mobile sand bed results in bed deformation.
- The shape depends on:
  - *Flow properties (Re, Fr, etc.)*
  - *Sand type*
  - *Amount of sand available*
- For unidirectional mean flow, high Reynolds numbers (rivers)
  ⇒ *Transverse dunes*

Dunes: Motivation

- Interaction of a flow field with a mobile sand bed results in bed deformation.
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  - *Flow properties (Re, Fr, etc.)*
  - *Sand type*
  - *Amount of sand available*
- For unidirectional mean flow, high Reynolds numbers (rivers)
  ⇒ *Transverse dunes*
- Limited sediment supply (desert)
  ⇒ *Barchan dunes*

Dunes: Motivation

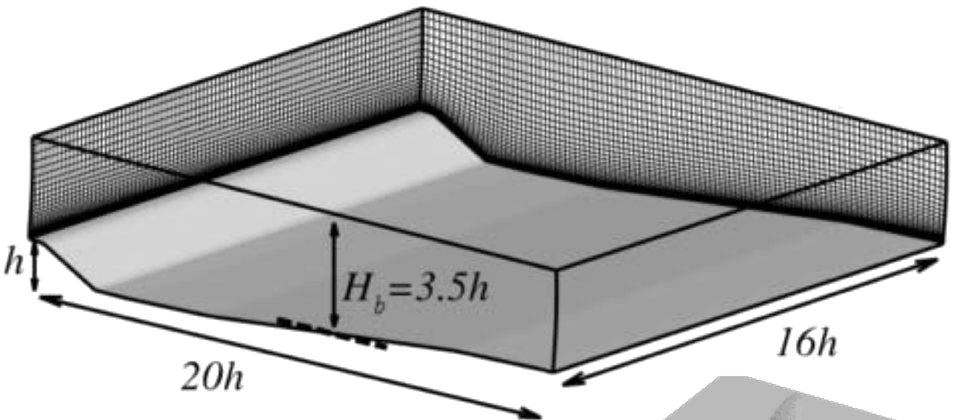
- In dunes, turbulence affects bed morphology and sediment transport.
- Field and laboratory experiments can highlight many of the important turbulent phenomena.
  - *Mean flow*
  - *Instantaneous flow structure*
- Experiments have limitations:
  - *Control of boundary conditions*
  - *Access to full field*
  - *Near-wall measurements*
- Improved numerical models are required to complement the experiments.

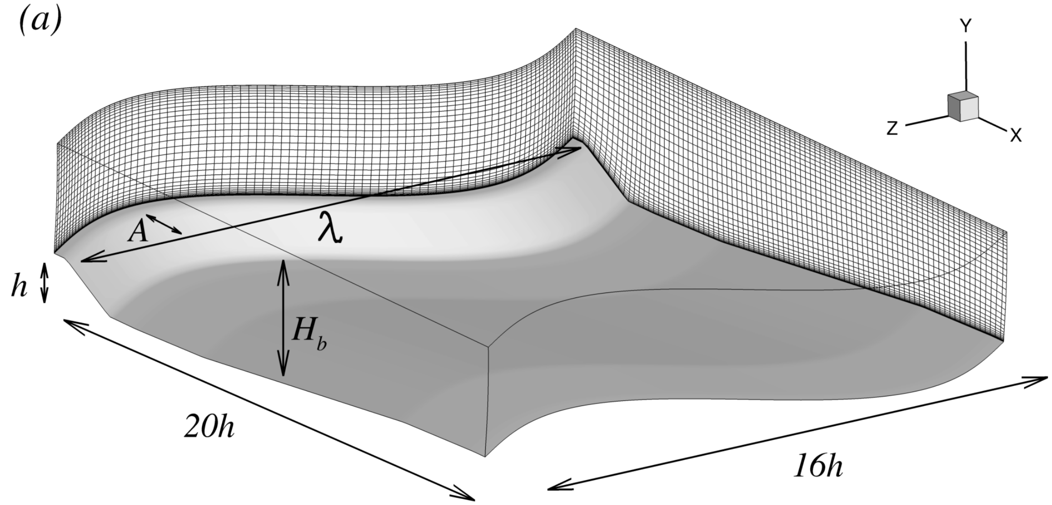
LES of Flows Over Dunes

- Curvilinear code
  - *2nd-order accurate in time and space.*
  - *Central differences on all terms*
  - *Lagrangian Dynamic subfilter-scale model*
- The model has been extensively validated in engineering and geophysical flows.
- Grids between $6x10^6$ and $41x10^6$ points.
  - *Up to 16,000 CPU-hours per simulation*

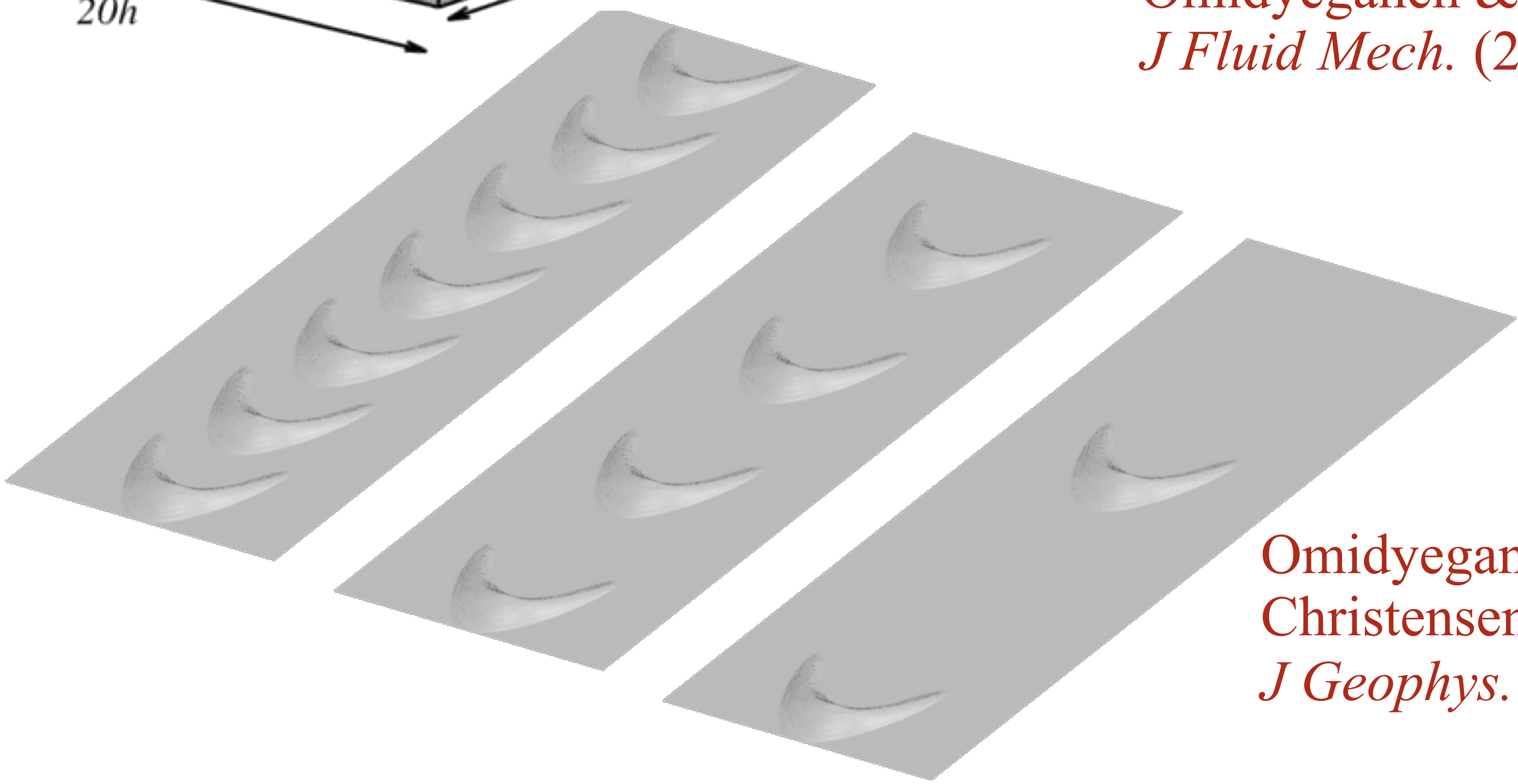
LES of Flows Over Dunes

Omidyeganeh & Piomelli
J of Turbulence (2010)





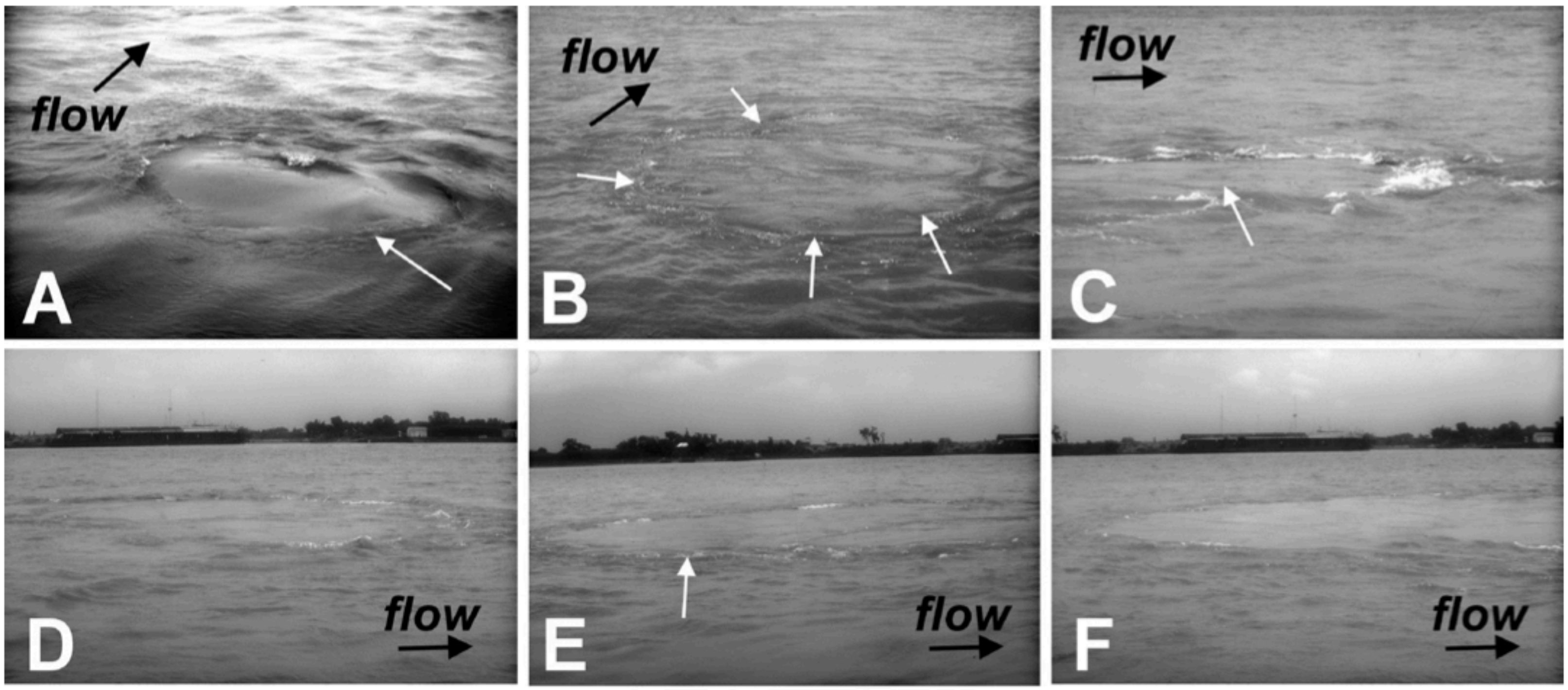
Omidyeganeh & Piomelli
J Fluid Mech. (2013a, 2013b)



Omidyeganeh, Piomelli, Christensen & Best
J Geophys. Res. (2013)

Boils

- “Boils” are eruptions at the water surface associated with large turbulent structures.



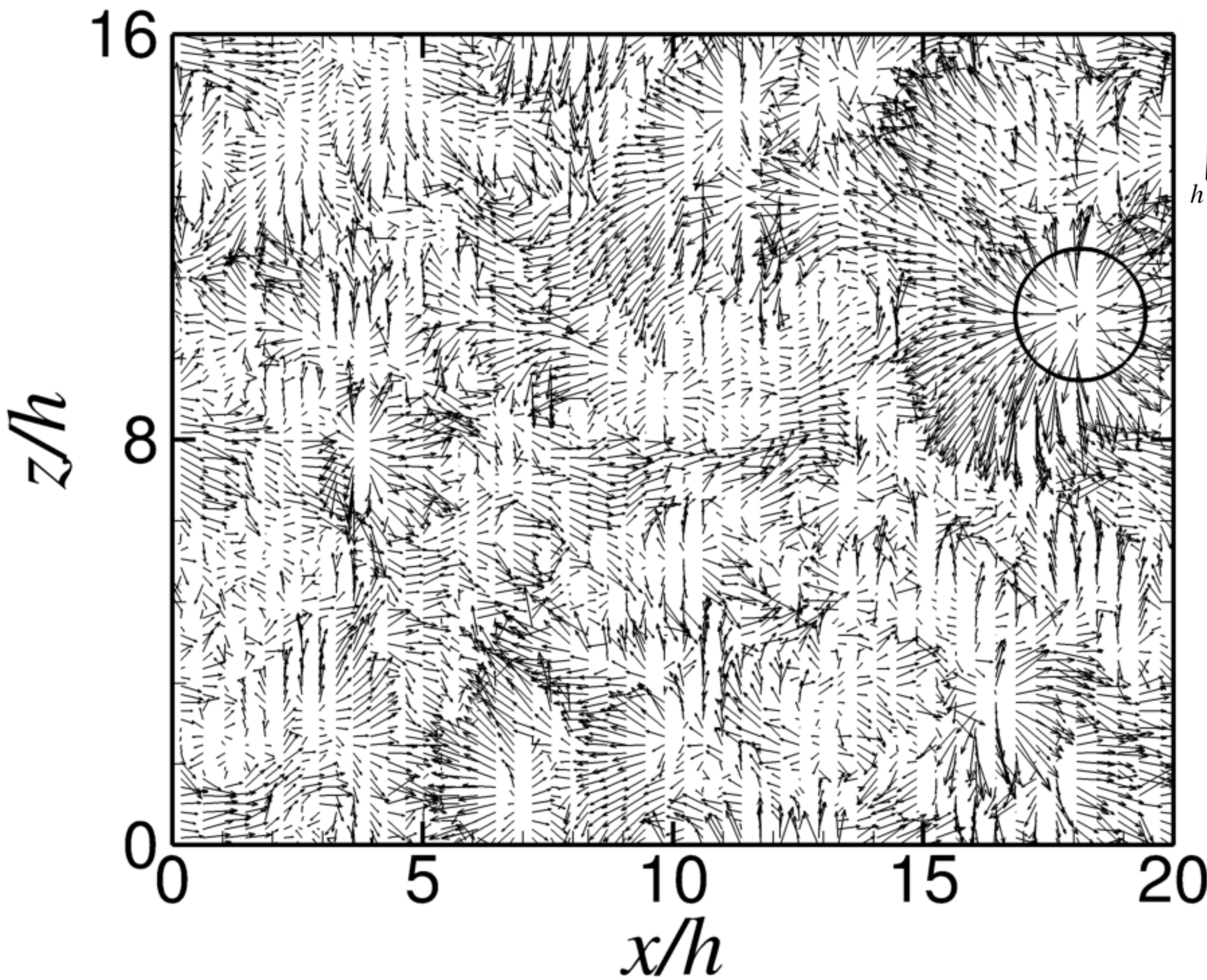
Photographs of vortex–free-surface interactions in the Jamuna River, Bangladesh. From Best (2005)

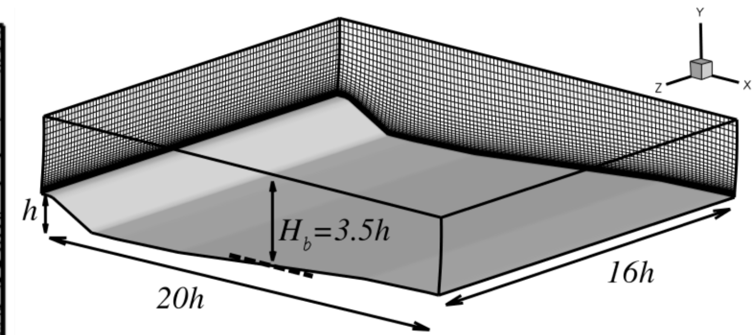
Boils

- "Boils" are eruptions at the water surface associated with large turbulent structures.
- Occur infrequently but generate significant Reynolds stress
- Are responsible for transport of fluid (sediment, nutrients….) from the bottom to the surface.
- Three conjectures on their generation:
  - *Oscillations of the reattachment line*
  - *Turbulent eddies from the stoss side*
  - *Eddies in the separated shear layer*
- Full-field, time dependent information is needed.

Boil Identification

- Boils can be identified in the numerical simulations

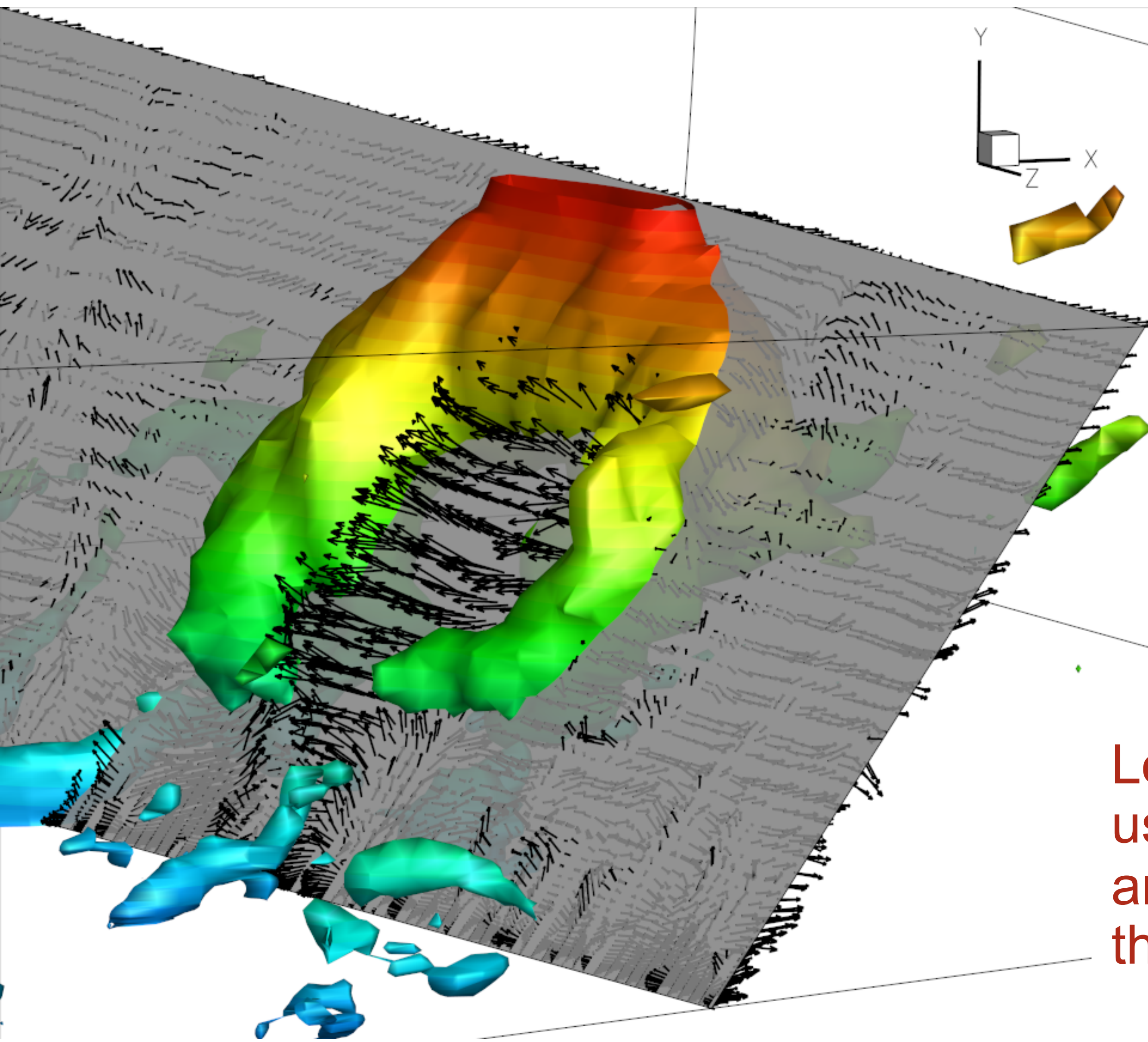


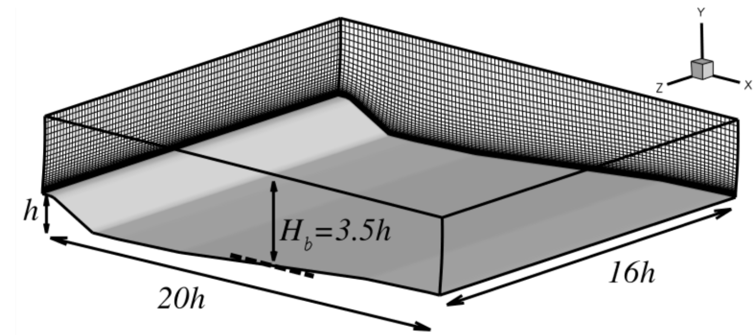


Velocity vectors at the water surface. 2D dunes

Boil Identification

- Boils can be identified in the numerical simulations and related too the vortical structures.



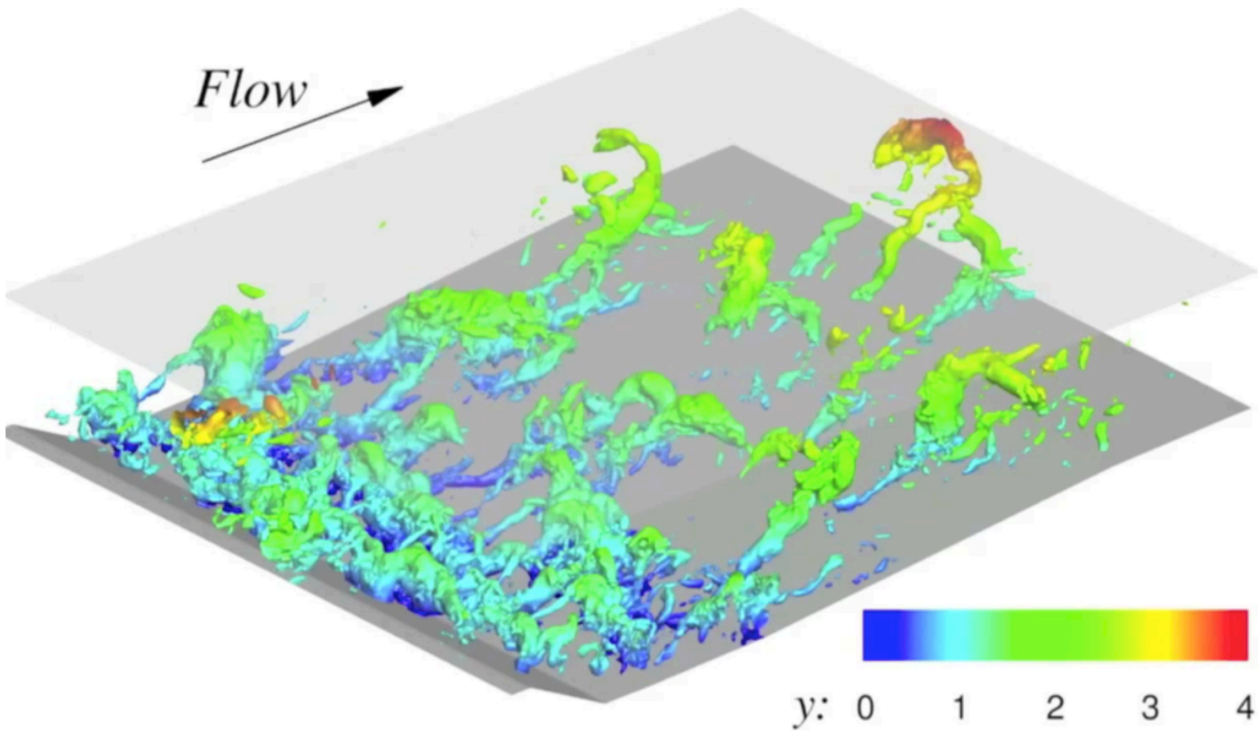


Visualization of a horseshoe vortex touching the water surface. 2D dunes.

Low-pressure isosurfaces are used to visualize the vortex, and are coloured by the distance to the free surface (0→4).

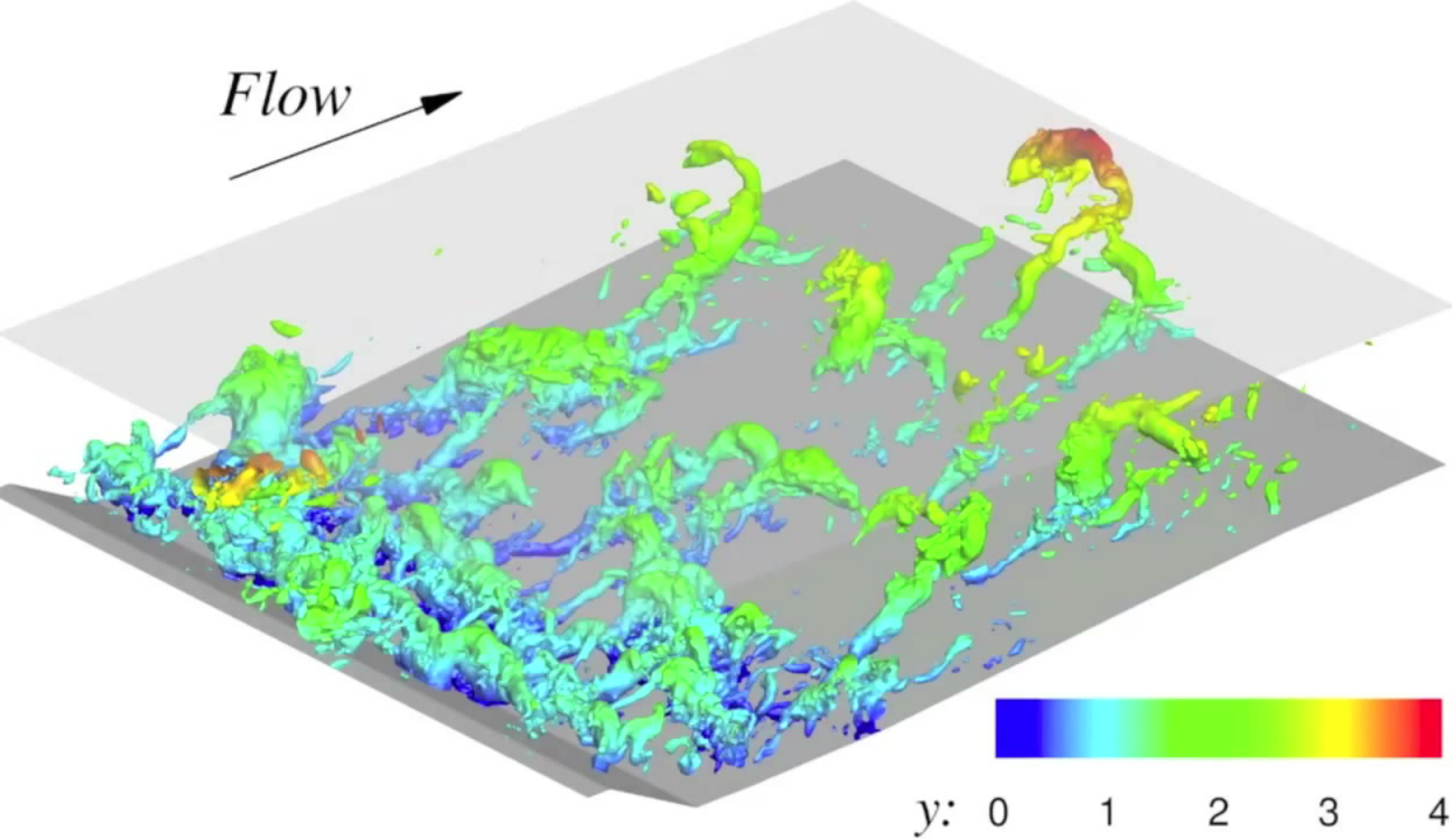
Boil Identification

- Boils can be identified in the numerical simulations and related too the vortical structures.
- Once the structures are identified, we can consider the full field.

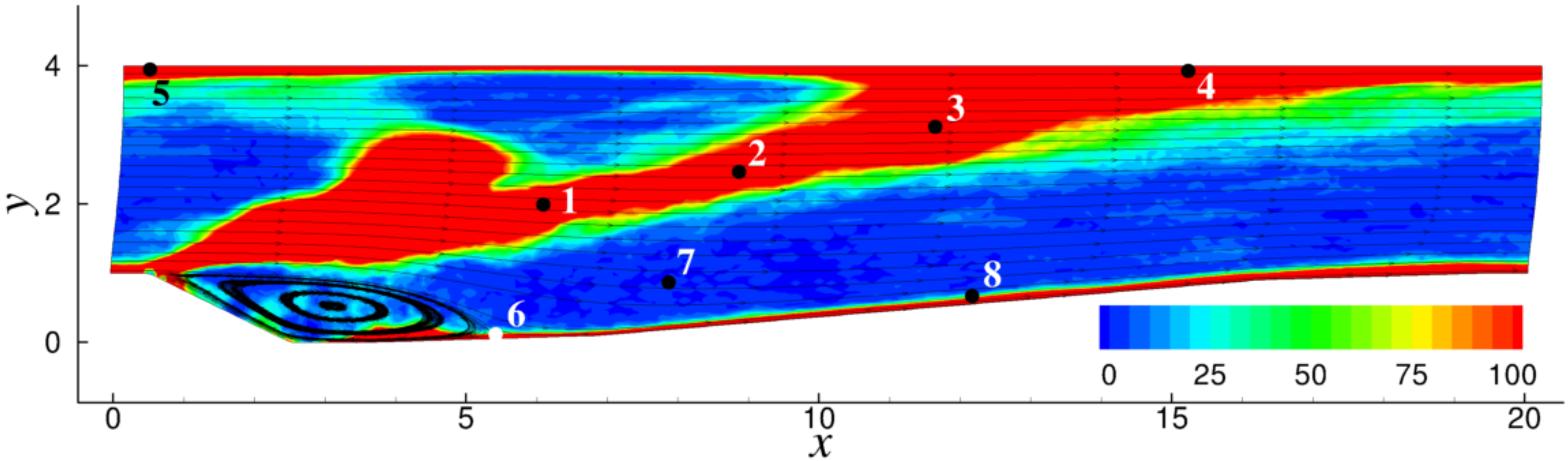


Low-pressure isosurfaces are used to visualize the vortex, and are coloured by the distance to the free surface (0→4).

Time History

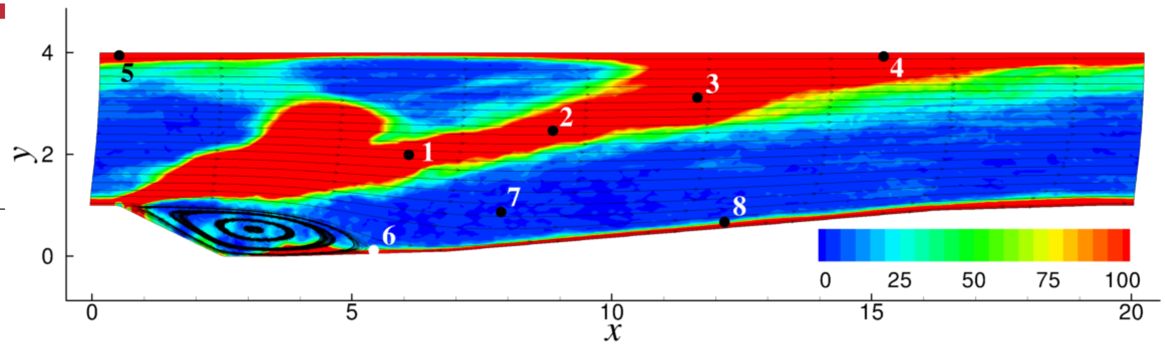


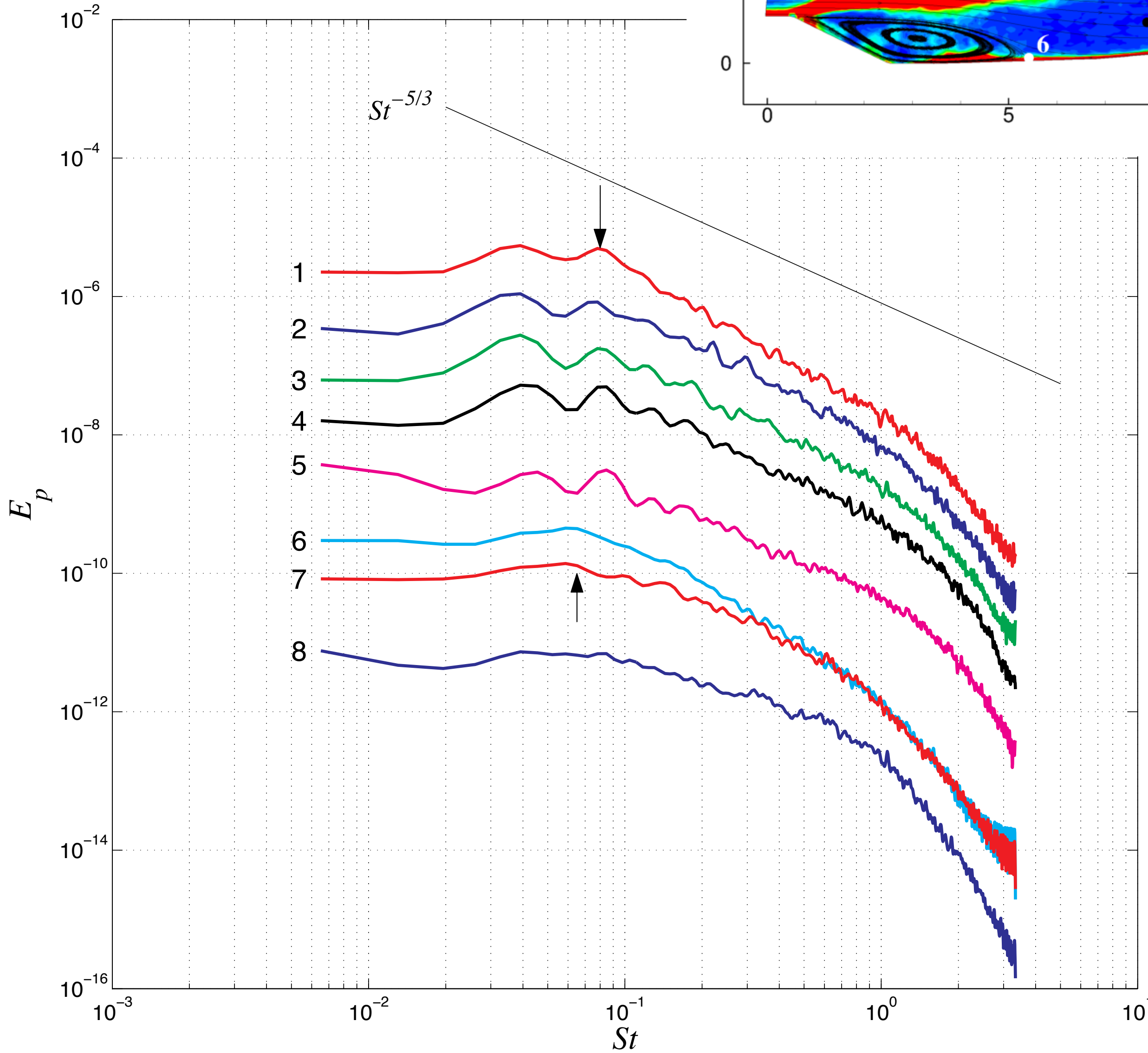
Quantitative Analysis



Frequency of horseshoe vortex appearance

Quantitative Analysis





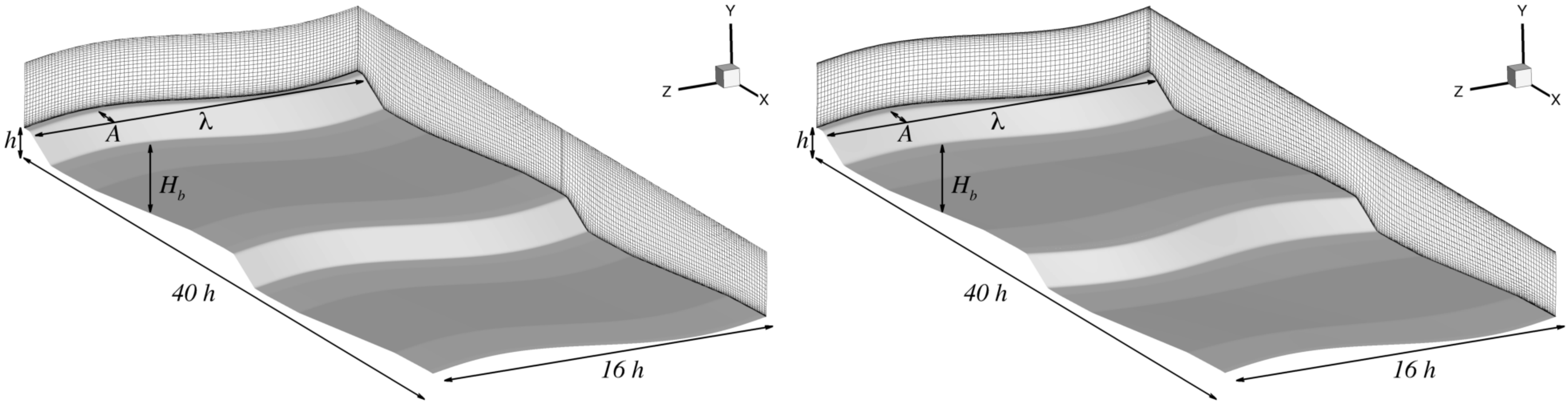
Power spectra of pressure

3D Dunes: Motivation

- Real world: Dunes are three-dimensional.
- Effects of three-dimensionality on flow resistance, sediment transport, and turbulence production are not well known.
- Experiments on three-dimensional dunes lack precise measurements of skin friction and form drag, as well as spatially-resolved turbulence stresses.

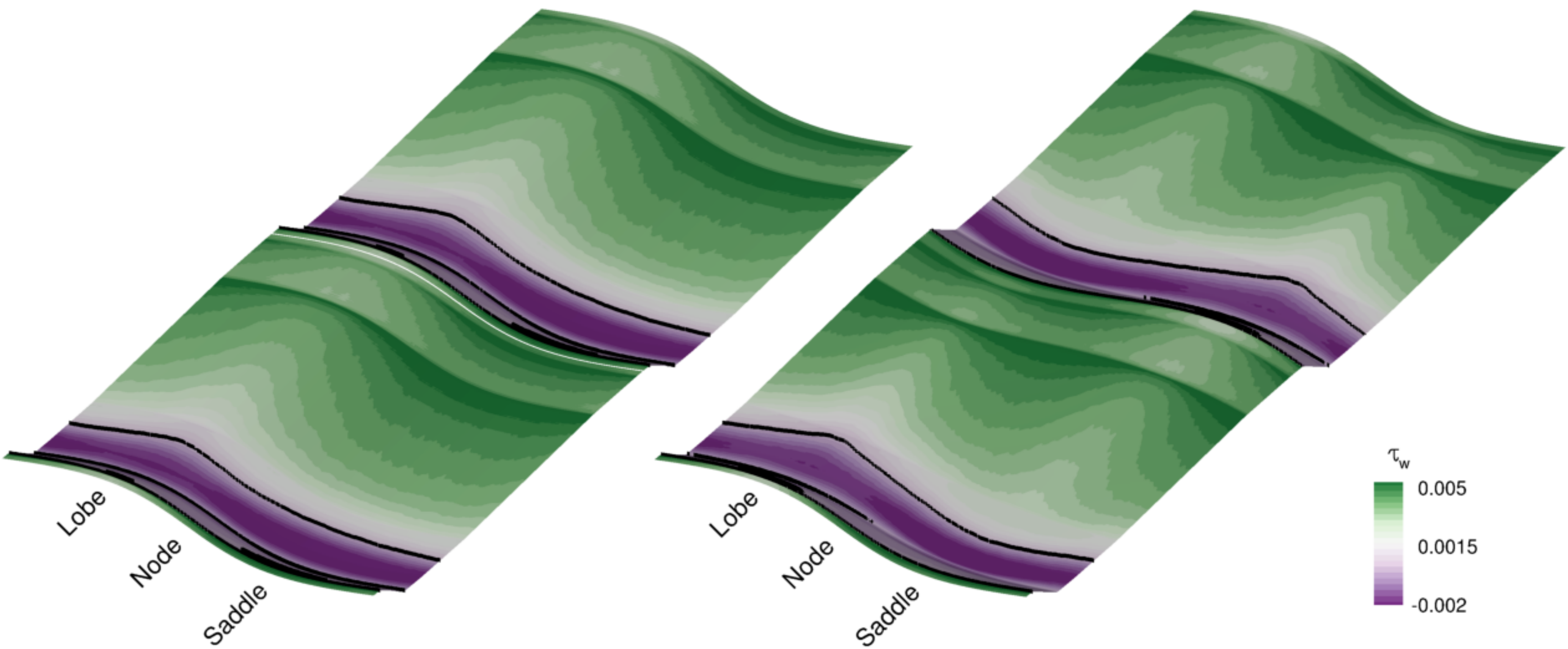
Setup

- Reynolds number: $Re = \dfrac{U_b H_b}{\nu} = 18,900$
- Two configurations: in-phase and staggered



Cases	A/h	λ/h	N_x x N_y x N_z	Δs^+	Δn^+	Δz^+
In-phase	1.0	16.0	512 x 96 x 256	22.0	0.7	18.1
Staggered	1.0	16.0	640 x 128 x 320	17.6	0.7	14.0

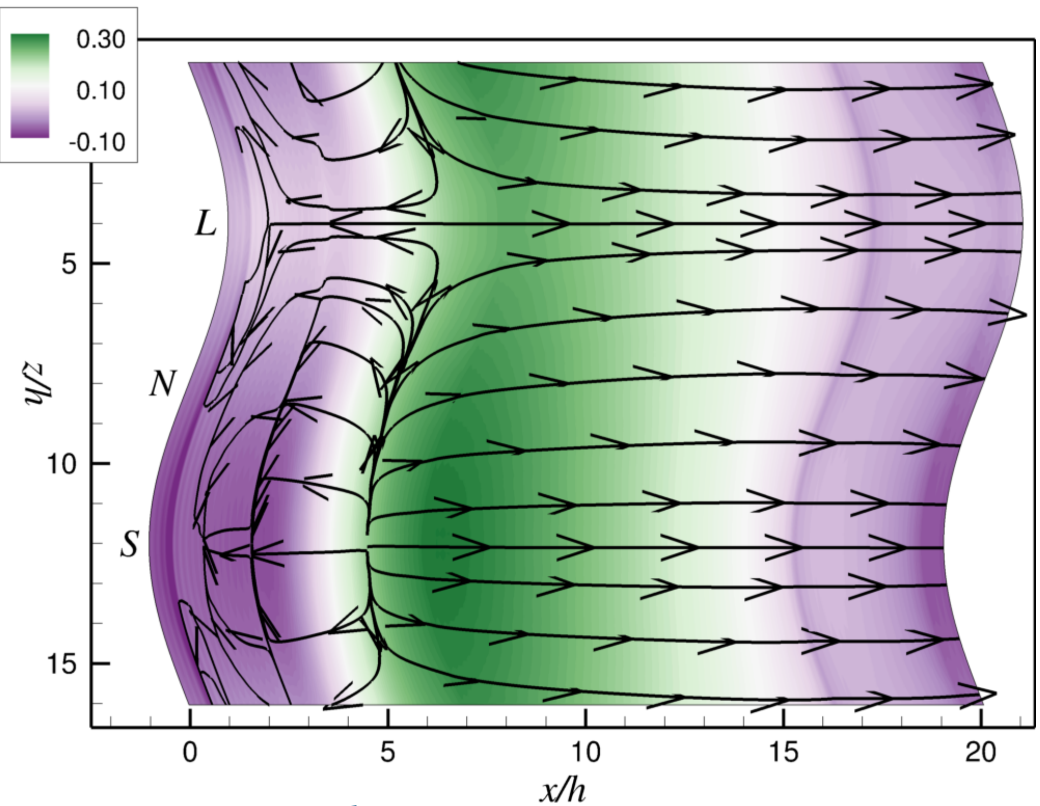
Wall stress τ_w



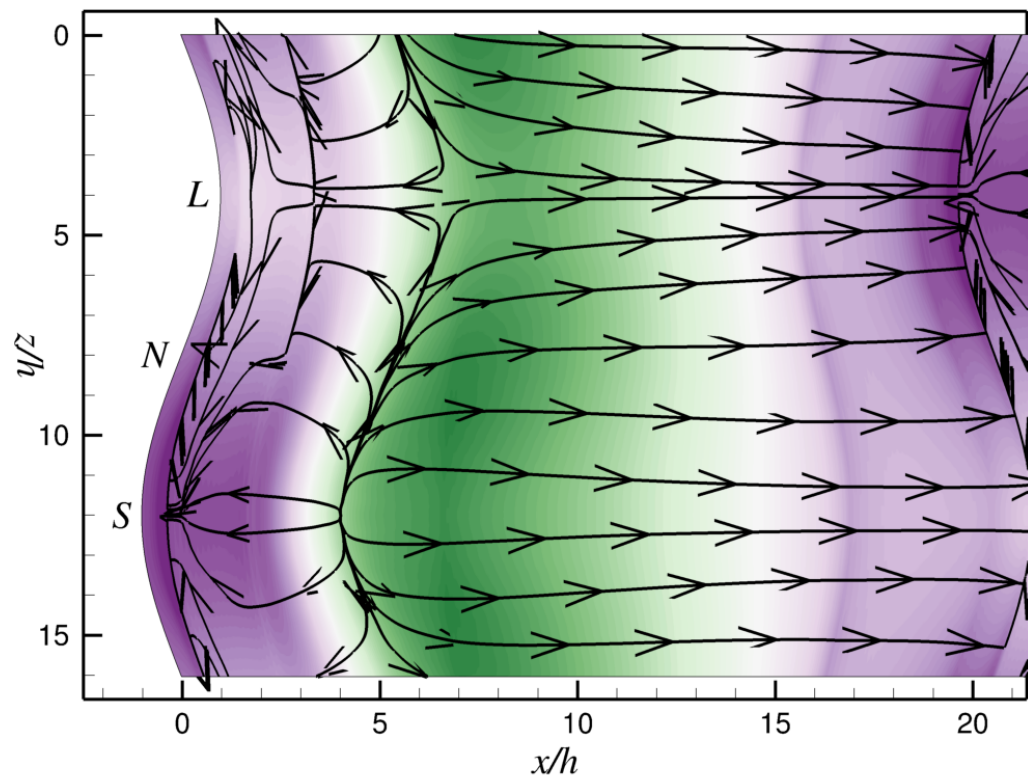
In-phase *Staggered*

- Lateral pressure gradient (from node to lobe) induces spanwise flow

Mean wall pressure and wall streamlines

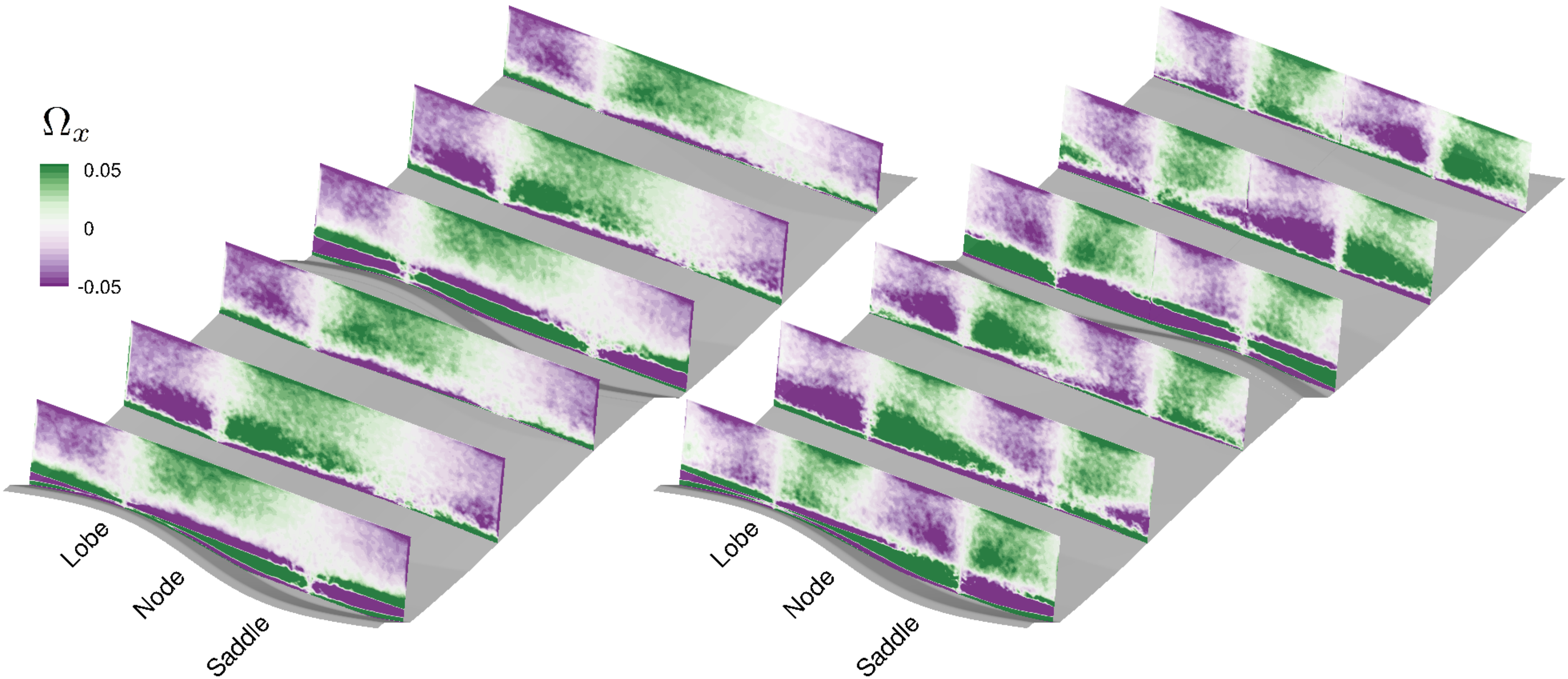


In-phase



Staggered

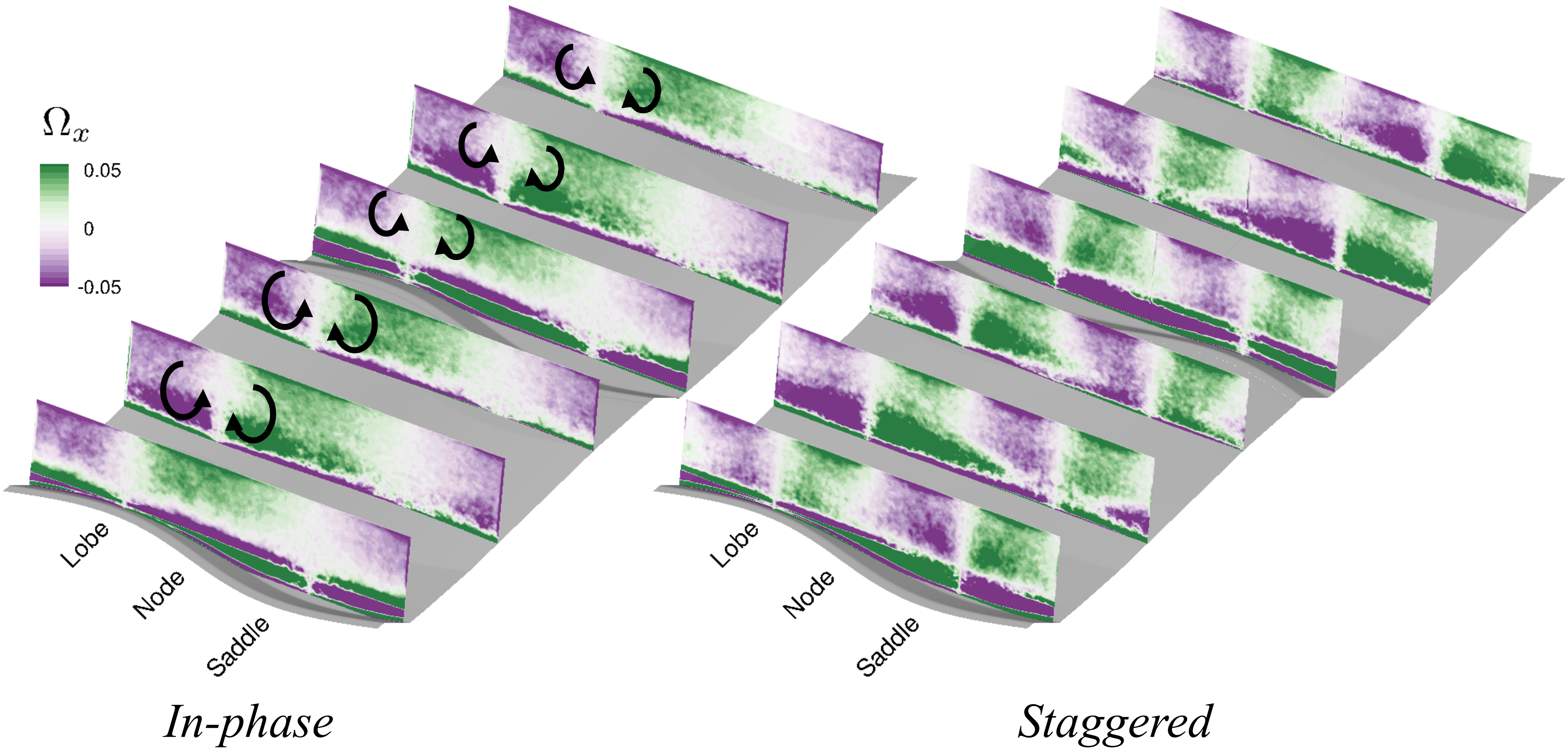
Mean streamwise vorticity Ω_x



In-phase

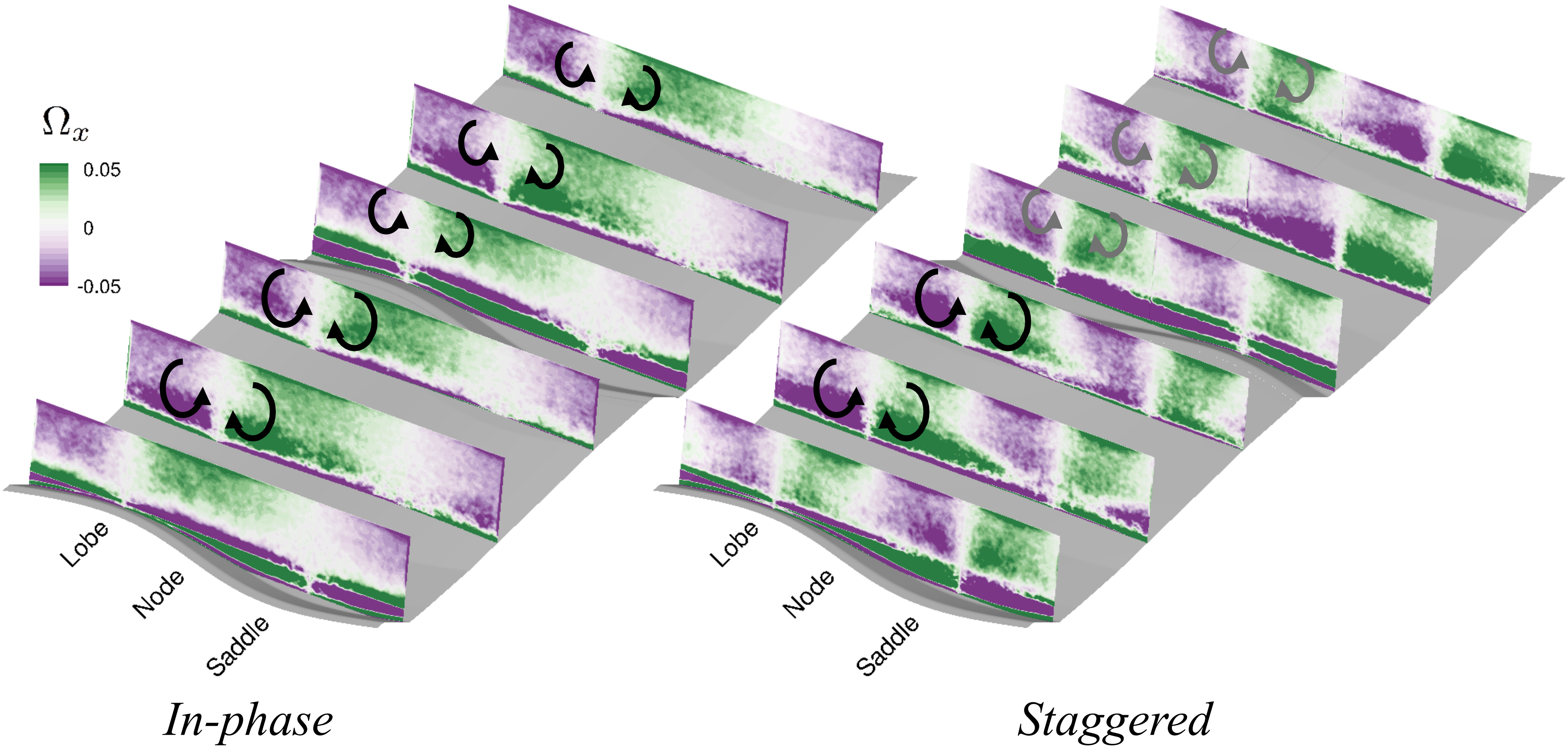
Staggered

Mean streamwise vorticity Ω_x

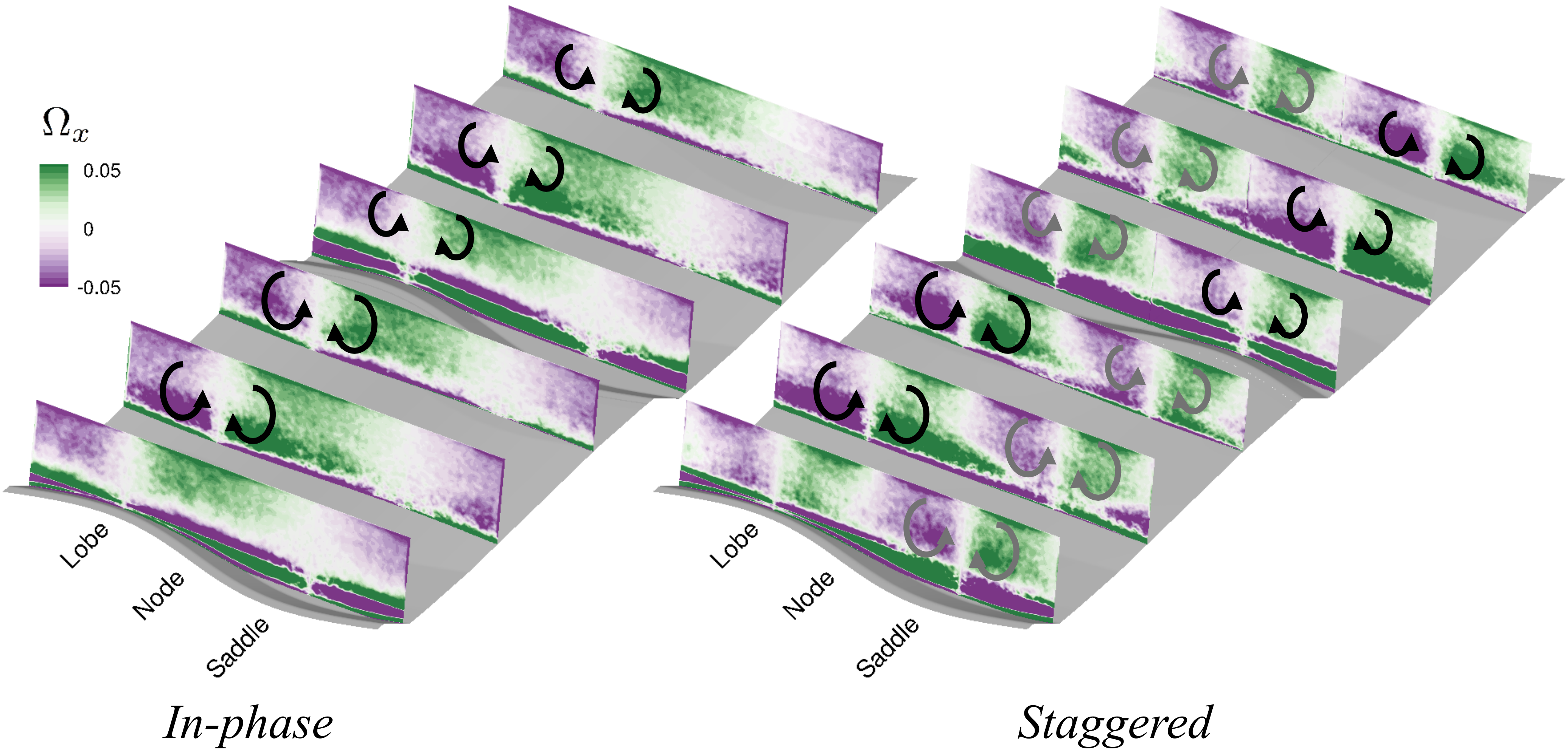


Results

Mean streamwise vorticity Ω_x

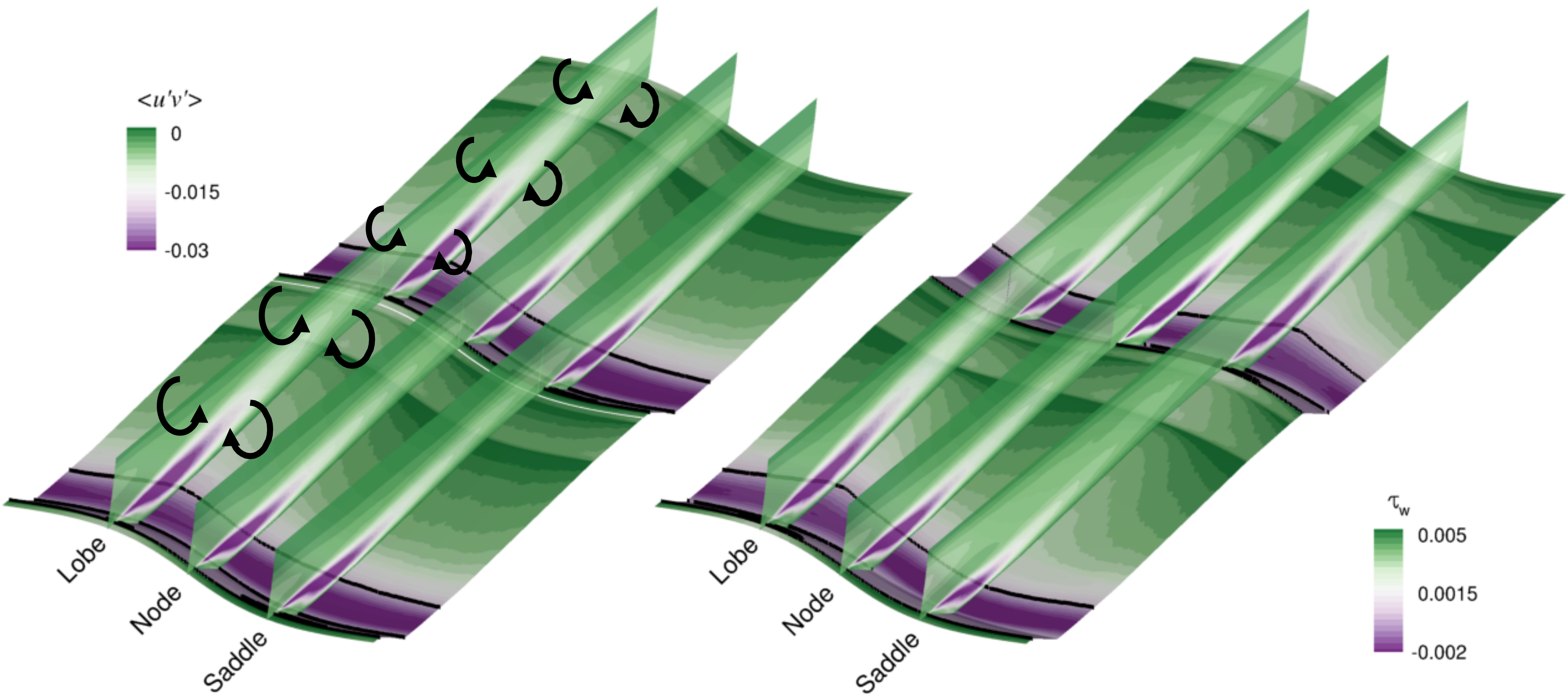


Mean streamwise vorticity Ω_x



Results

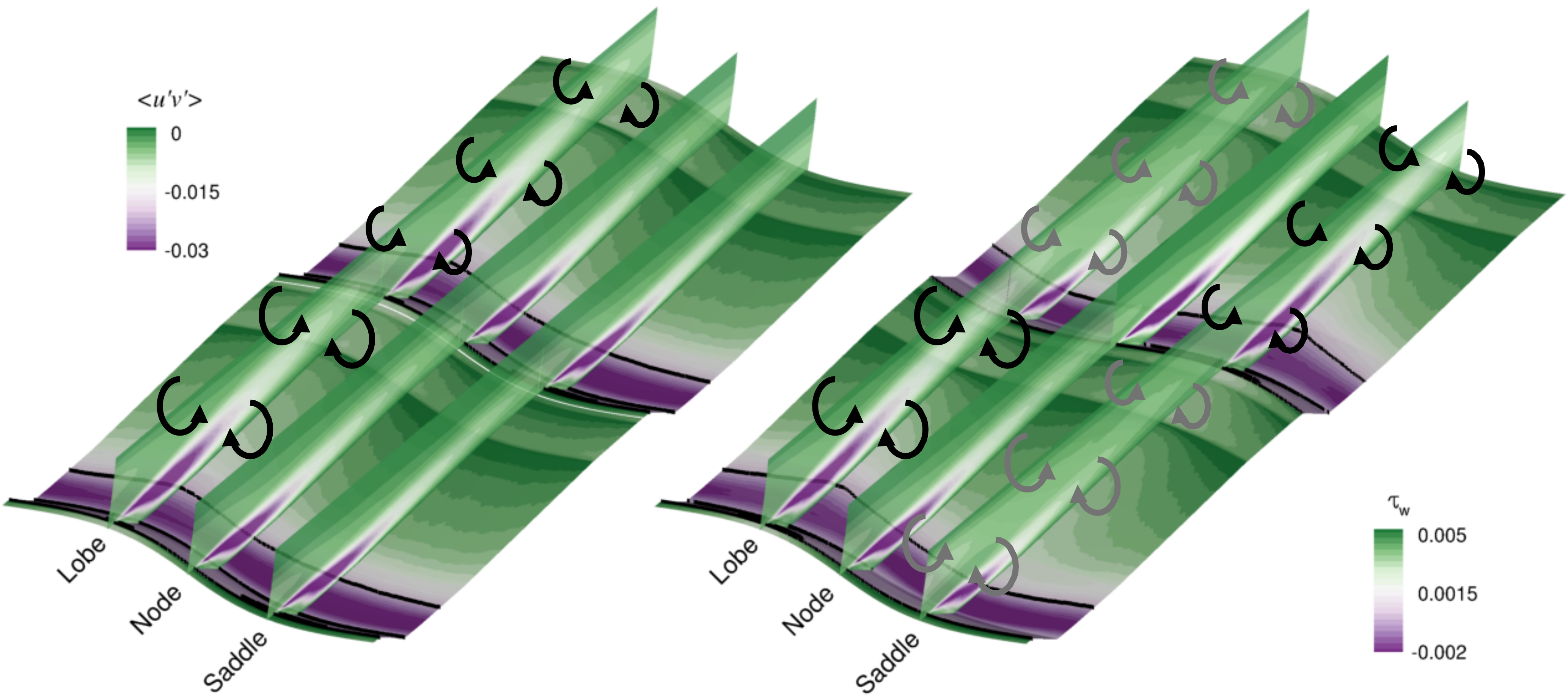
Reynolds shear stress $<u'v'>$ and wall stress τ_w



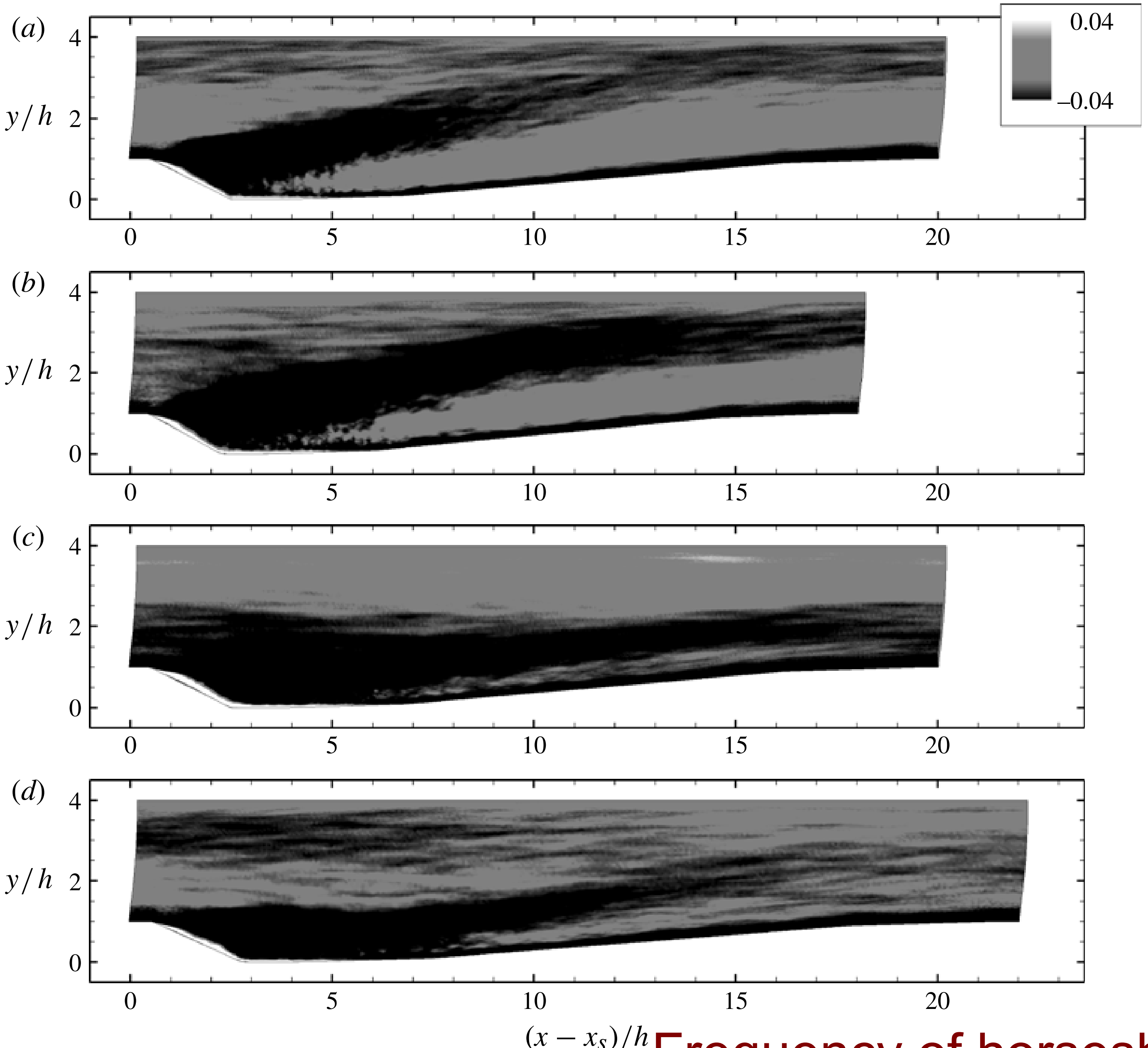
In-phase *Staggered*

RESULTS

Reynolds shear stress $<u'v'>$ and wall stress τ_w



In-phase *Staggered*



Lobe, in-phase

Lobe, staggered

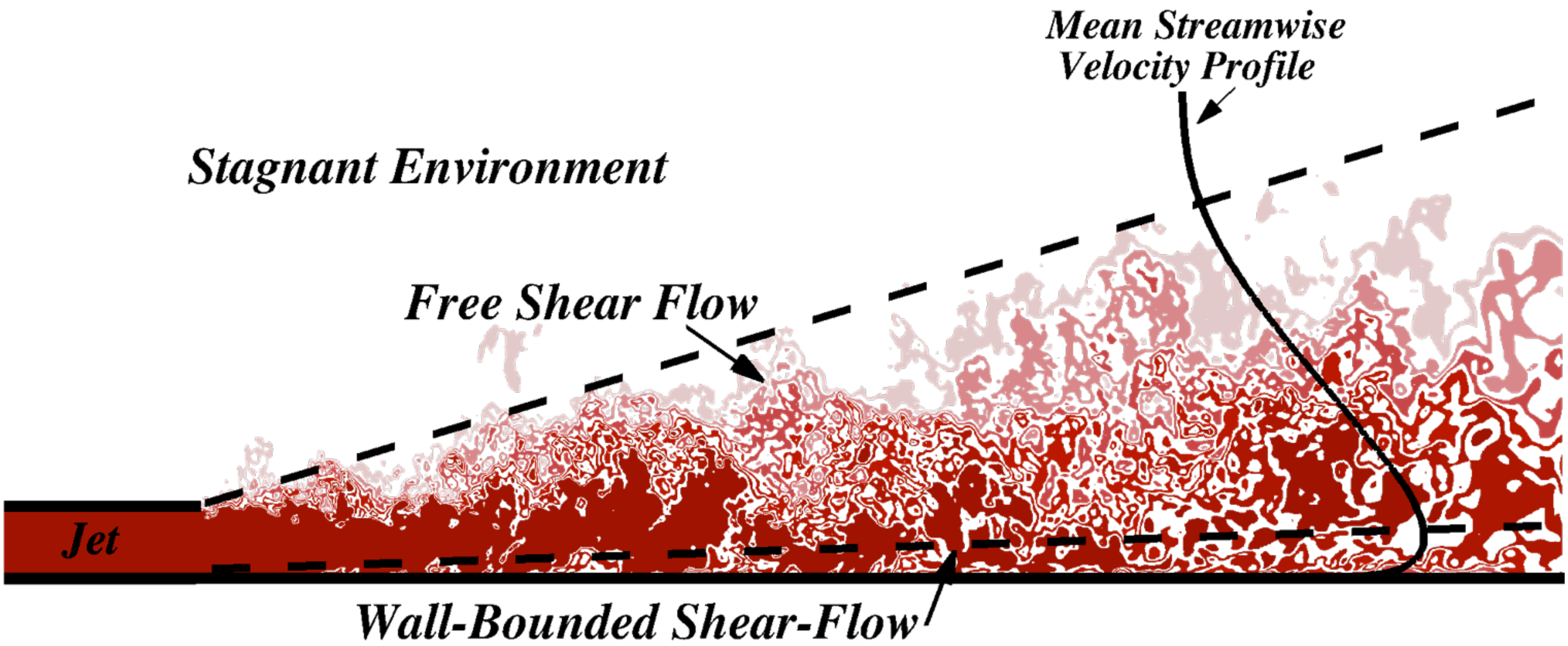
Saddle, in-phase

Saddle, staggered

Frequency of horseshoe vortex appearance

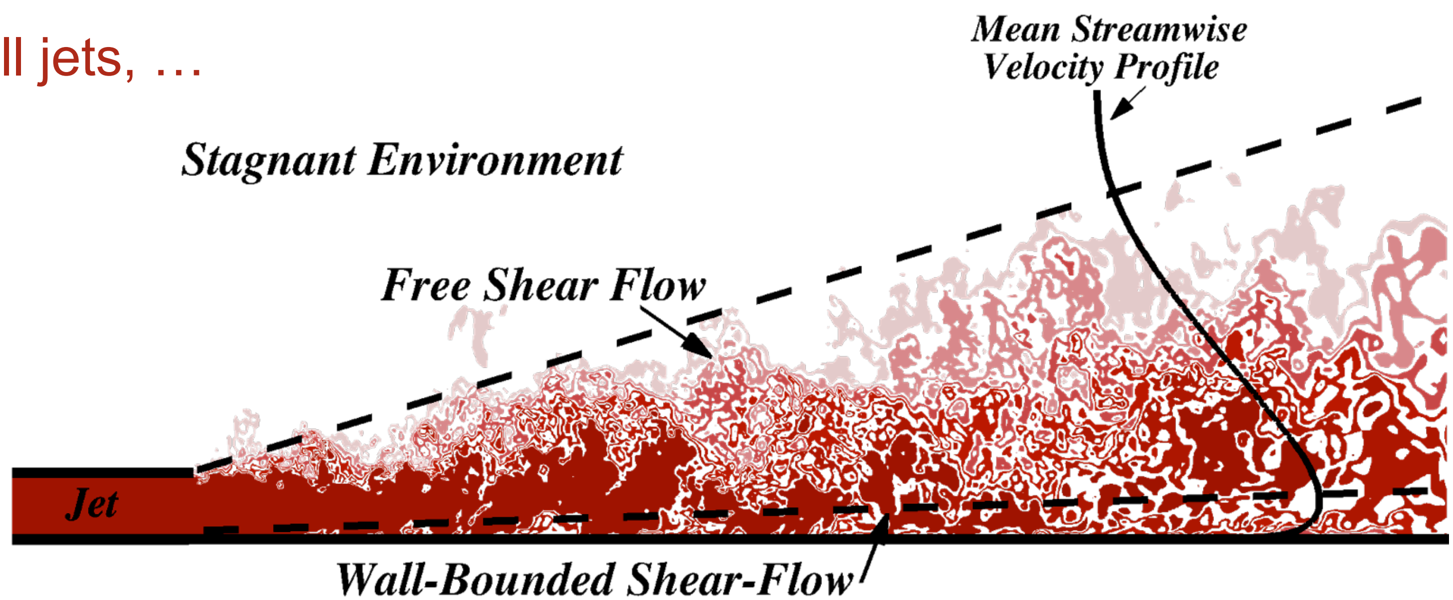
3D Dunes
Conclusions

- The three-dimensional bed form induces mean secondary flow in the streamwise direction.
  - *Low-momentum fluid close to the bed moves from the saddle plane toward the lobe plane, generating a vortex pair.*
  - *The secondary flow affects the whole flow depth.*
  - *In the staggered configuration, there are two vortex pairs, one formed at the lobe and one advected over the saddle from the previous dune.*
- The spatial distribution of the separated-shear layer alters the flow across the channel.
  - *The upwash of slow fluid enhances the flow deceleration and acceleration in the lobe plane.*
  - *Advection displaces the shear layer and the horseshoe vortices upwards.*



Mean Streamwise
Velocity Profile
Stagnant Environment
Free Shear Flow
Jet
Wall-Bounded Shear-Flow

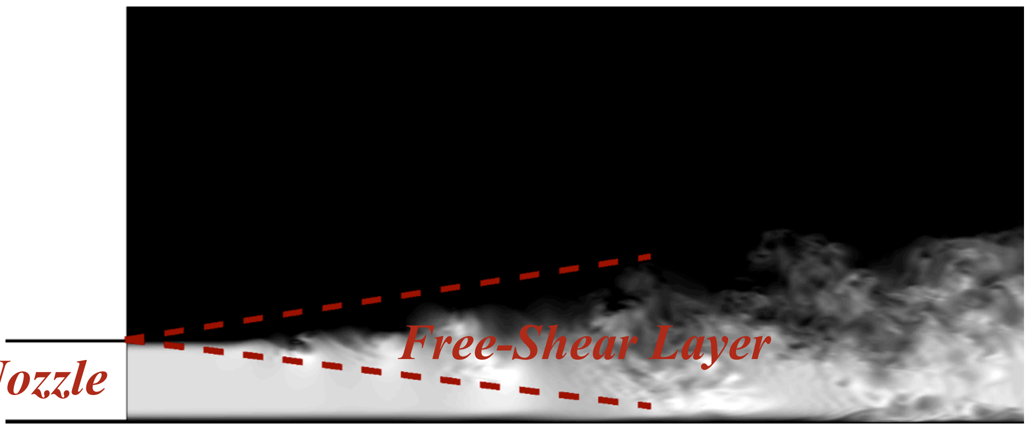
- Canonical turbulent shear-flows
  - *Wall-bounded shear flows*
    - Channel flows, pipes , boundary layers, …
  - *Free-shear flows*
    - Jets, wakes, …
- Hybrid turbulent shear-flows
  - *A combination of two or more of the canonical shear flows*
    - Wall jets, …



Introduction
Types of Wall-jets

- Two classes of wall-jets considered in this research
  - *Plane (two-dimensional) wall-jets*
  - *Radial wall-jets*

Plane wall-jet:



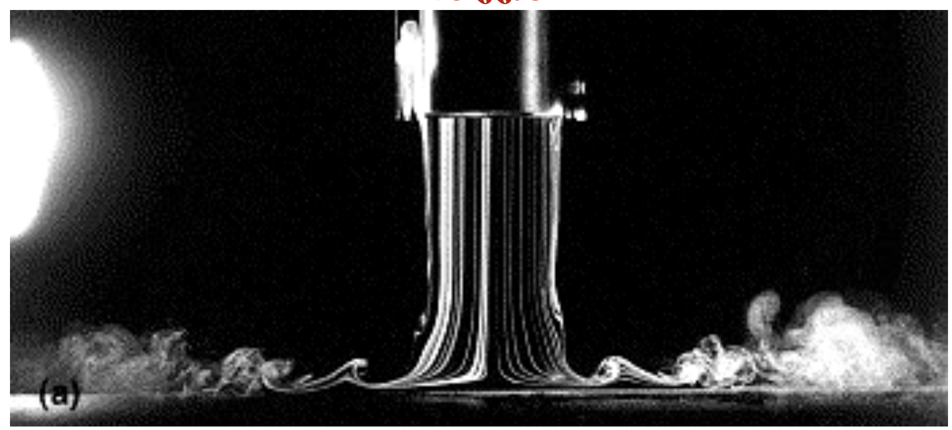
Nozzle

Fully-developed wall-jet region

Source: Current study

Radial wall-jet:

Nozzle

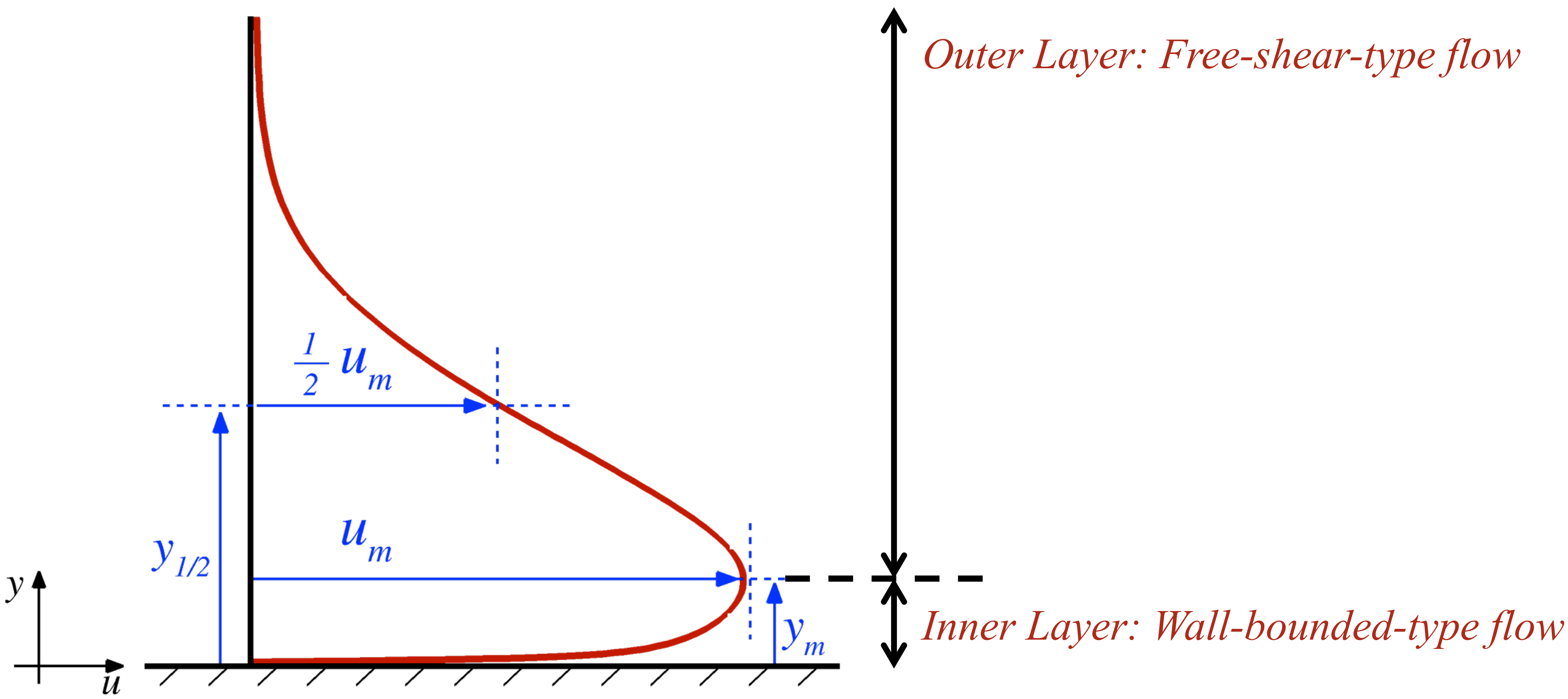


Impingement region

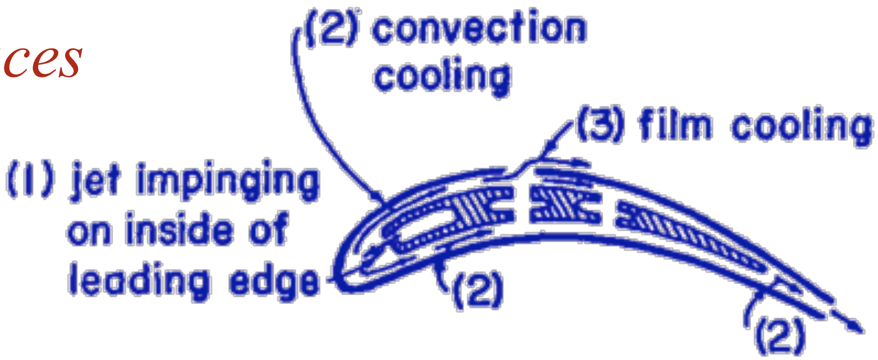
Fully-developed wall-jet region

Source: Cornaro et al., Exp. Therm. Fluid Sci., 1999

- Typical fully-developed mean streamwise velocity profile



- Wall-jets have diverse applications in the fields of
  - *Downwash of vertical take-off and landing aircrafts and helicopters*
  - *Heating, cooling, and drying of surfaces*
  - *Separation control over airfoils*
  - *Air-conditioning in the buildings*
  - *Water flow under sluice gates*
  - *Downburst of a tornado*



Source: J.L. Kerrebrock, Aircraft Engine and Gas Turbines (2nd Edition), MIT Press, 1992.

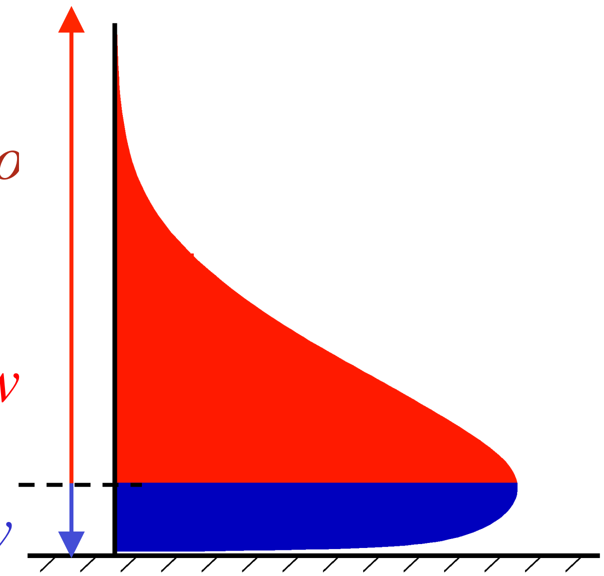
- The study of wall-jets will be useful in understanding
  - *More complex flows with two or more interacting shear layers*
  - *All sorts of asymmetric turbulent shear-flow*

Literature
Inner/outer layer interaction

- Two non-interacting shear layers
  - *Glauert (1956), Abrahamsson et al. (1994), Ro*

Outer Layer: Free-shear-type flow

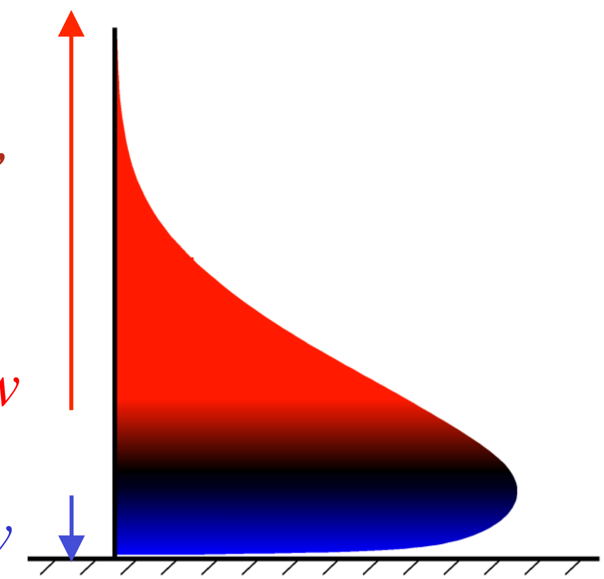
Inner Layer: Wall-bounded-type flow



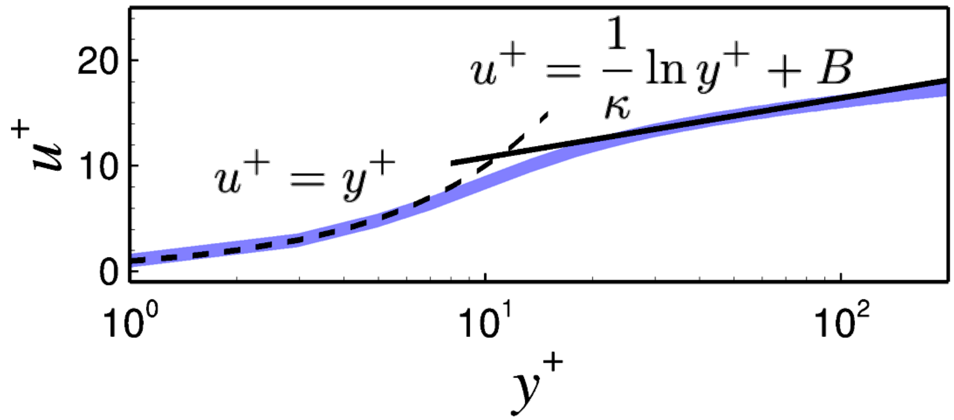
- Two interacting layers or three layers
  - *George et al. (1997), Barenblatt et al. (2005), (2004-2007), …*

Outer Layer: Free-shear-type flow

Inner Layer: Wall-bounded-type flow



- Logarithmic law of the wall



Reference	Wall-jet type	κ	B
Bradshaw and Gee, 1962	Plane	0.49	6.8
Patel, 1962	Plane	0.49 to 0.60	6.8
Wygnanski *et al.*, 1992	Plane	0.42	5.5 to 9.5
Ozdemir and Whitelaw, 1992	Radial	0.41	$1.292\, u_m/u_\tau - 6.2$
Abrahamsson *et al.*, 1994	Plane	0.41	5.0
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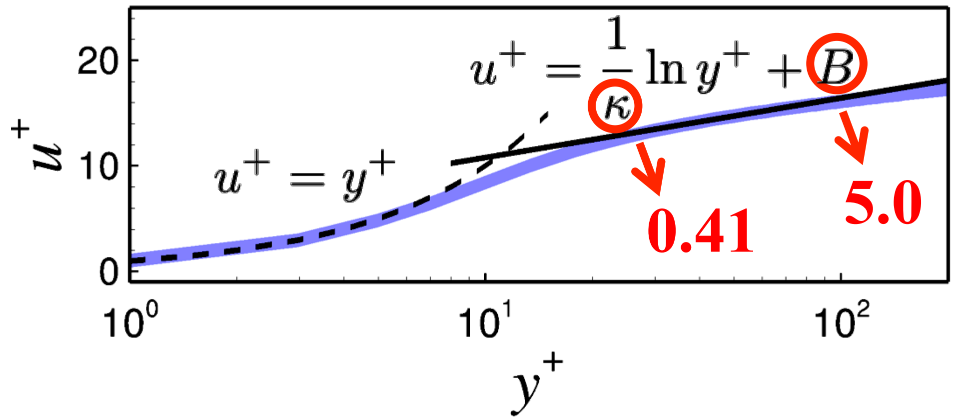
Tachie *et al.*, 2002	Plane	0.41	5.0
Dejoan and Leschziner, 2005	Plane	0.41	5.0
Smith, 2008	Plane	0.55	8.7
Uddin *et al.*,2013	Radial	0.41	$1.122\, u_m/u_\tau - 10.53$

- Logarithmic law of the wall



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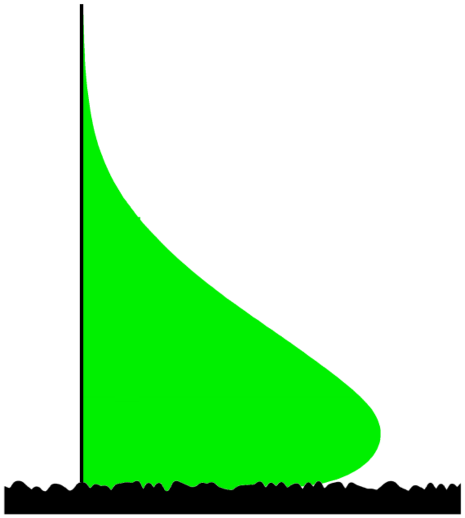
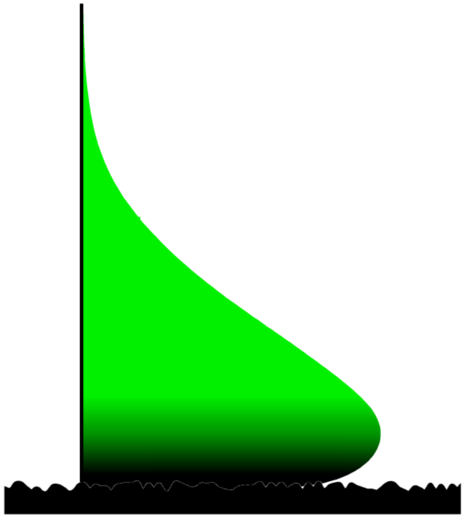
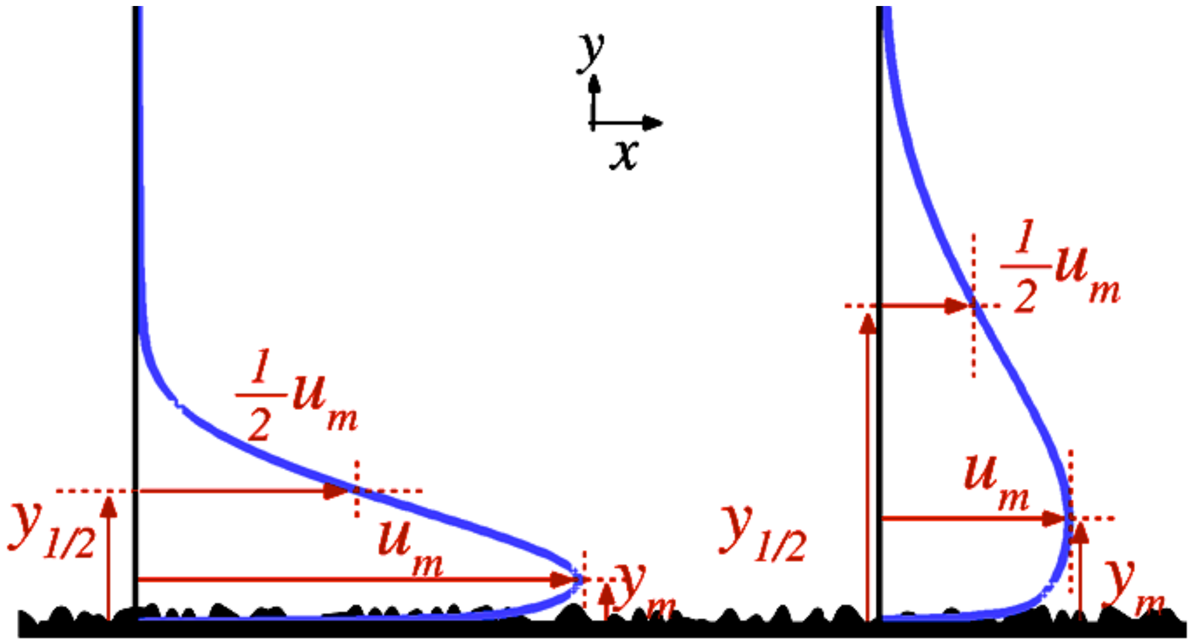
- Analysis of first order statistics
  - *Rajaratnam (1976), Hogg et al. (1997), ...*



- Analysis of the effects of roughness on different features of wall-jets
  - *Tachie et al. (2002-2004), Smith (2008), Rajaratnam and Mazurek (2005), Rostamy et al. (2011), ...*



- Rates of evolution of wall-jets



	u_m	y_m	$y_{1/2}$	$-du_m/dx$	dy_m/dx	$dy_{1/2}/dx$
Rajaratnam *et al.* (1981)	↓	-	-	-	↑	c
Tachie *et al.* (2004)	↓	↑	c	↑	↑	c
Smith (2008)	c	↑ or ≈c	c	c	-	c
Rostamy *et al.* (2011)	↓	↑	↑	↑	-	c

↑, increased; ↓, decreased; c, constant; -, not reported

Objectives

- The objective of this research is to
  - *Investigate some of the unknown aspects of plane and radial wall-jets*
    - The interaction of inner and outer layers
    - The extend to which the effects of roughness spreads away from the wall
    - The similarities and differences between plane and radial wall-jet
  - *Employ the results to answer some of the open questions in the literature*
    - Universality of the log-law constant
    - Effects of roughness on the rates of evolution of wall-jets

Formulation

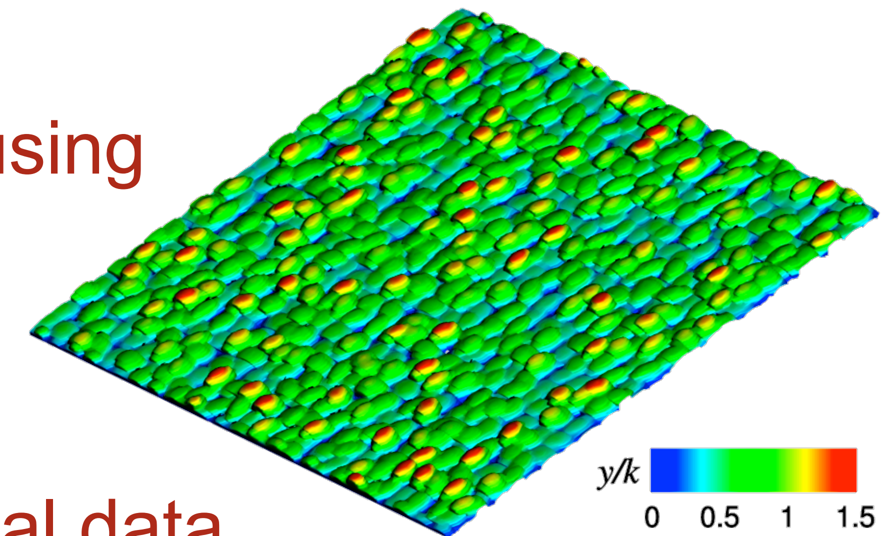
- Filtered continuity and Navier-Stokes equations

$$\nabla.\overline{\mathbf{u}} = 0 \qquad \frac{D\overline{\mathbf{u}}}{Dt} = -\nabla\overline{P} + 2\nabla.\left(\nu_{tot}\overline{\mathbb{S}}\right) \qquad \nu_{tot} = \frac{1}{Re} + \nu_t$$

- Eddy-viscosity assumption

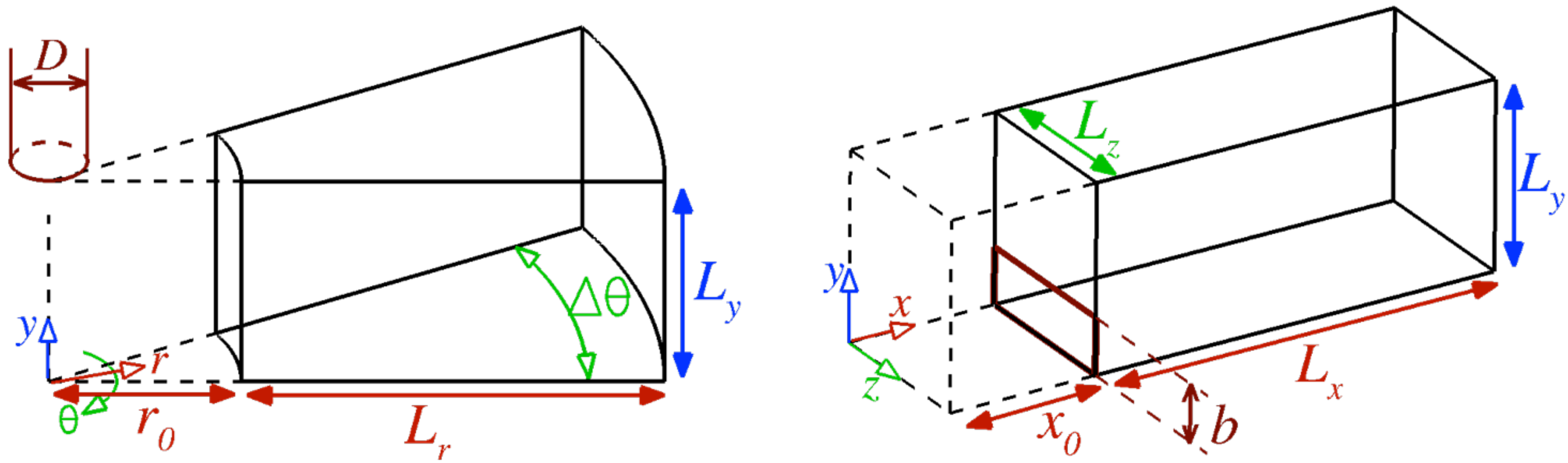
$$\nu_t = C\overline{\Delta}^2|\overline{\mathbb{S}}|$$

- Lagrangian dynamic eddy-viscosity model
- 2nd –order accurate in space and time, staggered finite difference code
- Random roughness is modeled using
  - *Virtual sandpaper model (Scotti 2006)*
  - *Immersed boundary method (IBM) based on volume of fluid (VOF) approach*
- Validated against the experimental data



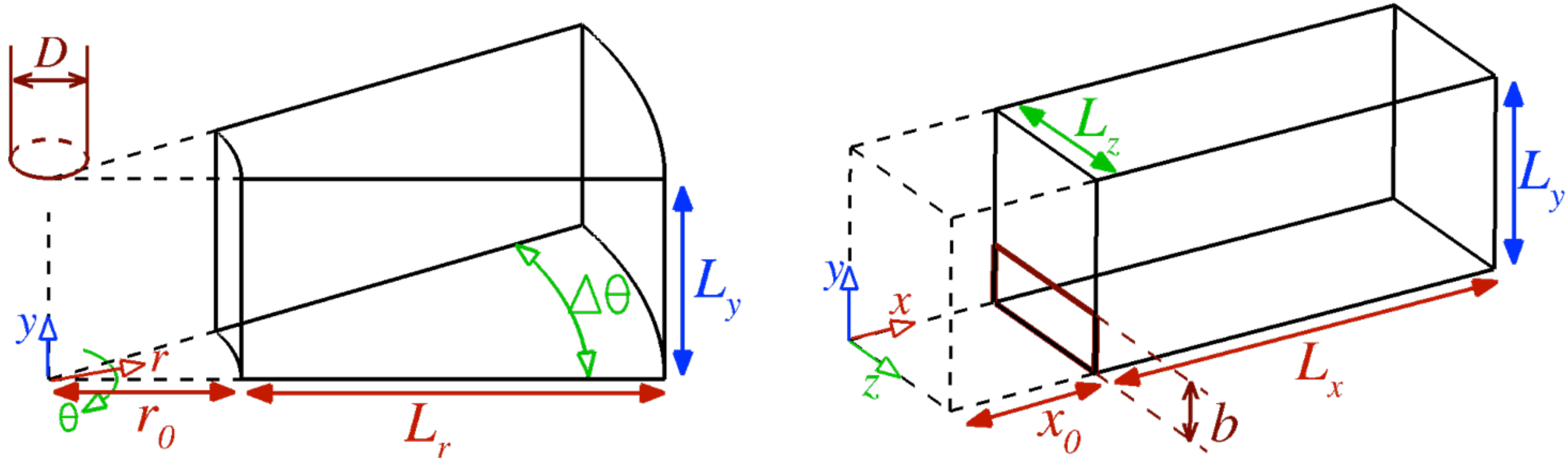
Setup

Effects of roughness

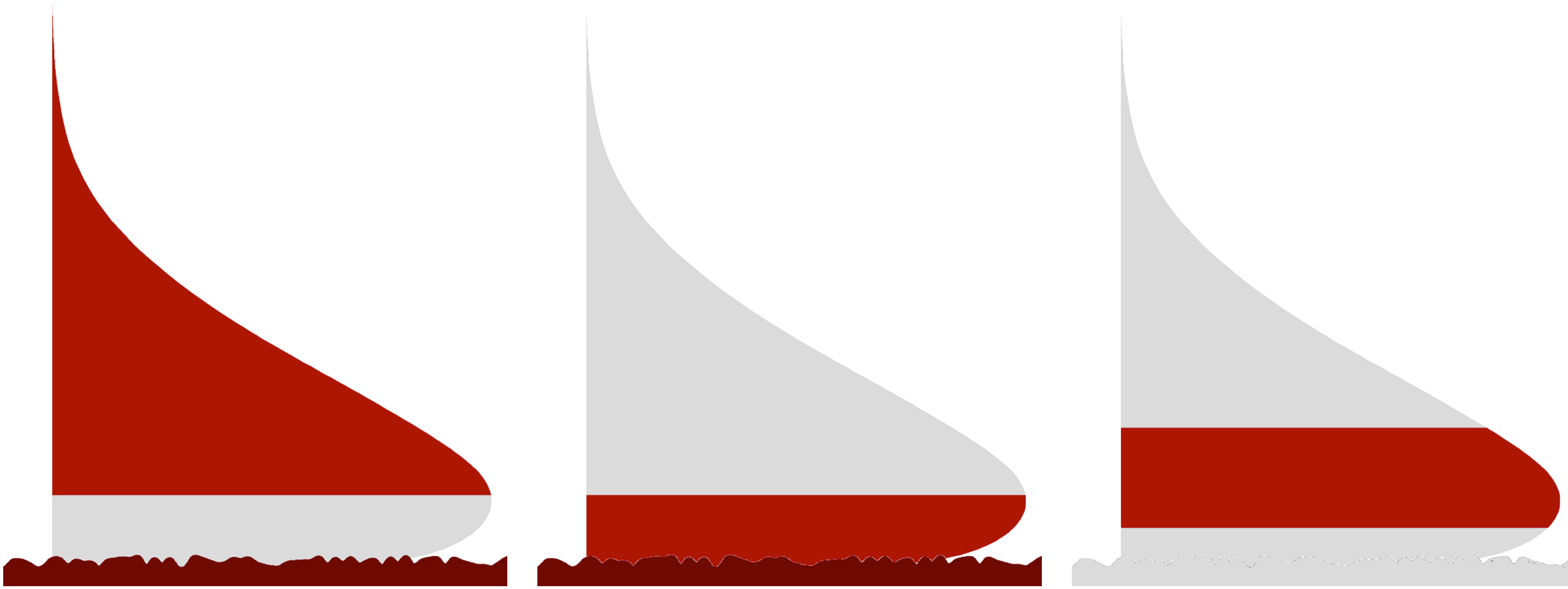


	x_0 or r_0	L_x or L_r	L_y	L_z or $\Delta\theta$	Re	Roughness	k_s^+
Plane-S2	0	40b	20b	10b	$u_b b/\nu = 7{,}500$	-	-
Plane-R	0	40b	20b	10b	$u_b b/\nu = 7{,}500$	10b to 35b	80 to 100
Radial-S	1D	29D	6D	$\pi/3$	$v_b D/\nu = 40{,}00$	-	-
Radial-R	1D	29D	6D	$\pi/3$	$v_b D/\nu = 40{,}00$	6D to 26D	20 to 40
Plane-S-I	1D	29D	10D	10D	$v_b D/\nu = 40{,}00$	-	-
Plane-S-L	1D	29D	10D	10D	$v_b D/\nu = 7{,}500$	-	-
Plane-S-H	0	30b	10b	5.5b	$u_b b/\nu = 40{,}000$	-	-

SETUP
INNER/OUTER LAYER INTERACTION



	x_0 or r_0	L_x or L_r	L_y	L_z or $\Delta\theta$	Re	Roughness	k_s^+
Plane-S2	0	$40b$	$20b$	$10b$	$u_b b/\nu = 7{,}500$	-	-
Plane-R	0	$40b$	$20b$	$10b$	$u_b b/\nu = 7{,}500$	$10b$ to $35b$	80 to 100
Radial-S	$1D$	$29D$	$6D$	$\pi/3$	$v_b D/\nu = 40{,}000$	-	-
Radial-R	$1D$	$29D$	$6D$	$\pi/3$	$v_b D/\nu = 40{,}000$	$6D$ to $26D$	20 to 40
Plane-S-I	$1D$	$29D$	$10D$	$10D$	$v_b D/\nu = 40{,}000$	-	-
Plane-S-L	$1D$	$29D$	$10D$	$10D$	$v_b D/\nu = 7{,}500$	-	-
Plane-S-H	0	$30b$	$10b$	$5.5b$	$u_b b/\nu = 40{,}000$	-	-



roughness
outer layer

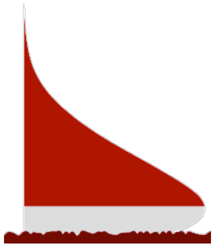
roughness
inner layer

Interaction inner
outer layers

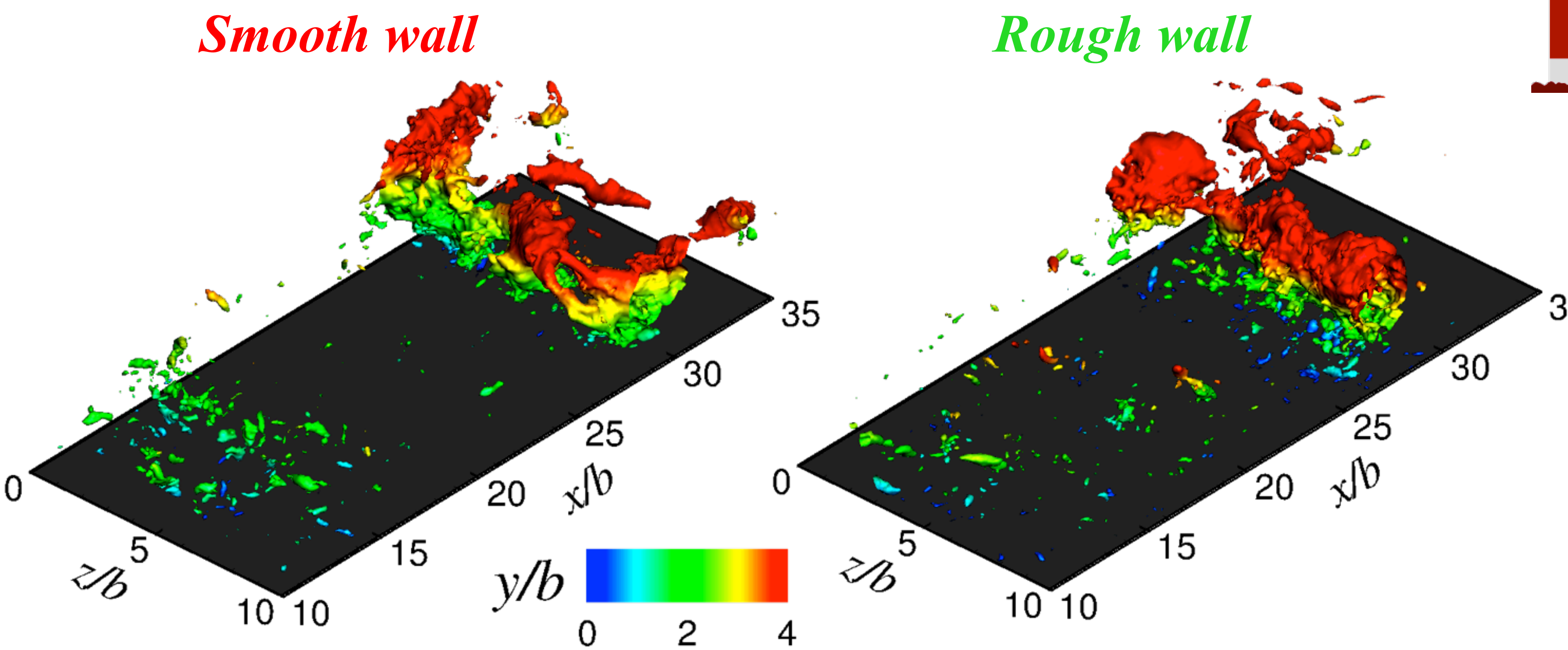
roughness
outer layer

Effects of roughness
on the inner layer

Interaction of inner
and outer layers



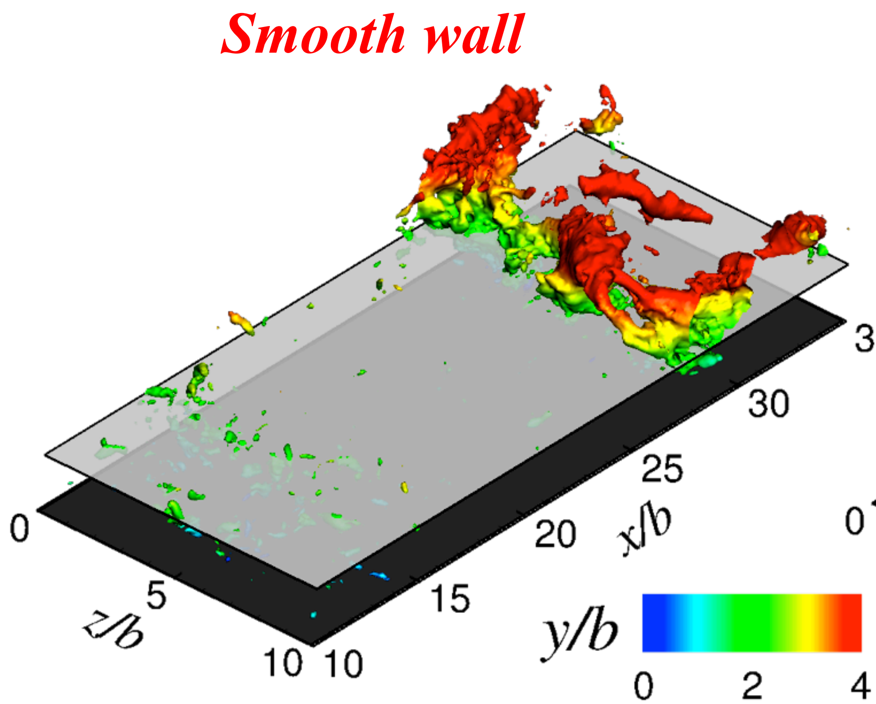
- Visualization of the instantaneous flow field

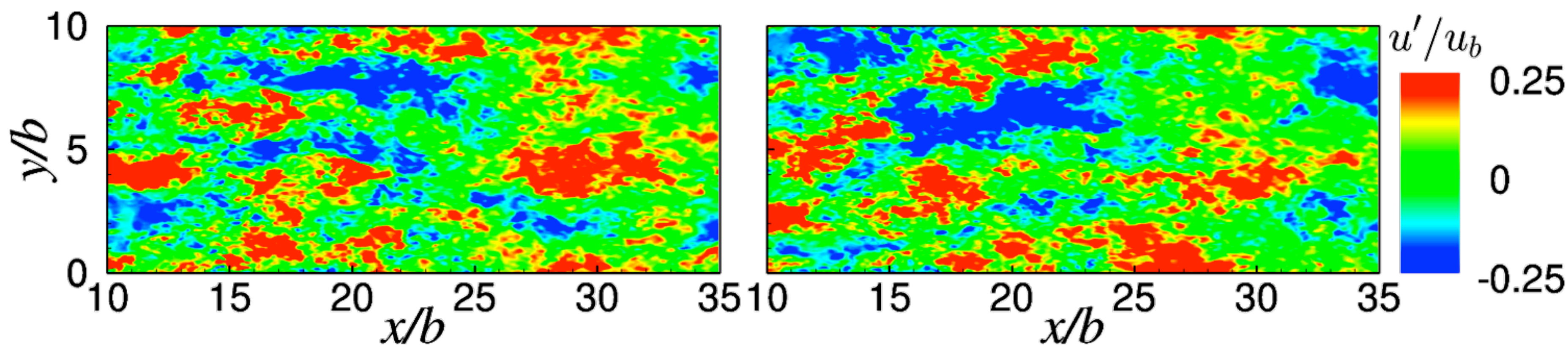


Results

Effect of roughness on the outer layer

- Visualization of the instantaneous flow field

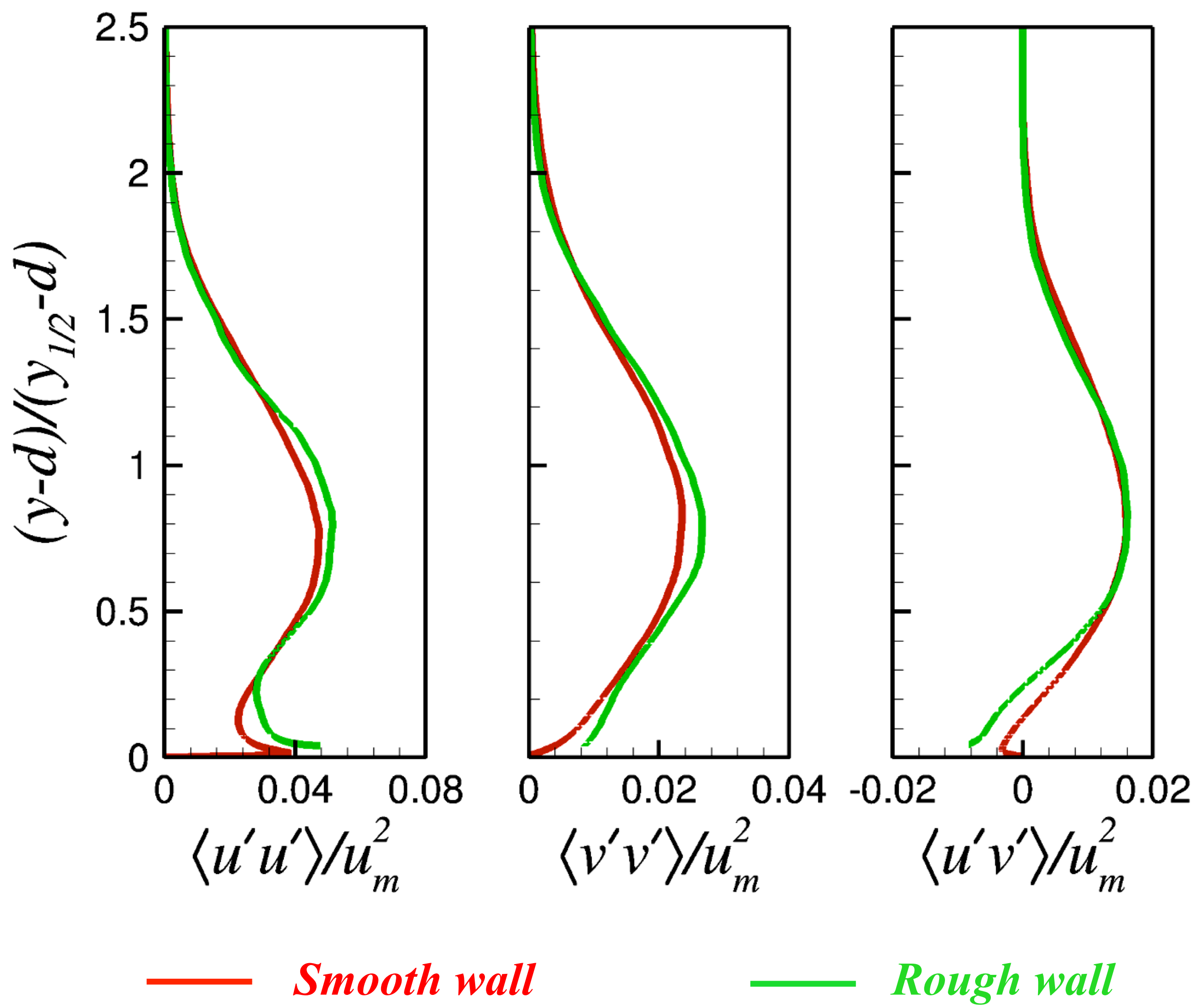




Results

Effect of roughness on the outer layer

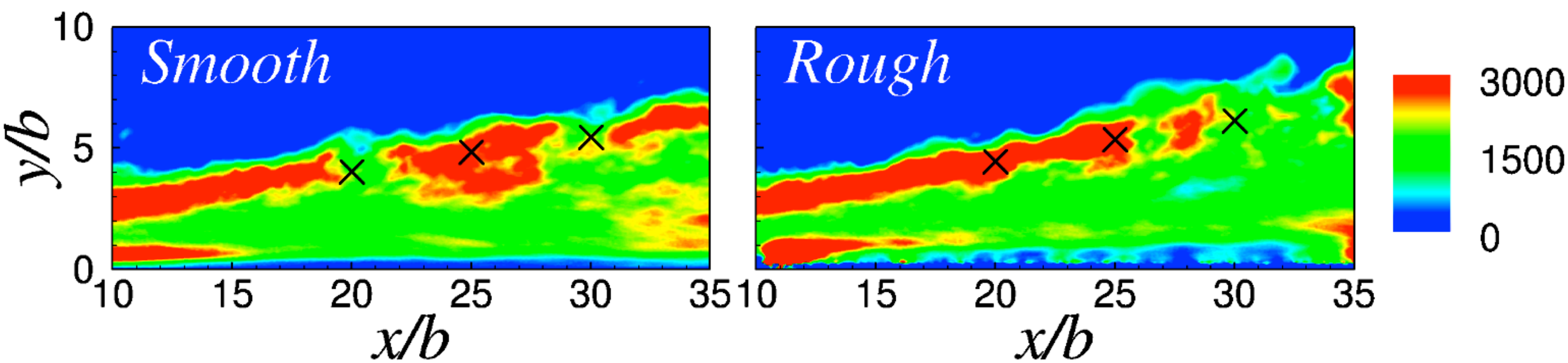
- Profiles of mean Reynolds stresses at $x = 20b$



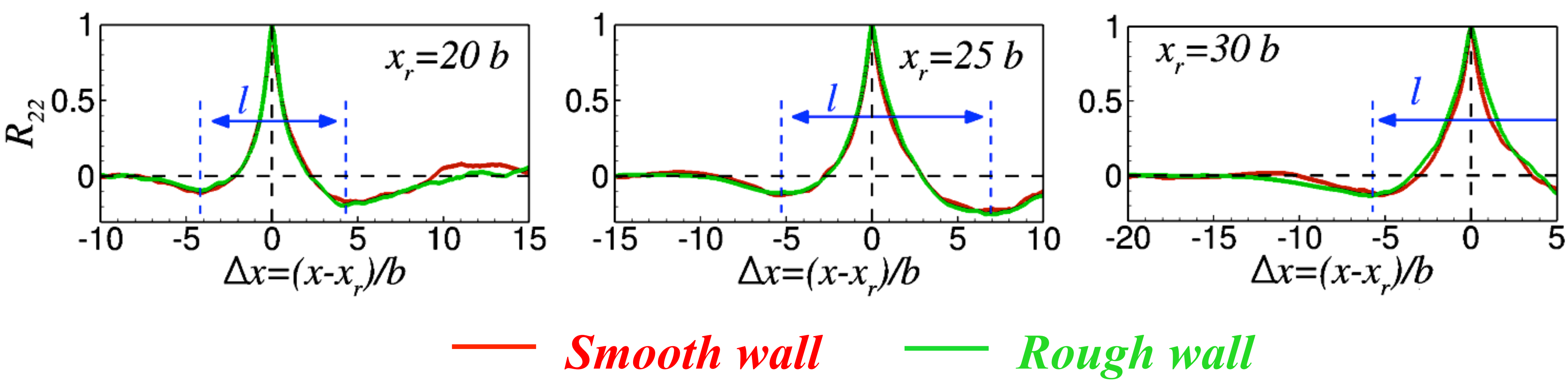
Results

Effect of roughness on the outer layer

- Contours of number of occurrence of $p' < -5\,p_{\mathrm{rms}}$ events



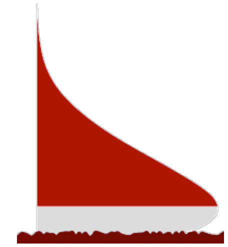
- Two-point autocorrelation: $R_{22}(x_r, y_r, x) = \frac{\langle v'(x_r,y_r).v'(x,y_r)\rangle}{\langle v'(x_r,y_r).v'(x_r,y_r)\rangle}$

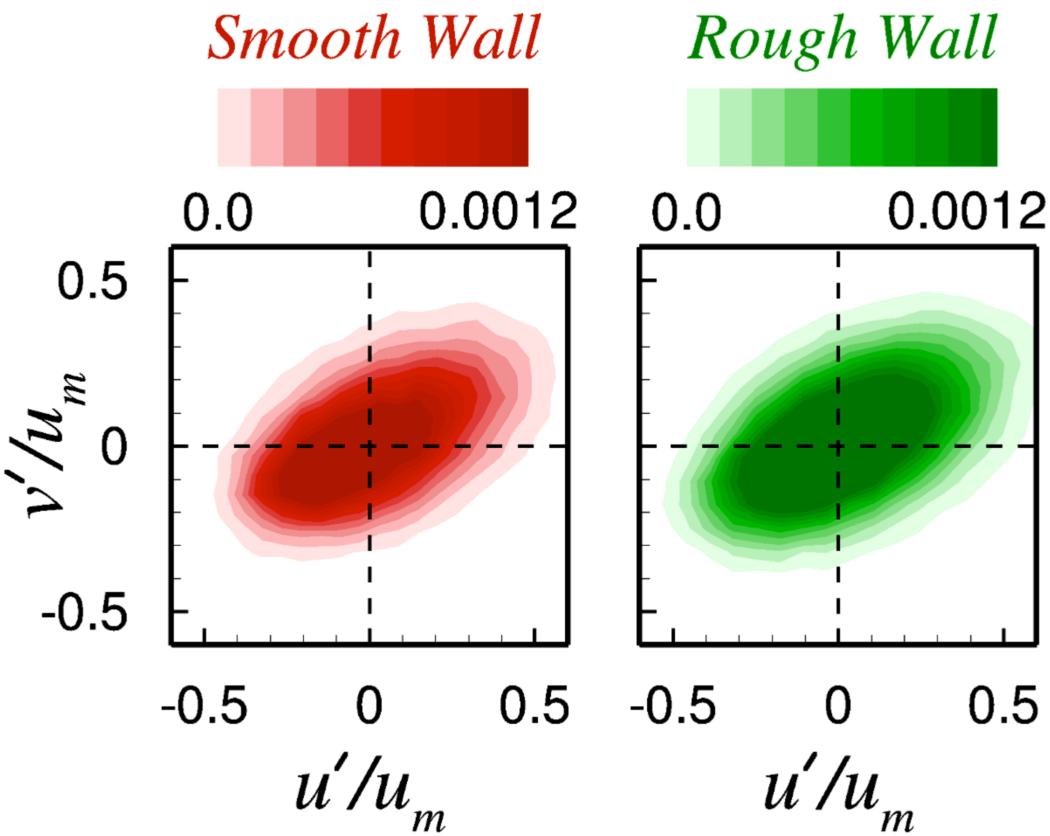


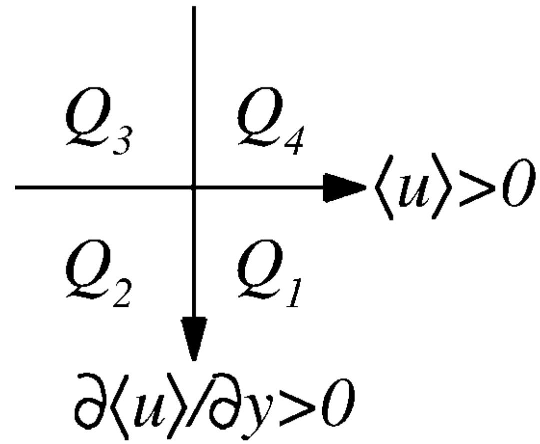
Results

Effect of roughness on the outer layer

- Joint Probability Density Function

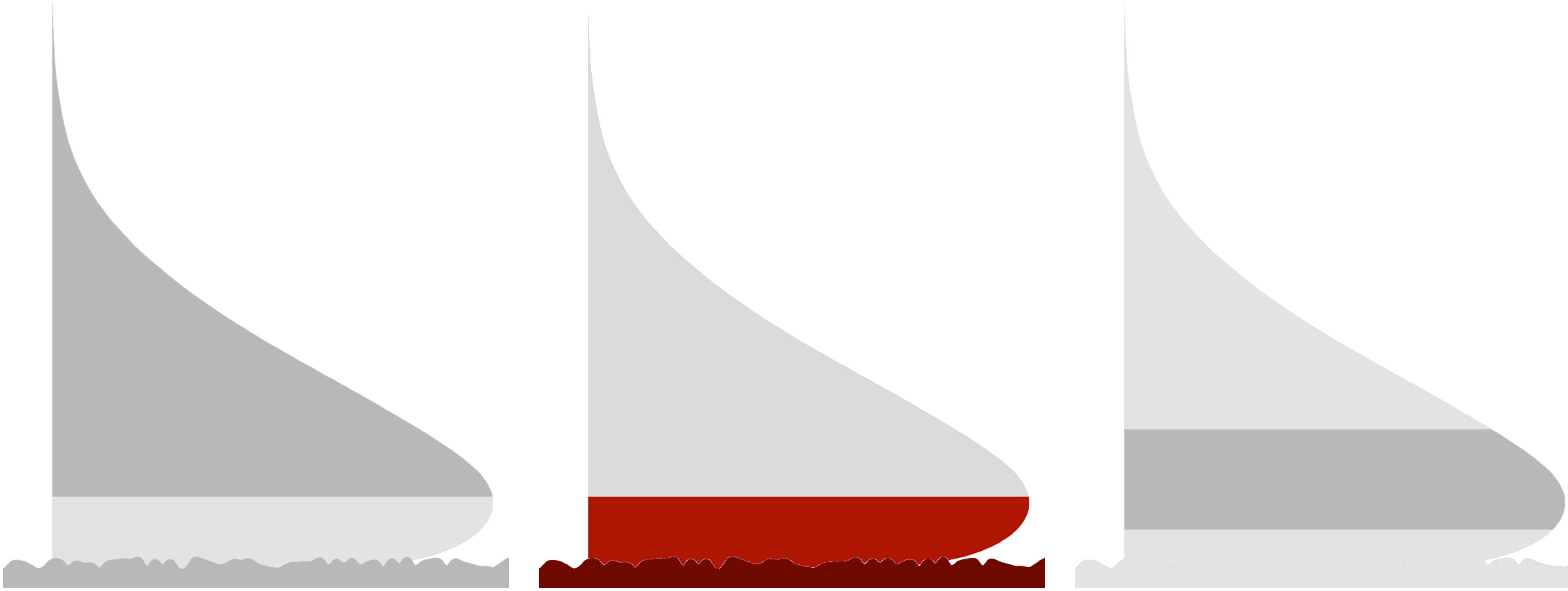






$y = y_{1/2}$ and $x/b = 30$

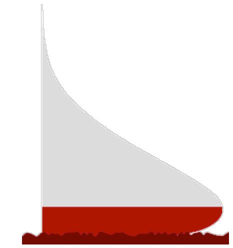
Results



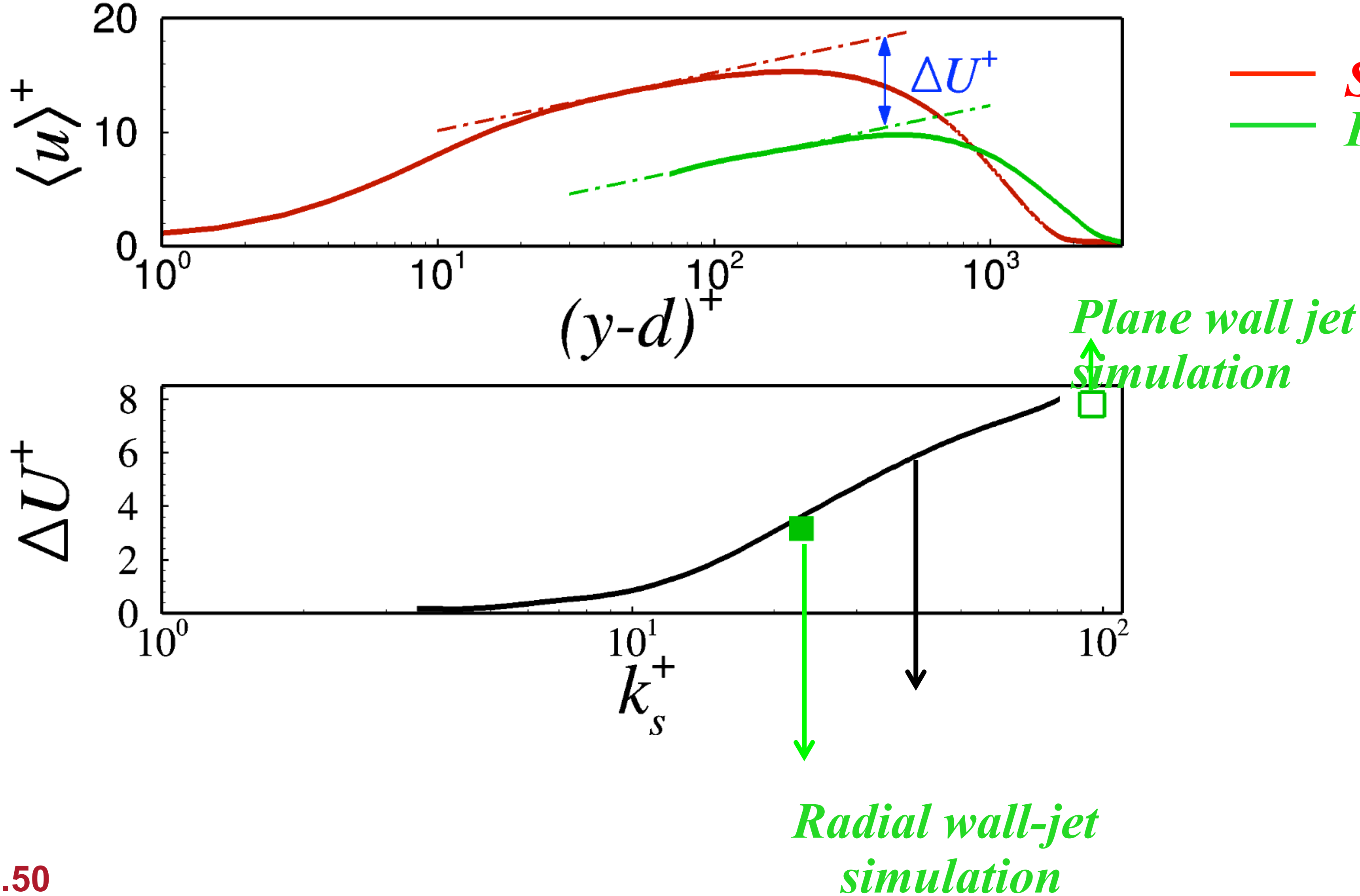
Effects of roughness on the outer layer

roughness inner layer

Interaction of inner and outer layers

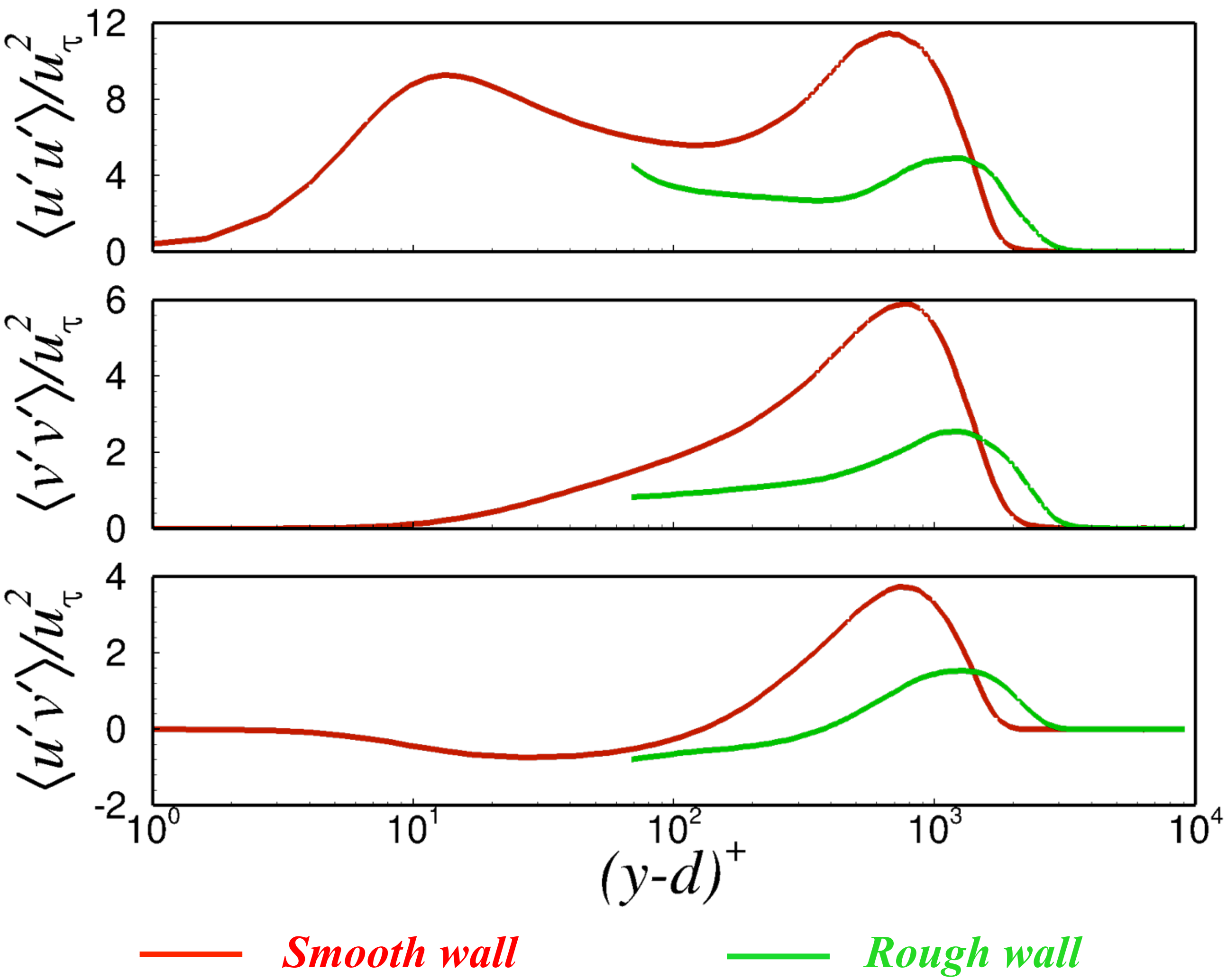


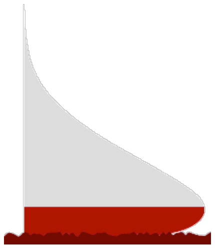
- Roughness function



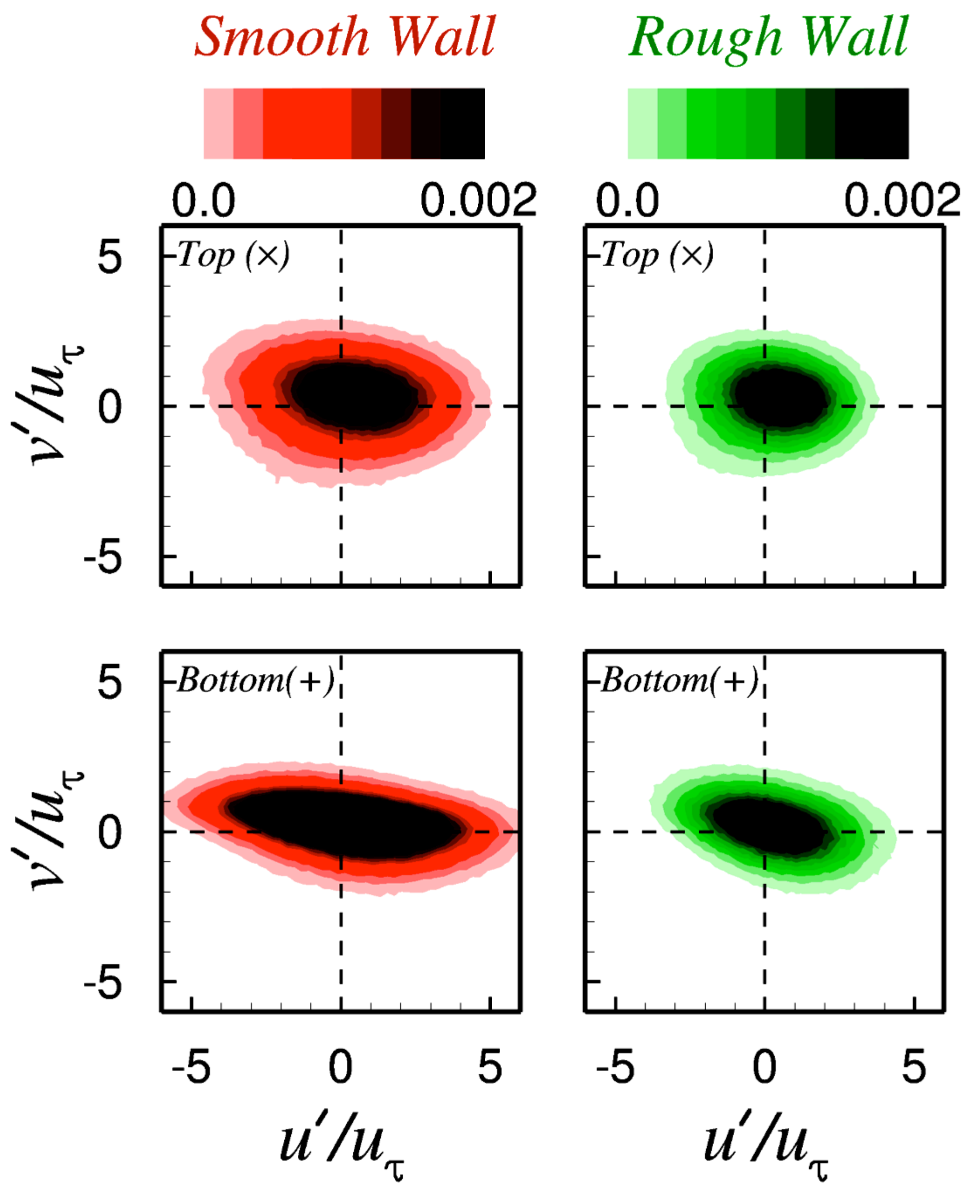
Results
Effect of roughness on the Inner layer

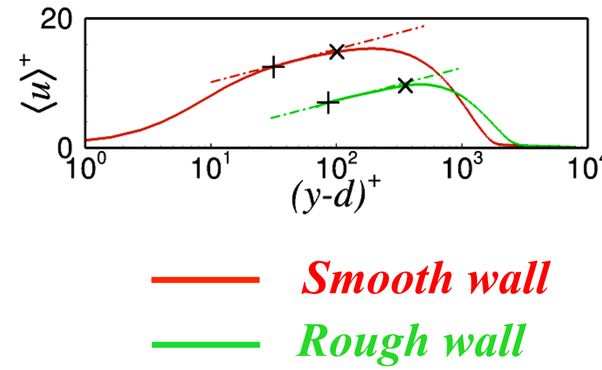
- Profiles of mean Reynolds stresses at $x = 20b$

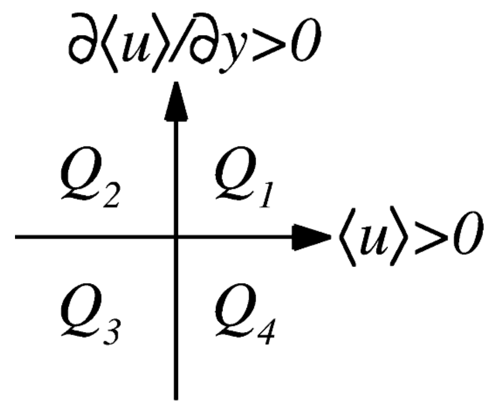


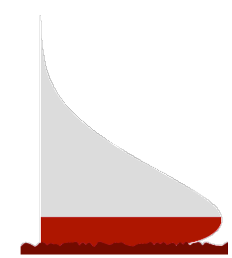


- Joint Probability Density Function, $P(u', v')$



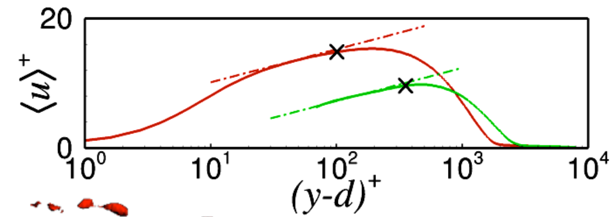


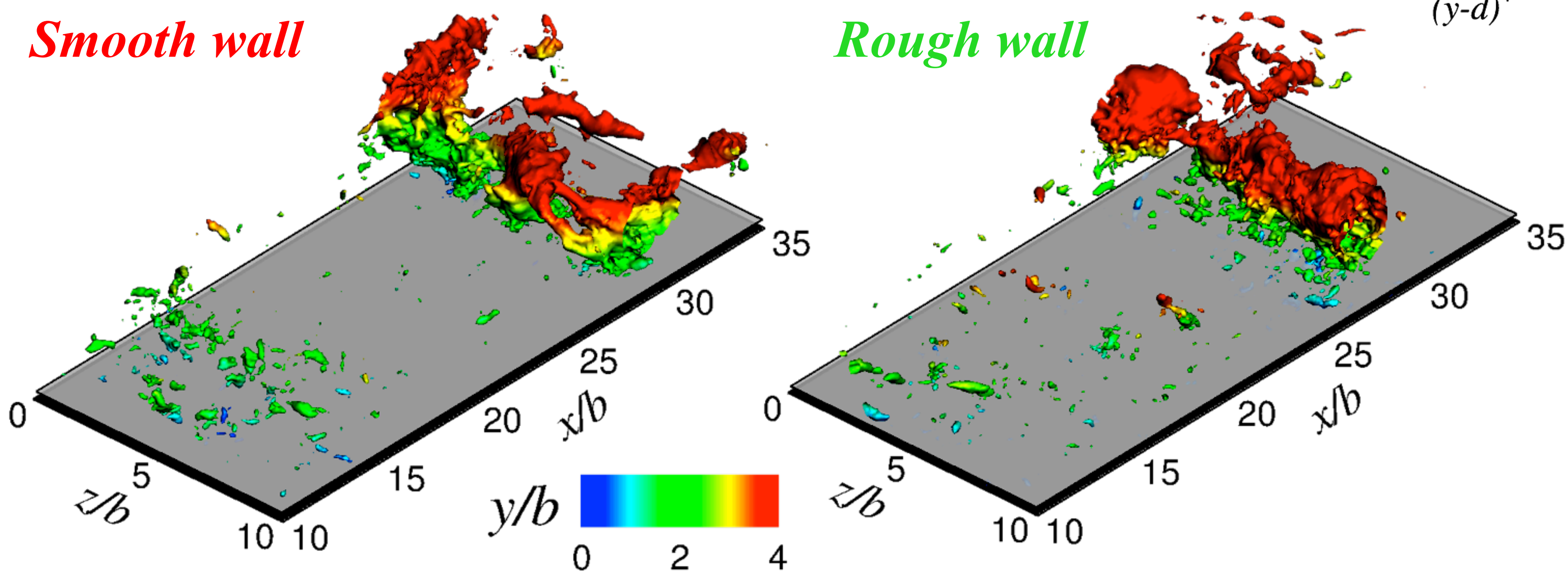


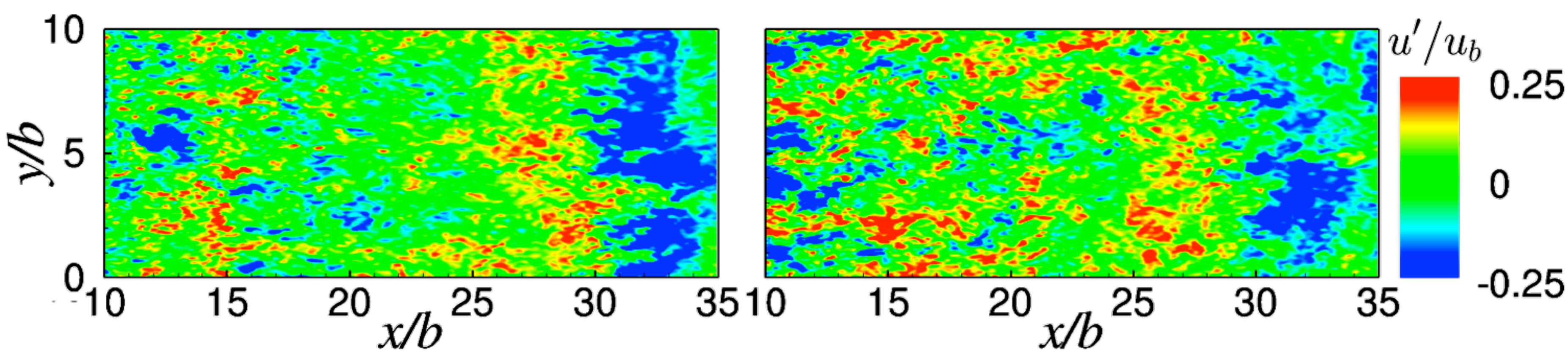


Results
Effect of roughness on the Inner layer

- Contours of radial velocity fluctuations



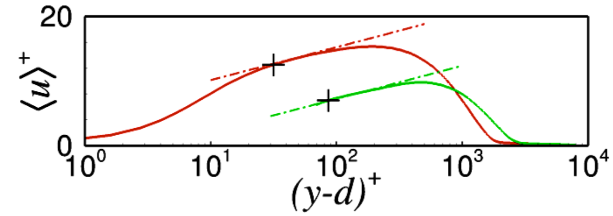


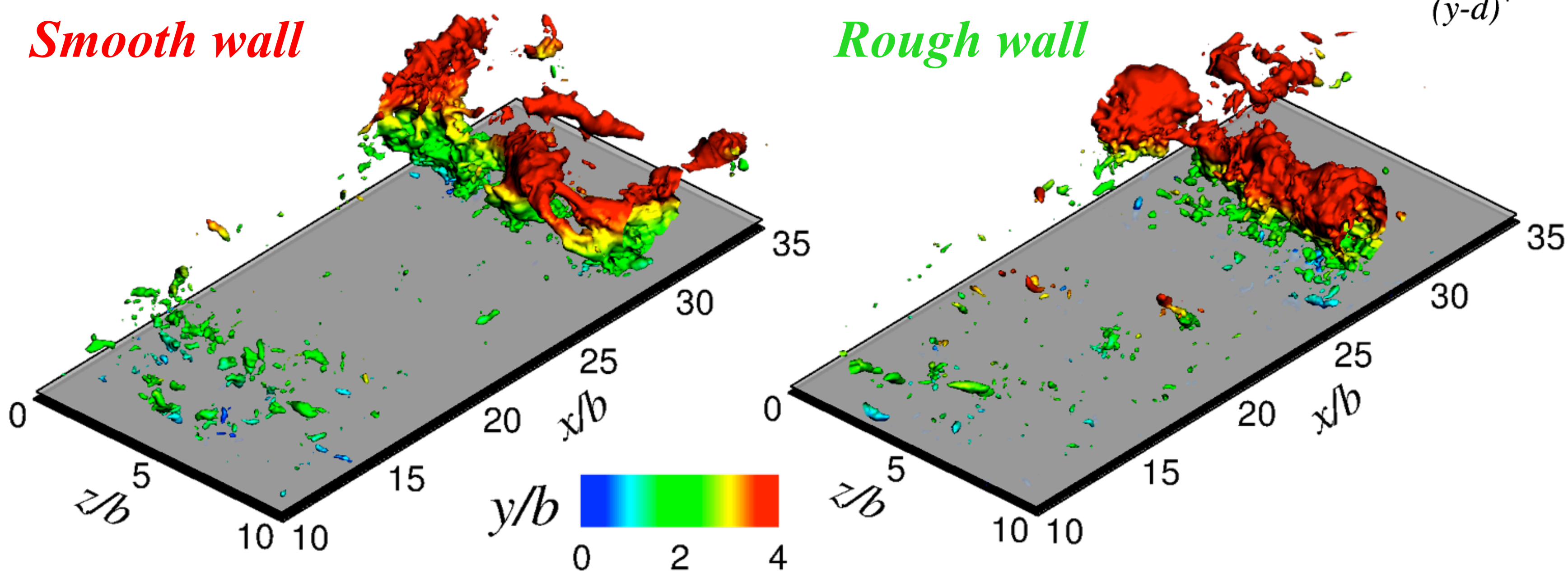


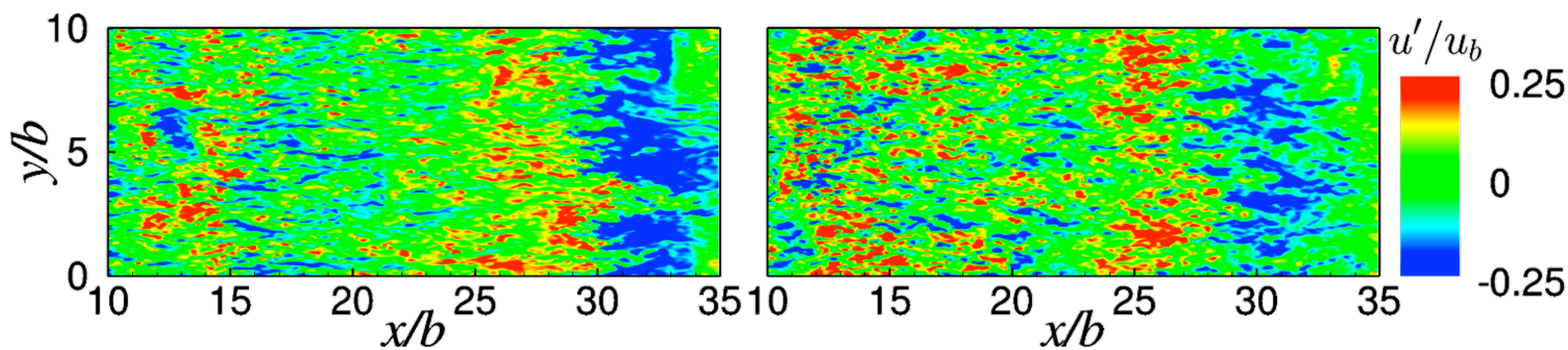
Results

Effect of roughness on the Inner layer

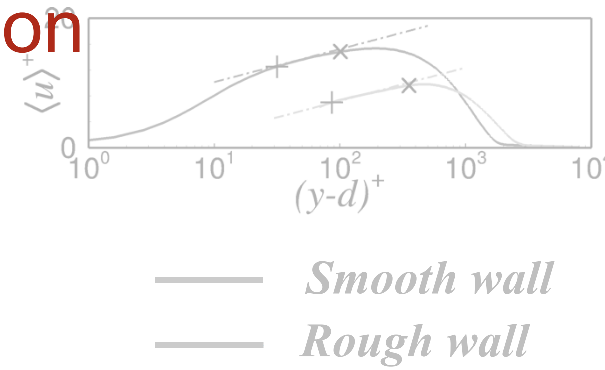
- Contours of radial velocity fluctuations



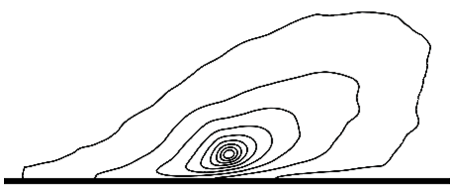


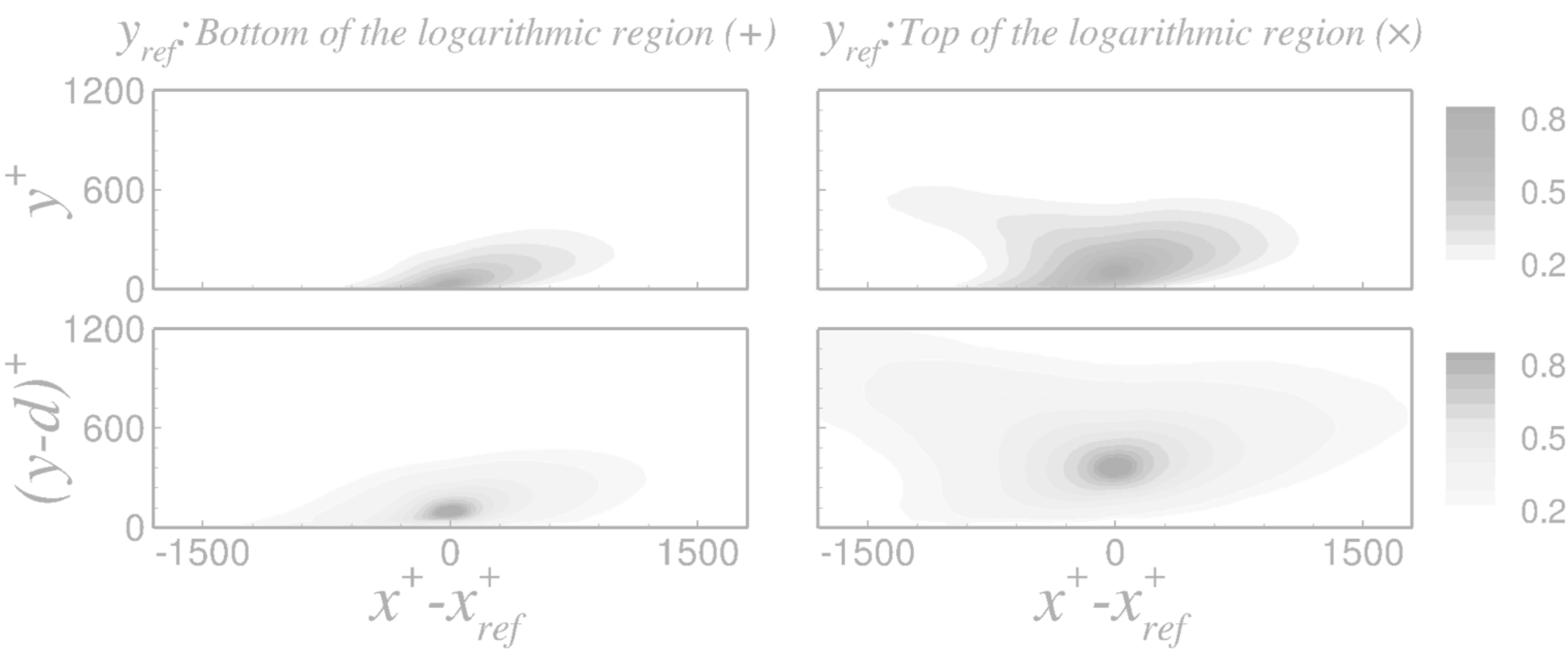


- Two-dimensional two-point autocorrelation

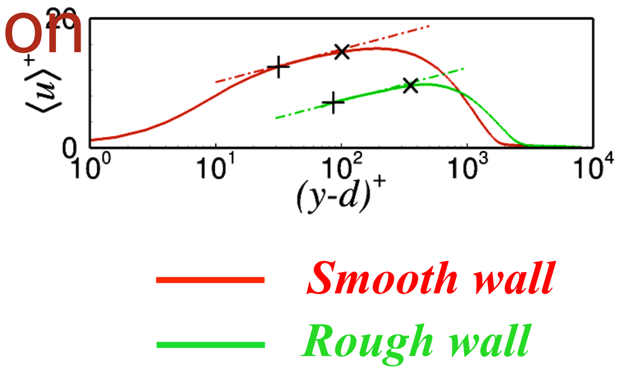


$$R_{11}\left(x_{ref},y_{ref},x,y\right)=\frac{\langle u'\left(x_{ref},y_{ref}\right).u'\left(x,y\right)\rangle}{\langle u'\left(x_{ref},y_{ref}\right).u'\left(x_{ref},y_{ref}\right)\rangle}$$

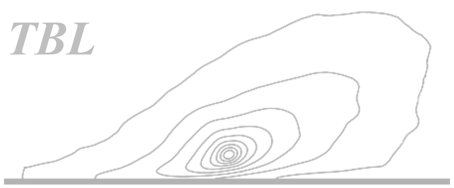


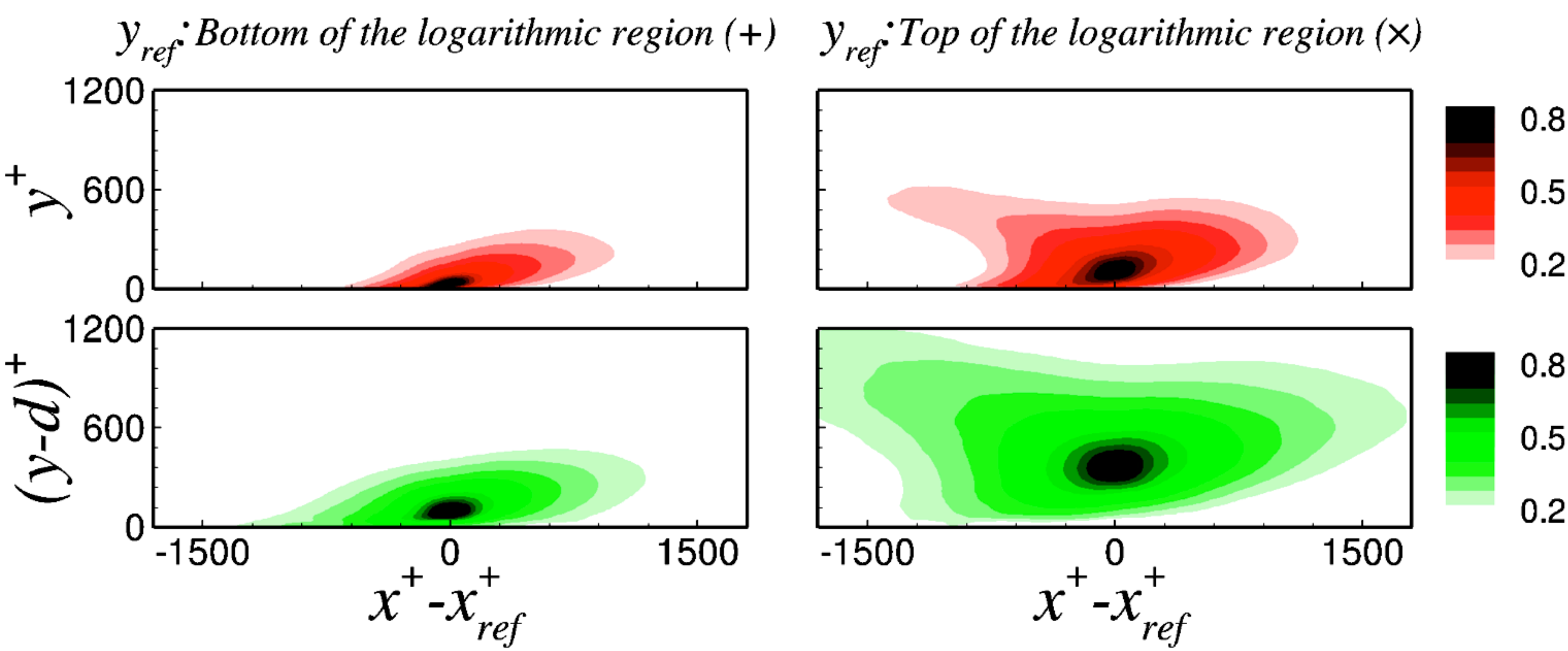


- Two-dimensional two-point autocorrelation

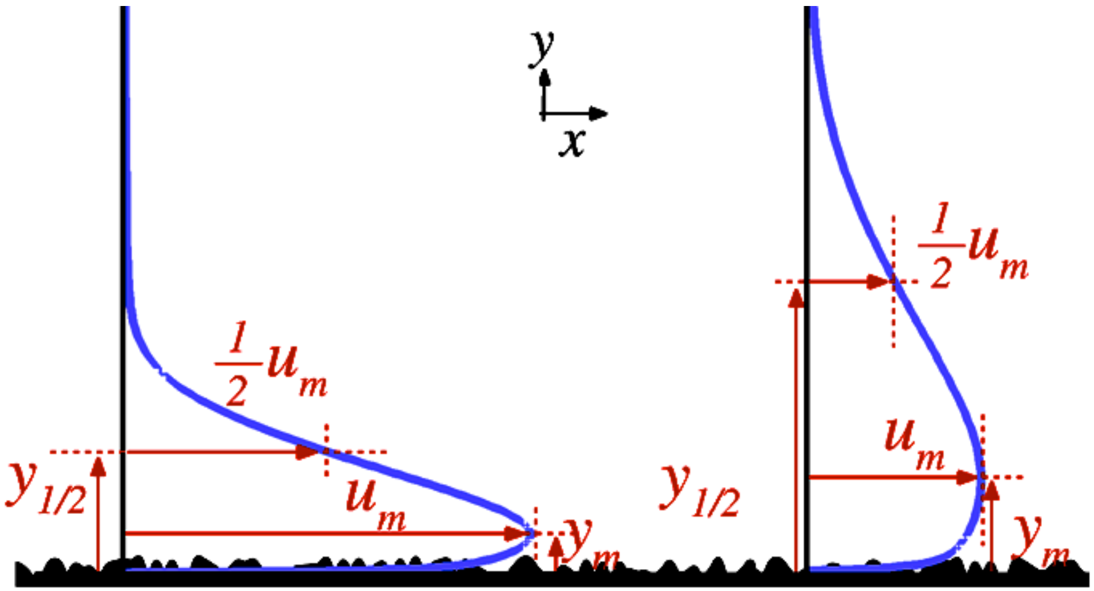


$$R_{11}\left(x_{ref}, y_{ref}, x, y\right) = \frac{\left\langle u'\left(x_{ref}, y_{ref}\right).u'\left(x, y\right)\right\rangle}{\left\langle u'\left(x_{ref}, y_{ref}\right).u'\left(x_{ref}, y_{ref}\right)\right\rangle}$$





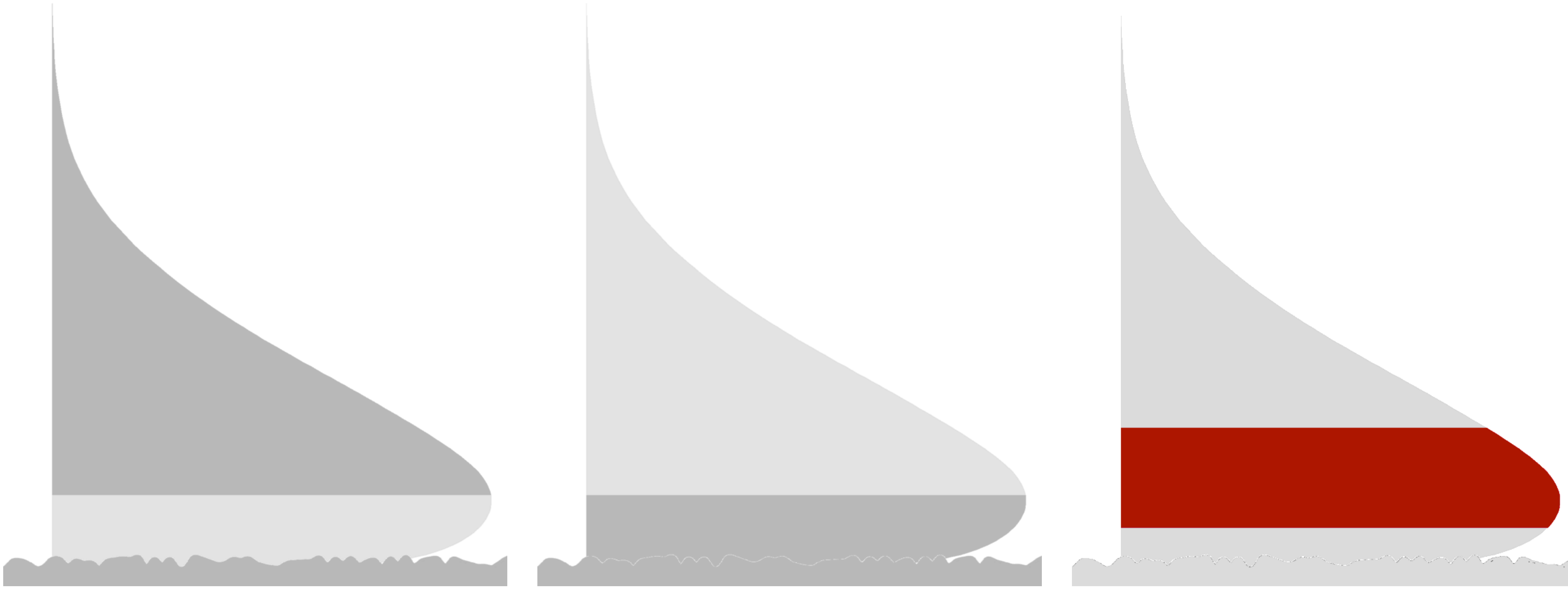
- Roughness effects are mostly confined to the inner layer of the flow



	u_m	y_m	$y_{1/2}$	$-du_m/dx$	dy_m/dx	$dy_{1/2}/dx$
Rajaratnam et al. (1981)	↓	-	-	-	↑	c
Tachie et al. (2004)	↓	↑	c	↑	↑	c
Smith (2008)	c	↑ or ≈c	c	c	-	c
Rostamy et al. (2011)	↓	↑	↑	↑	-	c
Current study	↓	↑	↑	↑	↑	≈c

↑, increased; ↓, decreased; c, constant; -, not reported

Results



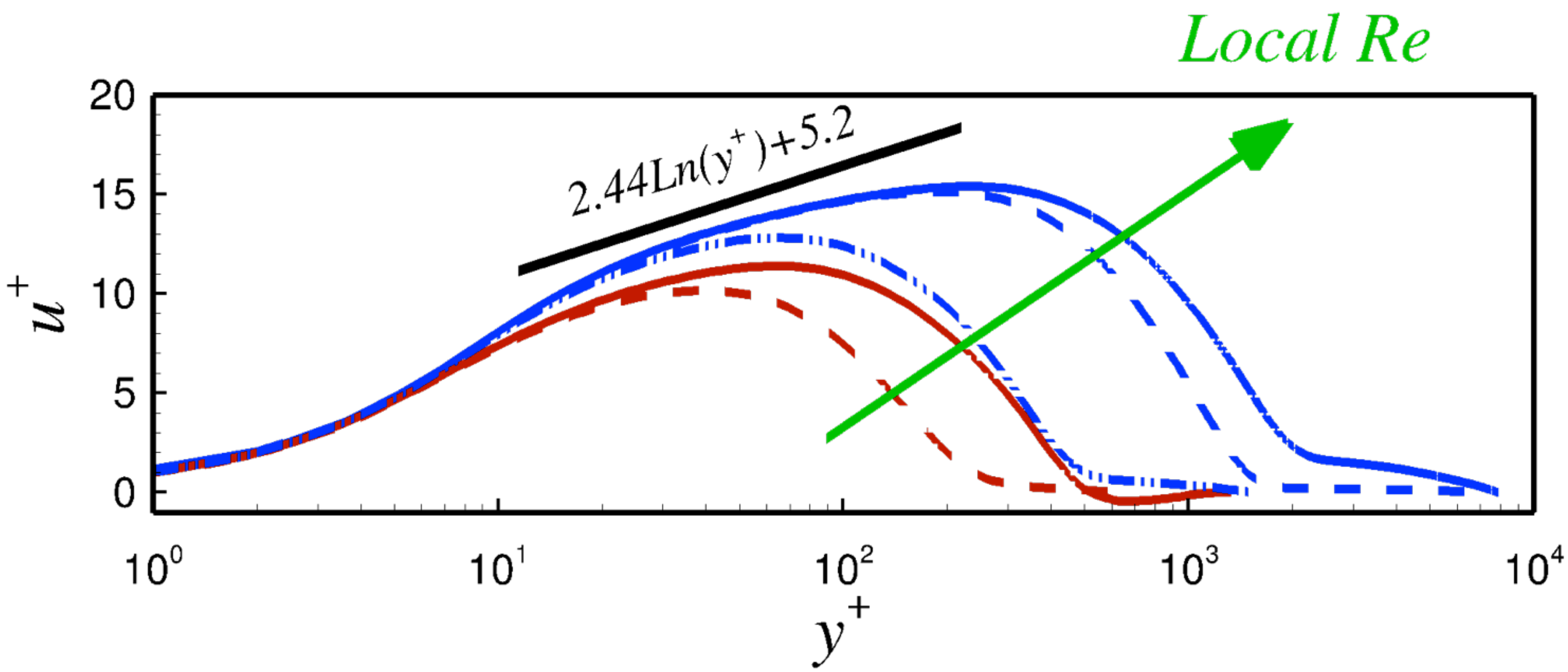
Effects of roughness on the outer layer

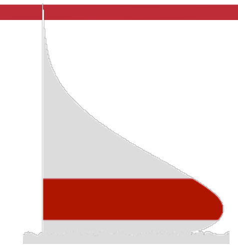
Effects of roughness on the inner layer

Interaction inner outer layers

Results

Inner/outer layer interaction





$$Re_m = u_m y_m/\nu,\ y^+_{1/2} = u_\tau y_{1/2}/\nu,\ Re^+ = u_\tau y_m/\nu$$

—— , Law of the wall

—— , $Re_m \approx 400$, Radial wall-jet

—— , $Re_m \approx 700$, Radial wall-jet

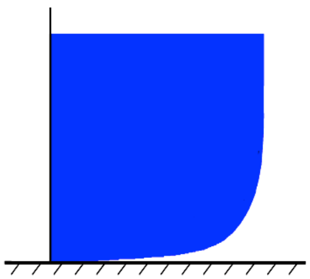
-·-·- , $Re_m \approx 850$, Plane wall-jet

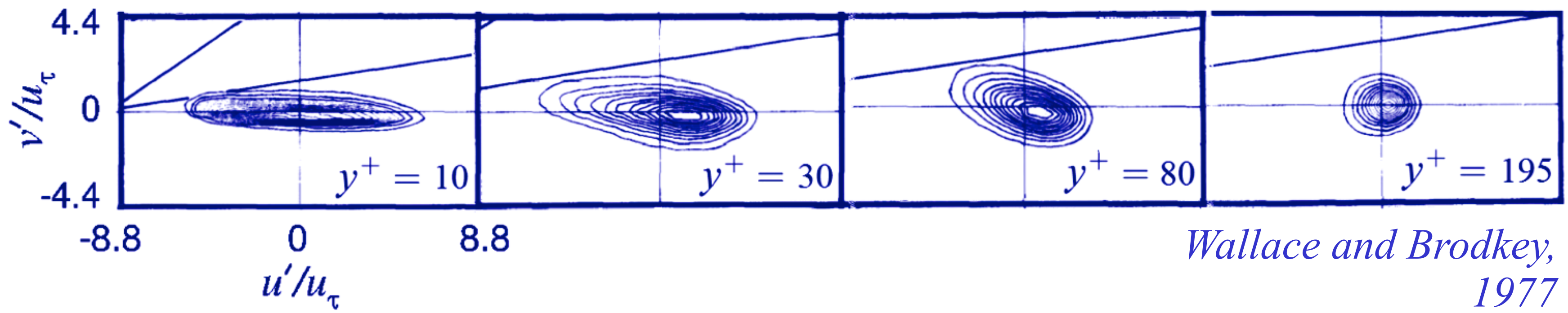
- - - , $Re_m \approx 2,900$, Plane wall-jet

—— , $Re_m \approx 3,600$, Plane wall-jet

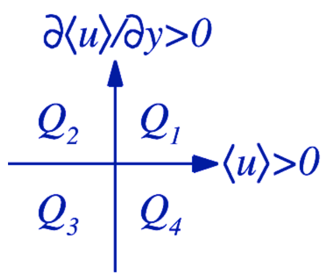
Results
Inner/outer layer interaction

- JPDF of streamwise and wall-normal velocity fluctuations

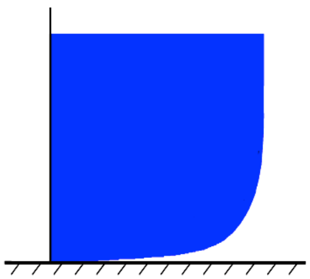


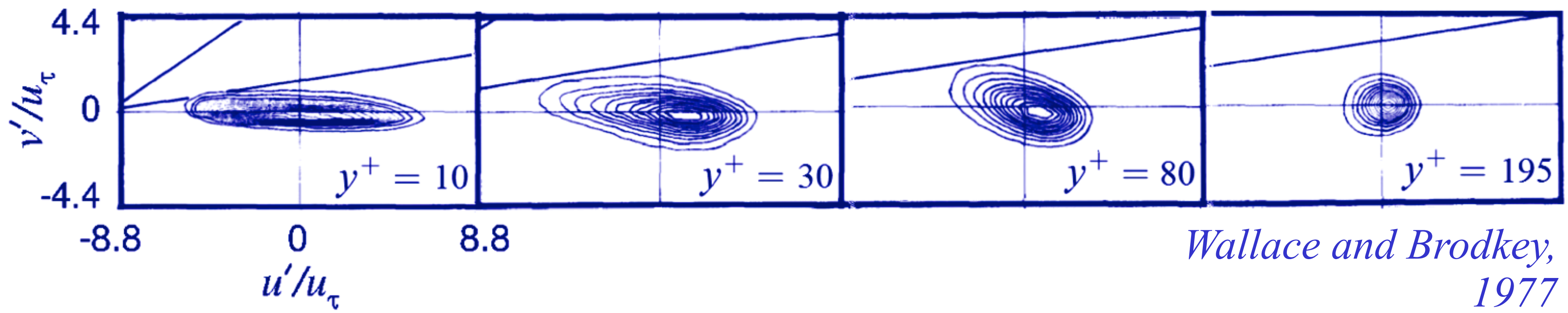


Wallace and Brodkey, 1977

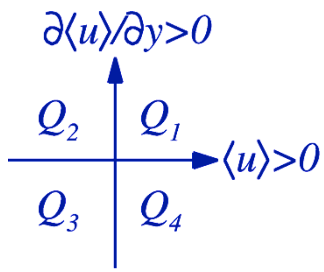


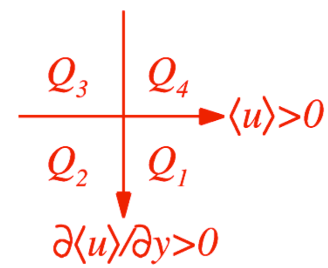
- JPDF of streamwise and wall-normal velocity fluctuations

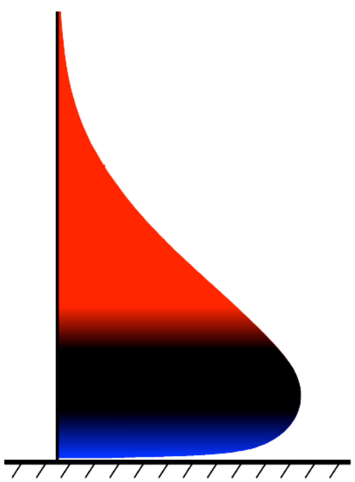


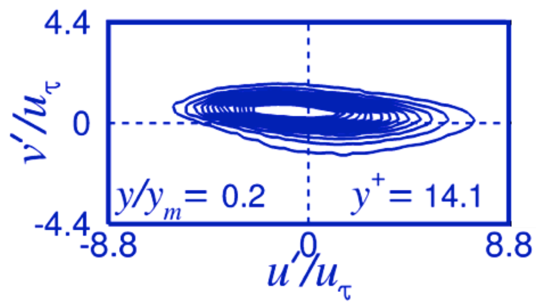


Wallace and Brodkey, 1977





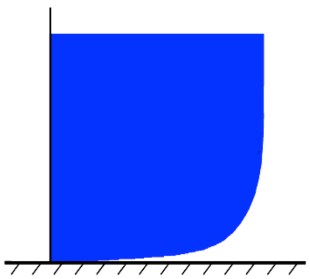


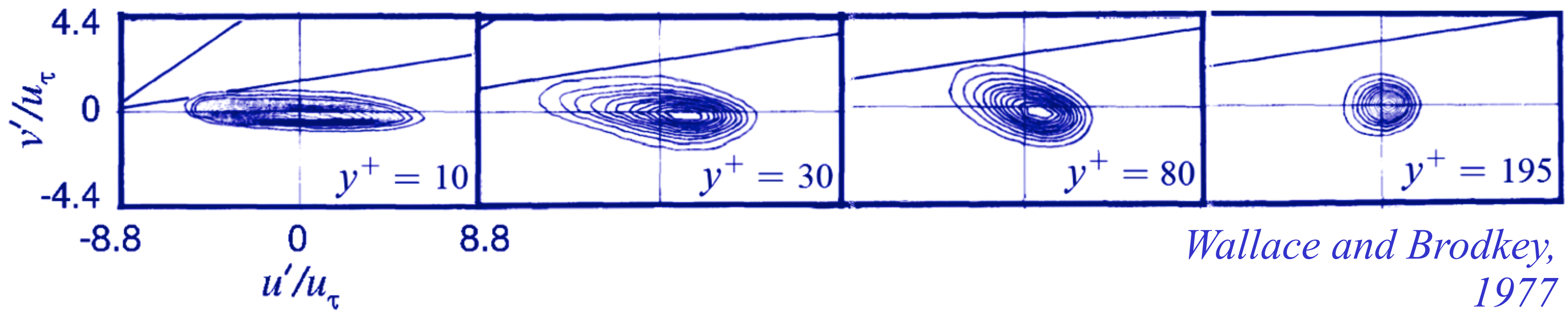


Results

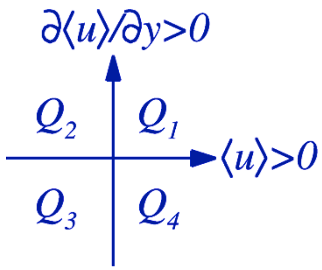
Inner/outer layer interaction

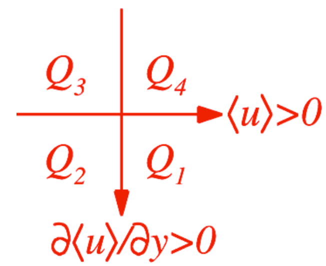
- JPDF of streamwise and wall-normal velocity fluctuations

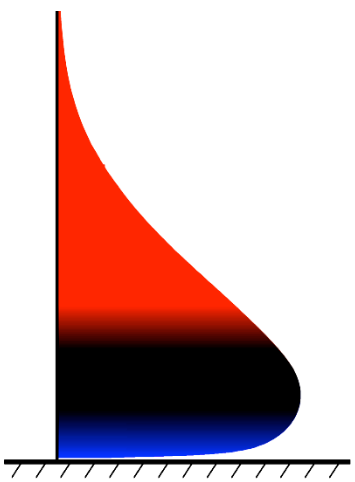


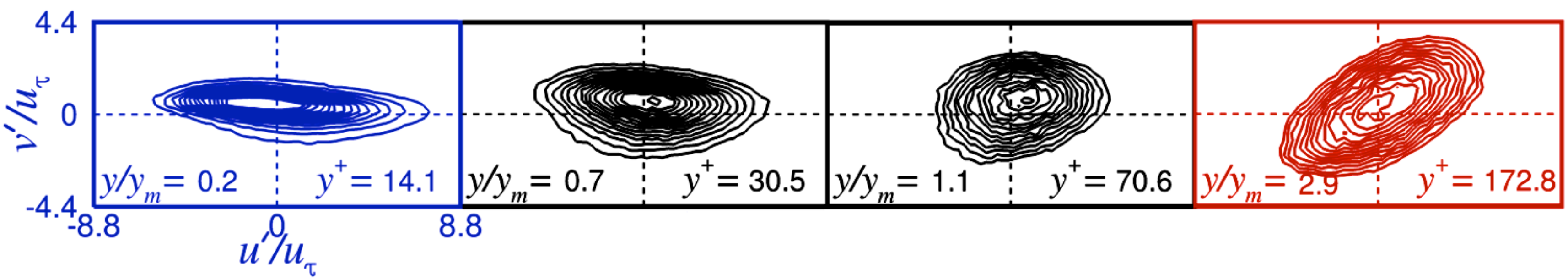


Wallace and Brodkey, 1977



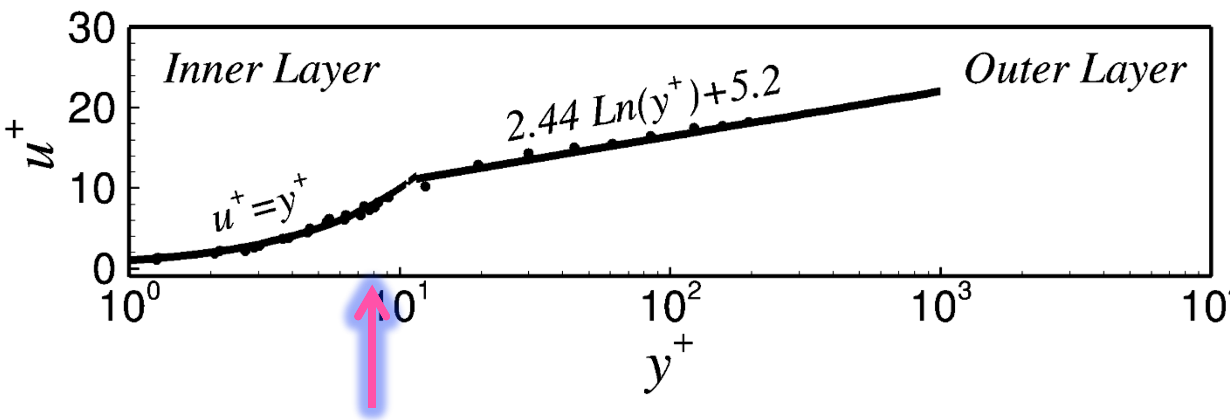


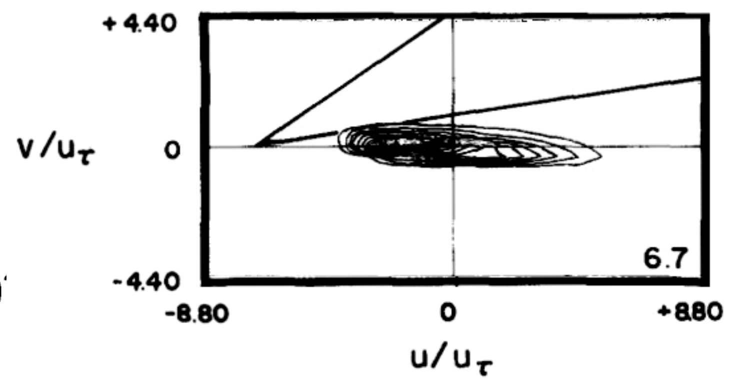




Results

Inner/outer layer interaction

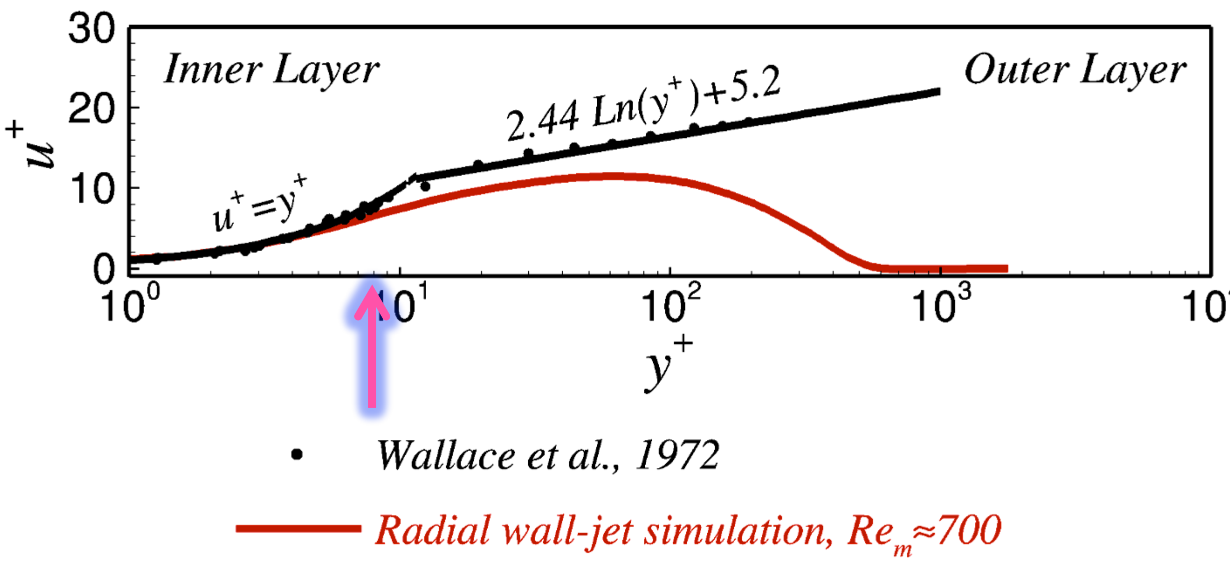


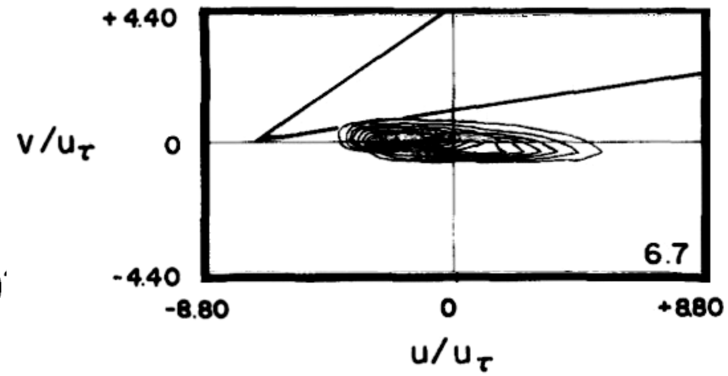


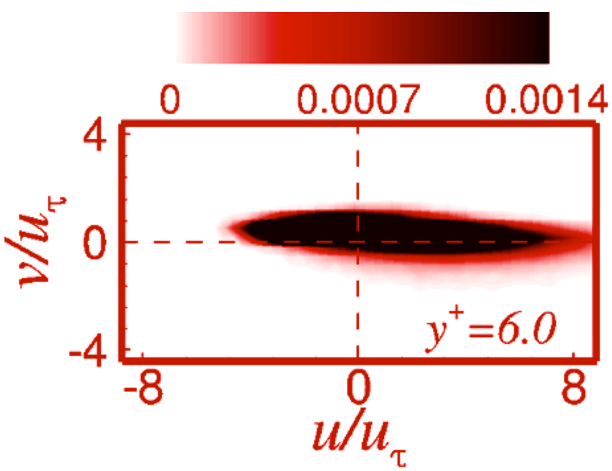
- *Wallace et al., 1972*

Results

Inner/outer layer interaction

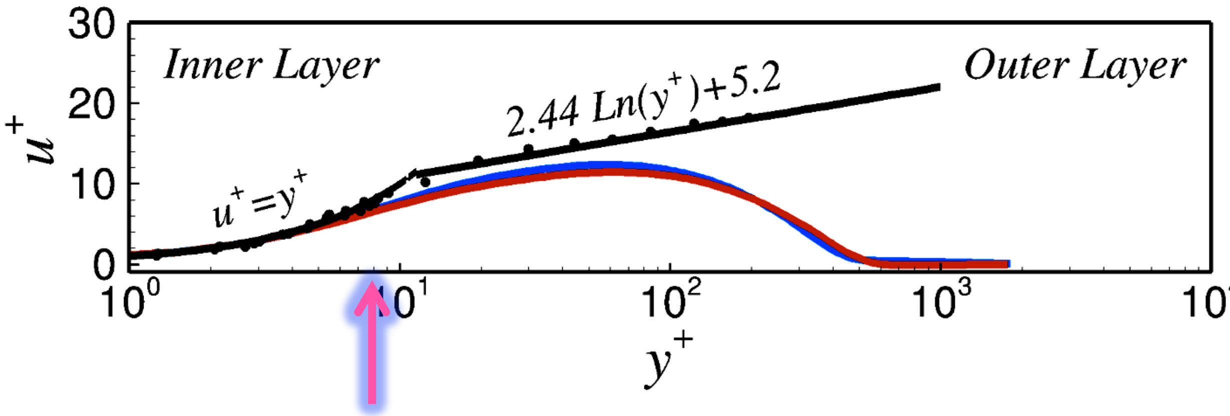






Radial wall-jet, $Re_m \approx 700$

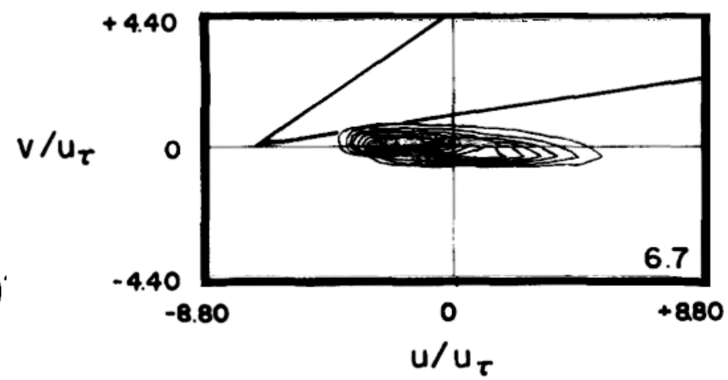
Results
Inner/outer layer interaction

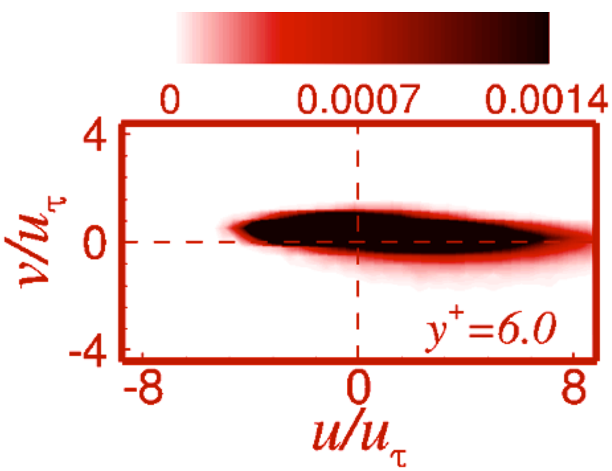


- Wallace et al., 1972

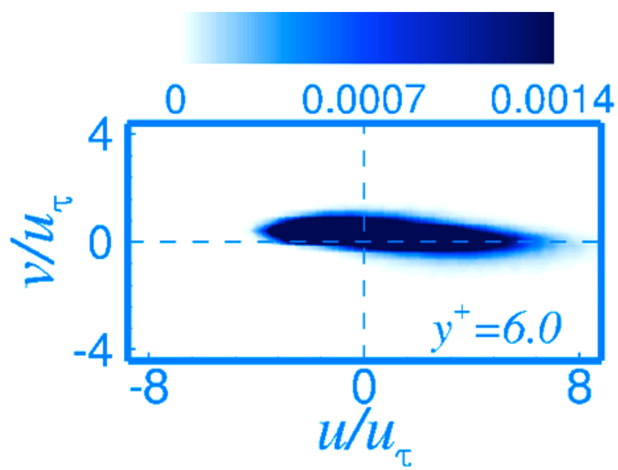
Radial wall-jet simulation, $Re_m \approx 700$

Plane wall-jet simulation, $Re_m \approx 700$





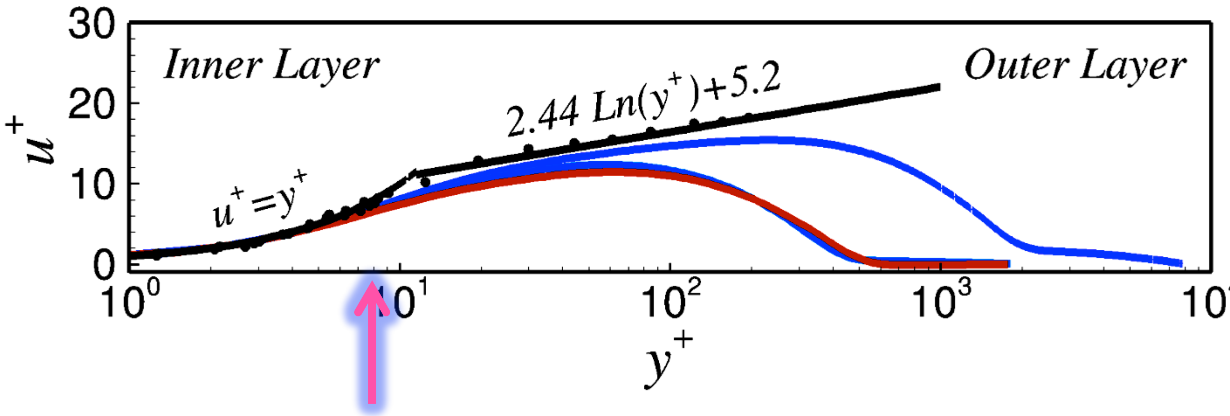
Radial wall-jet, $Re_m \approx 700$

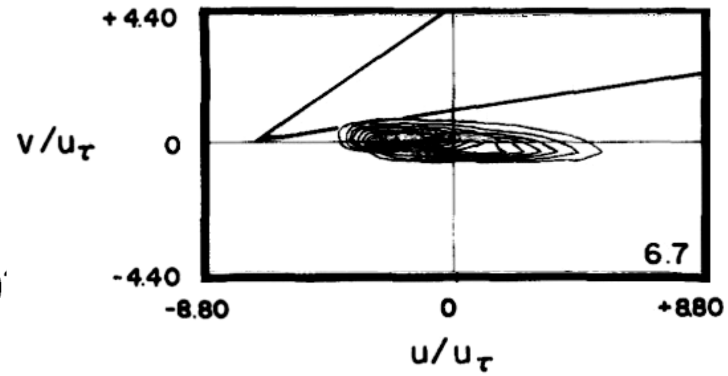


Plane wall-jet, $Re_m \approx 700$

Results

Inner/outer layer interaction



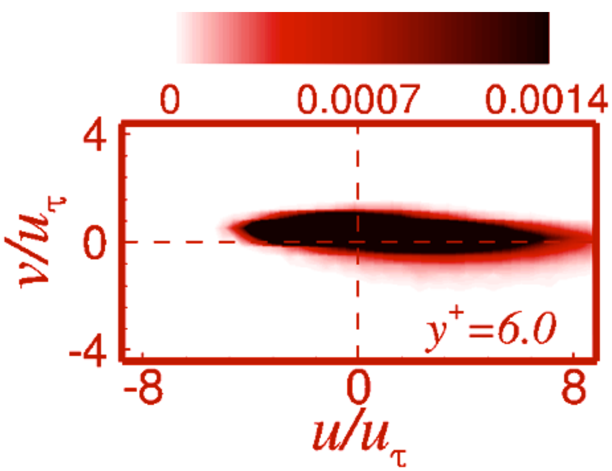


• *Wallace et al., 1972*

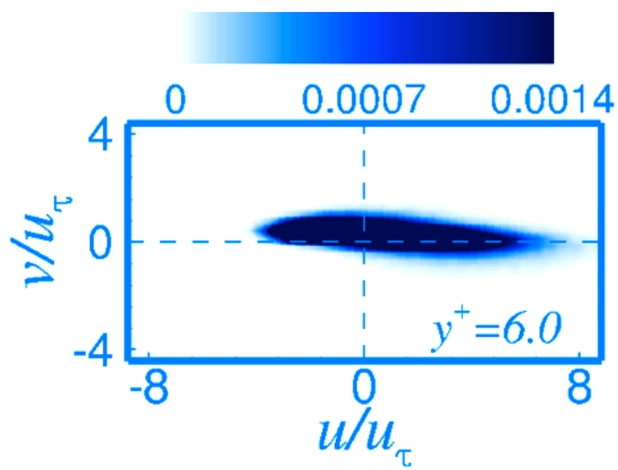
Radial wall-jet simulation, $Re_m \approx 700$

Plane wall-jet simulation, $Re_m \approx 700$

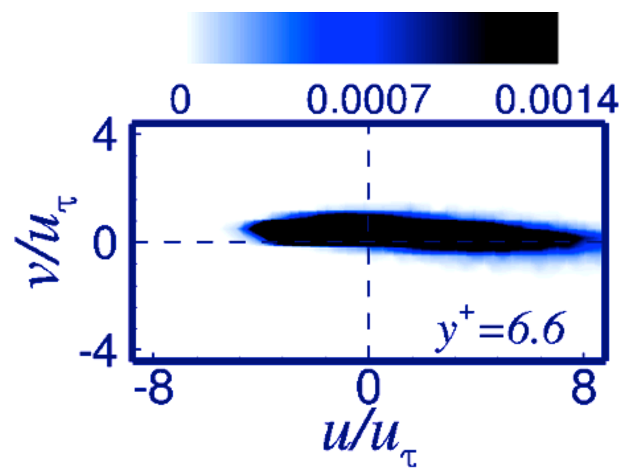
Plane wall-jet simulation, $Re_m \approx 3600$



Radial wall-jet, $Re_m \approx 700$



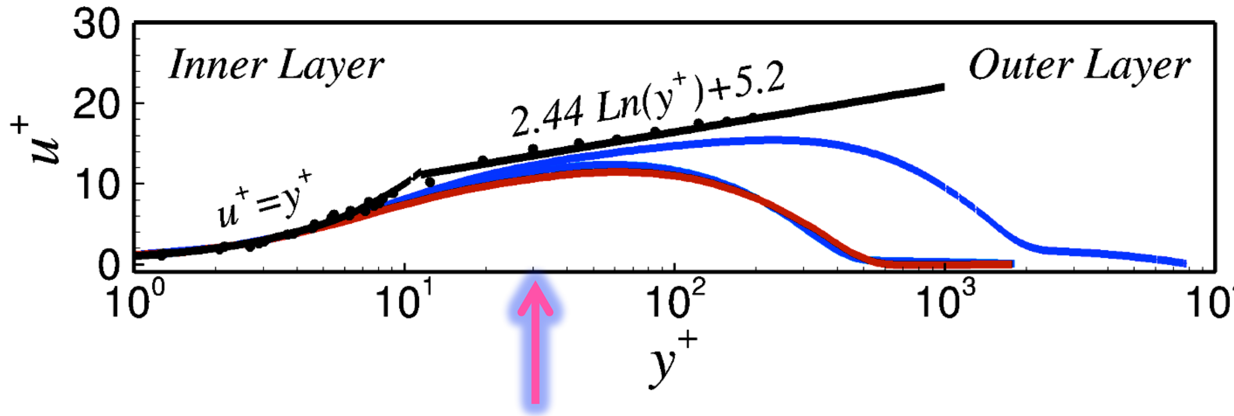
Plane wall-jet, $Re_m \approx 700$

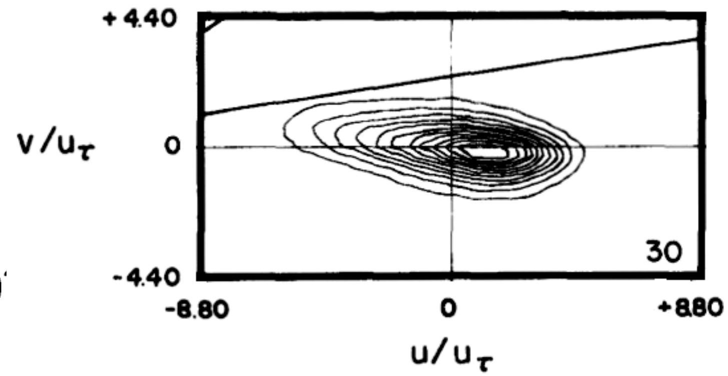


Plane wall-jet, $Re_m \approx 3600$

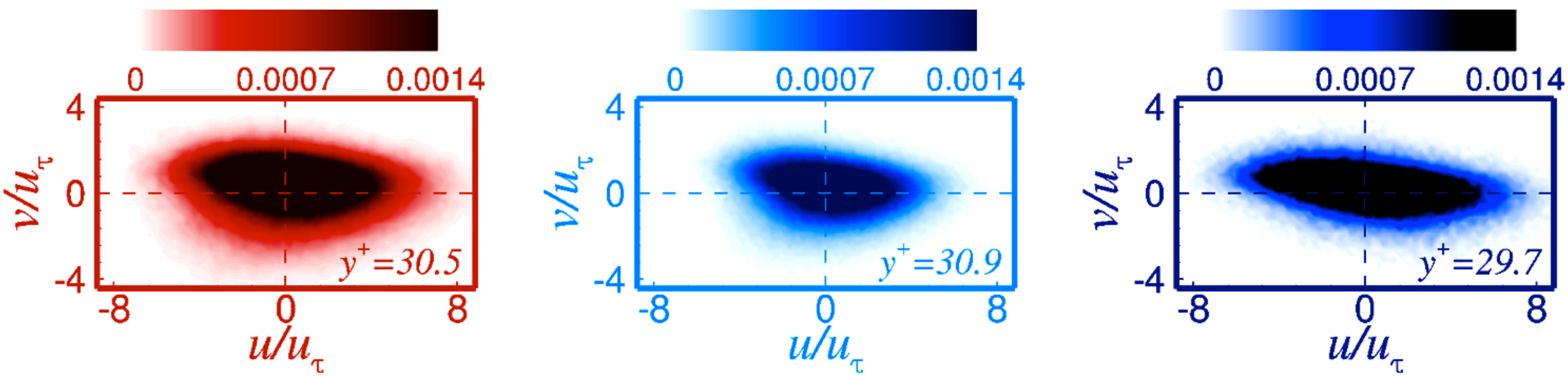
Results

Inner/outer layer interaction





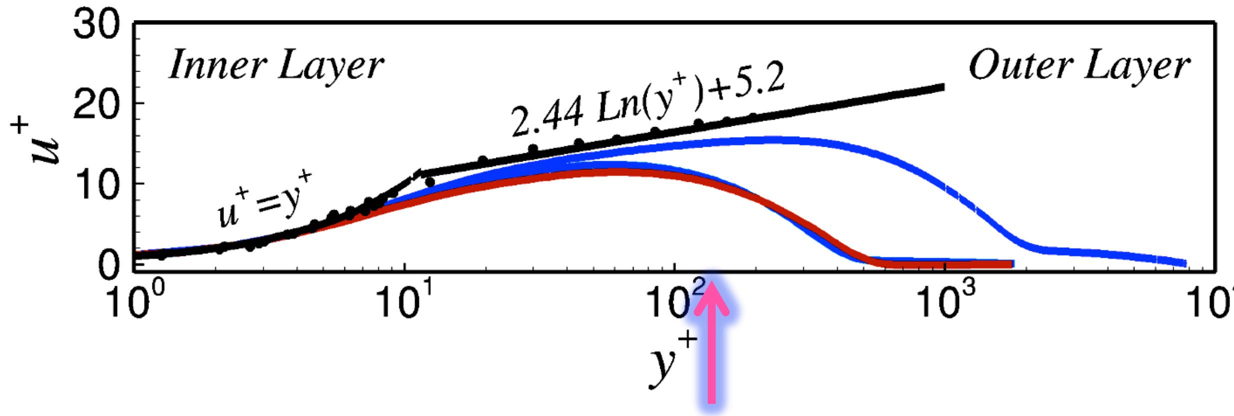
- • *Wallace et al., 1972*
- *Radial wall-jet simulation, $Re_m \approx 700$*
- *Plane wall-jet simulation, $Re_m \approx 700$*
- *Plane wall-jet simulation, $Re_m \approx 3600$*

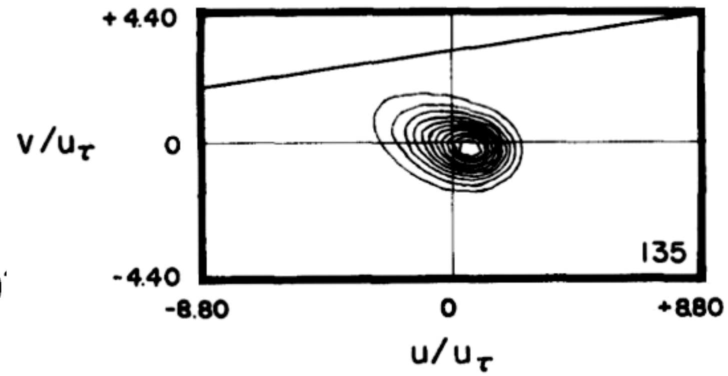


Radial wall-jet, $Re_m \approx 700$ | *Plane wall-jet, $Re_m \approx 700$* | *Plane wall-jet, $Re_m \approx 3600$*

Queen's University

Results
Inner/outer layer interaction



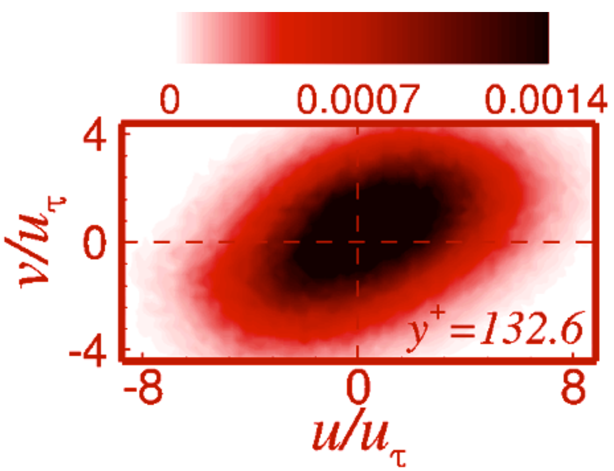


• *Wallace et al., 1972*

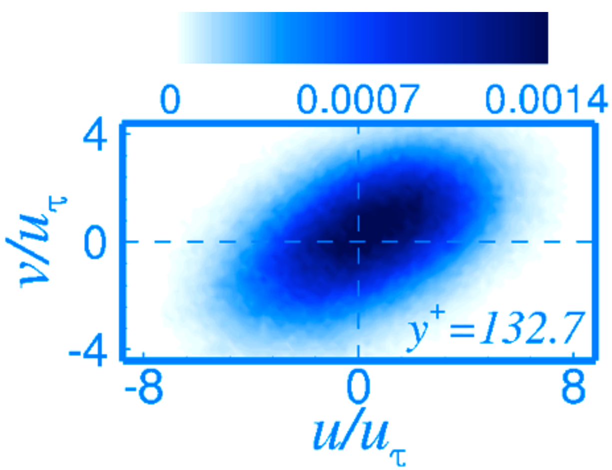
Radial wall-jet simulation, $Re_m \approx 700$

Plane wall-jet simulation, $Re_m \approx 700$

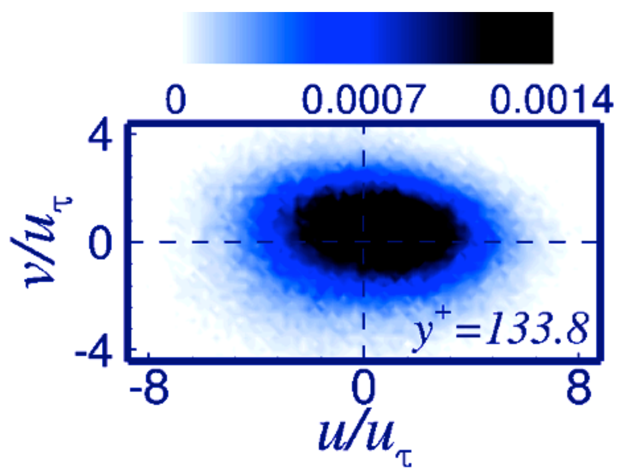
Plane wall-jet simulation, $Re_m \approx 3600$



Radial wall-jet, $Re_m \approx 700$



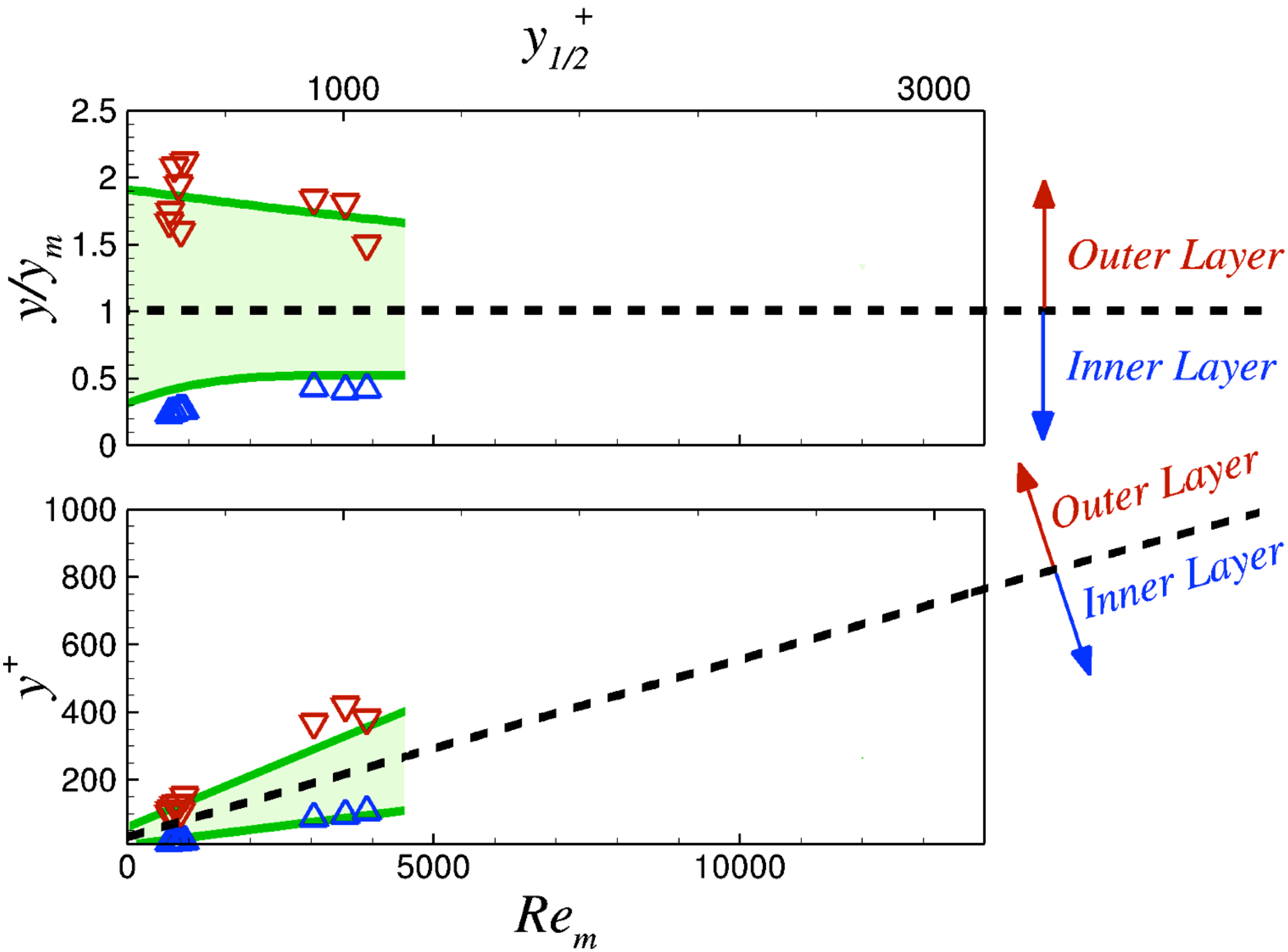
Plane wall-jet, $Re_m \approx 700$



Plane wall-jet, $Re_m \approx 3600$

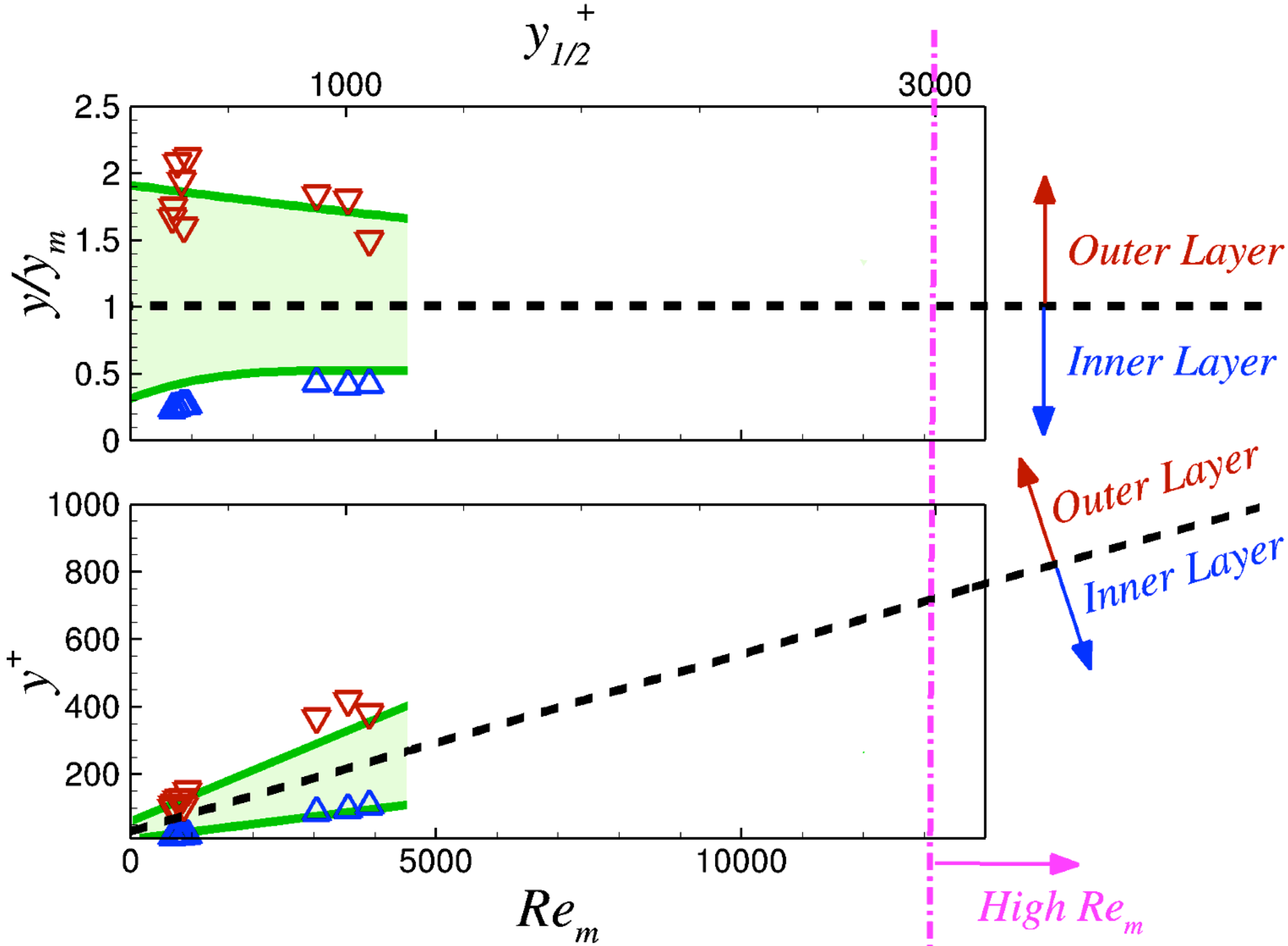
Results

Inner/outer layer interaction



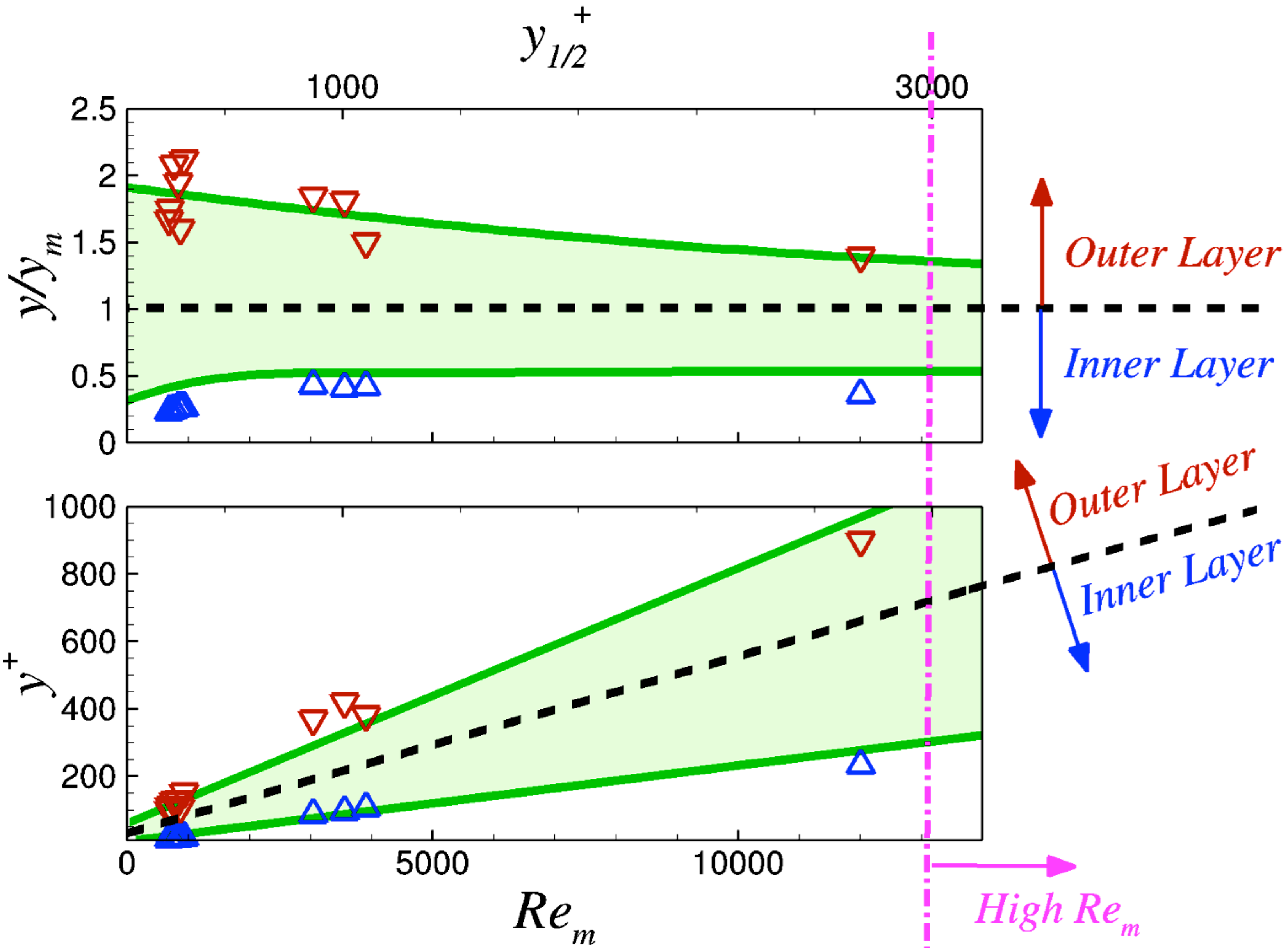
Results

Inner/outer layer interaction



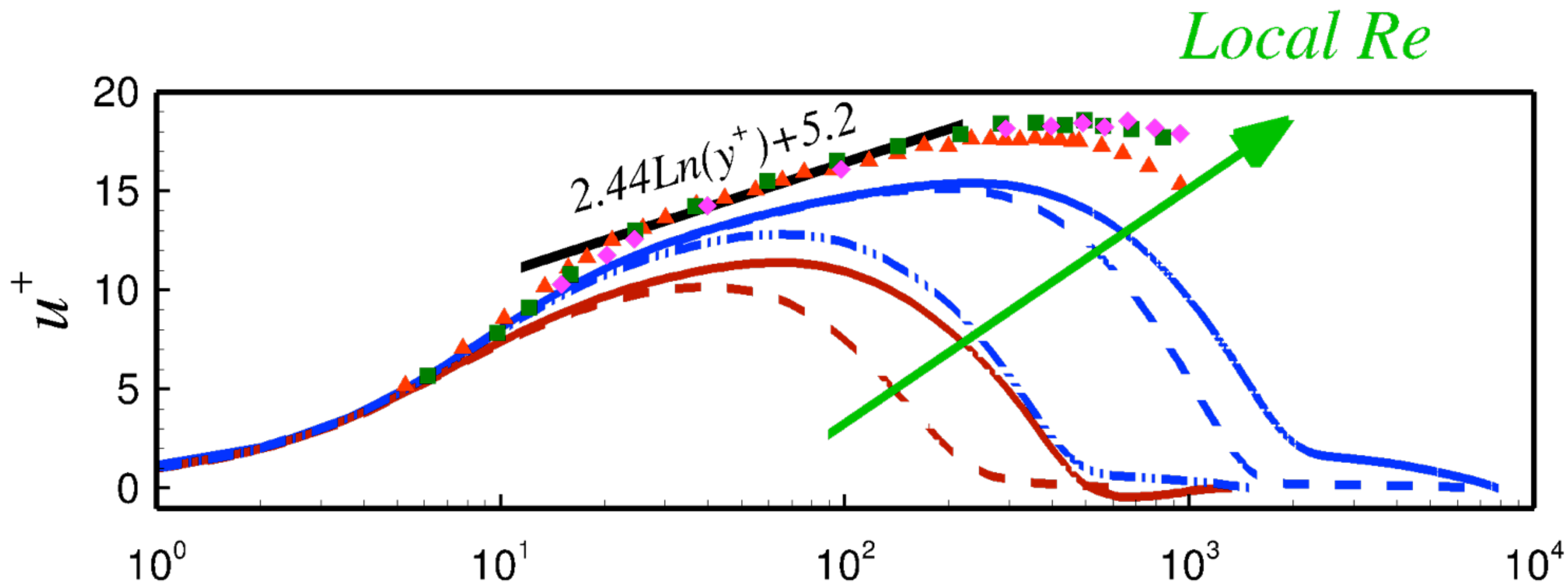
Results

Inner/outer layer interaction



Results

Inner/outer layer interaction



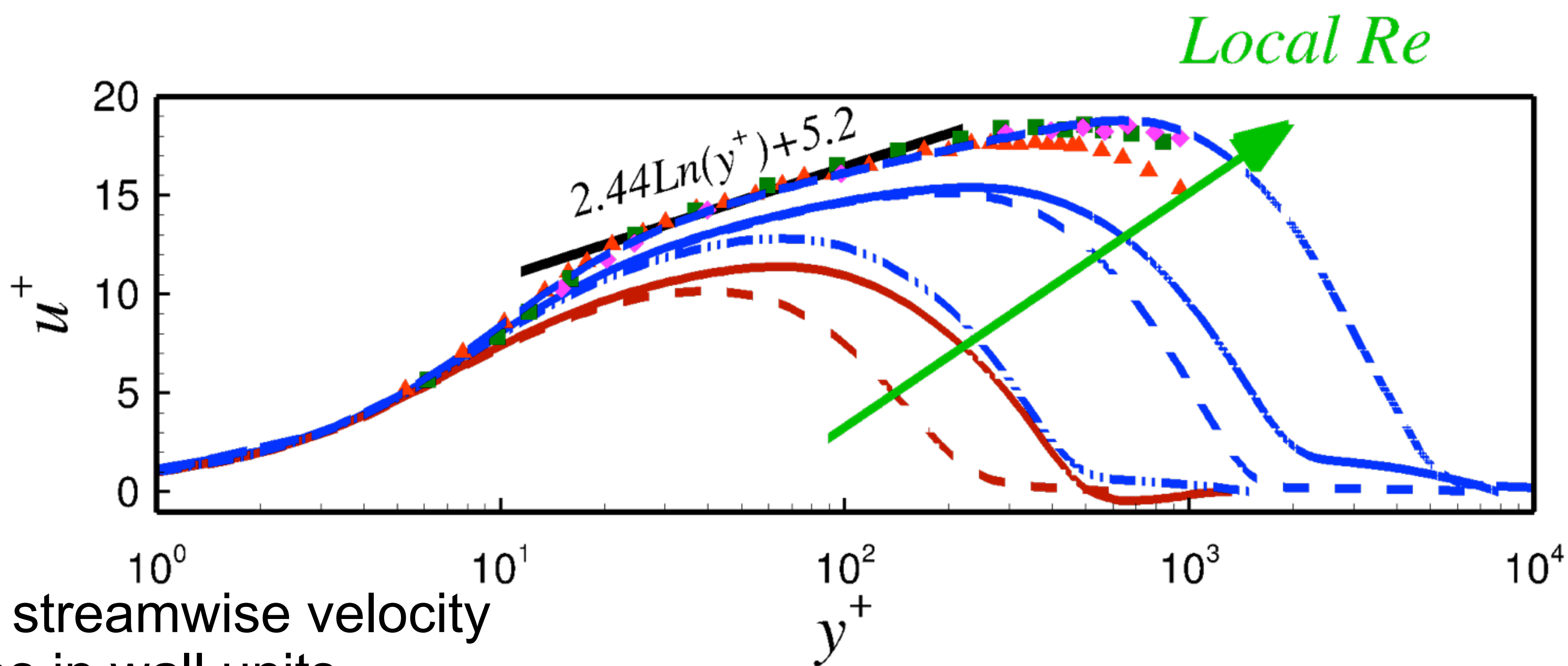
Mean streamwise velocity profiles in wall units

$$Re_m = u_m y_m/\nu,\ y^+_{1/2} = u_\tau y_{1/2}/\nu,\ Re^+ = u_\tau y_m/\nu$$

- —— , Law of the wall
- —— , $Re_m \approx 400$, Radial wall-jet
- —— , $Re_m \approx 700$, Radial wall-jet
- -··- , $Re_m \approx 850$, Plane wall-jet
- - - - , $Re_m \approx 2,900$, Plane wall-jet
- —— , $Re_m \approx 3,600$, Plane wall-jet
- ▲, $Re_m \approx 5,200$ (Eriksson *et al.*, 1998)
- ■, $Re_m \approx 8,600$ (Bradshaw and Gee, 1960)
- ◆, $Re_m \approx 12,200$ (Abrahamsson *et al.* 1994)

Results

Inner/outer layer interaction



Mean streamwise velocity profiles in wall units

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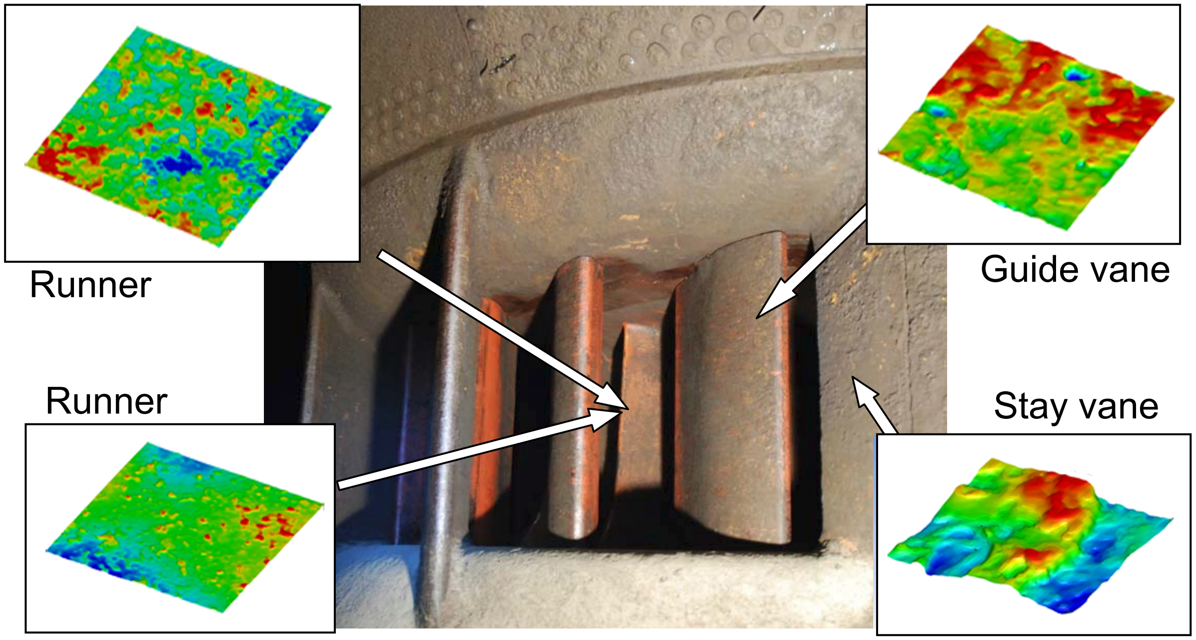
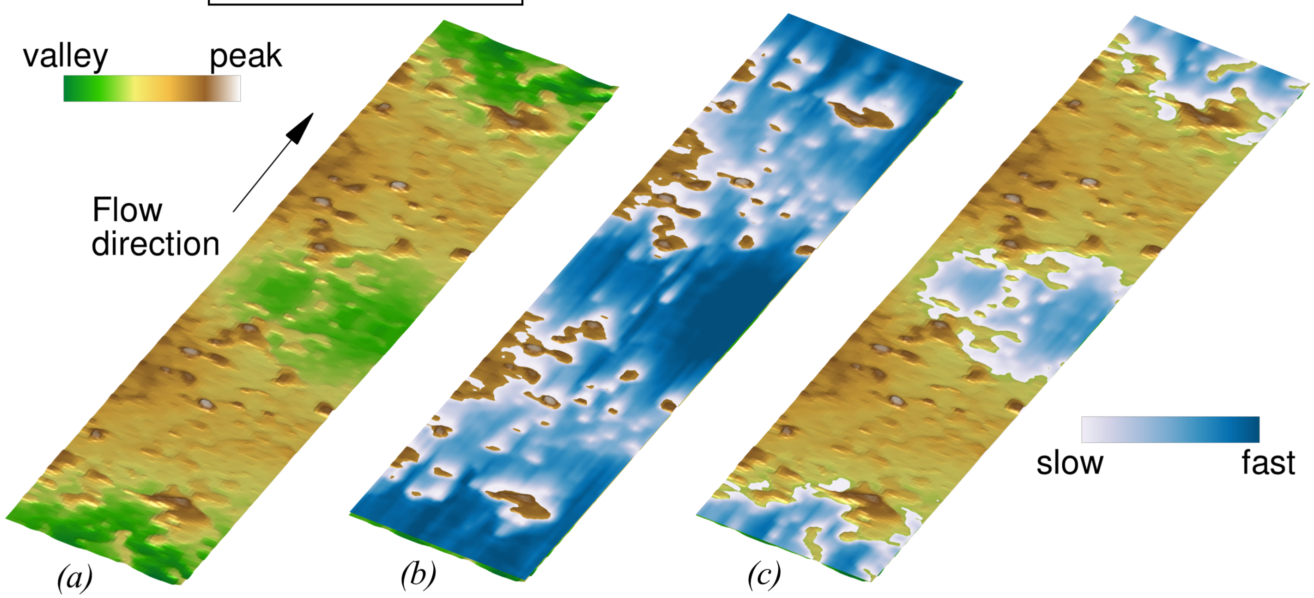
◆, $Re_m \approx 12,200$ (Abrahamsson *et al.* 1994)
— — , $Re_m \approx 12,100$

SUMMARY

- Conclusions
  - *The outer layer is not sensitive to roughness*
  - *The effects of roughness are confined to the inner layer*
  - *The interaction of inner and outer layers depends on the local Reynolds number*
  - *Plane and radial wall-jets behave the same at the same local Reynolds numbers*
- Future work
  - *Analysis of wall-jet at very high local Reynolds numbers*
  - *Systematic investigation of roughness height*
  - *Assessment of turbulence closure models for prediction of wall jets using LES and DNS databases.*

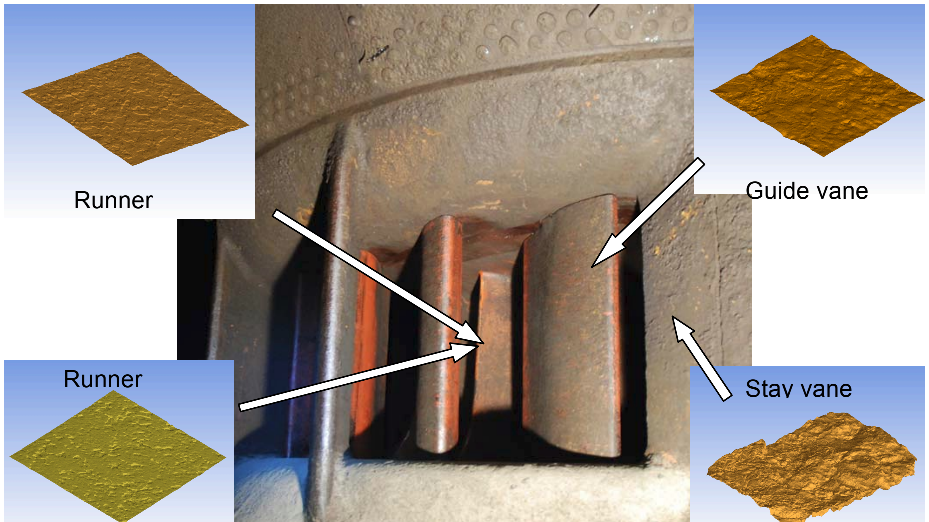
Applications
Flows over Rough Walls
(Junlin Yuan, Rabijit Dutta, Pouya Mottaghian)





Motivation

- Roughness occurs in many applications in engineering

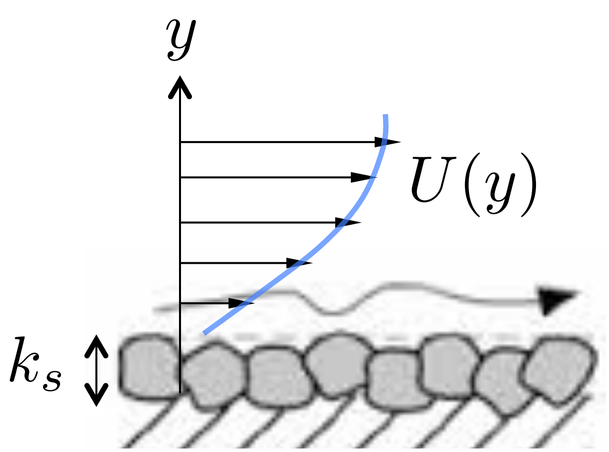


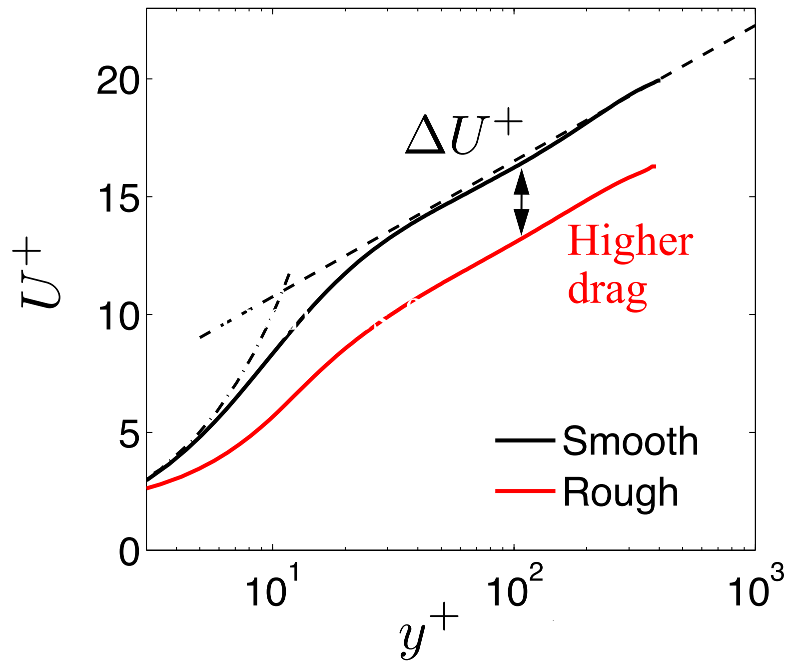
Motivation

- Roughness occurs in many applications in engineering and the natural sciences.

Roughness Effects

- Nikuradse (1933), Colebrook (1939); Raupach et al. (1991); Jiménez (2004);…
  - *Increased drag*





k_s: "equivalent sand-grain (SG) height"

$k_s^+ = k_s u_\tau / \nu$: "roughness Reynolds number"

Roughness Effects

- Nikuradse (1933), Colebrook (1939); Raupach et al. (1991); Jiménez (2004);…
  - *Increased drag*
  - *Increased wall-normal fluctuations*
  - *Decreased Reynolds stress anisotropy*
  - *Dependence on roughness detail is confined to a "Roughness layer"*

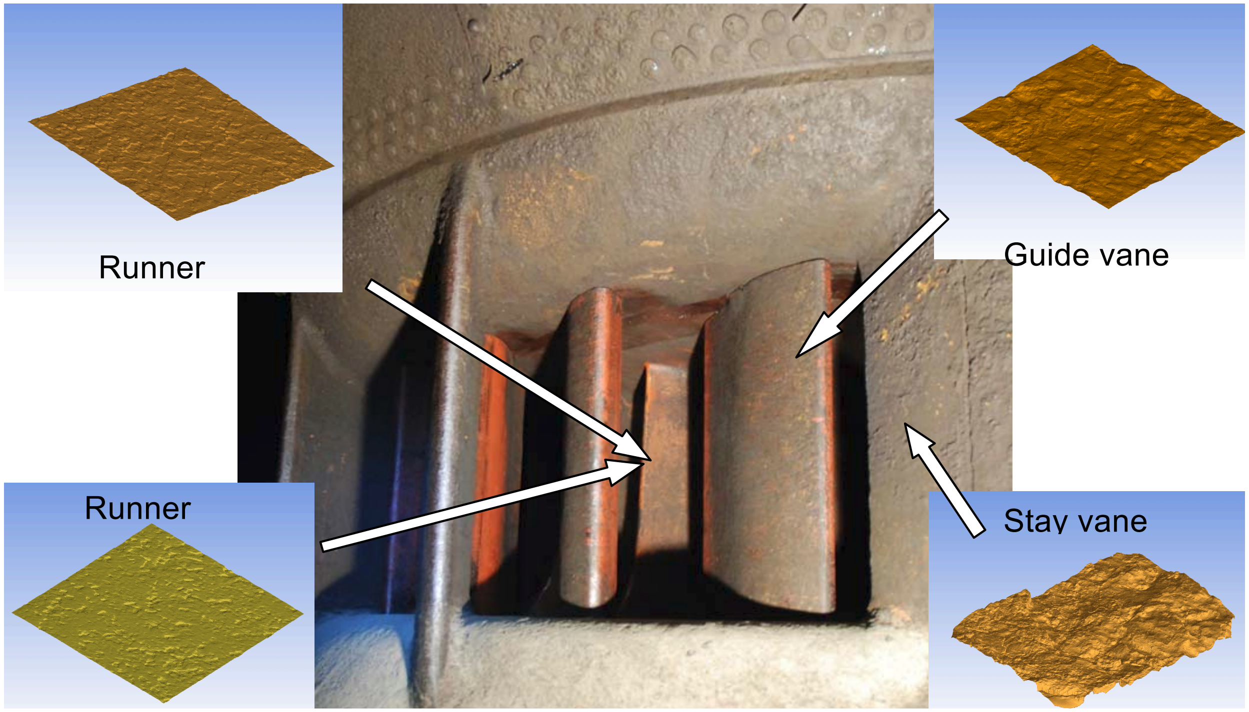
Methodology

- Solution of the filtered NS equations
  - *Either Cartesian or curvilinear code*
- Roughness is modelled using an Immersed-Boundary Method (IBM)
  - *Force added to RHS of NS equations to enforce no-slip condition off grid points*
  - *Grid must be finer than the typical roughness length scales* $\lambda_x,\ \overline{k},\ \lambda_z$ *:*

$$\Delta x < \lambda_x/6; \quad \Delta y < \overline{k}/10; \quad \Delta z < \lambda_z/6$$

Surface Characterization

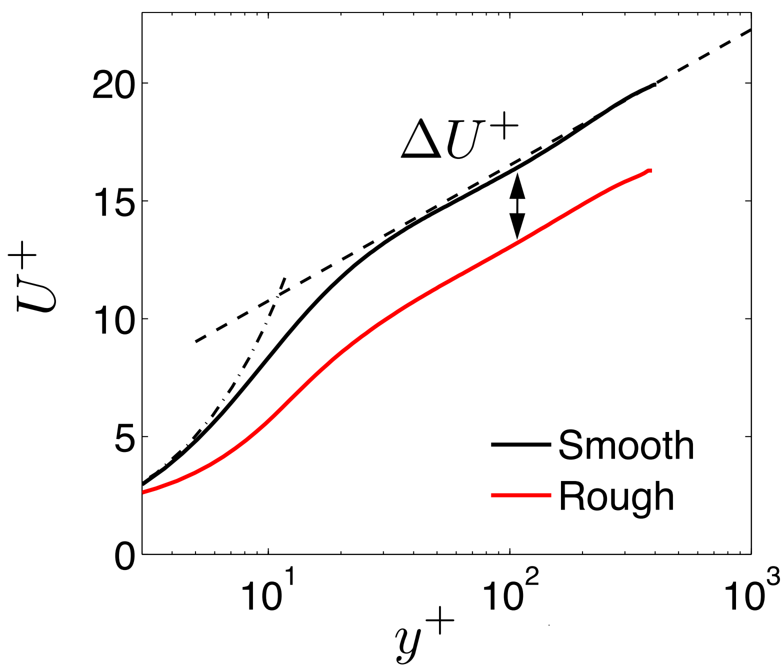
- Turbulence models for the RANS equations have been modified to include roughness
  - *Main parameter to describe the roughness in roughness extensions is k_s*
- Most realistic surfaces do not look like sandgrain roughness
- Need to determine the relationship between k and k_s for arbitrary surfaces.

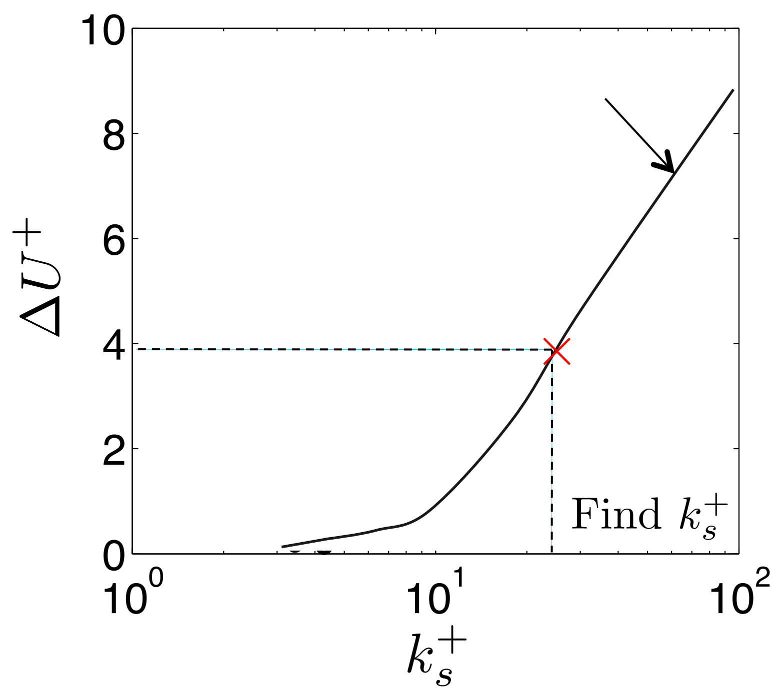


Surface Characterization

- Determine k_s for arbitrary surfaces

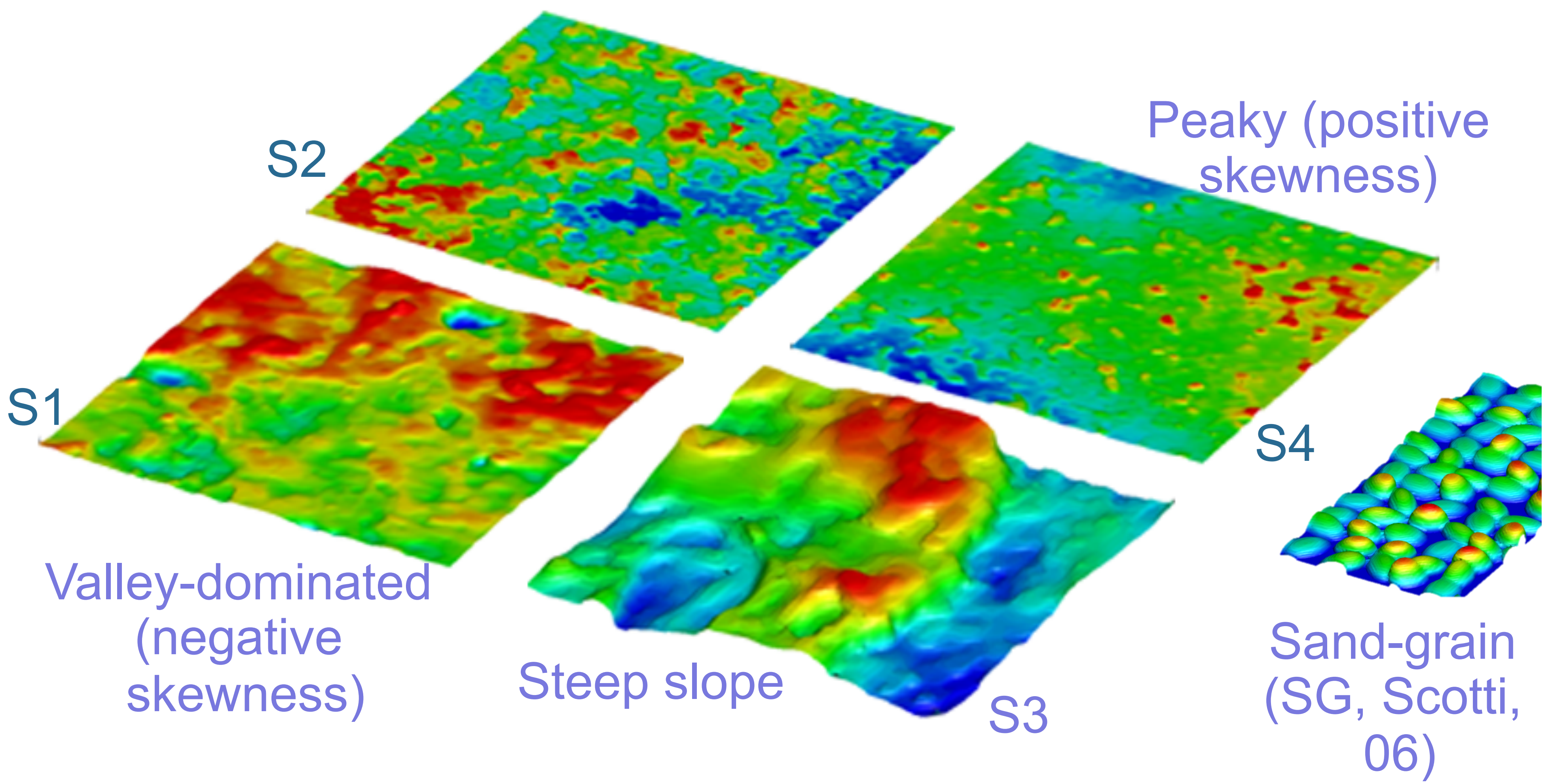
LES + resolved roughness → Measured roughness function → k_s





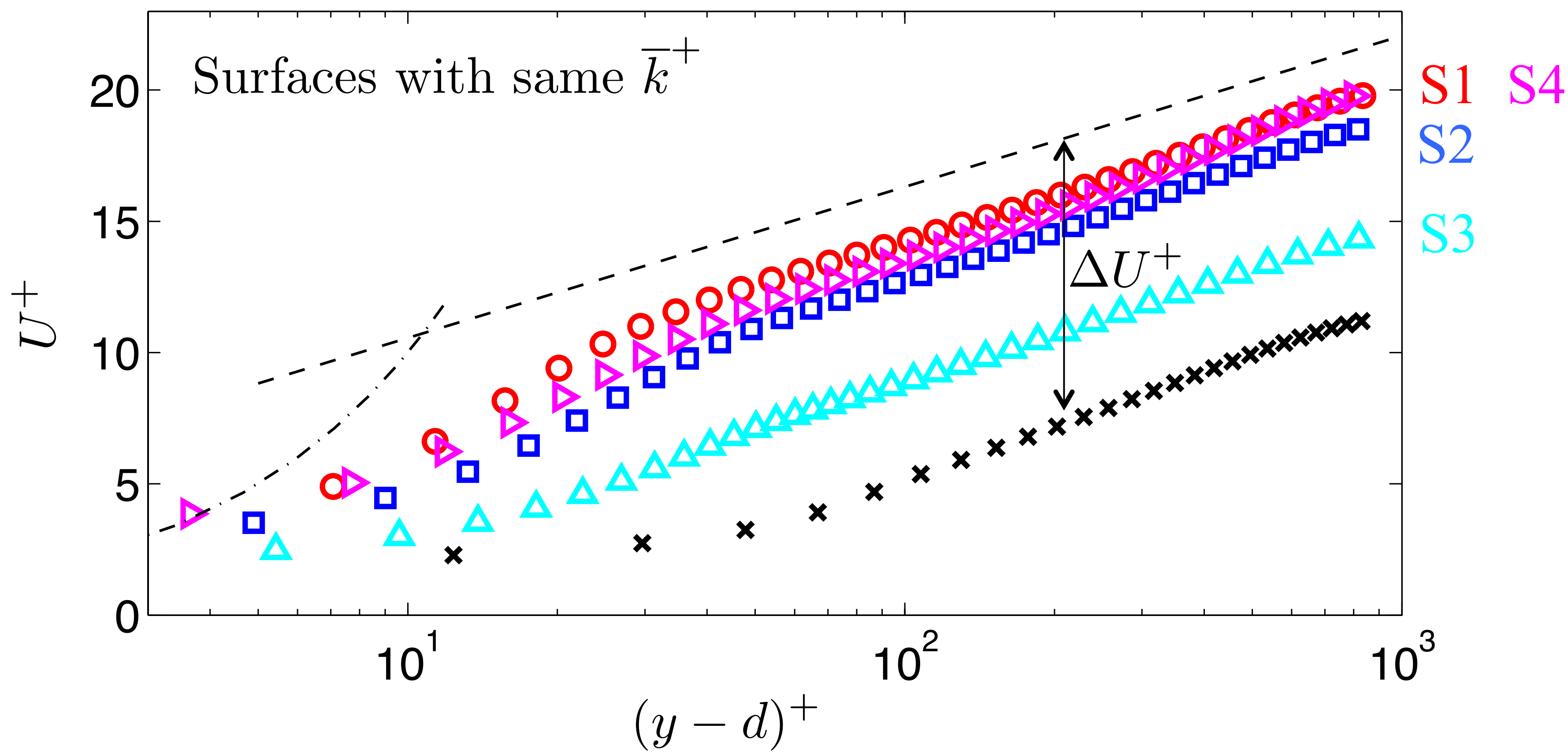
Surface Characterization

- Determine k_s for arbitrary surfaces



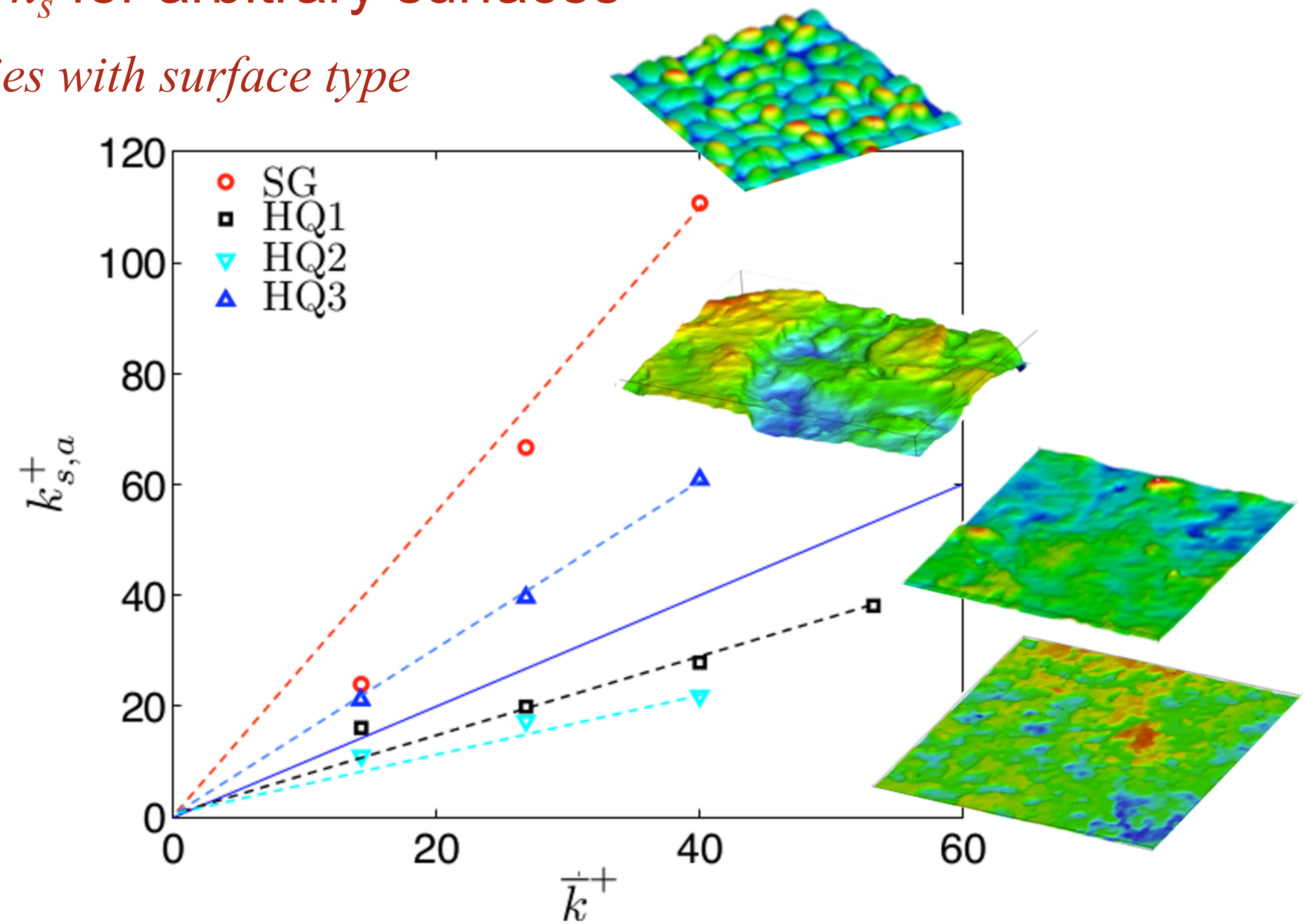
Surface characterization

- Determine k_s for arbitrary surfaces
  - *ΔU^+ varies with surface type*



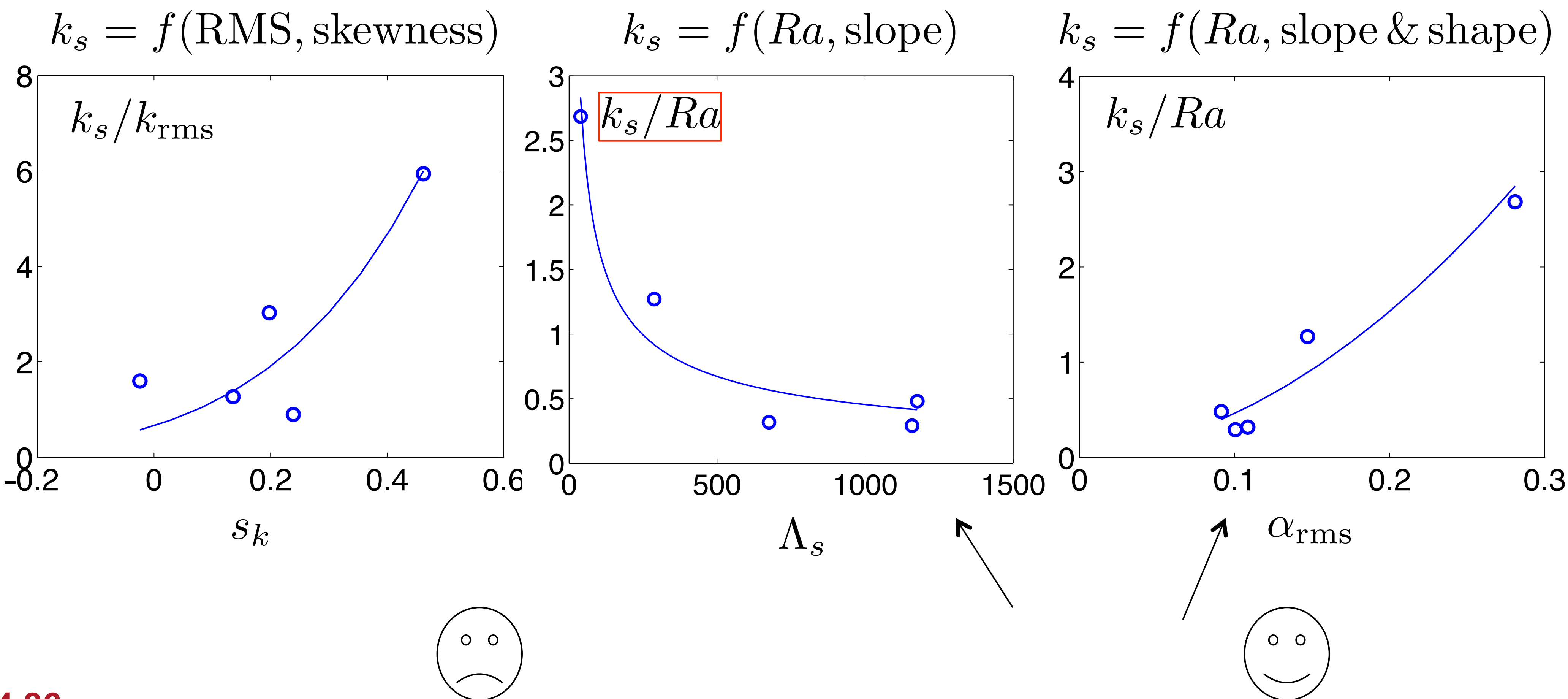
Surface characterization

- Determine k_s for arbitrary surfaces
  - $k_s/\overline{k}$ *varies with surface type*



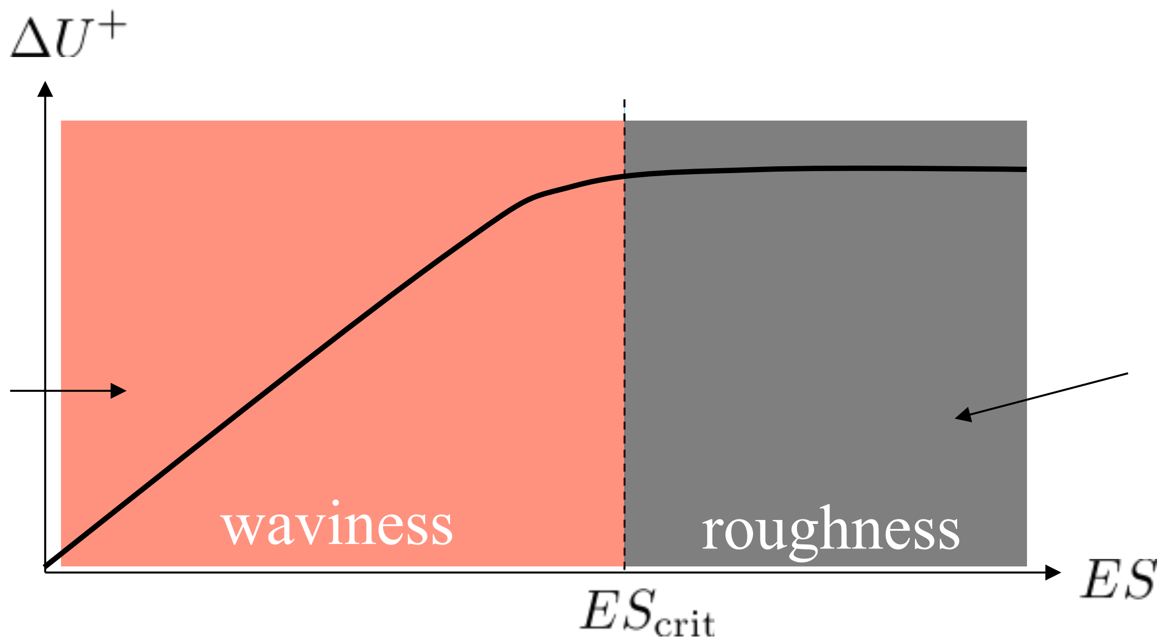
SURFACE CHARACTERIZATION

- Evaluation of k_s-correlations
  - *5 surfaces → 5 data points*
  - *Evaluate existing correlation to obtain the sandgrain roughness*



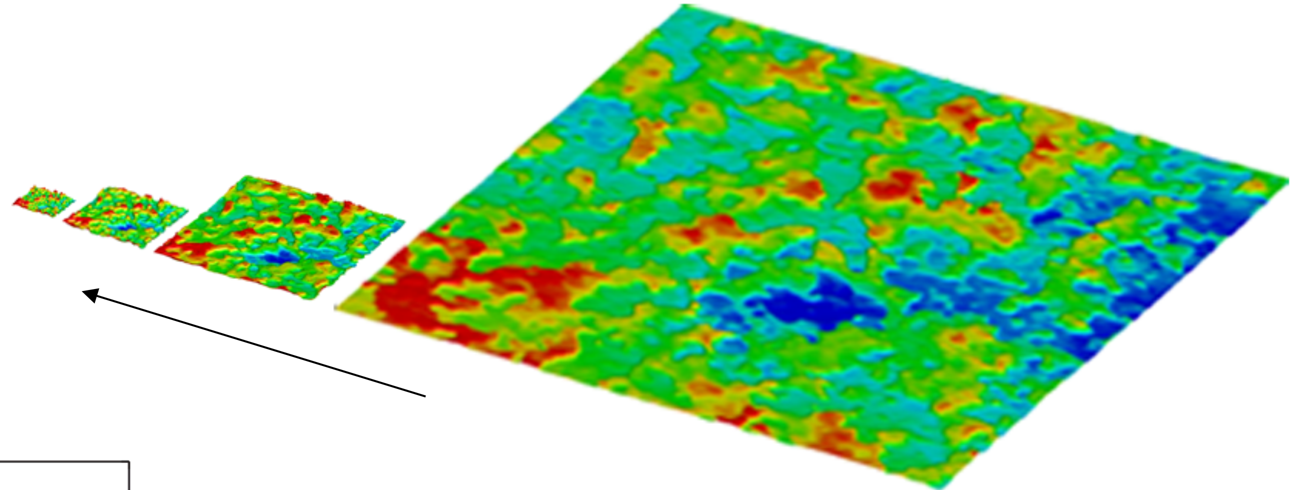
Surface Characterization

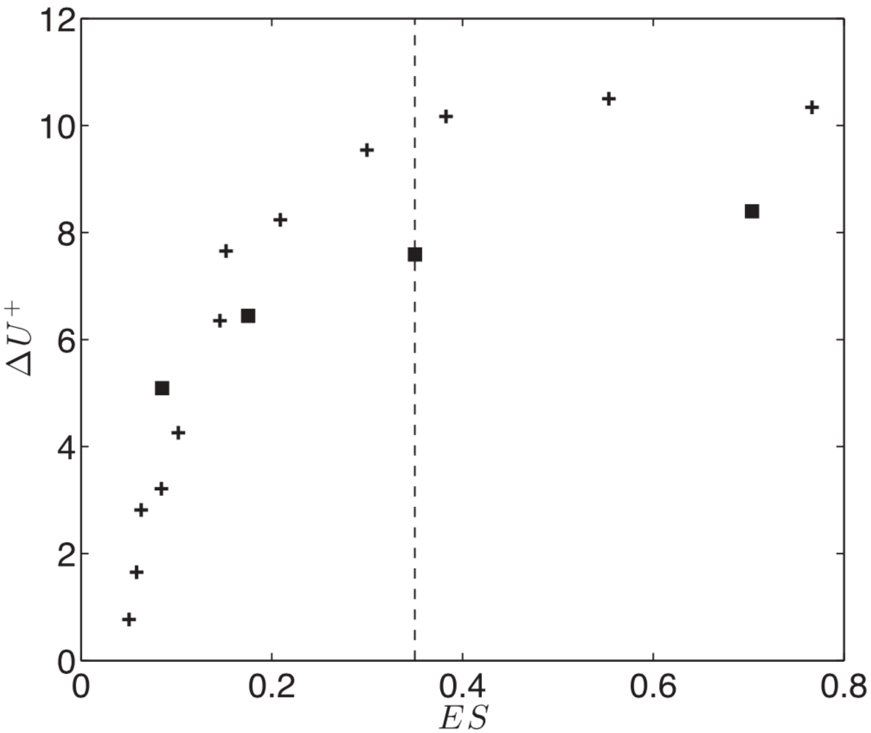
- Evaluations of k_s-correlations
  - *Effective slope* $ES = \frac{1}{L}\int_0^L \left|\frac{dk}{dx}\right| dx$
  - *"Slope is an important parameter only when ES is less than a critical value: 0.35, in 'waviness regime'."*



Surface Characterization

- Stretch one type of surface to change the slope without changing the other surface



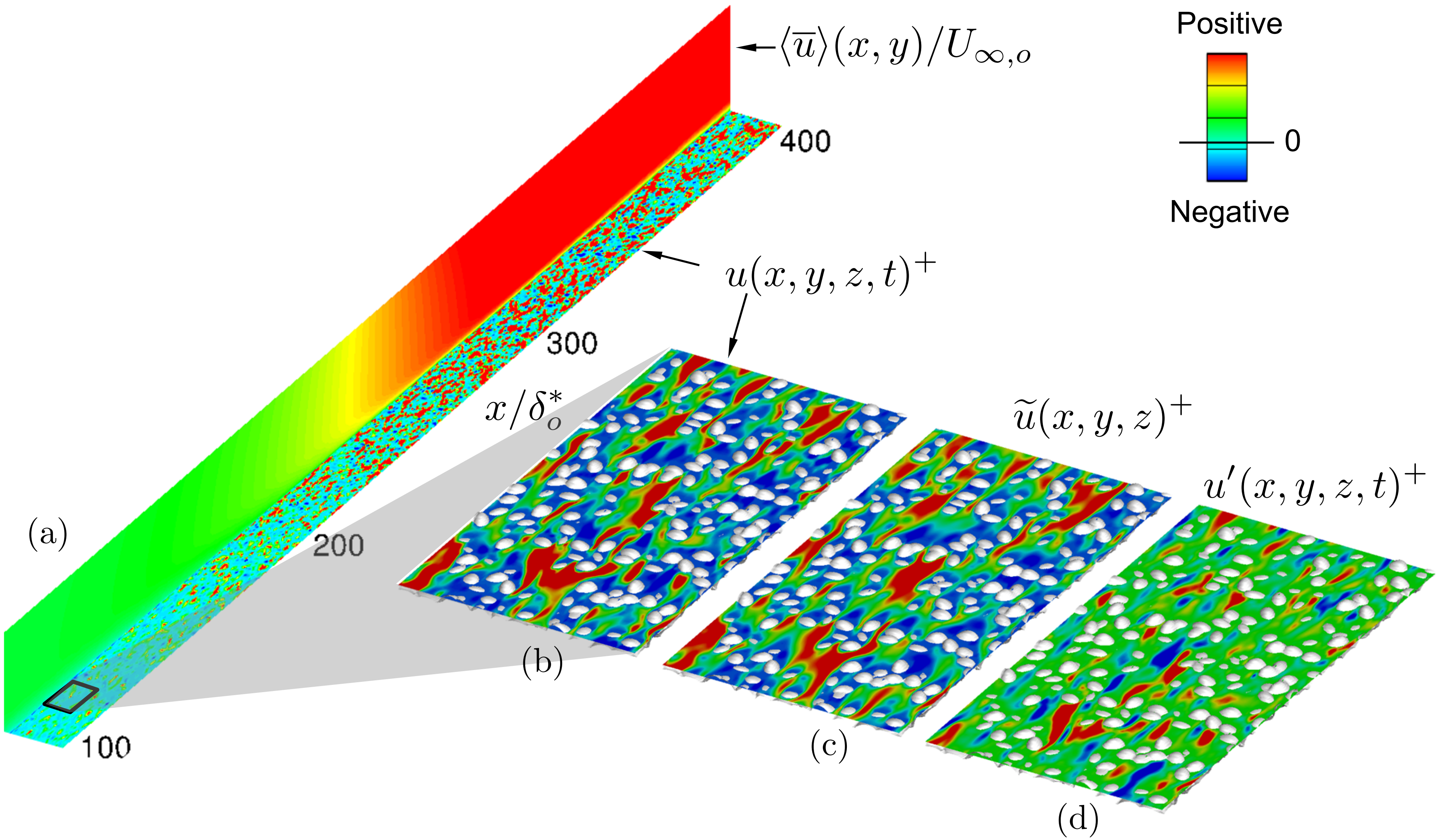


- The critical value of the ES depends on surface characteristics
- The surfaces considered are all in the "waviness" regime

J. Yuan and U. Piomelli. Estimation and prediction of the roughness function on realistic surfaces. J. Turbul., 15(6):350–365, 2014.

Applications: Wake/Turbulence interaction

June Yuan



Double averaging

$$\theta(x,y,z,t) = \langle\overline{\theta}\rangle(x,y) + \widetilde{\theta}(x,y,z) + \theta'(x,y,z,t).$$

Time and spanwise-averaged field

Spatial variation of the time-averaged field

Fluctuating field

Double averaging

$$\theta(x,y,z,t) = \langle\overline{\theta}\rangle(x,y) + \widetilde{\theta}(x,y,z) + \theta'(x,y,z,t).$$

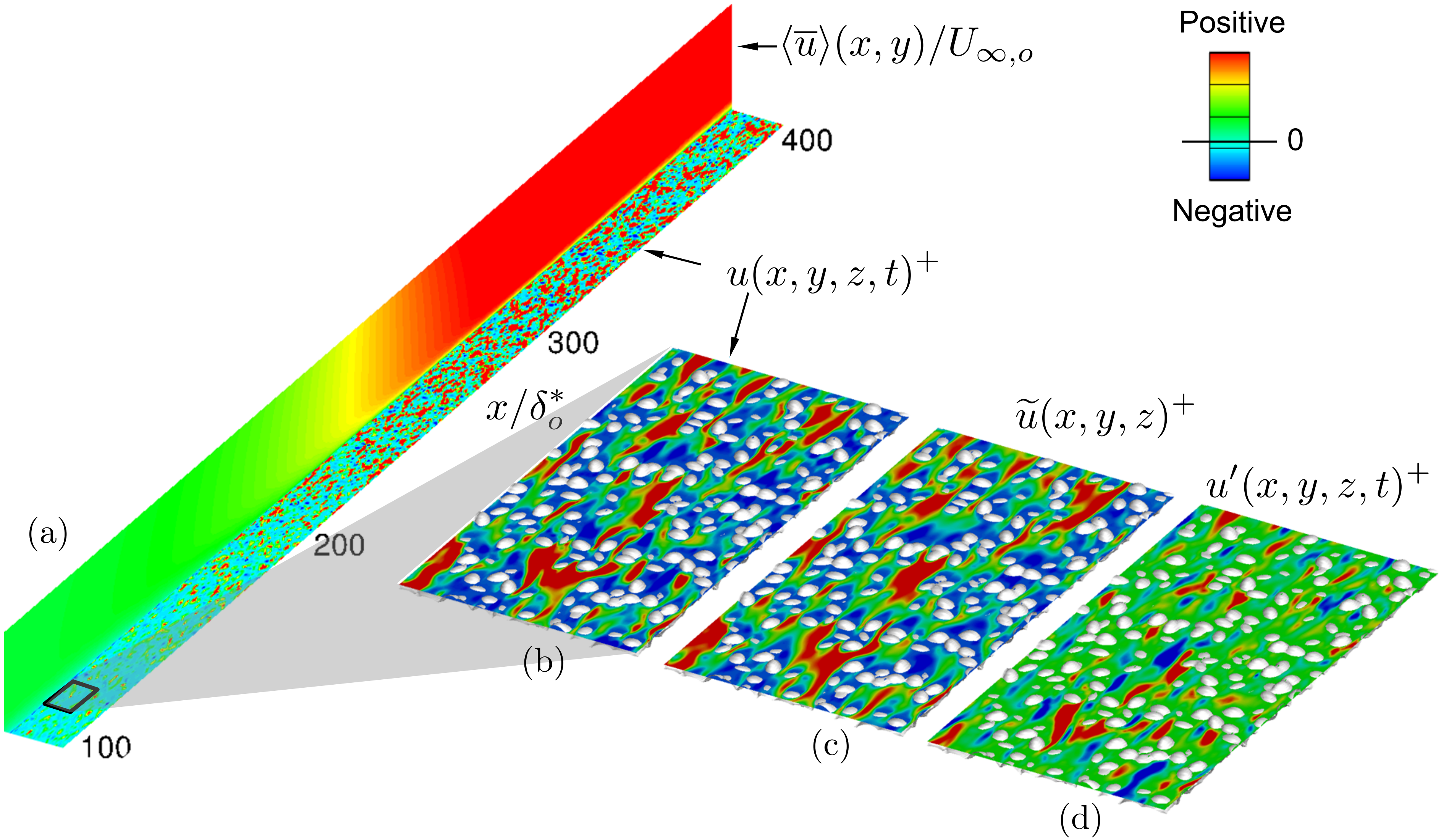
Time and spanwise-averaged field

Spatial variation of the time-averaged field
Wake field

Fluctuating field

Double averaging

$$\theta(x, y, z, t) = \langle\overline{\theta}\rangle(x, y) + \widetilde{\theta}(x, y, z) + \theta'(x, y, z, t).$$

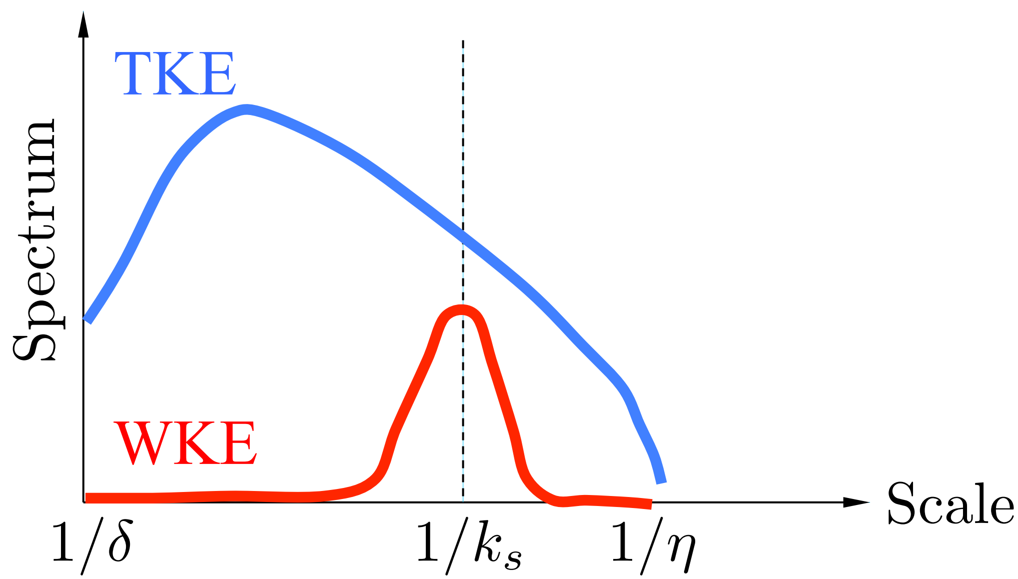


Wake-Turbulence Interaction

- Motivation
  - *Wake fluctuation must play a role in differentiating between roughness types.*
  - *v', w': important for near-wall instability & momentum transfer*
  - *Roughness → wake field → change in Reynolds-stress budgets*

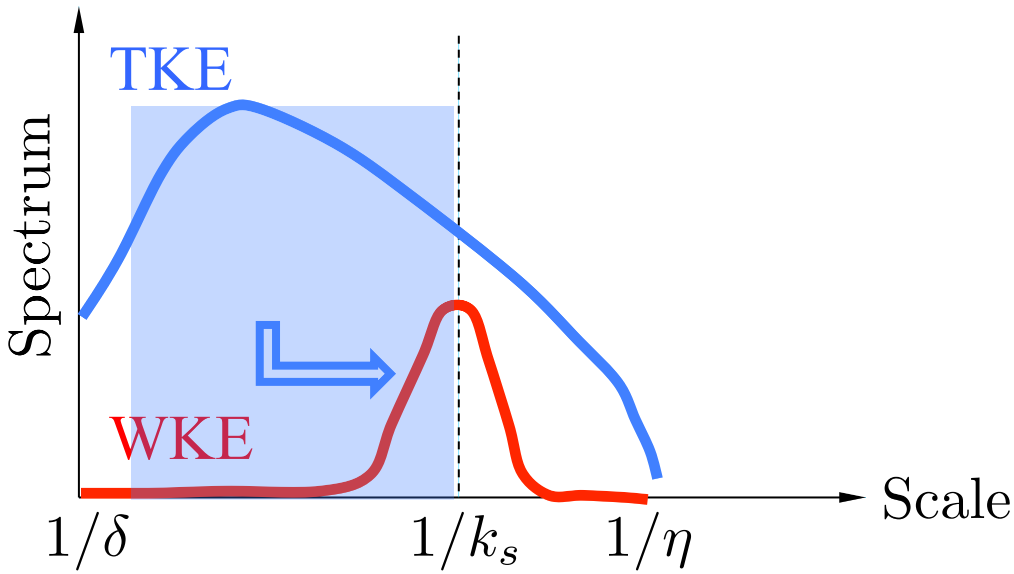
Wake-Turbulence Interaction

- Wake-turbulence interaction
  - *Two-way*



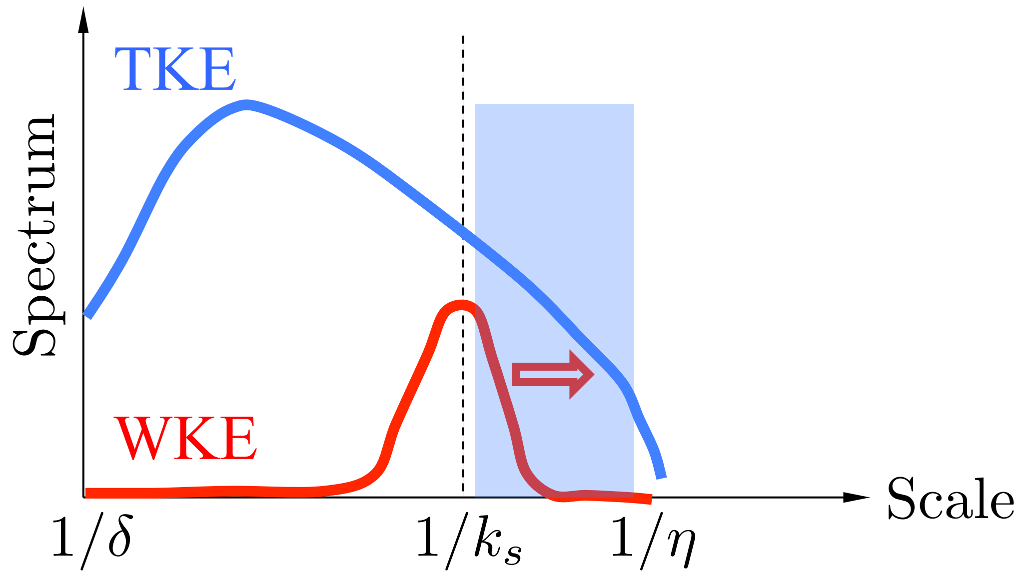
Wake-Turbulence Interaction

- Wake-turbulence interaction
  - *Two-way*



Wake-Turbulence Interaction

- Wake-turbulence interaction
  - *Two-way*



Wake-Turbulence Interaction

- Wake-turbulence interaction
  - *Two-way*
  - *Wake production, P_w*
    - Net conversion (Finnigan 00)
    - Shear term with opposite signs in budgets of WKE & TKE

$$P_w = -\left\langle \widetilde{u_i' u_j'} \frac{\partial \widetilde{u}_i}{\partial x_j} \right\rangle_s$$

Wake-turbulence interaction

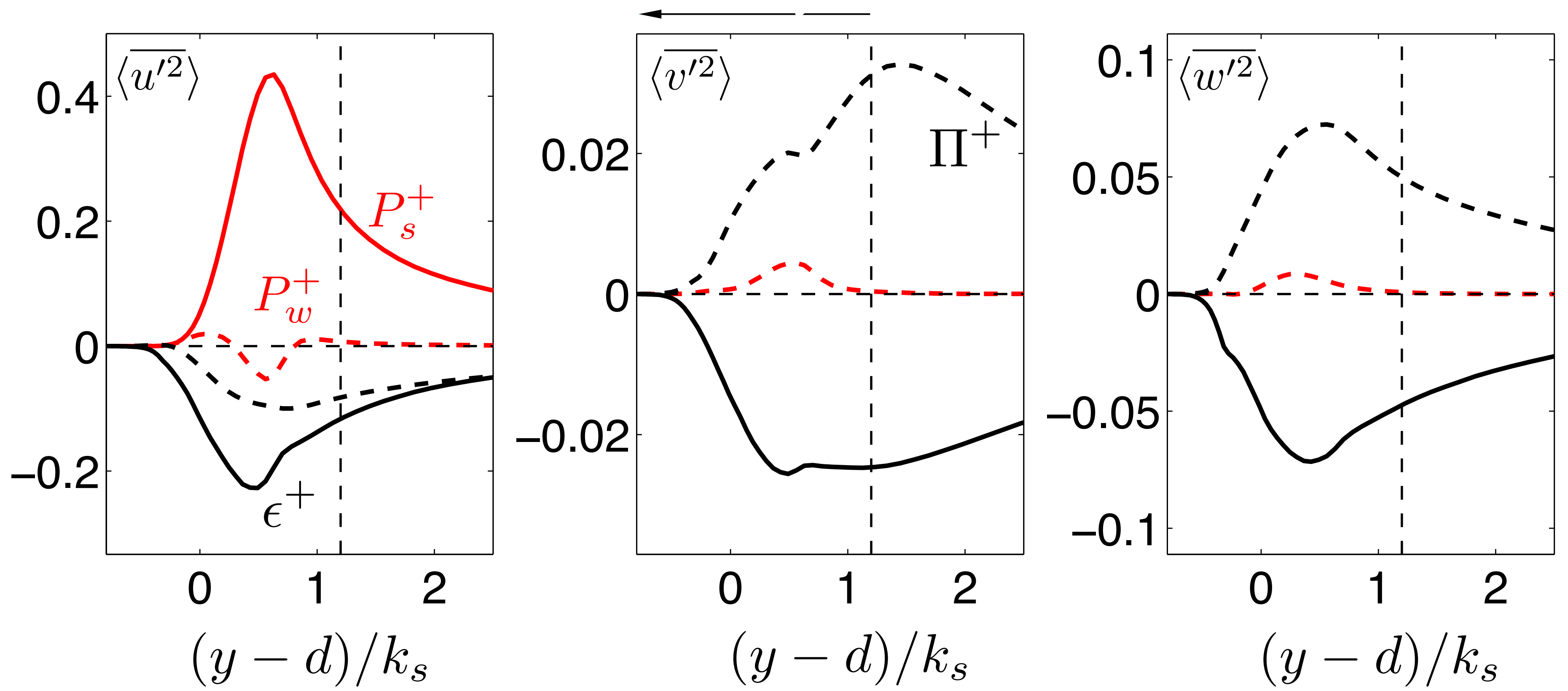
- Results
  - P_w *in ZPG boundary layer*
    - Reynolds-stress budgets

P_s: shear production

P_w: wake production

ϵ: dissipation

Π: pressure work

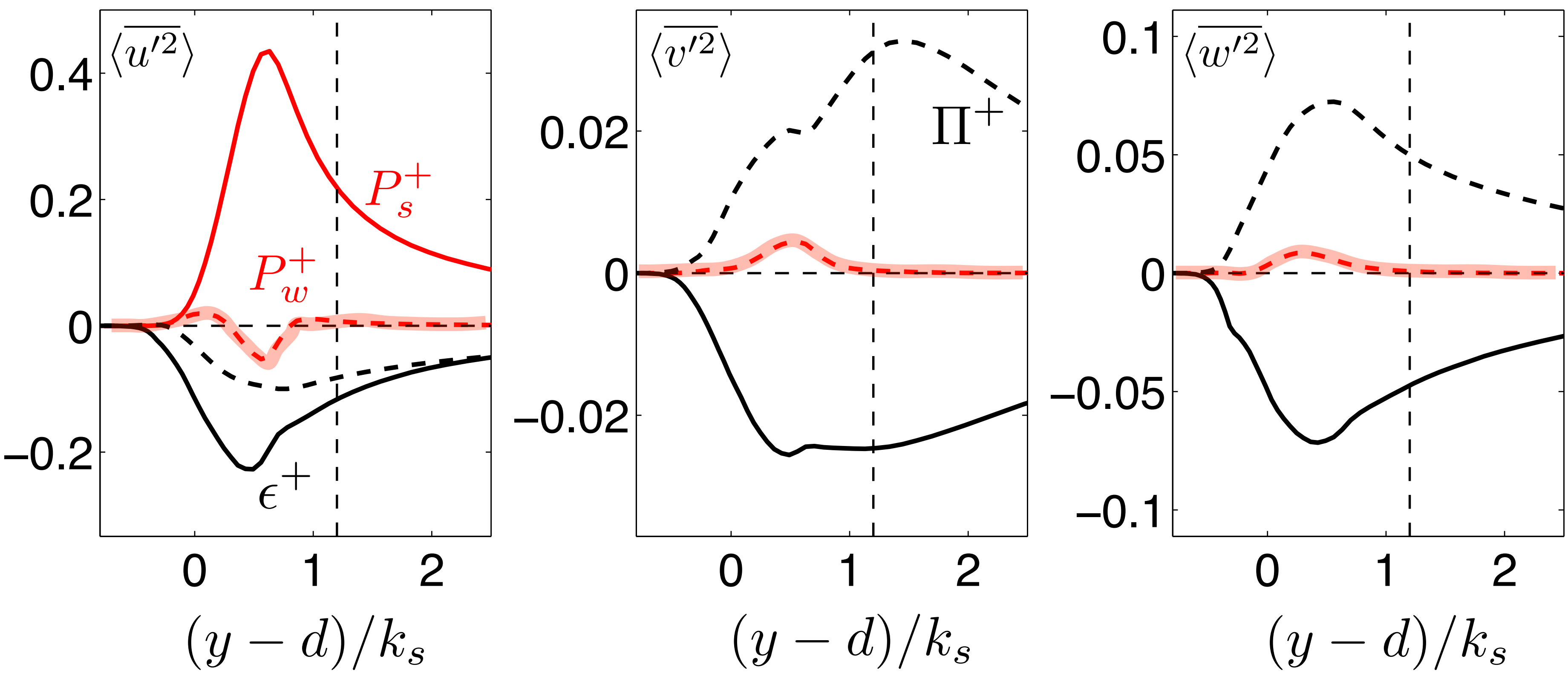


Wake-turbulence interaction

- Results
  - P_w *in ZPG boundary layer*
    - Reynolds-stress budgets
    - P_w → higher v', w', p'
    - TKE redistribution ⇒ less anisotropy

$P_w < 0$

$P_w > 0$



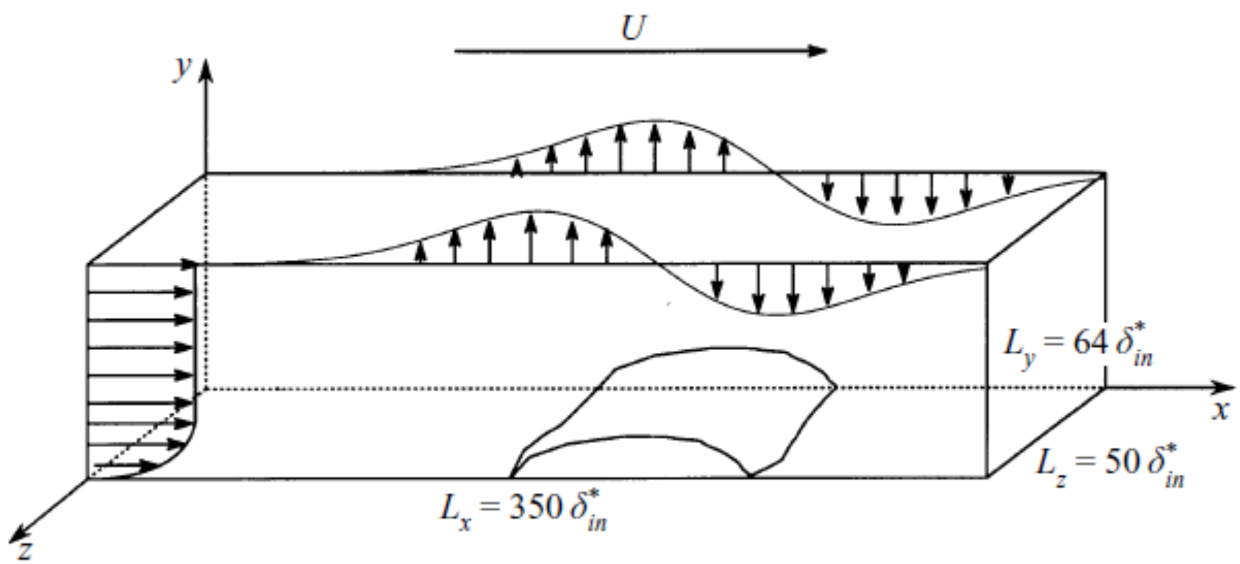
Junlin Yuan, Pouya Mottaghian

Motivation

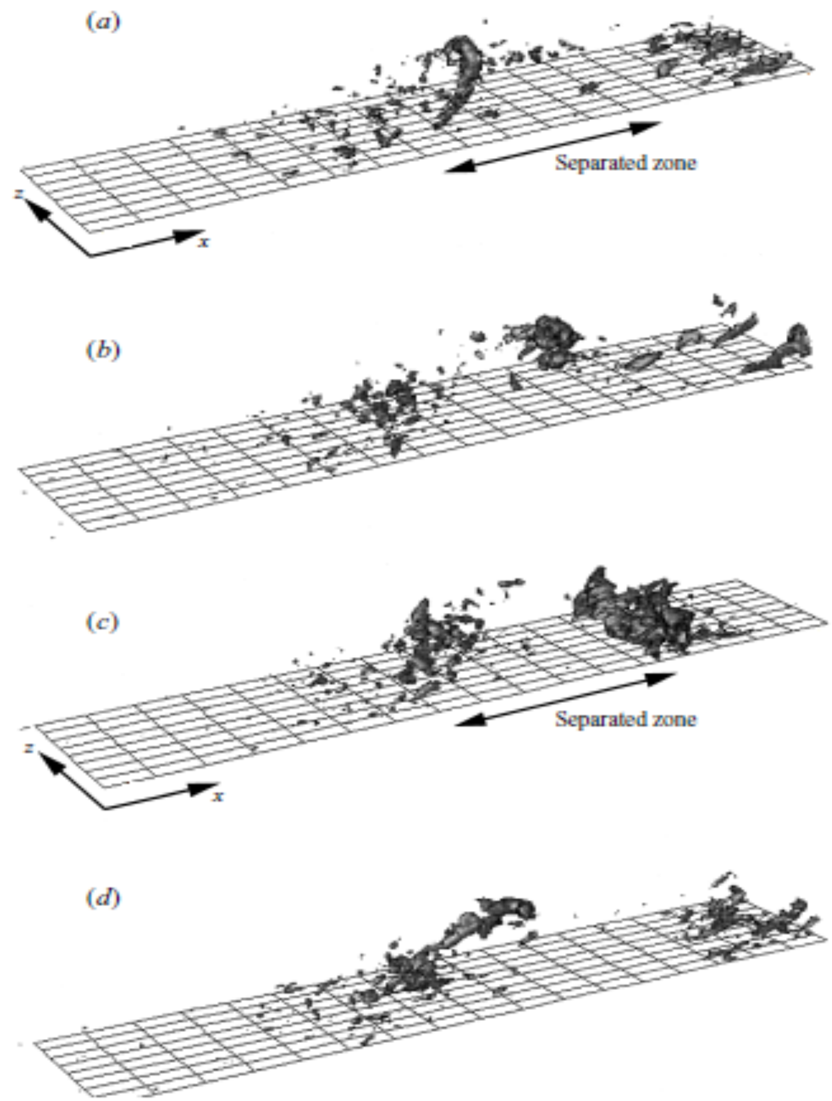
- Canonical boundary layers
  - *Zero pressure gradient*
  - *Smooth wall*
- Realistic boundary layers
  - *Acceleration - Favourable pressure gradient (FPG)*
  - *Deceleration - Adverse pressure gradient (APG)*
  - *Curvature*
  - *3D mean-flow*

Literature

- Flat-plate boundary layer with APG
  - *Laminar:*
    - Spalart & Strelets (2000),
  - *Turbulent*
    - Perry and co-workers (1966+), Na & Moin (1998), Skote & Henningson (2002), …

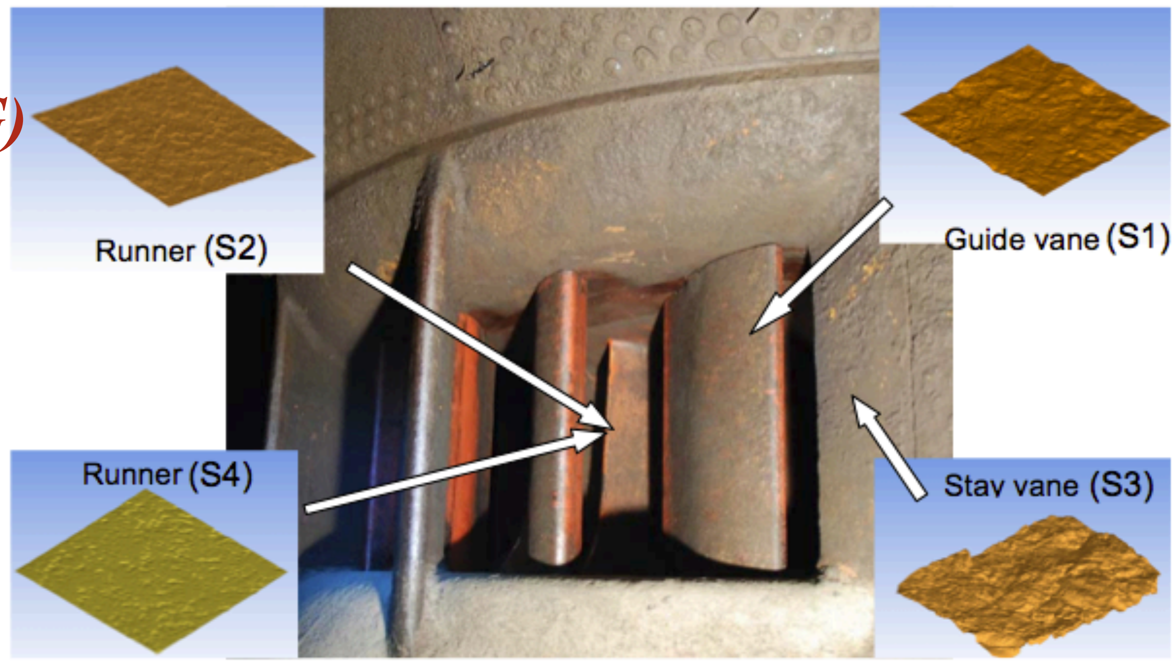


- Flat-plate boundary layer with APG
  - *Observations:*
    - Mean shear layer detachment
    - Unsteady separation bubble
    - Turbulent structures travel around the bubble.
    - Intensity increases inside shear layer.
    - Turbulence increases in intensity & homogeneity at reattachment.



Motivation

- Canonical boundary layers
  - *Zero pressure gradient*
  - *Smooth wall*
- Realistic boundary layers
  - *Acceleration - Favourable pressure gradient (FPG)*
  - ***Deceleration – Adverse pressure gradient (APG)***
  - *Curvature*
  - *3D mean-flow*
  - ***Surface roughness***



LITERATURE

- Rough-wall boundary layers with APG
  - *Song and Eaton (2002), Cao and Tamura (2005), Durbin et al. (2001), …*
  - *Geometry studied*
    - Flow around ramp
    - Flow over hill
  - *Earlier separation & larger separation bubble (vs. smooth case)*
    - Larger momentum deficit
  - *Limitations*
    - More complex geometry ⇒additional effects
    - Skin friction calculation
    - Flow structures
    - Accessibility of the roughness sublayer

QUESTION

- How do roughness (inner layer effect) and deceleration (outer layer effect) interact?

What we know

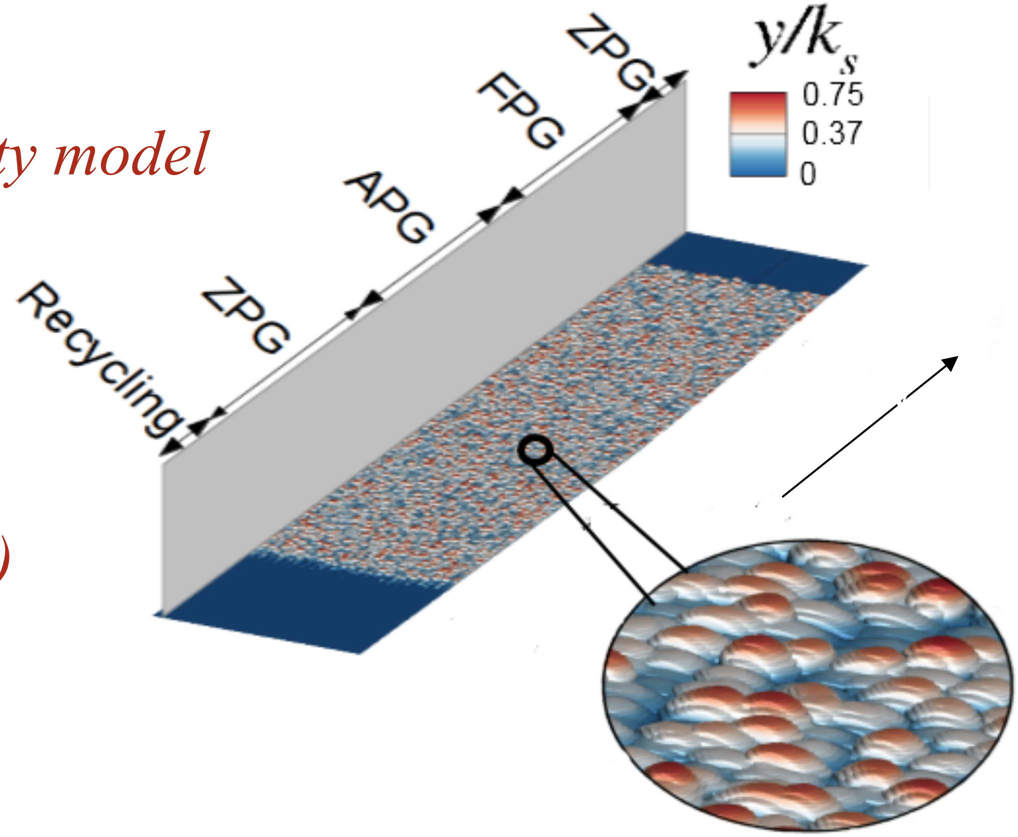
- Roughness promotes turbulence & mixing in the ZPG region
- Roughness increases drag
- APG (before separation) promotes turbulence & mixing

Questions

- Why earlier separation in rough case?
- Does separation communicate the roughness effects away from the wall?

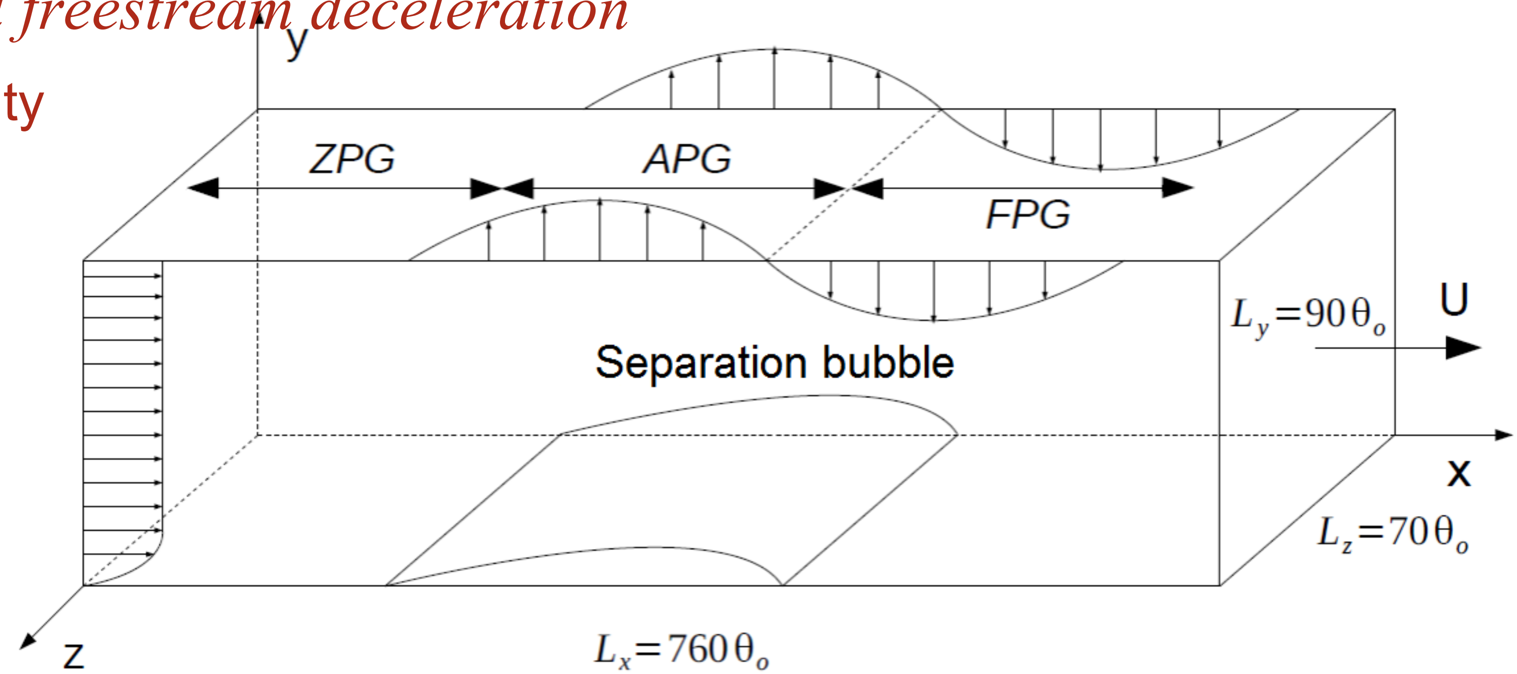
Problem Formulation

- Continuity and Navier-Stokes equations
- 2nd-order accurate in space and time, staggered finite difference code
- Crank-Nicolson + Adams-Bashforth time advancement
- Large-eddy simulation
  - *Lagrangian dynamic eddy-viscosity model*
- Resolved roughness
  - *Virtual sandpaper model*
    - Scotti (2006)
  - *Immersed boundary method (IBM)*
    - Volume of fluid approach



Problem Formulation

- Boundary conditions
  - *Spanwise: periodic boundary condition*
  - *Inlet: Recycling/Rescaling method of Lund et al. (1998)*
  - *Outlet: Convective outflow boundary condition*
  - *Top: imposed freestream deceleration*
    - Zero vorticity

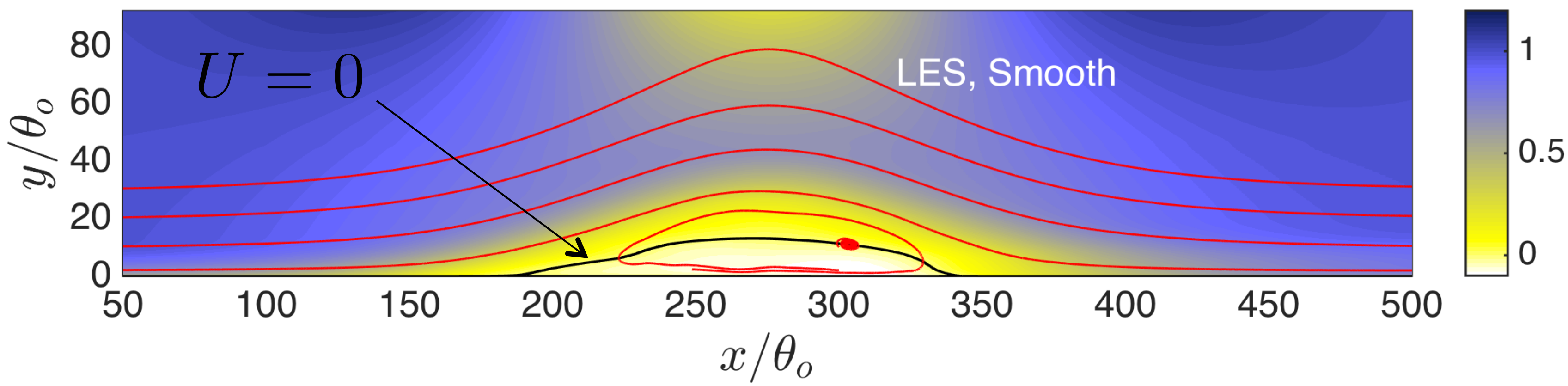


Problem Formulation

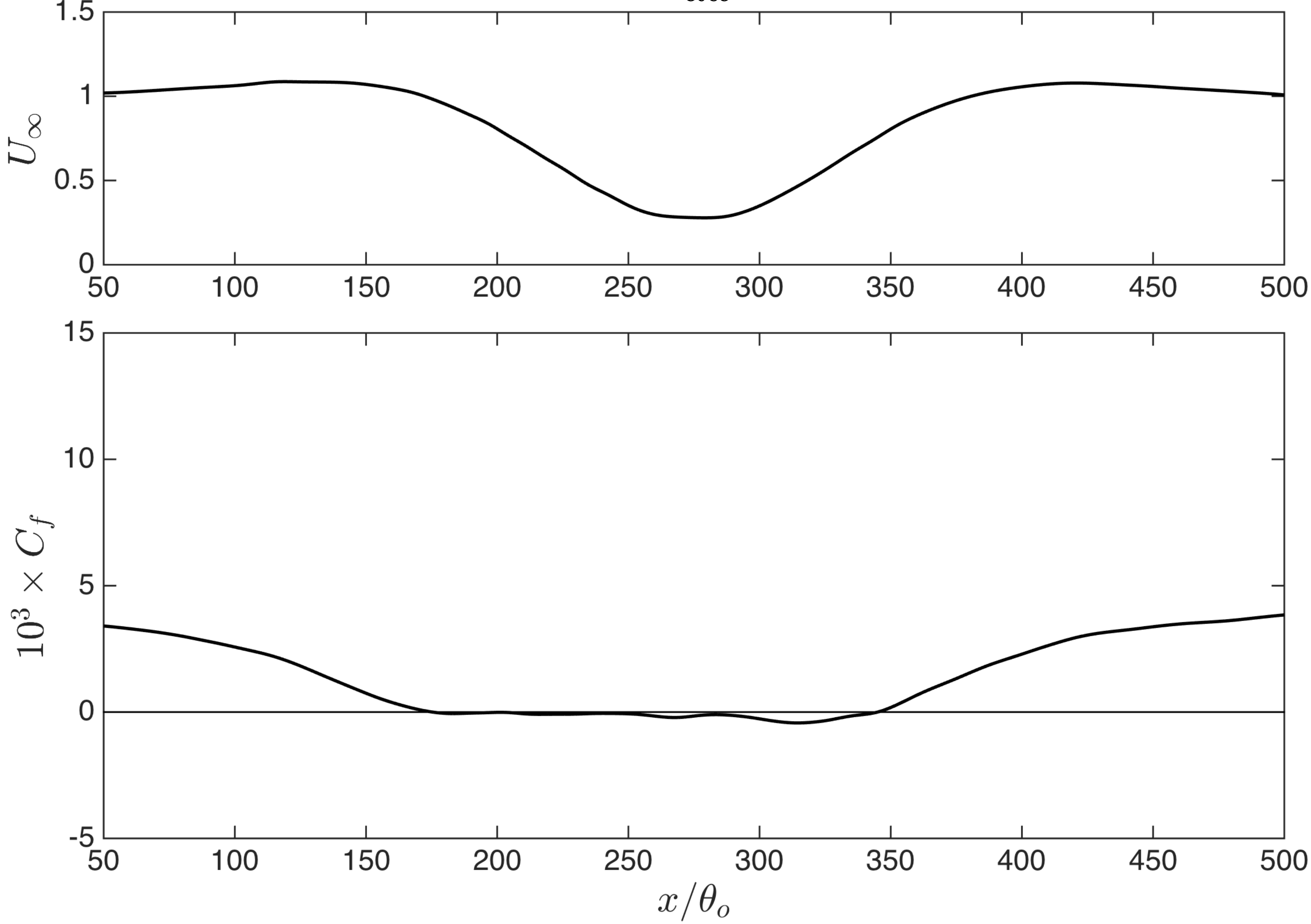
- Cases
  - *Reference scales:* $u_{\infty,o},\ \theta_o$

Case	$Re_{\theta,0}$	k/θ_0	Grid	Domain size	Δx^+_{max}	Δy^+_{min}	Δz^+_{max}
Smooth	300	0	640×192×128	$460\theta_0 \times 65\theta_0 \times 50\theta_0$	20	0.6	11
Smooth	2300	0	2560×384×384	$760\theta_0 \times 90\theta_0 \times 70\theta_0$	30	1	19
Rough	2300	0.47	2048×384×384	$760\theta_0 \times 90\theta_0 \times 70\theta_0$	50	1.5	26
Rough	2300	0.95	2560×432×384	$760\theta_0 \times 90\theta_0 \times 70\theta_0$	44	0.8	28

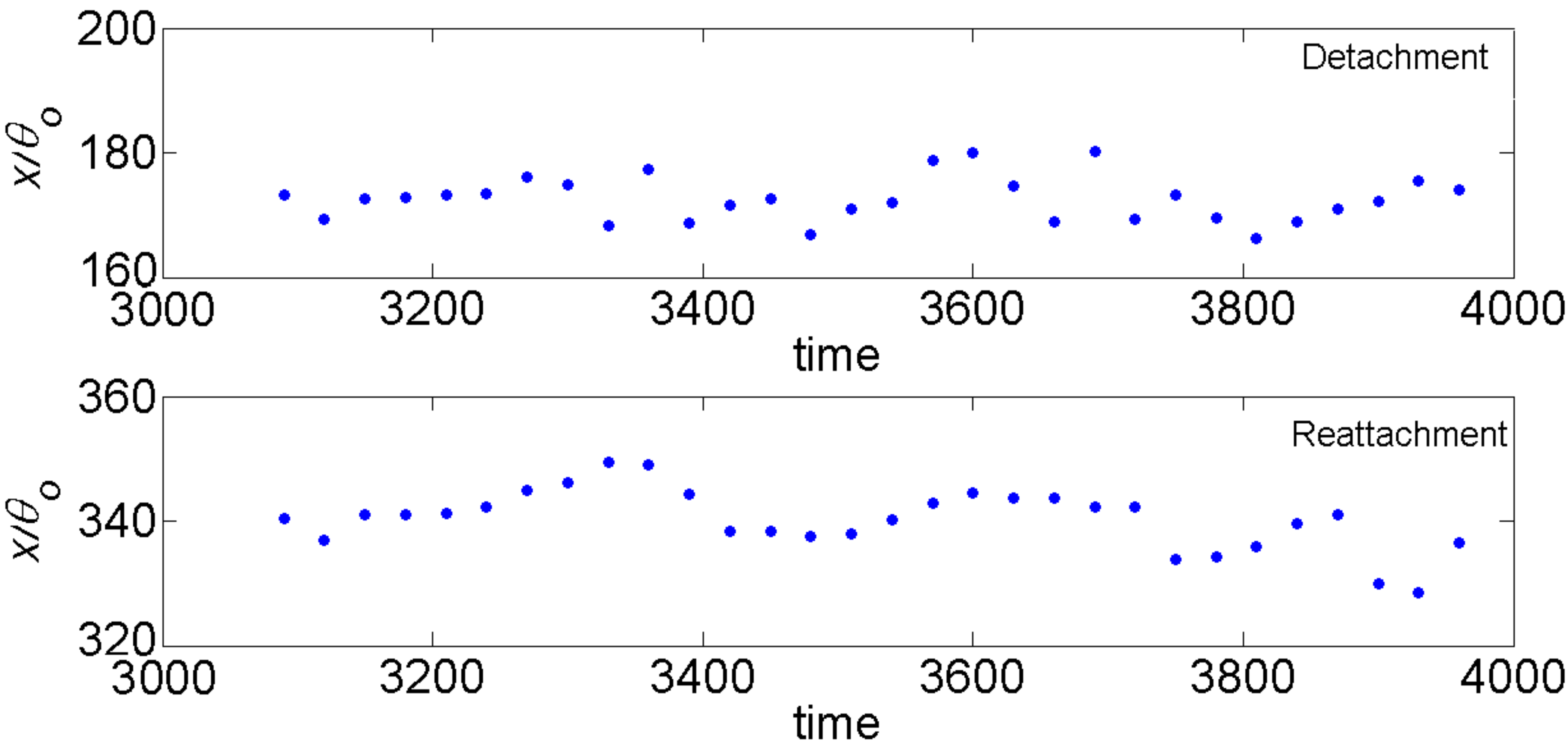
  - *Between 750 and 1500 grid volumes per roughness element.*


$U = 0$
LES, Smooth
80
60
40
20
0
50
100
150
200
250
300
350
400
450
500
y/θ_o
x/θ_o
1
0.5
0

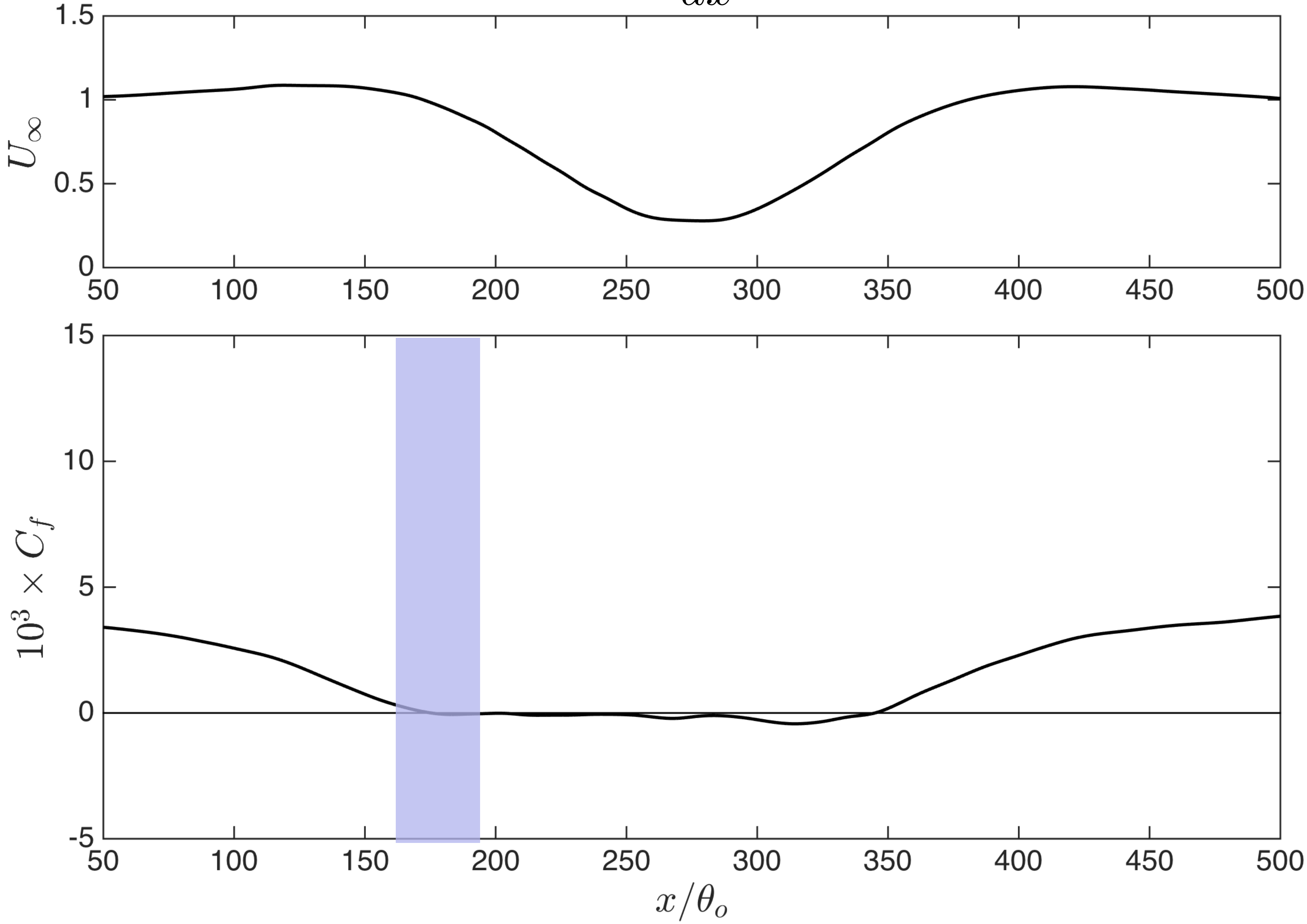
- Standard definition: $\tau_w = 0, \ \dfrac{d\tau_w}{dx} < 0$



- The separation point is unsteady

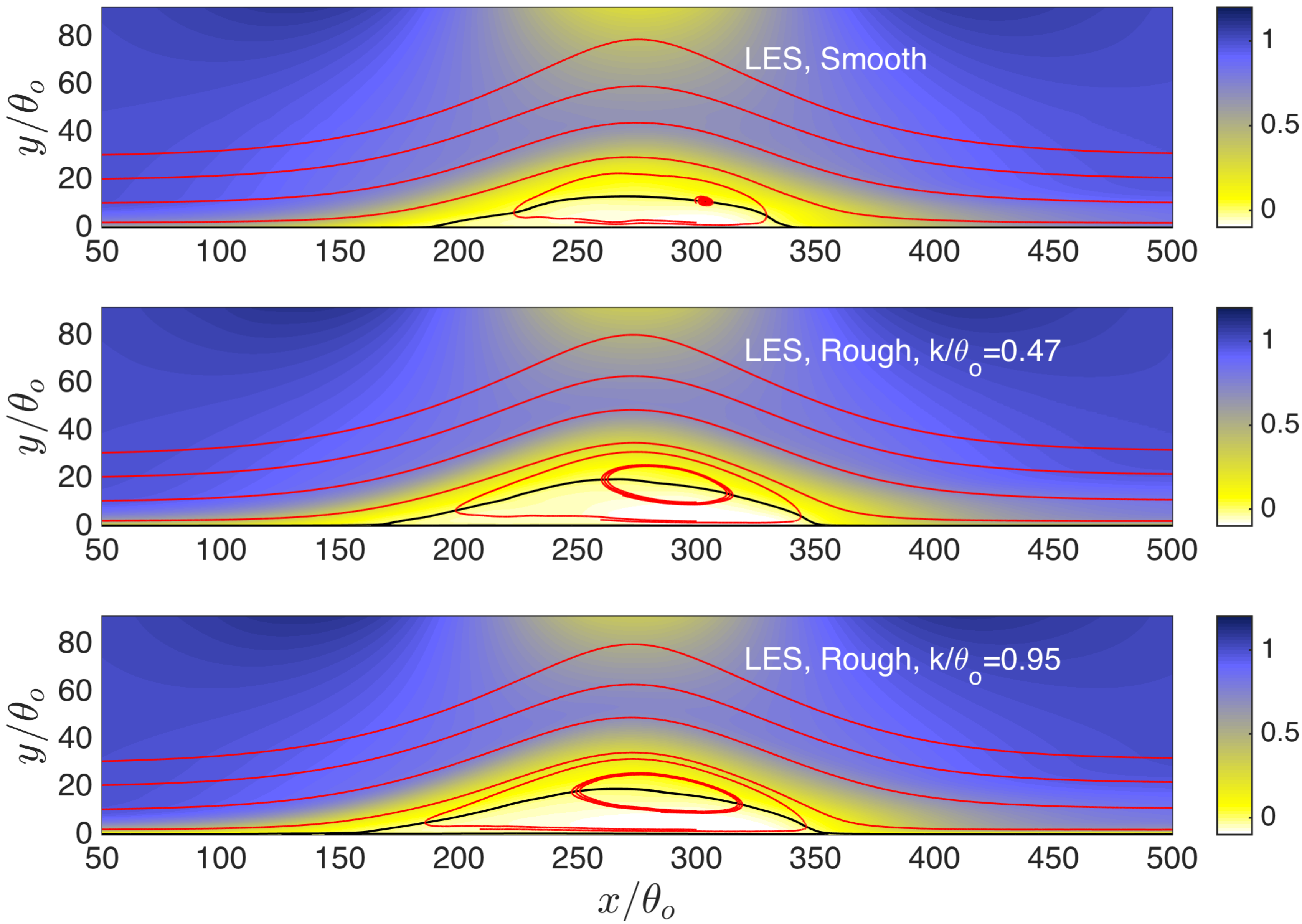


- Standard definition: $\tau_w = 0,\ \dfrac{d\tau_w}{dx} < 0$



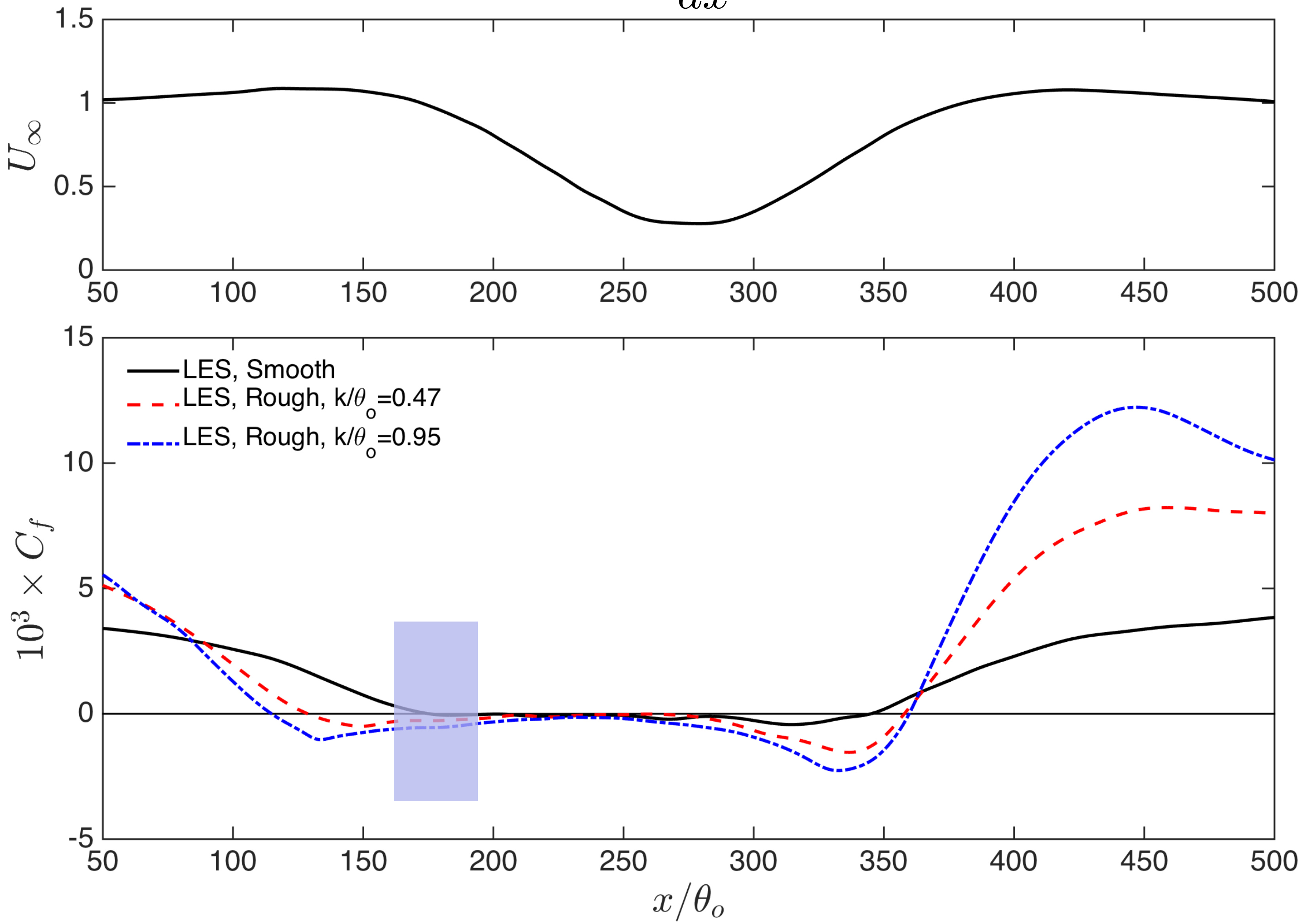
Results

Streamlines



- Standard definition: $\tau_w = 0, \; \dfrac{d\tau_w}{dx} < 0$
  - τ_w *includes viscous and form drag (from the wake of the roughness elements)*
  - *In the fully rough regime, form drag dominates*

- Standard definition: $\tau_w = 0, \ \dfrac{d\tau_w}{dx} < 0$

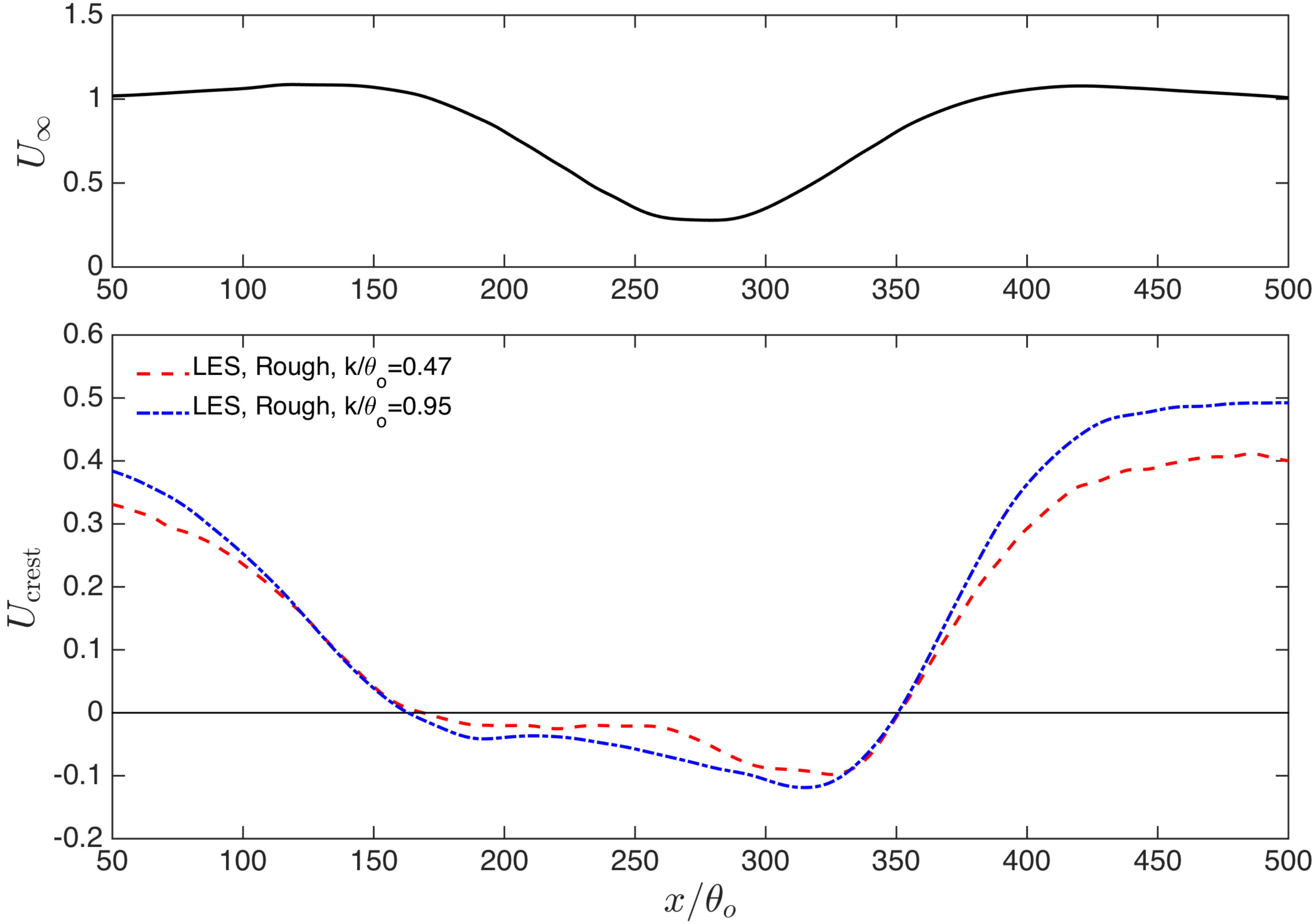


Results
Separation

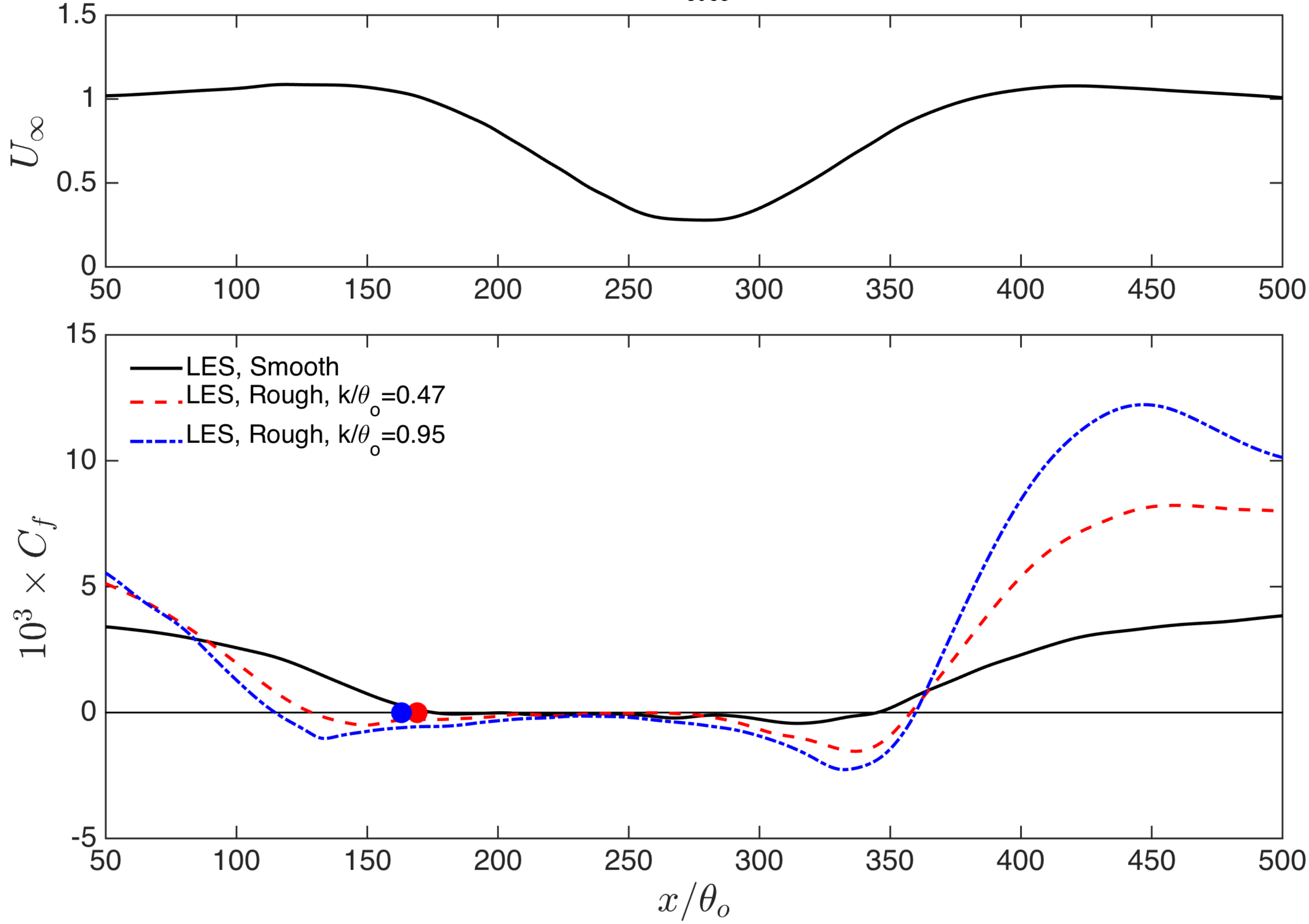
- Skin-friction coefficient crosses zero earlier for the rough cases

- Does this represent separation?
- How is separation determined in experiments?

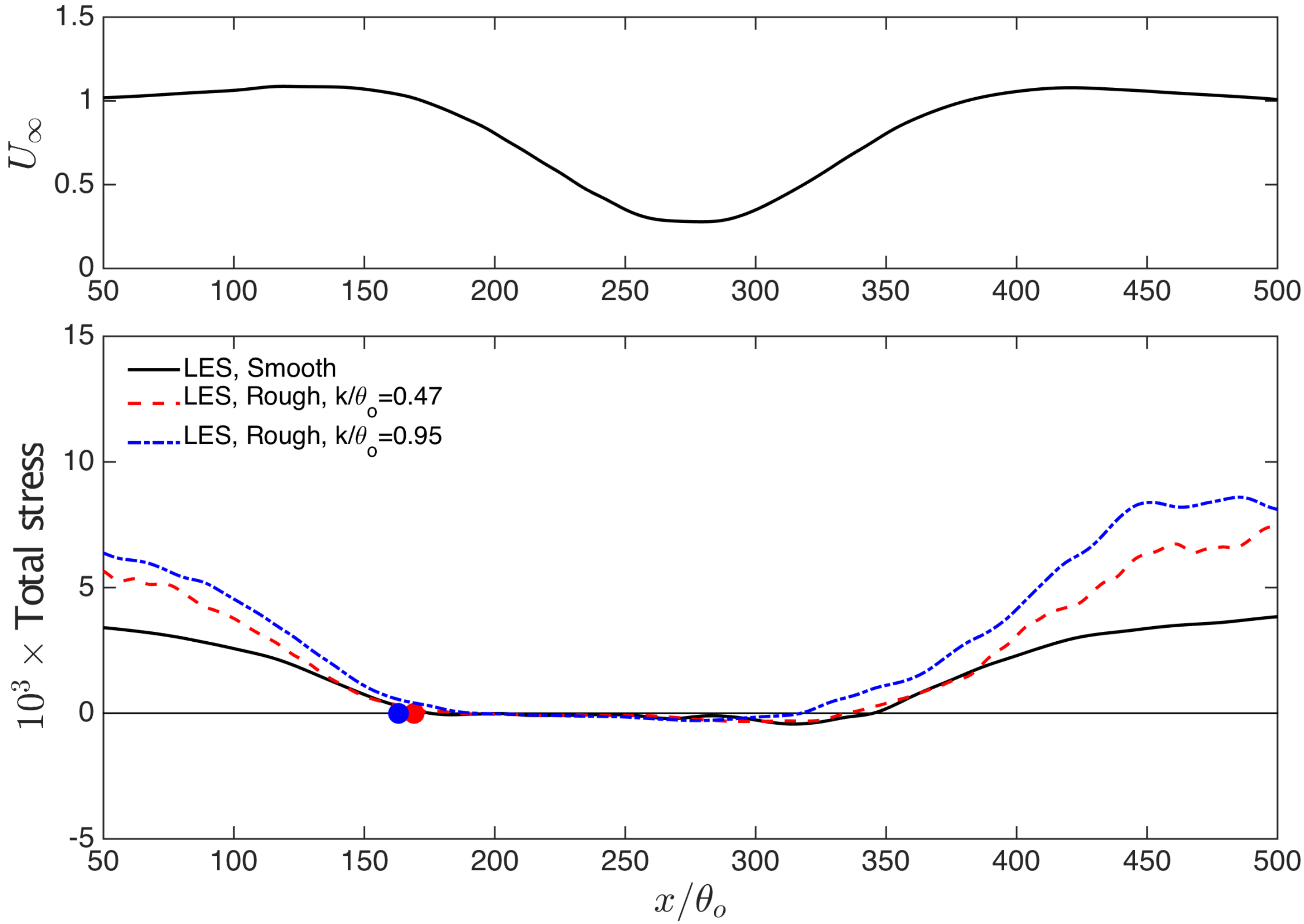
… From the velocity above the crest



- Standard definition: $\tau_w = 0, \ \dfrac{d\tau_w}{dx} < 0$



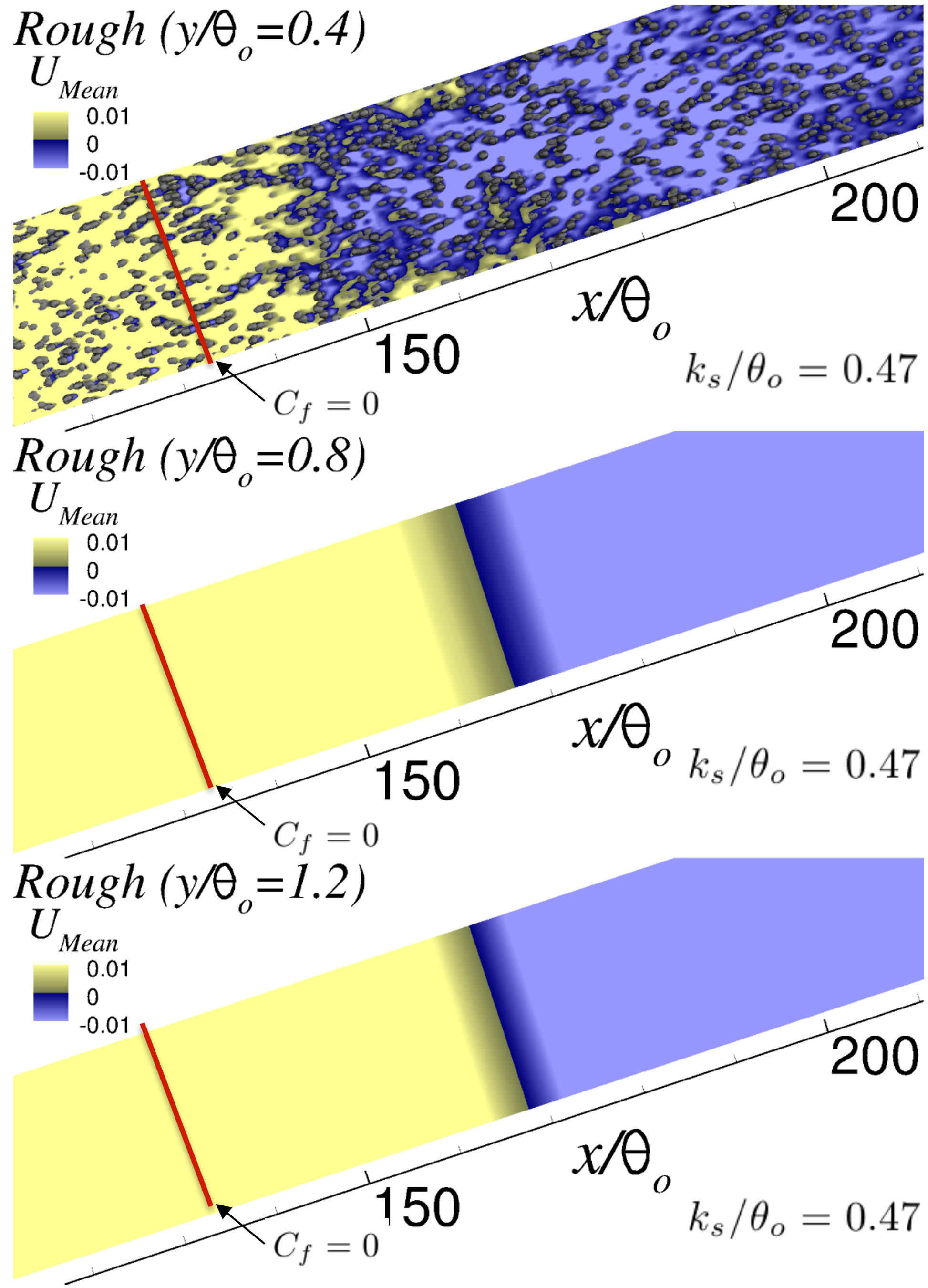
- Total stress above the crest: $\tau = -\langle u'v' \rangle + \frac{1}{Re}\left(\frac{dU}{dy} + \frac{dV}{dx}\right)$



Reversed flow in the Roughness layer

- Time-averaged velocity
  - *Flow reversal inside the roughness layer **even in the region where** $C_f>0$*
  - *The point where the mean velocity changes sign does not coincide with the point where C_f changes sign*

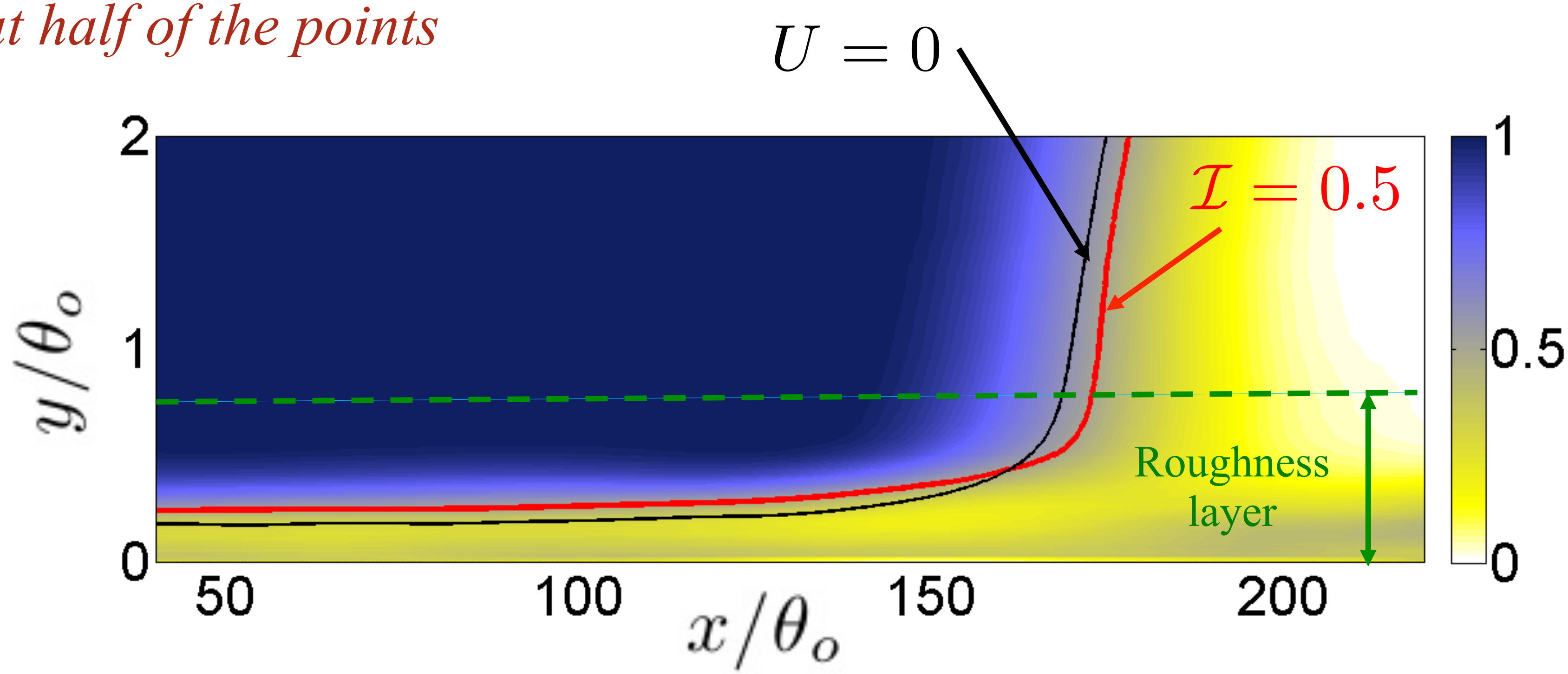
⇒ **Flow reversal without separation**



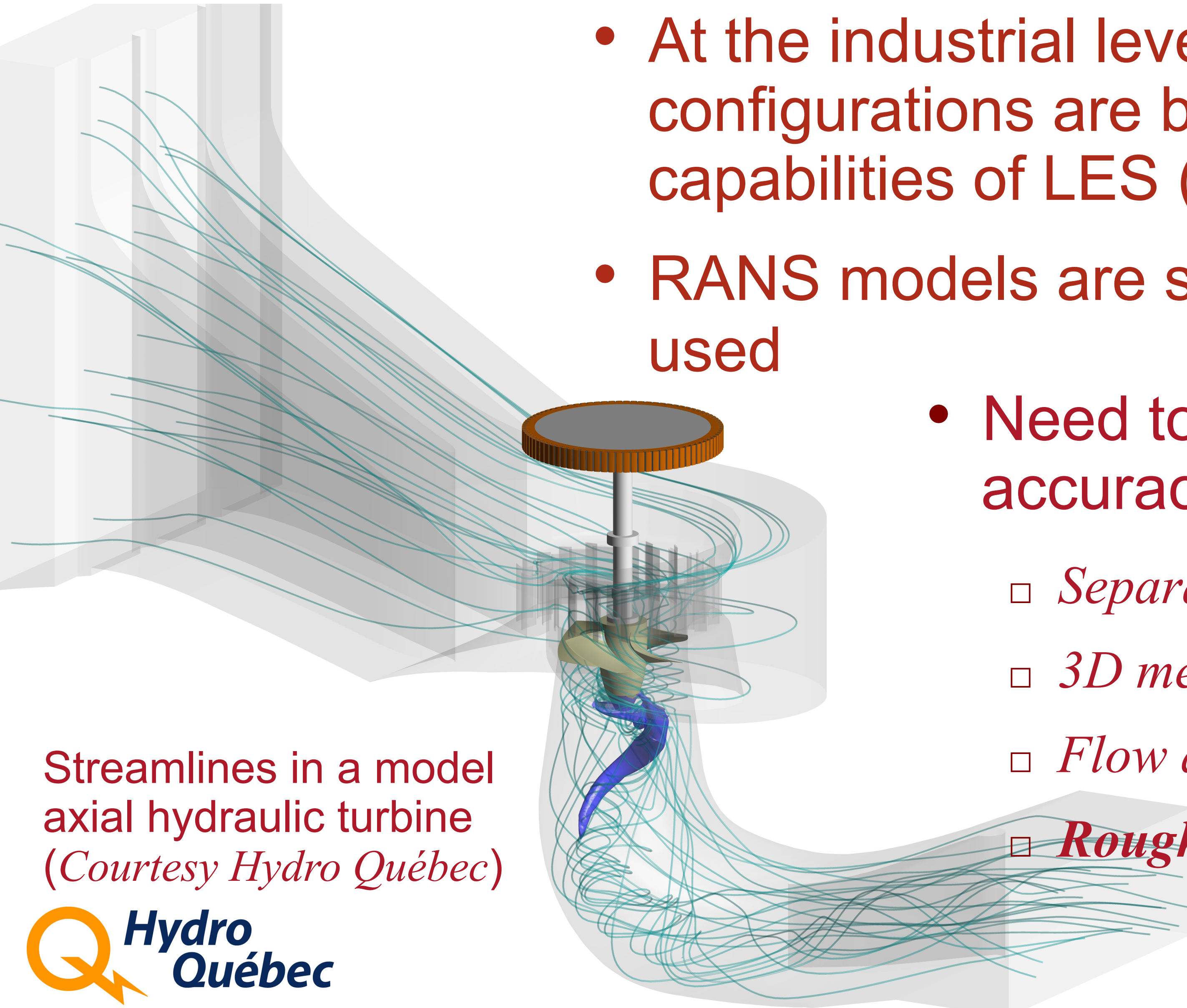
Results

Reversed flow in the Roughness layer

- Define intermittency function:
  - $\mathcal{I} = 1$ *when the time-averaged velocity is in the positive-x direction at all points*
  - $\mathcal{I} = 0$ *when the time-averaged velocity is in the negative-x direction at all points*
  - $\mathcal{I} = 0.5$ *when the time-averaged velocity is in the positive-x direction at half of the points*



- At the industrial level, complete configurations are beyond the capabilities of LES (cost)
- RANS models are still extensively used
  - Need to evaluate their accuracy in presence of
    - *Separation*
    - *3D mean flow*
    - *Flow acceleration*
    - ***Roughness***



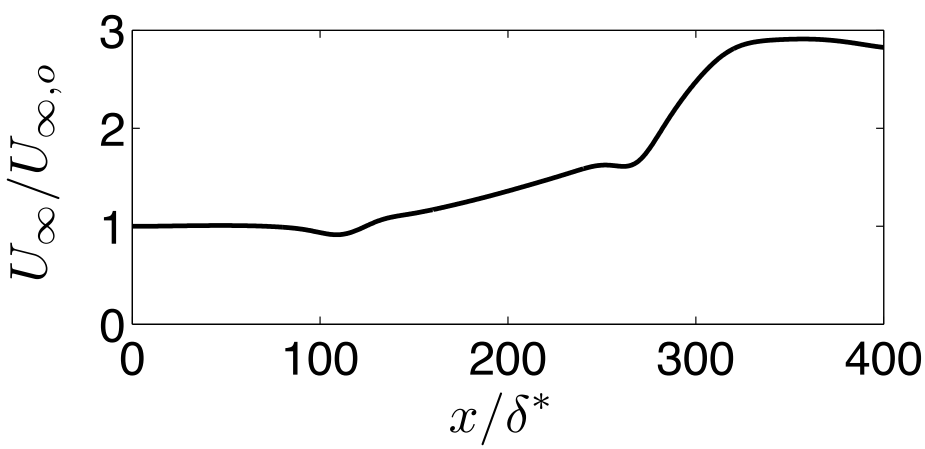
Streamlines in a model axial hydraulic turbine (*Courtesy Hydro Québec*)

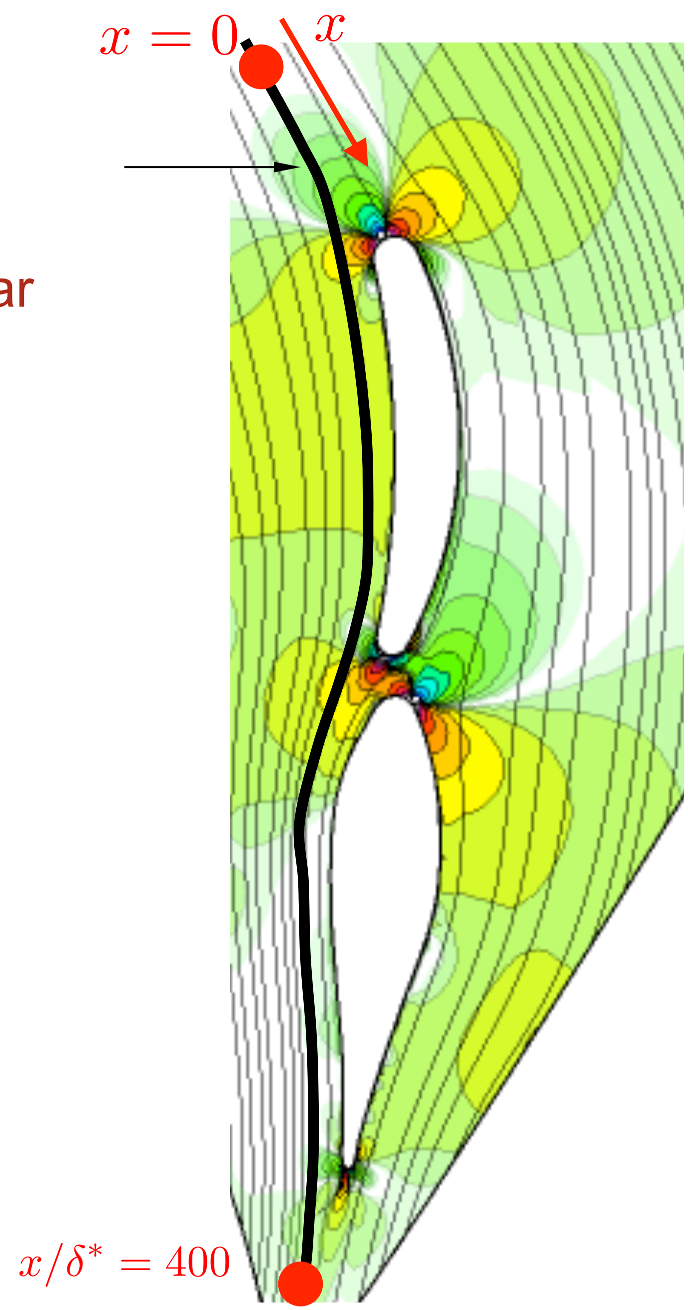
Hydro Québec

Applications:
RANS Models for Rough Wall BLs

- RANS models for rough-wall boundary layers correct the eddy viscosity ν_T based on the equivalent sand-grain roughness
- The equivalent sand-grain roughness must be prescribed.
    - *Several correlations exist to relate the actual geometric scale of the roughness to k_s (Yuan and Piomelli, JoT 2014).*
- Roughness modifications act by
    - *Shifting the velocity (eddy viscosity, TKE,…) profiles by an offset to account for the increased momentum near the wall.*
    - *Modifying the boundary conditions for the transported quantities (ν_T, $\mathcal{K}$, ε, ω…)*

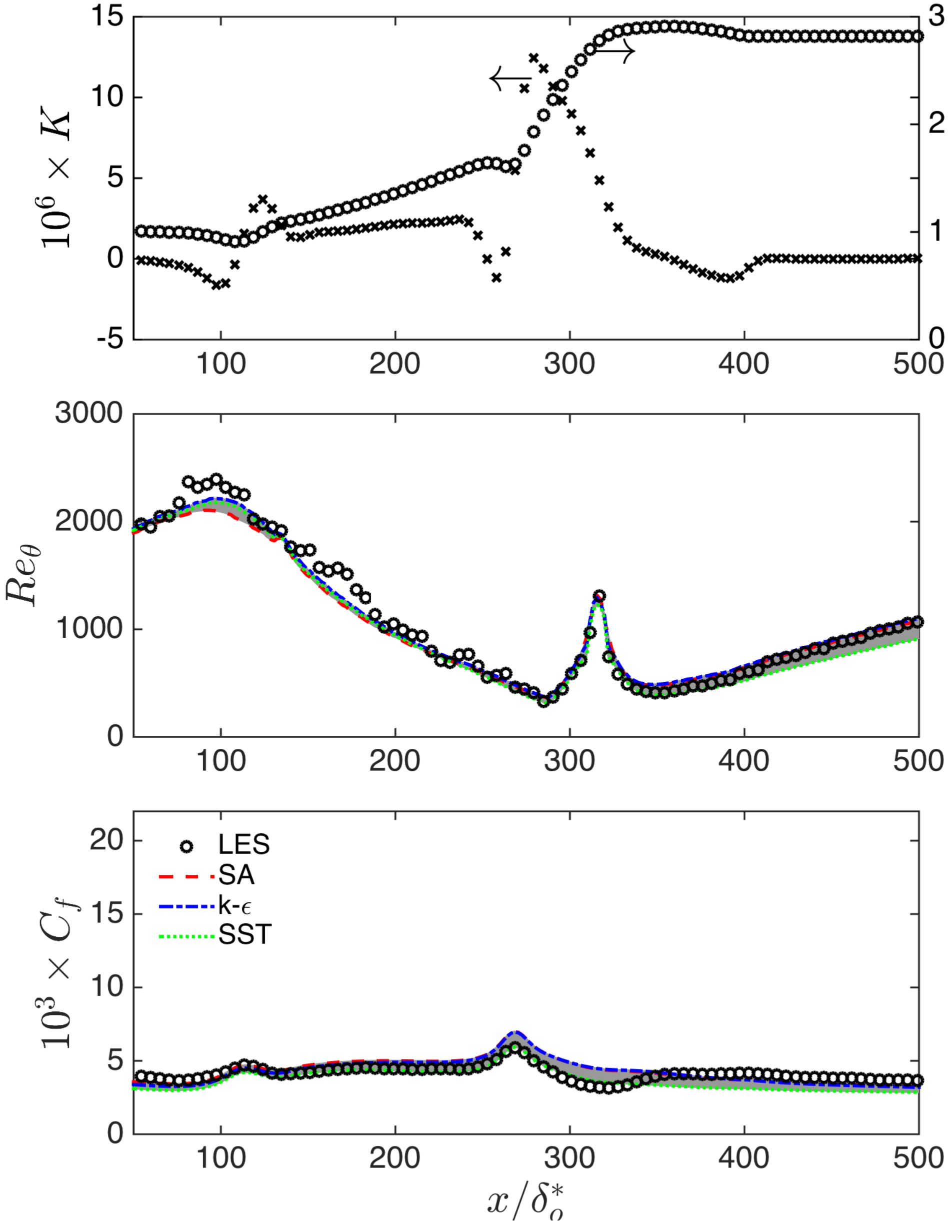
- Flow in a hydraulic turbine
  - *Streamline selection:*
    - Outside the boundary layer, but not too far away: *2-3*δ away from wall
    - Extensive non-ZPG region



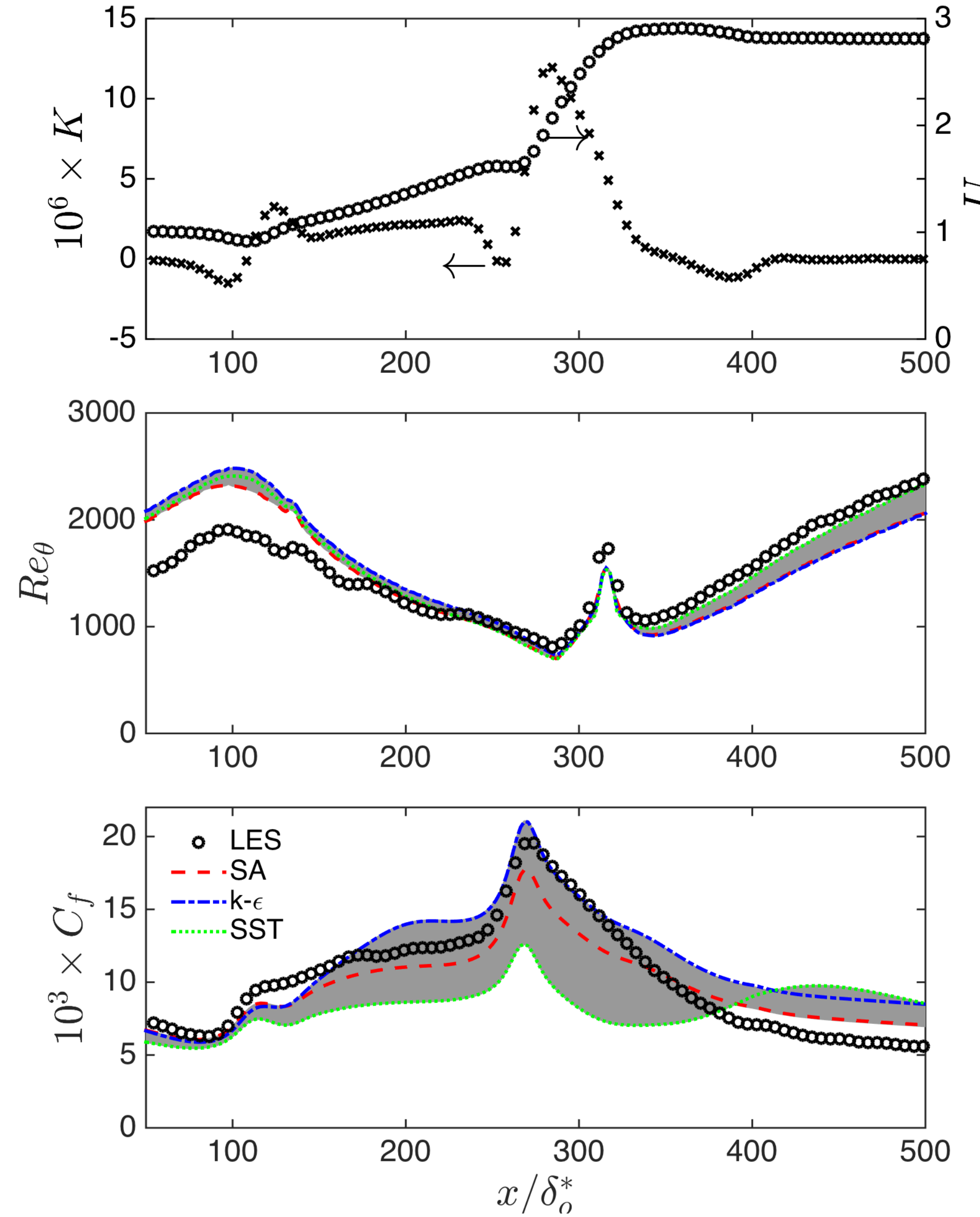


APPLICATIONS:
RANS MODELS FOR ROUGH WALL BLS

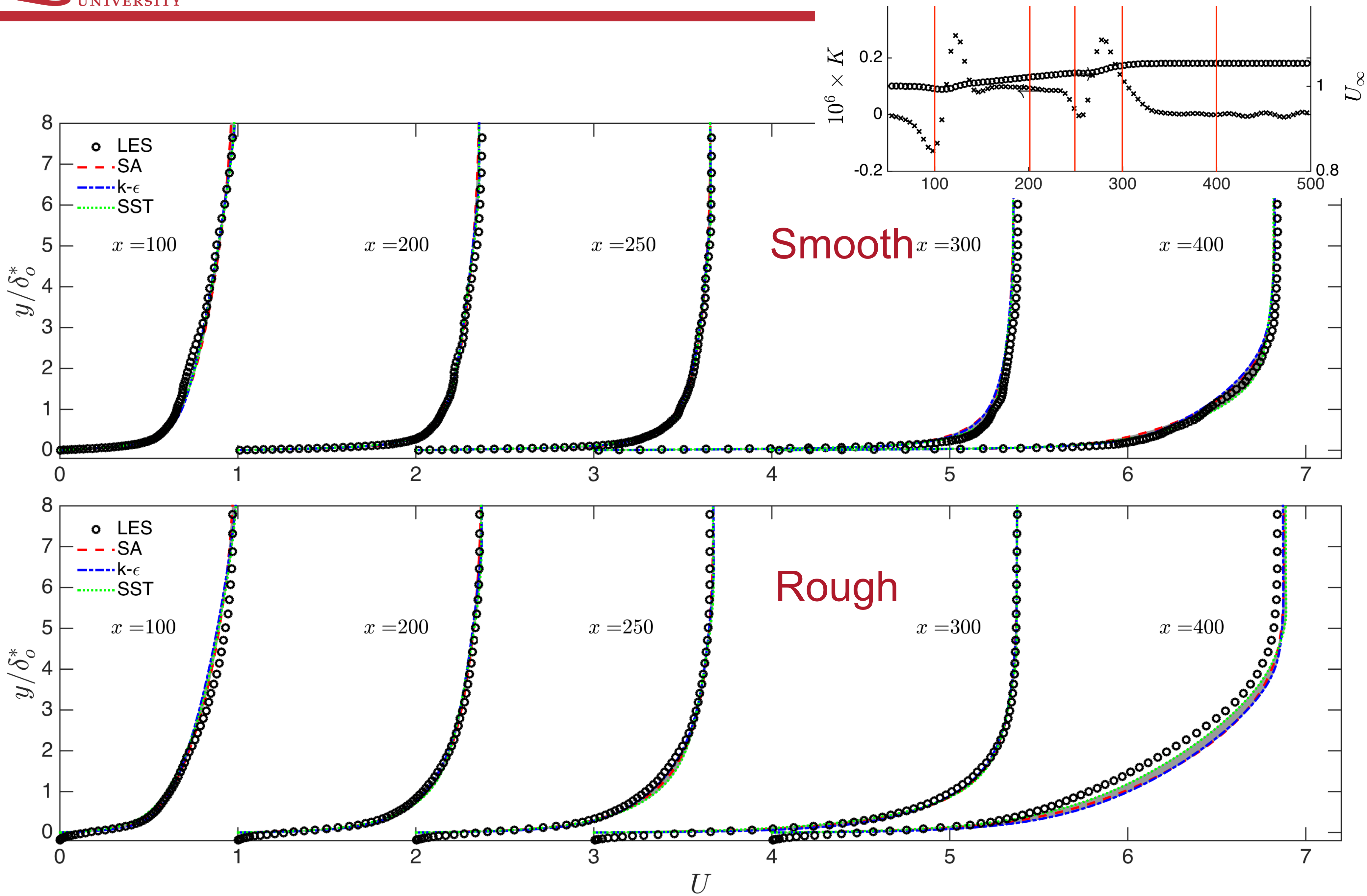
Smooth



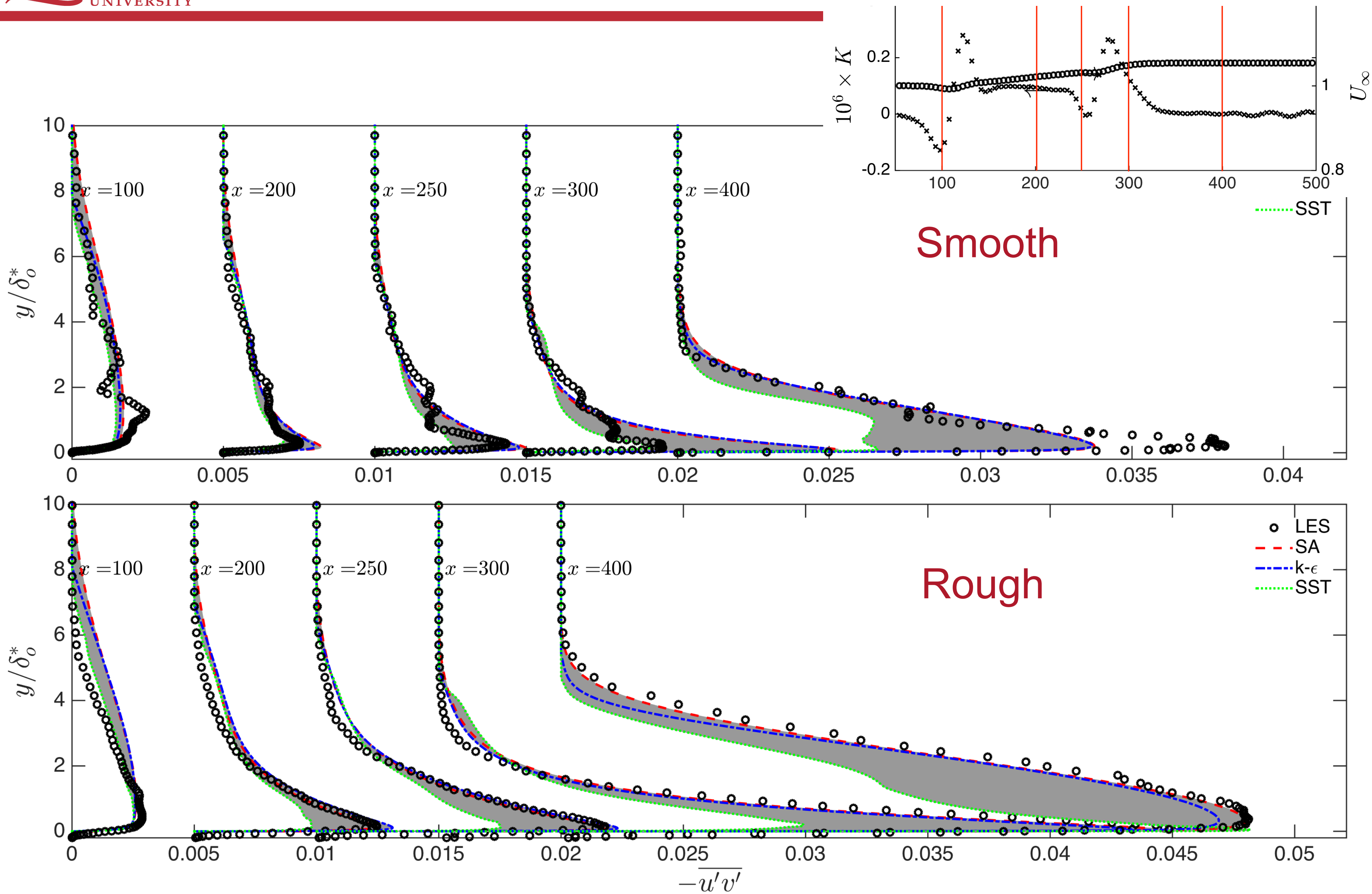
Rough

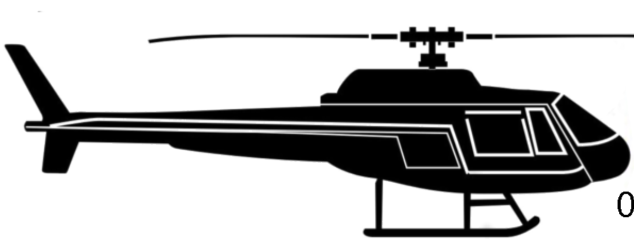


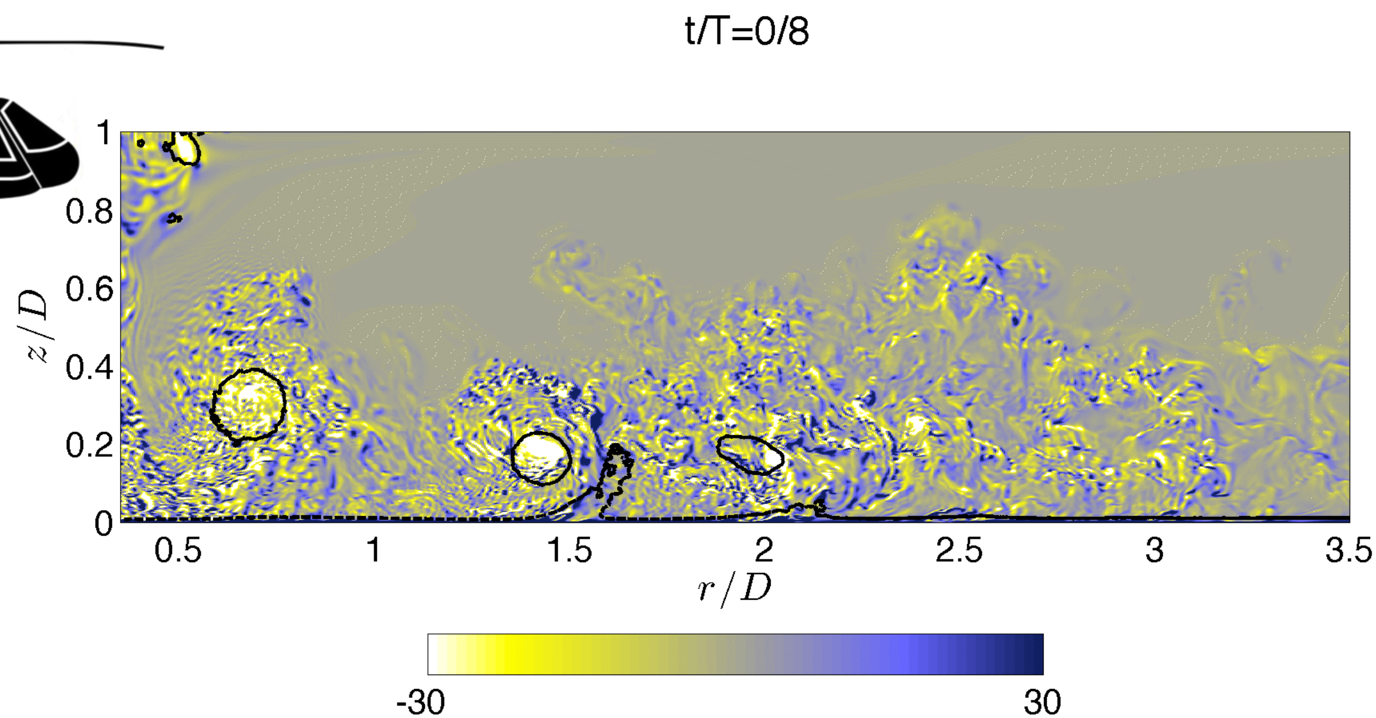
Applications:
RANS Models for Rough Wall BLs



Applications:
RANS Models for Rough Wall BLs





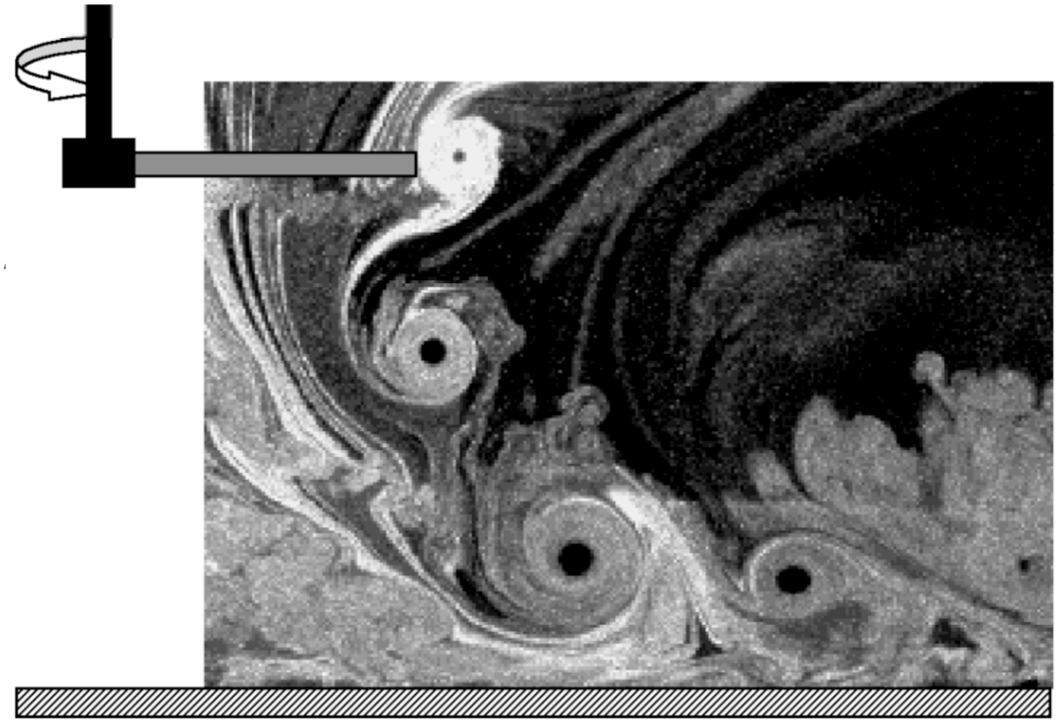


t/T=0/8
1
0.8
0.6
0.4
0.2
0
z/D
0.5
1
1.5
2
2.5
3
3.5
r/D
-30
30

Motivation

- Impinging jets occur in
  - *Heat transfer applications*
  - *Meteorology (downdrafts)*
  - *Helicopter aerodynamics*

- The interaction of the vortices with the ground
  - *Changes the turbulent flow field*
  - *Results in the development of secondary vortices, which interact with the primary ones*
  - *Changes the vortex development*
  - *May result in particle lifting and suspension.*



From T. Lee, J. G. Leishman, and M. Ramasamy, (2008)

- It is important to develop models that relate the impinging jet (i.e., rotor wake) and vortex characteristics to the particle dynamics.
  - *Existing models are usually inviscid (vortex line)*

Objectives

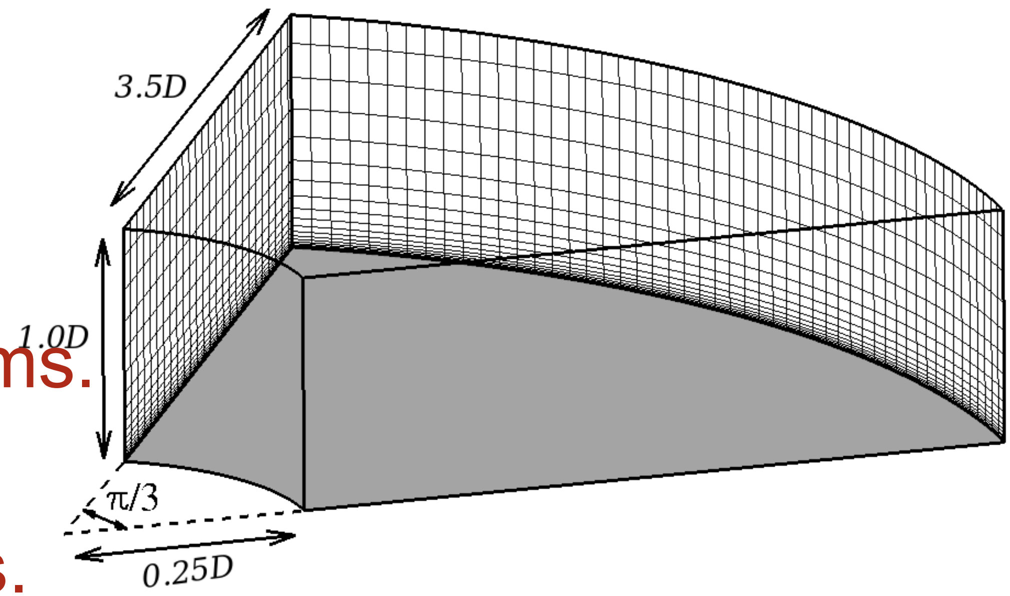
- Study the interaction between the vortices and the near-wall turbulence.
  - *Moderate Reynolds number*
- Quantify the vortex decay in a turbulent wall-bounded flow.
  - *Moderate Reynolds number*
  - *High Reynolds number*
- Understand the physical mechanisms responsible for vortex decay.
- Develop lower level models that account for both viscous and turbulent effects.

Methodology

- Strategy:
  - *Develop a vortex-generation method that is*
    - Non-intrusive, Controllable
  - *Simulate increasingly realistic configurations*
    - 2D impingement
    - Axisymmetric impingement
    - Axisymmetric wall jet
  - *Perform hierarchical model validation:*
    - LES to validate Hybrid RANS
  - *Extend to high Re*
  - *Study decay laws and develop lower-level models*

Methodology

- Numerical solution of the filtered Navier-Stokes equations.
- Staggered grid.



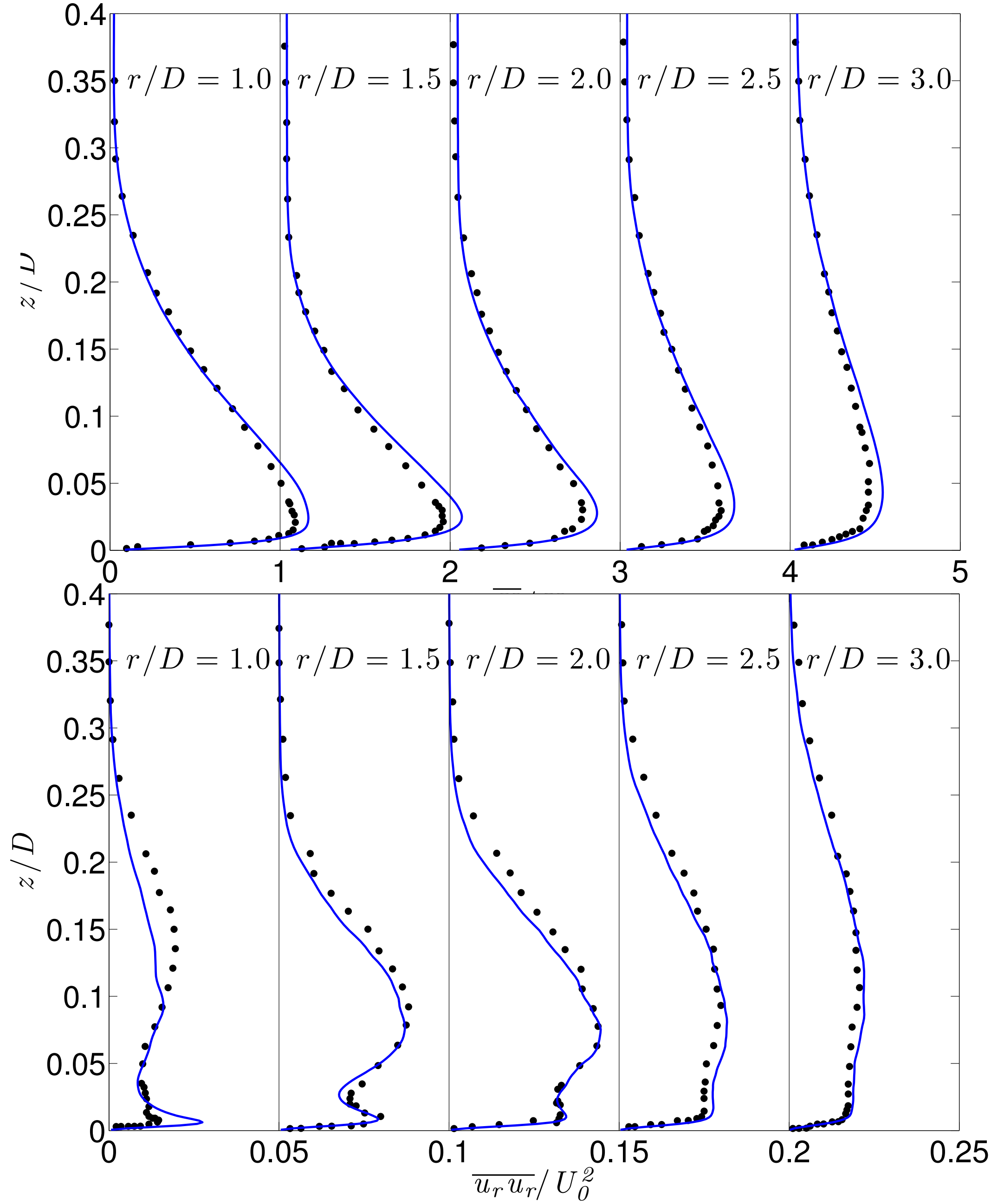
- Second-order accurate in space and time.
- Central differences on all terms.
- Axi-symmetric configuration that does not include the axis.
- Inlet condition:

$$\langle U_{jet} \rangle = U_o + A \sin(2\pi t/T)$$

- Synthetic turbulence added at the jet exit and the inner radial boundary.
- $Re = U_{jet} D_{jet} / \nu = 66,000$

VALIDATION

- Impinging jet experiment by Cooper, Jackson, Launder & Liao (1993)
- Verified grid requirements
- Verified domain size



Phase Averaging

$$\langle f(\mathbf{x},\phi)\rangle = \frac{1}{N}\sum_{n=1}^{N} f(\mathbf{x}, t_n + T)$$

$$f = \overline{F} + \widetilde{f} + f' = \langle f \rangle + f'$$

Time average

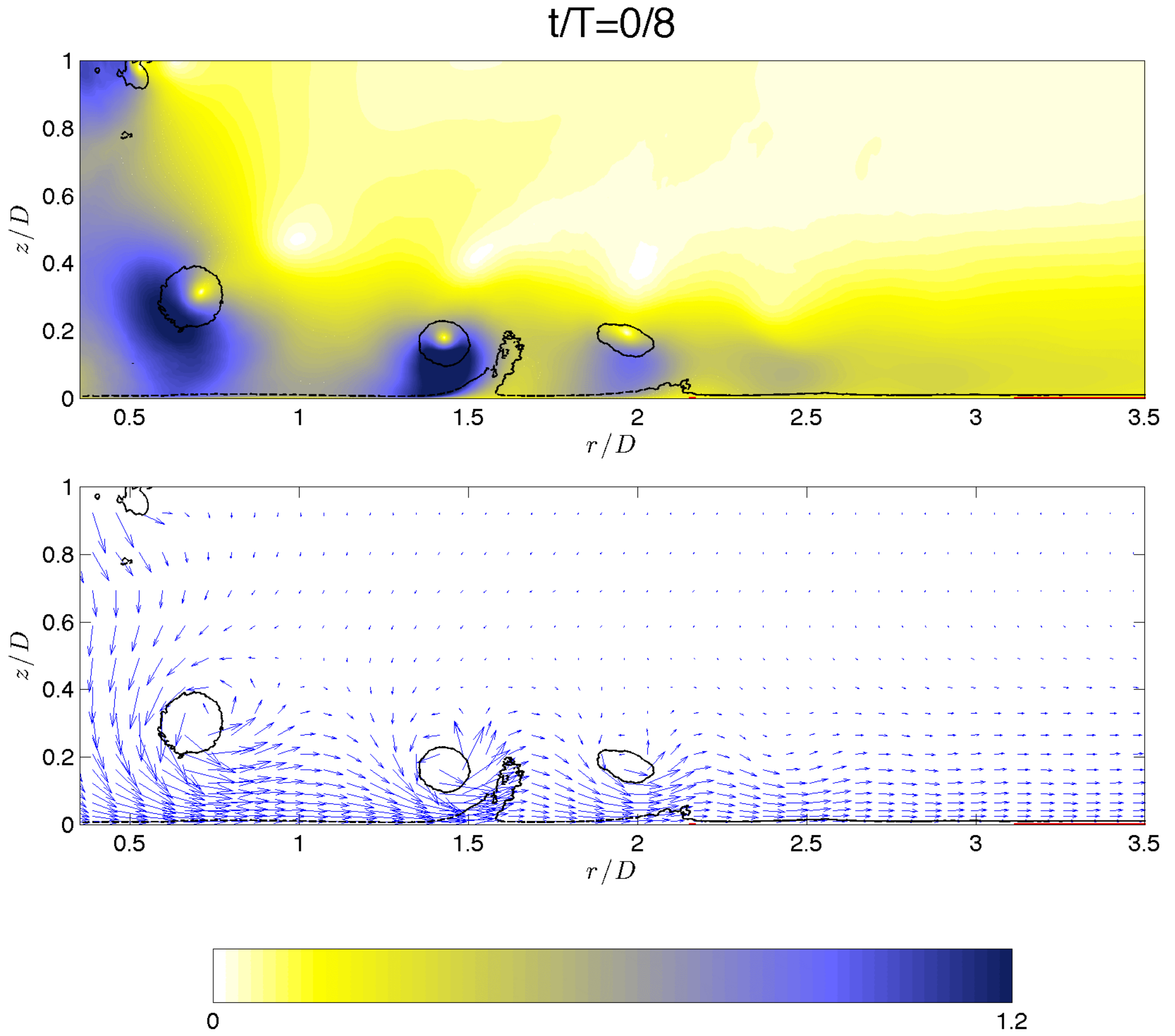
Periodic

Stochastic

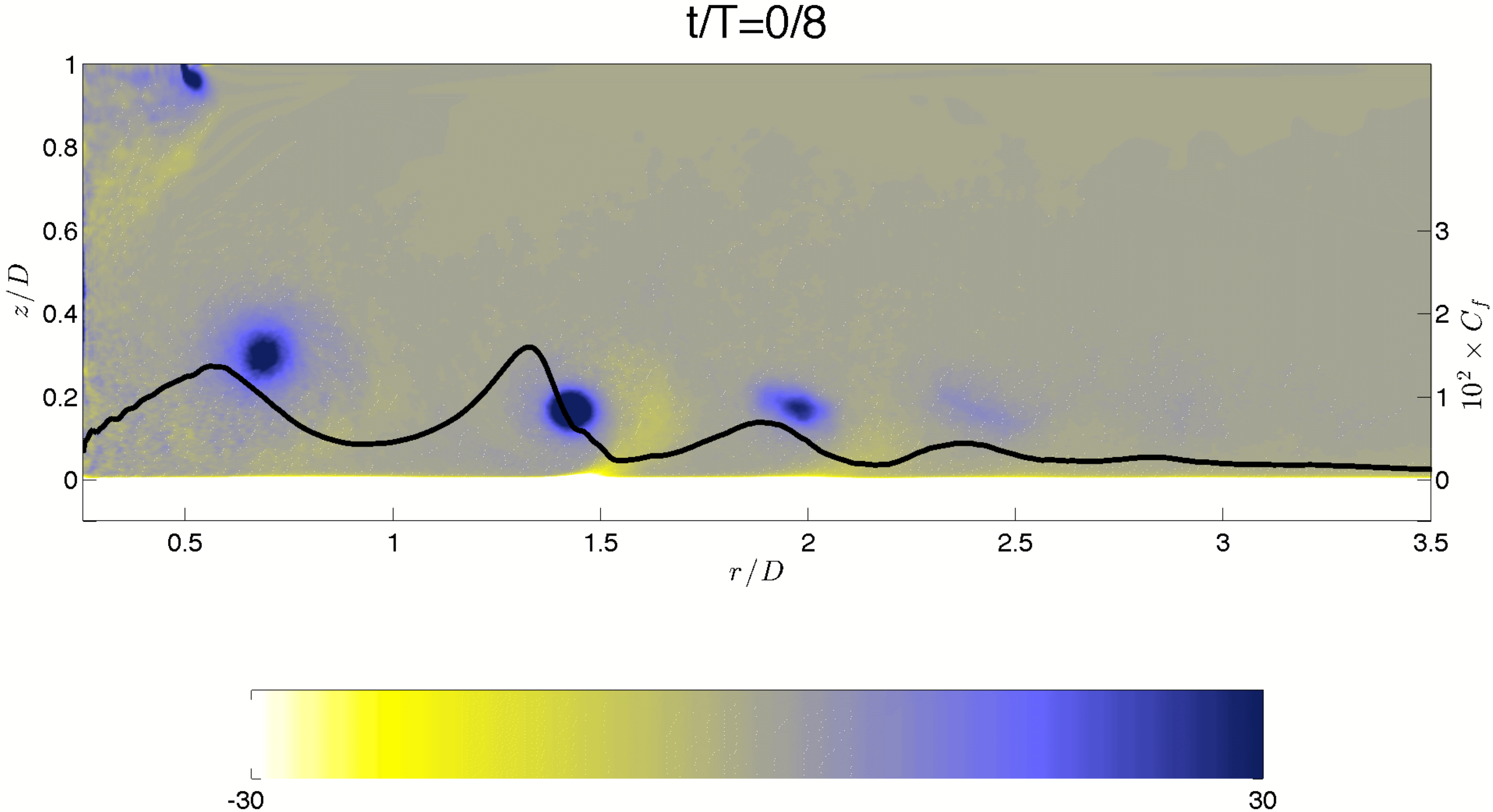
Phase average

$$\langle u_i u_j \rangle - U_i U_j = \langle u_i \rangle \langle u_j \rangle + \langle u_i' u_j' \rangle$$

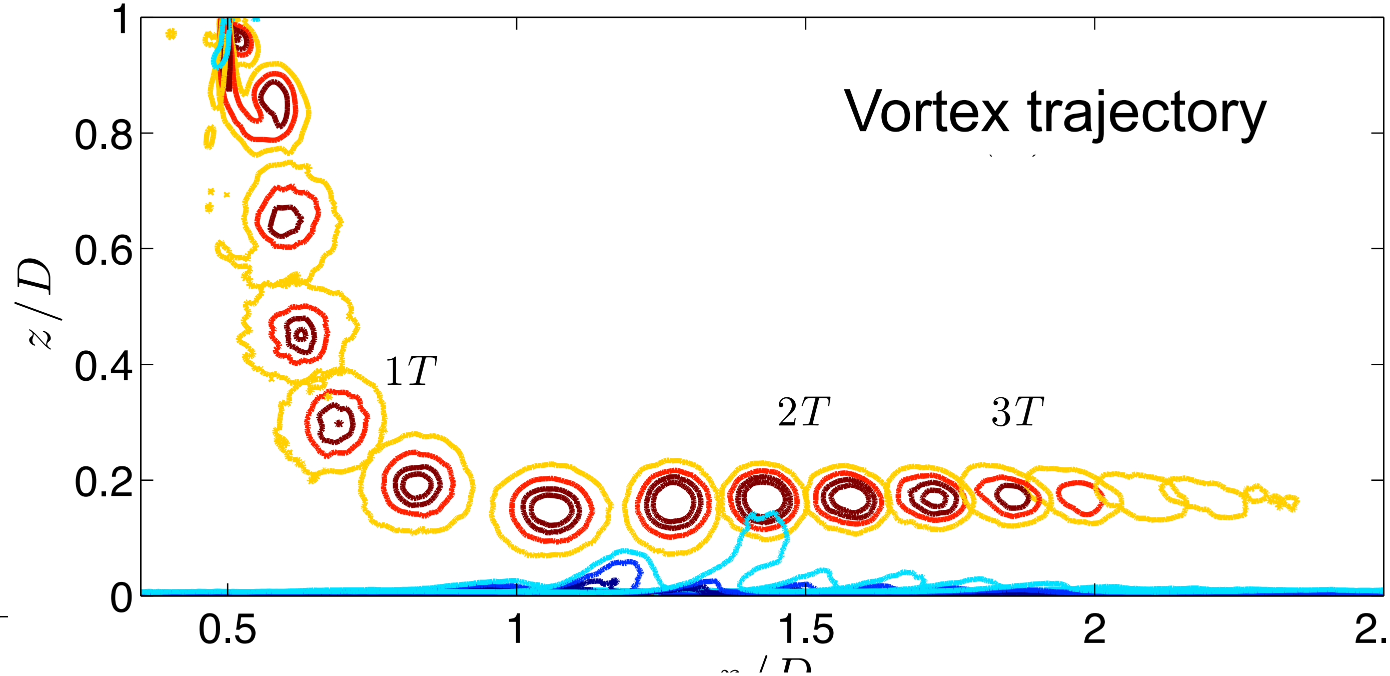
Phase-averaged Velocity

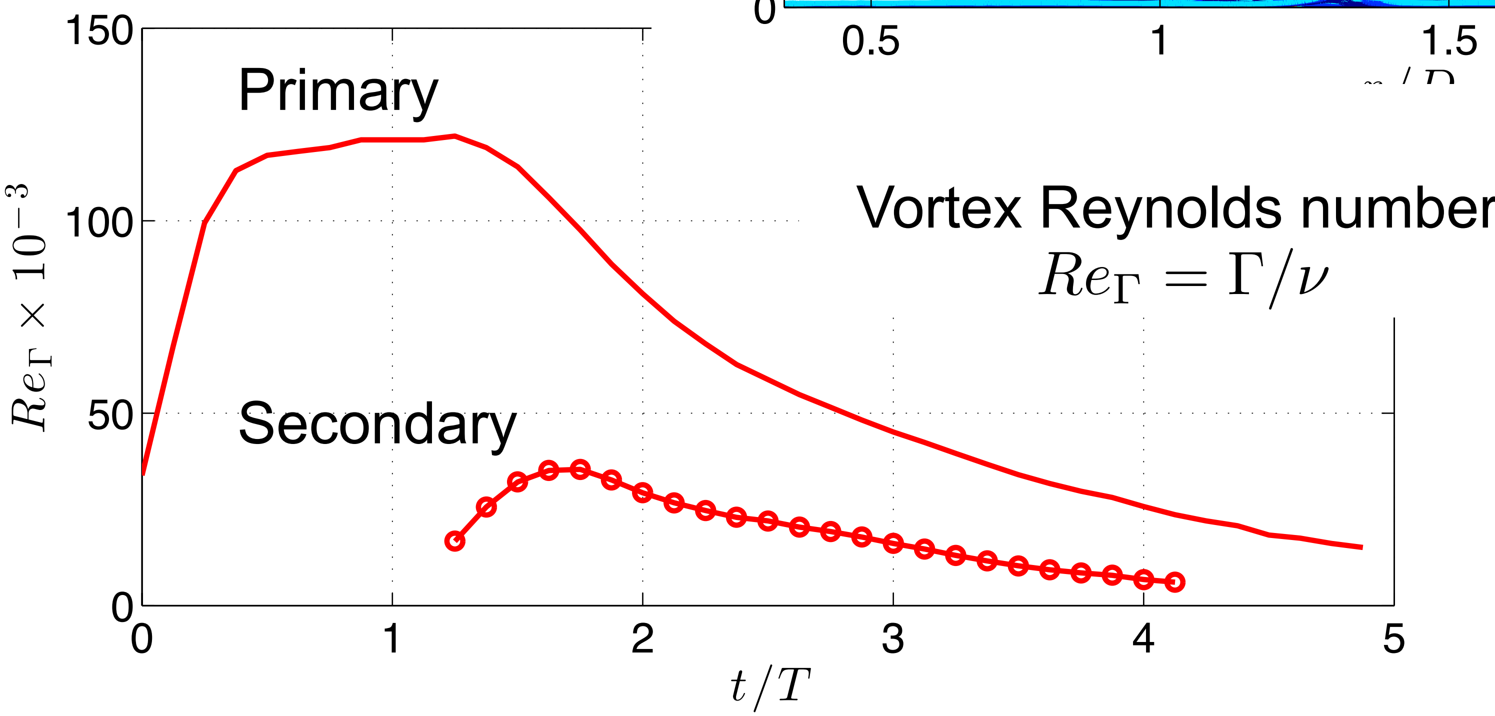


Phase-averaged Vorticity and C_f

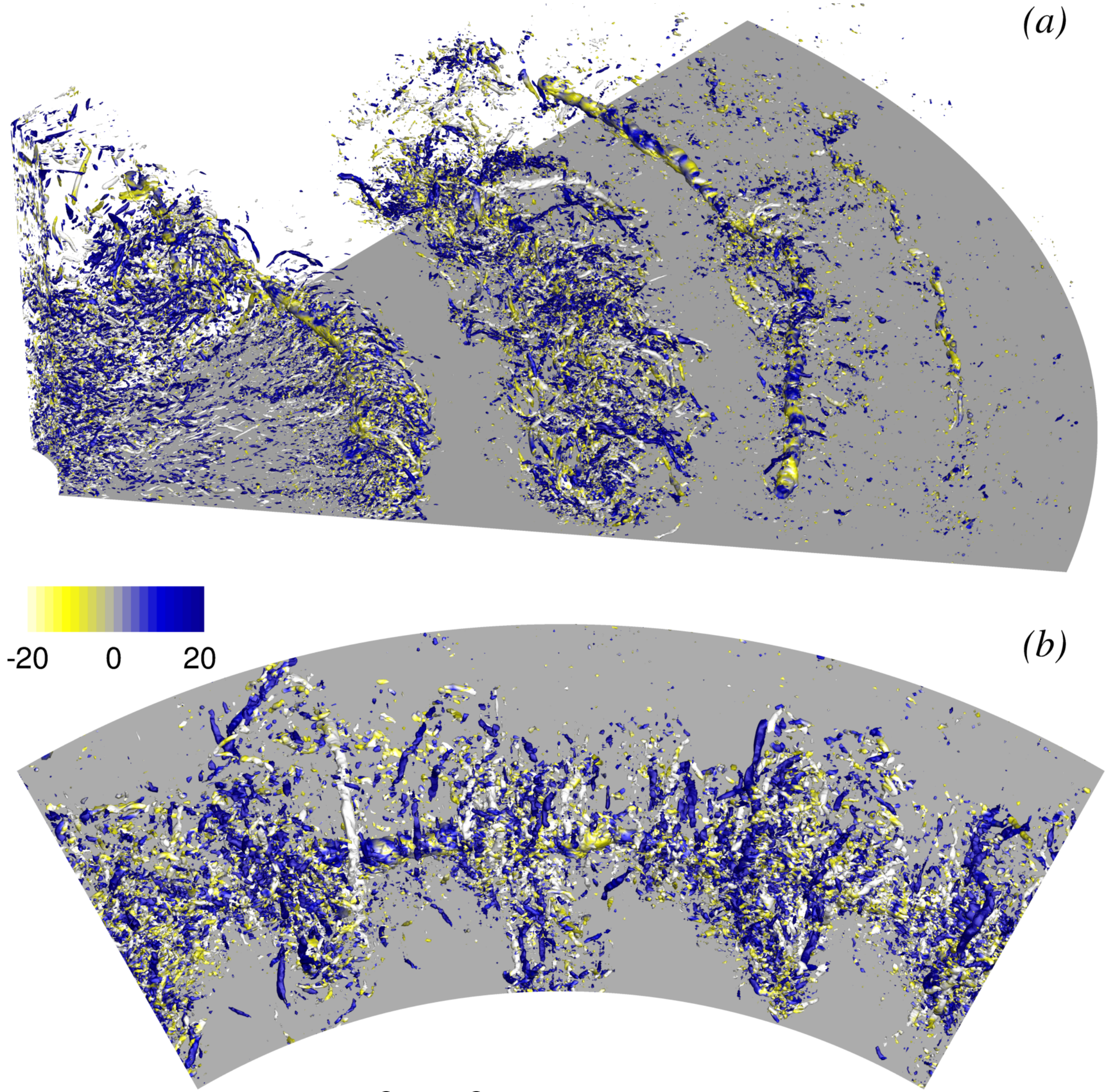


Phase-averaged vorticity and C_f



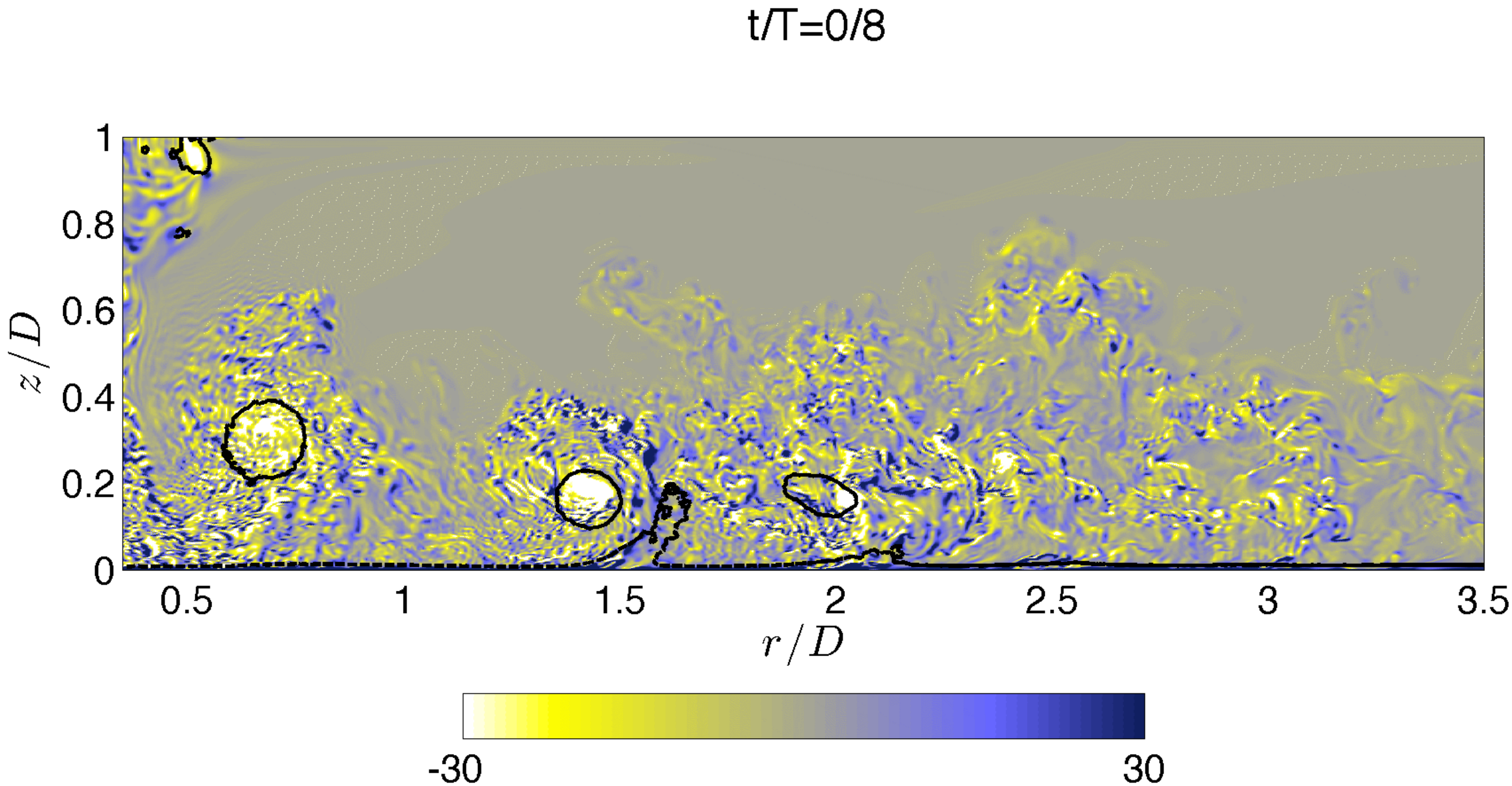


Instantaneous Field



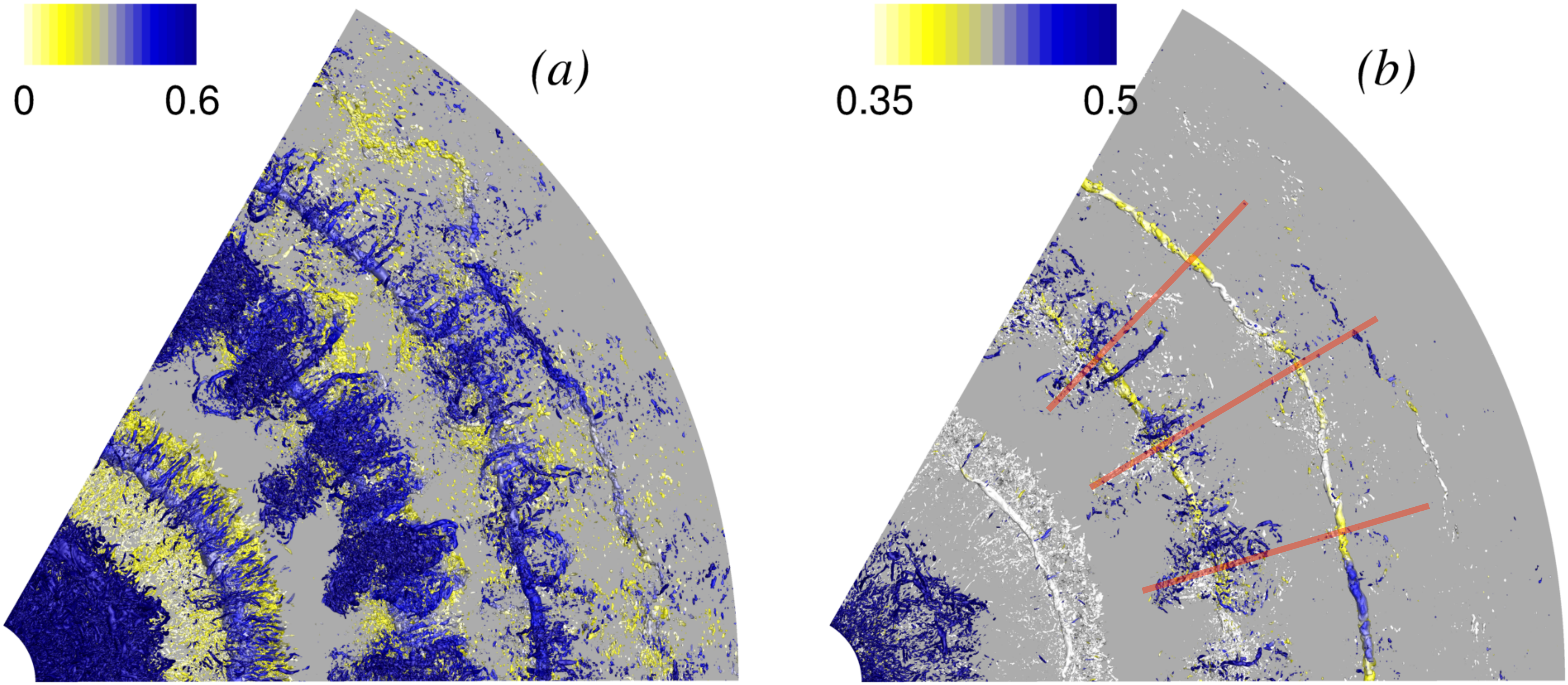
Contours of $Q = -\frac{1}{2}\frac{\partial \overline{u}_j}{\partial x_i}\frac{\partial \overline{u}_i}{\partial x_j} = \frac{1}{2}\left(\overline{\Omega}_{ij}\overline{\Omega}_{ij} - \overline{S}_{ij}\overline{S}_{ij}\right)$ coloured by ω_r

Stochastic Azimuthal vorticity ω'_θ



Instantaneous Field

- Crow instability.
  - *Contributes to development of three-dimensionality.*



Contours of $Q = -\frac{1}{2}\frac{\partial \overline{u}_j}{\partial x_i}\frac{\partial \overline{u}_i}{\partial x_j} = \frac{1}{2}\left(\overline{\Omega}_{ij}\overline{\Omega}_{ij} - \overline{S}_{ij}\overline{S}_{ij}\right)$ coloured by z

Budget of $\langle \omega_\theta \rangle$

$$\frac{D\langle \boldsymbol{\omega} \rangle}{Dt} = \frac{\partial \langle \boldsymbol{\omega} \rangle}{\partial t} + \langle \mathbf{u} \rangle \cdot \nabla \langle \boldsymbol{\omega} \rangle =$$

$$\langle \boldsymbol{\omega} \rangle \cdot \nabla \langle \mathbf{u} \rangle$$

Vortex stretching by phase-averaged flow

$$+\nabla \times (\nabla \cdot \langle \boldsymbol{\tau}_{tot} \rangle)$$

Viscous and SGS diffusion

$$+\langle \boldsymbol{\omega}' \cdot \nabla \mathbf{u}' \rangle - \nabla \cdot \langle \mathbf{u}' \boldsymbol{\omega}' \rangle$$

Vortex stretching by fluctuating field

Turbulent vorticity diffusion

$$-\nabla \times (\nabla \cdot \langle \mathbf{u}' \mathbf{u}' \rangle)$$

Budget of $\langle \omega_\theta \rangle$

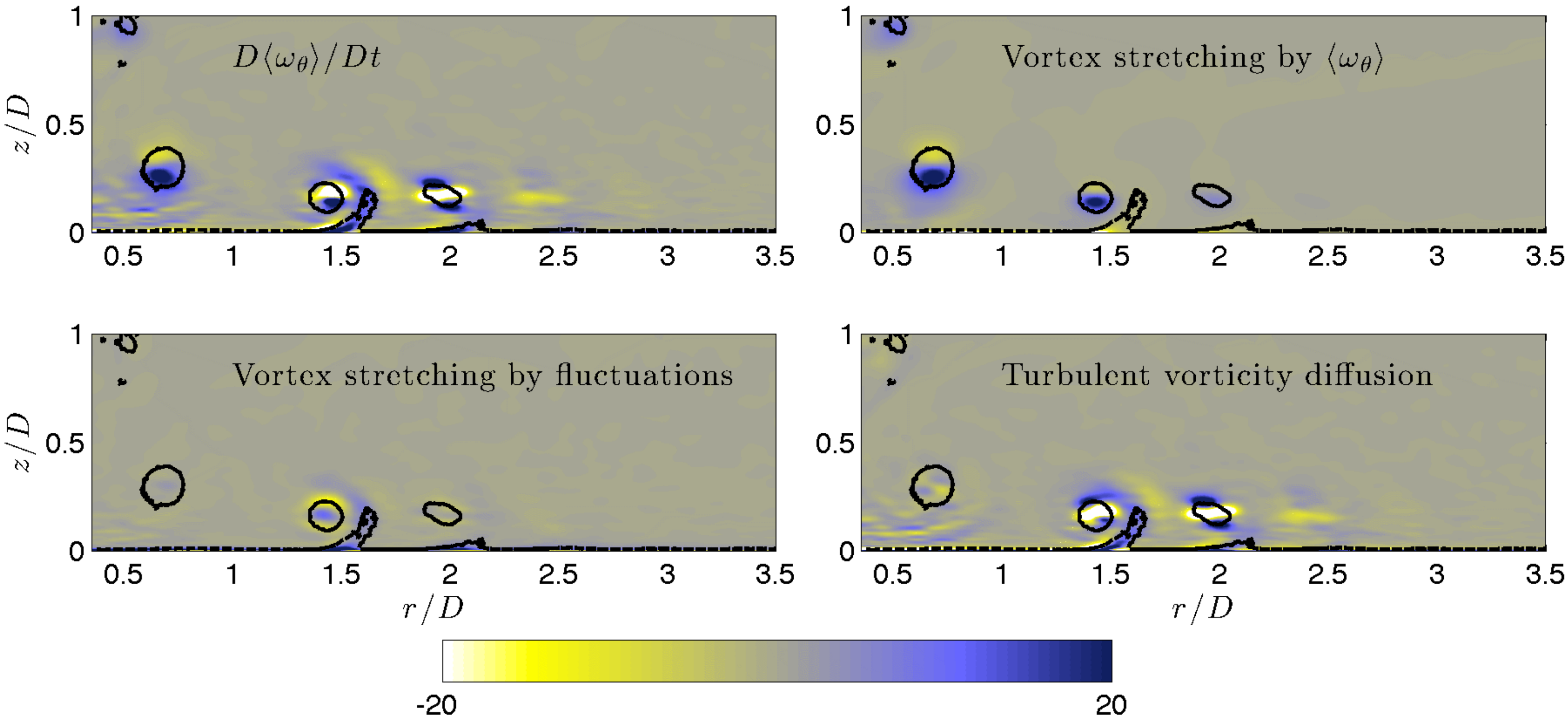
$$\frac{D\langle \boldsymbol{\omega} \rangle}{Dt} =$$ Vortex stretching by phase-averaged flow + ~~Viscous and SGS diffusion~~

+ Vortex stretching by fluctuating field + Turbulent vorticity diffusion

BUDGET OF $\langle \omega_\theta \rangle$

$$\frac{D\langle \boldsymbol{\omega} \rangle}{Dt} = \text{Vortex stretching by phase-averaged flow} + \text{Vortex stretching by fluctuating field} + \text{Turbulent vorticity diffusion}$$

t/T=0/8

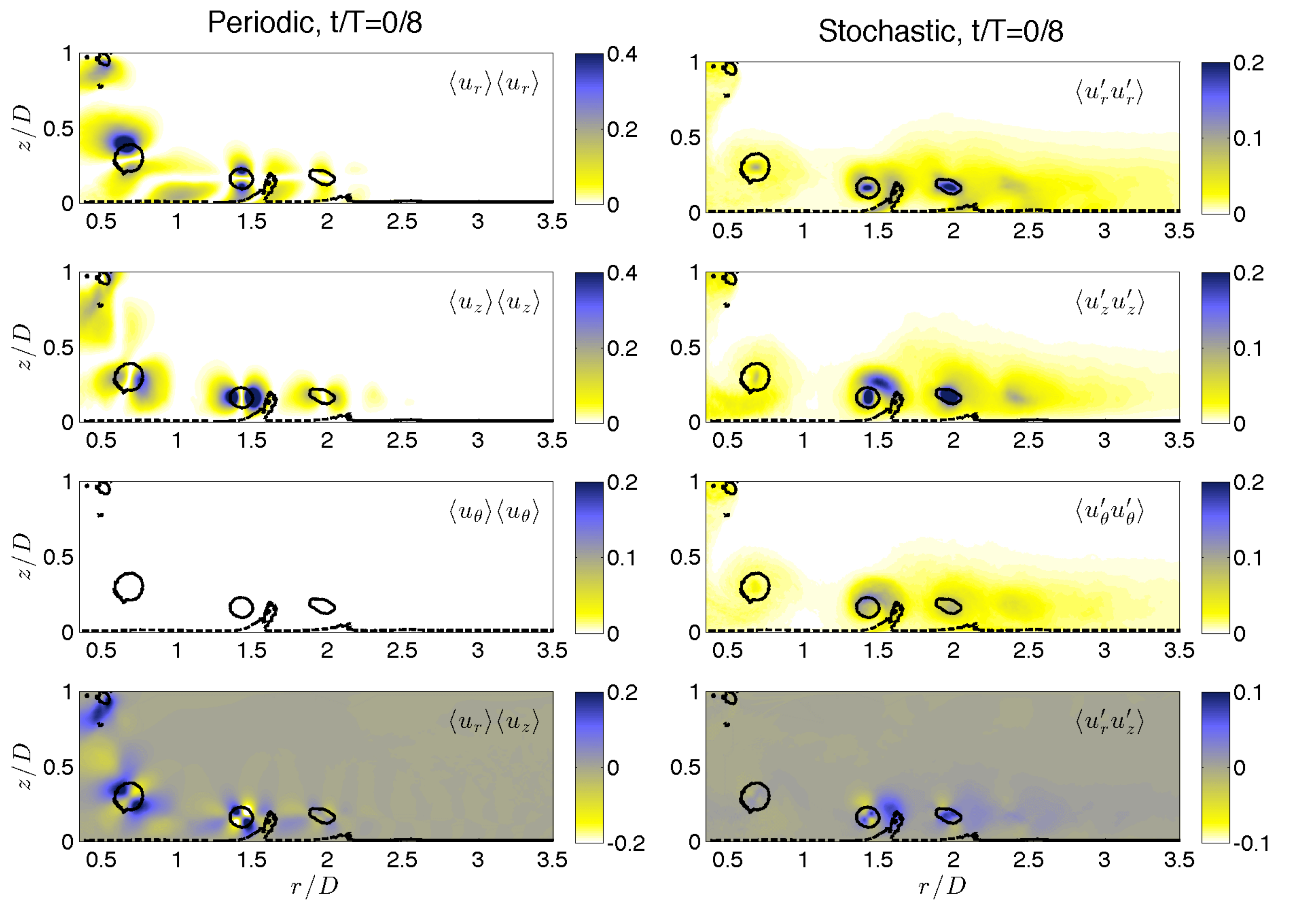


Budget of $\langle \omega_\theta \rangle$

$$\frac{D\langle \boldsymbol{\omega} \rangle}{Dt} =$$ Vortex stretching by phase-averaged flow + Vortex stretching by fluctuating field + Turbulent vorticity diffusion

$$-\nabla \times (\nabla \cdot \boldsymbol{u}'\boldsymbol{u}')|_\theta = \frac{\partial}{\partial z \partial r}\left(\langle u_r' u_r' \rangle - \langle u_z' u_z' \rangle\right) - \frac{1}{r}\frac{\partial}{\partial z}\langle u_\theta' u_\theta' \rangle + \left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial r^2}\right)\langle u_r' u_z' \rangle$$

Phase-Averaged Reynolds Stresses



OUTLINE

- Motivation
- Governing equations for LES
- Boundary conditions
- Subfilter-scale modelling
- Validation of an LES
- Applications
- **Hybrid RANS/LES methods**
- Challenges
- Conclusions