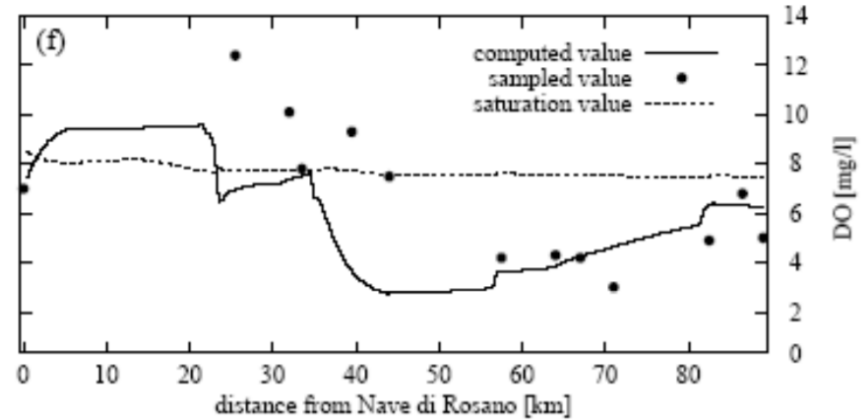
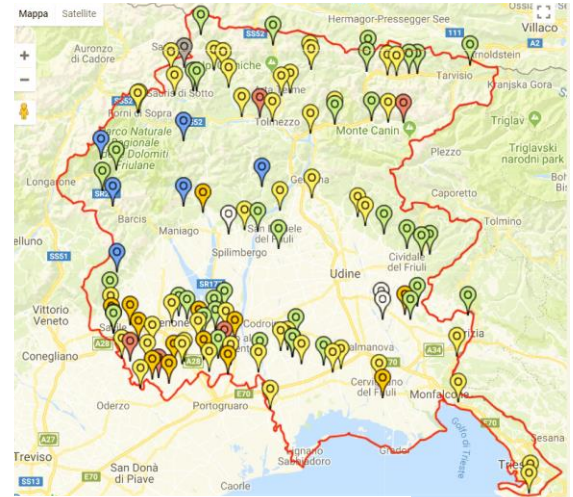
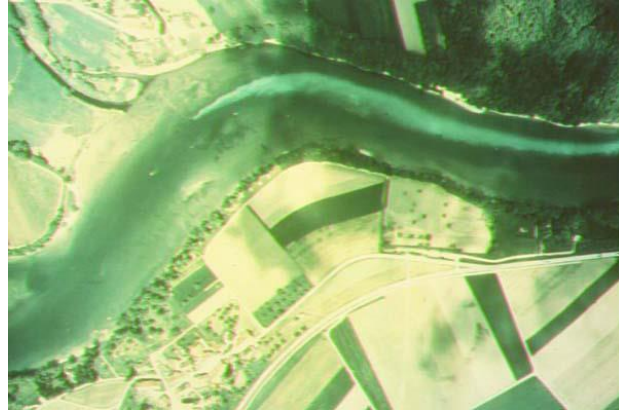


Modelli per l'analisi della qualità delle acque

A.A. 2017/2018

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Qualità dell'acqua in corpi idrici superficiali

1. Dimensionalità dominio
 $H \ll W \ll L$

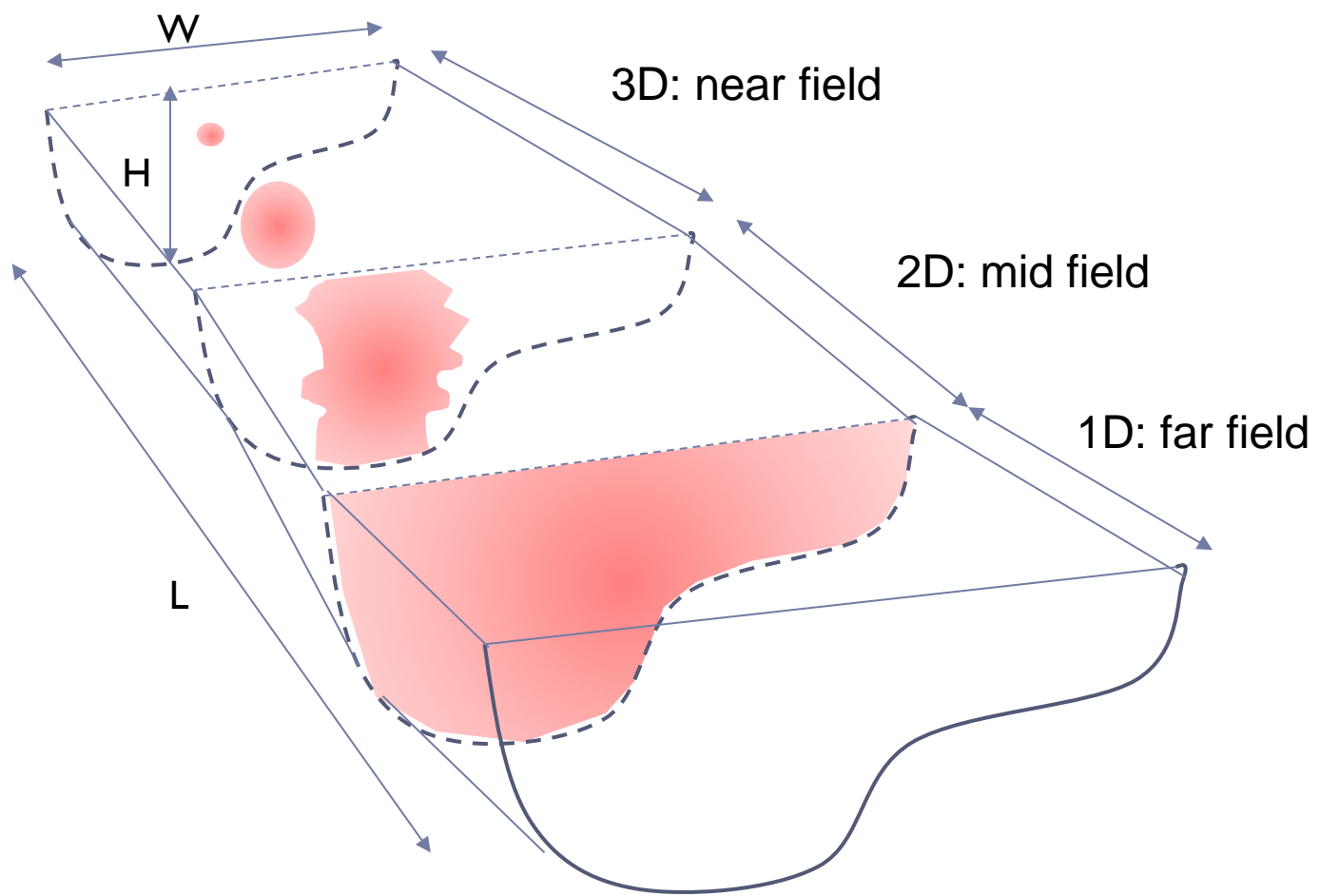
$C(x,y,z,t)$ in near field

$C(x,z,t)$ in mid field
 C =depth averaged concentration

$C(x,t)$ in far field
 C = section averaged concentration

2. Trasformazioni locali significative
(chimiche, fisiche, biologiche)

3. Interdipendenza dei parametri



Equazioni dei modelli di qualità: idraulica, trasporto e trasformazione

Conservazione massa liquido

$$\frac{\partial A}{\partial t} = -\frac{\partial Q}{\partial x} + q_{ws}$$

Conservazione quantità di moto liquido

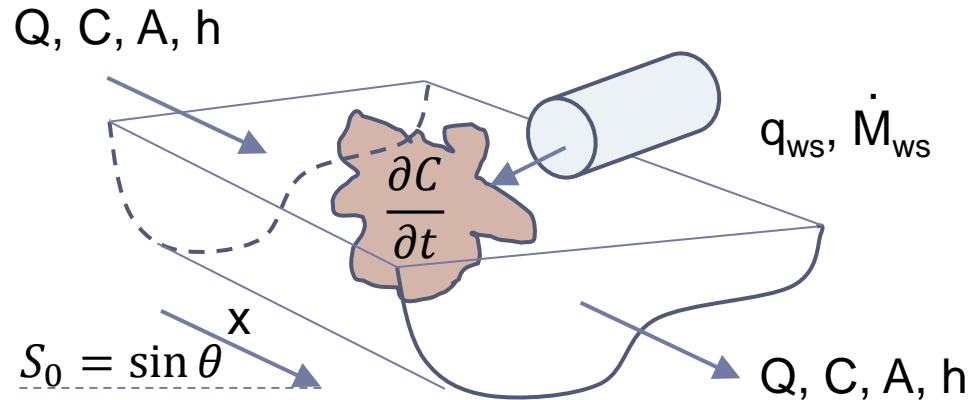
$$\frac{\partial Q}{\partial t} = -\frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + gAS_0 - gAS_f - gA \frac{\partial h}{\partial x}$$

Conservazione massa specie

$$\frac{dVC}{dt} = -\frac{\partial QC}{\partial x} dx + \frac{\partial}{\partial x} \left(DA \frac{\partial C}{\partial x} \right) dx + M_{ws} + V \frac{\partial C}{\partial t}$$

Shallow water equations

ID ADRE



q_{ws} = portata volumetrica scarico

gAS_f = resistenza del fondo

$\frac{\partial C}{\partial t}$ = trasformazione chimica/fisica/biologica

\dot{M}_{ws} = portata in massa specie scaricata

Modelli biologici: idraulica semplificata

Ipotesi: regime stazionario, moto permanente

$$\frac{\partial Q}{\partial x} = q_{ws}$$

$$gAS_0 = gAS_f$$

$$gAS_f = \frac{gQ^2}{AC^2R_i}$$

$$gAS_f = \frac{gn^2Q^2}{AR_i^{4/3}}$$

Chezy

Manning

$$\frac{\partial Q}{\partial x} = q_{ws}$$

$$h(Q) = \alpha Q^\beta$$

$$u(Q) = aQ^b$$

Rating curves

Exponent	Typical value	Range
b	0.43	0.4–0.6
β	0.45	0.3–0.5

MATERIAL	n
Man-made channels	
Concrete	0.012
Gravel bottom with sides:	
concrete	0.020
mortared stone	0.023
riprap	0.033
Natural stream channels	
Clean, straight	0.025-0.04
Clean, winding and some weeds	0.03-0.05
Weeds and pools, winding	0.05
Mountain streams with boulders	0.04-0.10
Heavy brush, timber	0.05-0.20

Dispersione longitudinale

$$D = k \cdot n \cdot u \cdot h^{5/6}$$

Costituenti della qualità dell'acqua

Specie non interagenti (conservative): $\frac{\partial C}{\partial t} = 0$

Specie non conservative:

$$\frac{\partial C}{\partial t} = k_1 C - k_2 C - k_3$$

↑ Formazione/crescita ↙ Consumo/morte ← decantazione

Specie interagenti, non conservative:

$$\frac{\partial C_1}{\partial t} = -k_{11}C_1 + k_{12}C_2 - k_{13}$$
$$\frac{\partial C_2}{\partial t} = +k_{21}C_1 - k_{22}C_2 - k_{23}$$

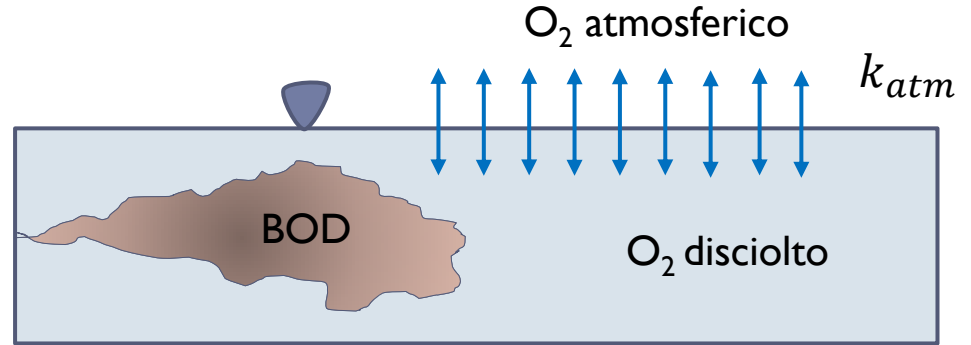
$$C_1 \rightleftharpoons C_2$$

Specie interagenti, non conservative:

$$\frac{\partial C_3}{\partial t} = \mu(C_1, C_2)C_3$$

Formazione/crescita
dipendente da altre specie

Modello biologico semplice: Streeter-Phelps

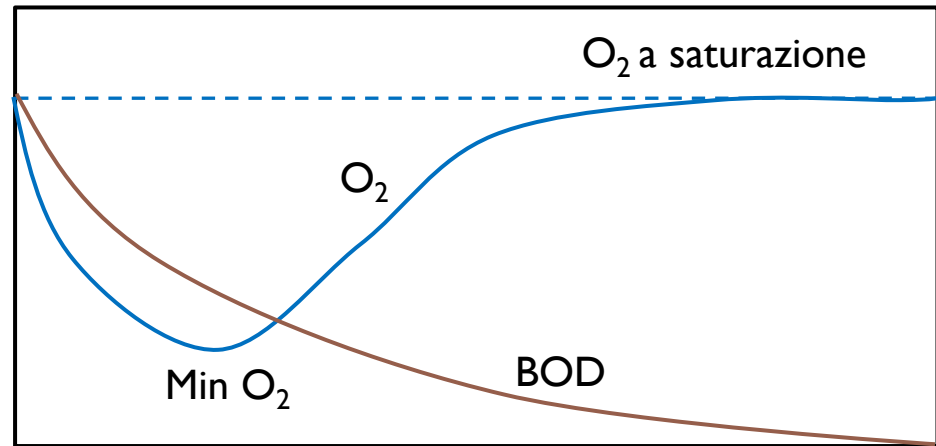


Degradazione sostanza organica

$$\frac{\partial BOD}{\partial t} = -k_{BOD} BOD$$

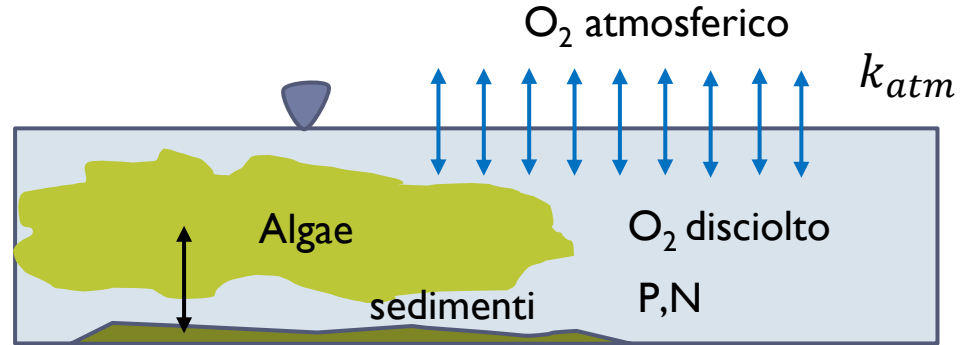
Bilancio ossigeno: reazione+degradazione

$$\frac{\partial O_2}{\partial t} = k_{atm}(O_{sat} - O_2) - k_{BOD} BOD$$



$x=ut$, distanza a valle dello scarico

Modello biologico più complesso: specie algali



Dinamica algale

$$\frac{\partial A}{\partial t} = \mu A - \rho A - \sigma A$$

Tasso di accrescimento

$$\mu = \mu_{max}$$

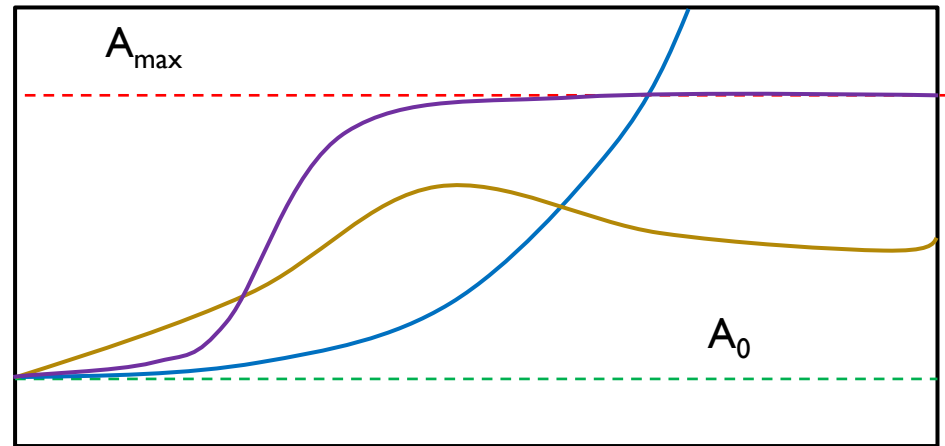
Exponential growth

$$\mu = \mu_{max} \frac{N}{N + K_N} \cdot \frac{P}{P + K_P}$$

Nutrient limited

$$\mu = \mu_{max} \left(1 - \frac{A}{A_{max}} \right)$$

Limited by max carrying capacity



$x=ut$, distanza a valle dello scarico

Equazioni modello

$$\frac{\partial O_2}{\partial t} = k_2(O_{sat} - O_2) - k_1 BOD + (\alpha_3\mu - \alpha_4\varrho)A - \alpha_5\beta_1NH_3 - \alpha_6\beta_2NO_2$$

$$\frac{\partial BOD}{\partial t} = -k_1 BOD$$

Streeter-Phelps

Crescita Alghe vs nutrienti N e P

$$\frac{\partial A}{\partial t} = \mu A - \varrho A - \frac{\sigma_1}{h} A$$

Alghe

$$\mu = \mu_{max} \frac{(NH_3 + NO_3)}{(NH_3 + NO_3) + K_N} \cdot \frac{DissP}{DissP + K_P}$$

$$\frac{\partial OrgN}{\partial t} = \alpha_1\varrho A - \beta_3 OrgN - \sigma_4 OrgN$$

$$\frac{\partial NH_3}{\partial t} = \beta_3 OrgN - \beta_1 NH_3 - \alpha_1 F \mu A$$

$$\frac{\partial NO_2}{\partial t} = \beta_1 NH_3 - \beta_2 NO_2$$

$$\frac{\partial NO_3}{\partial t} = \beta_2 NO_2 - \alpha_1 \mu (1 - F) A$$

Ciclo Azoto

$$\frac{\partial OrgP}{\partial t} = -\beta_4 OrgP - \sigma_5 OrgP + \alpha_2 \varrho A$$

$$\frac{\partial DissP}{\partial t} = \beta_4 OrgP - \alpha_2 \mu A$$

Ciclo Fosforo