8

Cyclonic Devices

8.1 INTRODUCTION

Cyclone separators have been used in the United States for about 100 yr, and are still one of the most widely used of all industrial gas-cleaning devices. The main reasons for the widespread use of cyclones are that they are inexpensive to purchase, have no moving parts, and can be constructed to withstand harsh operating conditions. Cyclonic devices by themselves are generally not adequate to meet stringent particulate emission standards, but they serve an important purpose. Their low capital cost and their maintenance-free operation make them ideal for use as precleaners for more expensive control devices such as fabric filters or electrostatic precipitators.

In the past, cyclones were regarded as low-efficiency collectors. However, efficiency varies greatly with particle size and with cyclone design. Recently, advanced design work has greatly improved cyclone performance. They can now achieve efficiencies greater than 98% for particles larger than 5 μm. In general, as efficiencies increase, operating costs increase, primarily because of the resulting higher pressure drops.

Cyclones use the centrifugal force created by a spinning gas stream to separate particles from a gas. Figure 8.1 shows a tangential inlet, reverse flow cyclone separator. The particulate-laden gas enters tangentially near the top of the device. The cyclone's shape and the tangential entry force the gas flow into a downward spiral. Centrifugal force and inertia cause the particles to move outward, collide with the outer wall, and then slide downward to the bottom of the device. Near the bottom of the cyclone, the gas reverses its downward spiral and moves upward in a smaller inner spiral. The "clean" gas exits from the top through a vortex-finder tube, and the particles exit from the bottom of the cyclone through a pipe sealed by a spring-loaded flapper valve or a rotary valve.

The centrifugal force is proportional to the square of the tangential velocity and inversely proportional to the radius of curvature of the gas trajectory. Therefore, the efficiency of a cyclone increases as the diameter of the device is reduced. To achieve higher efficiencies dictates the use of smaller cyclones. However, the pressure drop through the cyclone increases rapidly as the tangential velocity increases. A way to maintain high efficiencies with a moderate pressure drop is to use a large number of small cyclones placed in parallel.

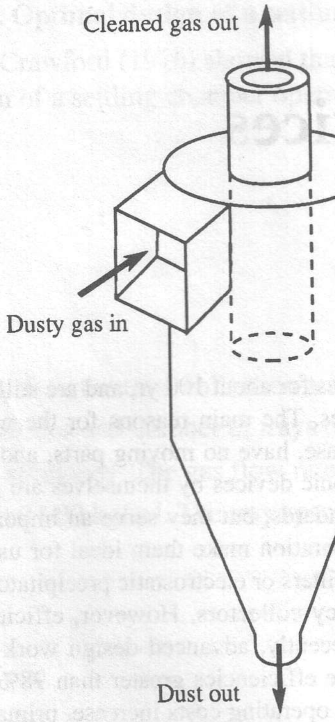

Figure 8.1 Schematic diagram of a tangential inlet reverse flow cyclone

This chapter covers the sizing and costing of single cyclones based on theoretical and empirical considerations, and the optimal design of systems of multiple cyclones to minimize TAC. Because cyclonic devices are usually used as precleaners for more sophisticated particulate control equipment, such as ESP or fabric filters, it is important to be able to characterize the particle size distribution of the aerosol that penetrates the cyclones and enters the next device. This chapter presents computational techniques for that purpose.

8.2 CYCLONIC FLOW

Your objectives in studying this section are to

1. Develop a fractional collection efficiency equation for particulate matter in ideal, laminar cyclonic flow.
2. Develop a fractional collection efficiency equation for particulate matter in ideal, turbulent cyclonic flow.
3. Use a semiempirical practical design equation to predict cyclone performance under real conditions.

Consider a particle entering tangentially onto a horizontal plane of a spinning gas stream at r_3, as shown in Figure 8.2. Because of the centrifugal force, the particle will follow a path outward across the the flow streamlines. Its velocity vector will have a tangential component (u_θ) and a radial component (u_r). The velocity of the spinning gas is assumed to have only a tangential component, v_θ, with $v_r = 0$. Tangential gas flows of this type usually are of the form $v_\theta r^m$ = constant. For an ideal, inviscid fluid in such a vortex flow $m = 1$, although in real flows the value of m may range downward to 0.5. The analysis of cyclone performance that follows begins with ideal, laminar flow. Then, it considers ideal, turbulent flow. Because both of these represent idealized cases that are not attained in real cyclones, it turns finally to a semiempirical theory that has been widely used in practical cyclone design.

8.2.1 Ideal, Laminar Cyclonic Flow

The so-called ideal laminar cyclonic flow refers to a frictionless flow in which the streamlines follow the contours of the cyclone. When the flow enters through a rectangular slot of area $W(r_2 - r_1)$, the gas velocity components are (Crawford 1976):

$$v_r = 0, \qquad v_\theta = v = \frac{Q}{Wr\ln\left(\frac{r_2}{r_1}\right)} \tag{8.1}$$

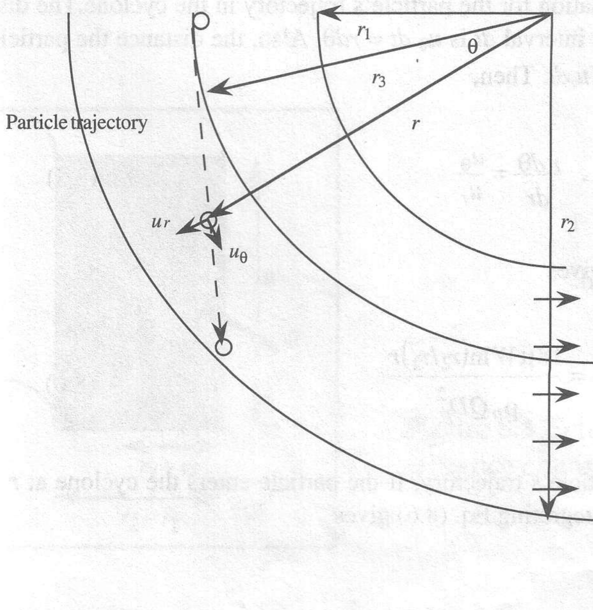

Figure 8.2 Trajectory of a particle in laminar cyclonic flow

To determine the collection efficiency consider a particle entering the cyclone at a radial position r_3 that strikes the wall at an angular position θ_f. The particle's velocity components at any point on its trajectory are u_r and u_θ. The radial velocity component is the terminal velocity of the particle when the centrifugal force, $F_c = m_p u_\theta^2/r$, acts on it. When the drag force can be given by Stokes's law, this velocity is

$$u_r = \frac{F_c}{3\pi\mu D_p} \tag{8.2}$$

Since the θ-component of the particle's velocity is that of the fluid, $u_\theta = v_\theta$, and

$$F_c = \frac{\pi}{6}\rho_p D_p^3 \frac{Q^2}{W^2 r^3 (\ln r_2/r_1)^2} \tag{8.3}$$

Combine Eqs. (8.2) and (8.3) to obtain:

$$u_r = \frac{\rho_p Q^2 D_p^2}{18\mu r^3 W^2 (\ln r_2/r_1)^2} \tag{8.4}$$

The next step is to obtain an equation for the particle's trajectory in the cyclone.The distance traveled in the θ-direction in a time interval dt is $u_\theta\, dt = r d\theta$. Also, the distance the particle moves in the r-direction in time dt is $dr = u_r dt$. Then,

$$\frac{r\,d\theta}{dr} = \frac{u_\theta}{u_r} \tag{8.5}$$

Substituting Eqs. (8.1) and (8.4) in (8.5) gives

$$\frac{d\theta}{dr} = \frac{18\mu W \ln(r_2/r_1) r}{\rho_p Q D_p^2} \tag{8.6}$$

a differential equation describing the particle's trajectory. If the particle enters the cyclone at $r = r_3$ and hits the outer wall at $\theta = \theta_f$, then integrating Eq. (8.6) gives

$$\theta_f = \frac{9\mu W \ln(r_2/r_1)}{\rho_p Q D_p^2}\left(r_2^2 - r_3^2\right) \tag{8.7}$$

An alternative is to solve Eq. (8.7) for r_3 to find the entrance position of a particle that hits the outer wall at $\theta = \theta_f$,

$$r_3 = \left[r_2^2 - \frac{\rho_p Q D_p^2 \theta_f}{9\mu W \ln(r_2/r_1)}\right]^{1/2} \tag{8.8}$$

To obtain an expression for the collection efficiency of a cyclone, assume that the entering particle concentration and gas velocity are uniform across the entrance cross section (see Figure 8.3). If the cyclone has an angle θ_f, all particles that enter the device at $r \geq r_3$ hit the wall over $0 \leq \theta \leq \theta_f$. The collection efficiency is just that fraction of the particles in the entering flow that hit the outer wall before $\theta = \theta_f$. Therefore,

$$\eta(D_p) = \frac{W(r_2 - r_3)}{W(r_2 - r_1)} \tag{8.9}$$

Substituting Eq. (8.8) into (8.9) and rearranging

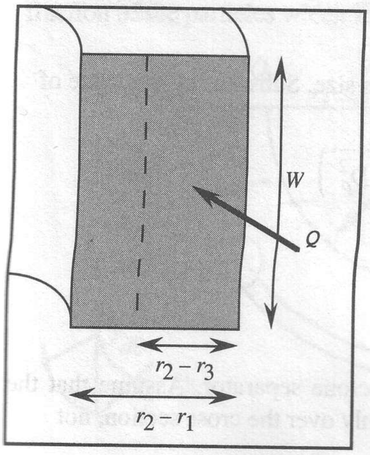

Figure 8.3 Schematic diagram of a cyclone entrance channel

$$\eta(D_p) = \frac{1 - \left[1 - \frac{\rho_p Q D_p^2 \theta_f}{9 \mu W r_2^2 \ln(r_2/r_1)}\right]^{1/2}}{1 - r_1/r_2} \tag{8.10}$$

The value of θ_f at which $\eta(D_p) = 1$ is given by

$$\theta_f = \left[\frac{\rho_p Q D_p^2}{9 \mu W \ln(r_2/r_1)}\right]^{-1} \left(r_2^2 - r_1^2\right) \tag{8.11}$$

Example 8.1 Ideal, Laminar Cyclonic Flow

Air at 298 K and 1 atm flows at the rate of 5.0 m^3/s and carries with it particulate matter with a density of 1,500 kg/m^3. The stream enters a cyclonic region with $r_1 = 0.2$ m and $r_2 = 0.4$ m in ideal, laminar flow. Through what angle must the flow turn in the cyclone if the efficiency is to be unity for 30-μm particles? The height of the channel, W, is 1 m. Plot the efficiency as a function of particle size for this angle.

Solution

$$\theta_f = \frac{9\left(1.84 \times 10^{-5}\right)(1)\ln 2}{1{,}500(5.0)\left(9 \times 10^{-10}\right)}\left(0.4^2 - 0.2^2\right) = 2.041 \text{ rad} = 117°$$

Equation (8.10) gives the efficiency as a function of particle size. Substituting the value of qf calculated earlier,

$$\eta(D_p) = 2\left(1 - \sqrt{1 - 0.000833 D_p^2}\right)$$

where D_p is in μm. Figure 8.4 is a plot of this equation.

8.2.2 Ideal, Turbulent Cyclonic Flow

Figure 8.5 shows the model of the turbulent flow cyclone separator. Assume that the effect of the turbulent eddies is to distribute the particles uniformly over the cross section, not

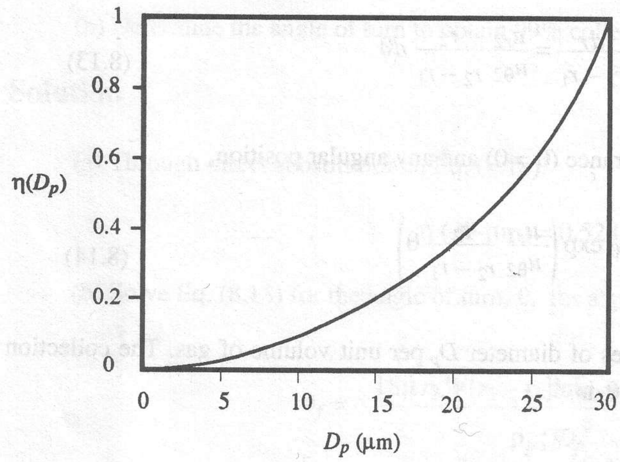

Figure 8.4 Efficiency curve for Example 8.1

only at the entrance, but at any given angle θ. This is a conservative assumption because the centrifugal force effects may serve to damp out the turbulent eddies which naturally occur in turbulent duct flow. Relatively little is known about this area.

Consider the effect of a laminar layer next to the outer edge of the cyclone, as shown in Figure 8.5. Once a particle, vigorously mixed in the core of the flow, enters this layer, it travels to the outer wall and is removed. The distance the particle travels in the θ direction in the laminar sublayer over a time interval dt is $u_{\theta 2}\, dt = r_2\, d\theta$, where $u_{\theta 2}$ is evaluated at $r = r_2$. The thickness of the laminar sublayer is

$$dr = u_{r2}\, dt = u_{r2} \frac{r_2\, d\theta}{u_{\theta 2}} \tag{8.12}$$

where u_{r2} is also evaluated at $r = r_2$. The fractional diminution of particles over the angle $d\theta$ is the fraction of the particles which lies in the boundary layer, or

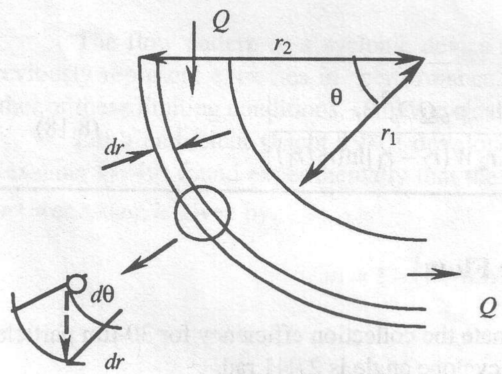

Figure 8.5 Turbulent cyclonic flow

$$-\frac{dn}{n} = \frac{dr}{r_2 - r_1} = \frac{u_{r2}}{u_{\theta 2}} \frac{r_2}{r_2 - r_1} d\theta \tag{8.13}$$

Integrating this equation between the entrance ($\theta = 0$) and any angular position,

$$n = n_0 \exp\left(-\frac{u_{r2}}{u_{\theta 2}} \frac{r_2}{r_2 - r_1} \theta\right) \tag{8.14}$$

where n_0 is the initial number of particles of diameter D_p per unit volume of gas. The collection efficiency of a cyclone that has an angle θ_f is

$$\eta(D_p) = 1 - \frac{n(\theta_f)}{n_0} = 1 - \exp\left[-\frac{u_{r2} r_2 \theta_f}{u_{\theta 2}(r_2 - r_1)}\right] \tag{8.15}$$

For lack of a better approximation, use the inviscid gas velocity components given by Eq. (8.1) to represent the fluid velocity field in the turbulent flow cyclone. Therefore,

$$u_{\theta 2} = \frac{Q}{W r_2 \ln\left(\frac{r_2}{r_1}\right)} \tag{8.16}$$

$$u_{r2} = \frac{\rho_p Q^2 D_p^2}{18 \mu r_2^3 W^2 (\ln r_2/r_1)^2} \tag{8.17}$$

and the collection efficiency of the cyclone is:

$$\eta(D_p) = 1 - \exp\left[-\frac{\rho_p Q D_p^2 \theta_f}{18 \mu r_2 W (r_2 - r_1) \ln(r_2/r_1)}\right] \tag{8.18}$$

Example 8.2 Ideal, Turbulent Cyclonic Flow

(a) Consider the data of Example 8.1. Estimate the collection efficiency for 30-μm particles assuming that the flow is turbulent and the cyclone angle is 2.041 rad.

(b) Determine the angle of turn to obtain 99% collection efficiency for the 30-μm particle.

Solution

(a) Through direct substitution in Eq. (8.18):

$$\eta\ (30\text{-}\mu\text{m}) = 0.528\ (52.8\%)$$

(b) Solve Eq. (8.18) for the angle of turn, θ_f, for a given efficiency.

$$\theta_f = -\frac{18\mu r_2 W(r_2 - r_1)\ln(r_2/r_1)\ln(1-\eta)}{\rho_p Q D_p^2}$$

Substituting numerical values, the angle for 99% collection efficiency of the 30-μm particles is $\theta_f = 12.53$ rad = 2 full turns.

Comments

A comparison of the results of Examples 8.1 and 8.2 shows that turbulence reduces dramatically the efficiency of a cyclone. A precise criterion for transition from laminar to turbulent flow in a cyclone does not exist (Flagan and Seinfeld 1988). Experimentally determined collection efficiency curves appear to conform more closely to turbulent than to laminar flow conditions.

8.2.3 Practical Cyclone Design Equation

The flow pattern in a cyclonic device is a complex one and the two models presented previously represent extremes in performance. Because operating cyclones do not conform to either of these limiting conditions, semiempirical design equations predict their performance.

Leith and Licht (Licht 1980) developed a theory useful in practical cyclone design. Alexander (1949) found experimentally that the exponent in the fluid tangential velocity profile, $v_\theta r^m$ = constant, is given by:

$$m = 1 - \left(1 - 0.67 D_c^{0.14}\right)\left(\frac{T}{283}\right)^{0.3} \tag{8.19}$$

where D_c is the cyclone body diameter in meters and T is the gas temperature in K. The collection efficiency, according to the model by Leith and Licht, is given by

$$\eta(D_p) = 1 - \exp\left(-\psi D_p^M\right) \tag{8.20}$$

where

$$M = 1/(m+1)$$

$$\psi = 2\left[\frac{KQ\rho_p C_c(m+1)}{18\mu D_c^3}\right]^{M/2} \tag{8.21}$$

where K is a dimensionless geometric configuration parameter and C_c is the Cunningham correction factor. *Be consistent with the units in Eqs.* (8.20) *and* (8.21).

Example 8.3 Leith-Licht Model for Efficiency of Cyclone

Consider the gas stream of Examples 8.1 and 8.2. It flows through a cyclone with a body diameter of 2.0 m and a value of K, the geometric configuration parameter, of 551.3. Estimate the removal efficiency of the cyclone for 10-μm particles.

Solution

For a cyclone body diameter of 2.0 m and a gas temperature of 298 K, Eq. (8.19) yields m = 0.734, M = 0.577. From Eq. (8.21), Ψ = 1041. For 10-μm particles, Eq. (8.20) yields η = 0.742 (74.2%).

8.3 STANDARD CYCLONE CONFIGURATIONS

Your objectives in studying this section are to

1. Understand the concept of a cyclone of standard proportions.
2. Calculate the efficiency of various standard cyclone configurations.
3. Estimate the pressure drop through cyclones of standard proportions.

Extensive work has been done to determine in what manner the relative dimensions of cyclones affect their performance. A number of configurations have been proposed and studied sufficiently to be regarded as "standards." In these *standard cyclone configurations* all dimensions are related to the cyclone body diameter. Table 8.1 presents the dimension ratios of these, along with values of the geometric configuration parameter, K, and a constant, N_H, relating the pressure drop through the cyclone to the inlet velocity head. Figure 8.6 illustrates the various dimensions in Table 8.1.

Table 8.1 Standard Cyclone Configurations

Term	Description	Stairmand[a]	Swift[b]	Lapple[c]
$K_a = a/D_c$	inlet height	0.5	0.44	0.5
$K_b = b/D_c$	inlet width	0.2	0.21	0.25
S/D_c	outlet length	0.5	0.5	0.625
D_e/D_c	outlet diameter	0.5	0.4	0.5
h/D_c	cylinder height	1.5	1.4	2.0
H/D_c	overall height	4.0	3.9	4.0
B/D_c	dust outlet	0.375	0.4	0.25
N_H	Eq. (8.22)	6.4	9.24	8.0
K	Eq. (8.21)	551.3	699.2	402.9

[a]Stairmand (1951) [b]Swift (1969) [c]Shepherd and Lapple (1939).

The other major consideration in cyclone specification, besides collection efficiency, is pressure drop. While forcing the gas through the cyclone at higher velocities results in improved removal efficiencies, to do so increases the pressure drop and the operating costs. There is ultimately an economic trade-off between efficiency and operating cost. Several methods have been proposed to estimate the total pressure drop in the flow of gas through a standard cyclone. Unfortunately, there is no definitive study to determine which is the best to use. Most methods agree to express the pressure drop in terms of a multiple of the inlet velocity head, or

$$\Delta P = N_H \rho_f v_E^2/2 \tag{8.22}$$

where

N_H is a constant which depends on the cyclone configuration (see Table 8.1)

ρ_f is the gas density

v_E is the gas velocity in the cyclone inlet duct $= Q/ab$.

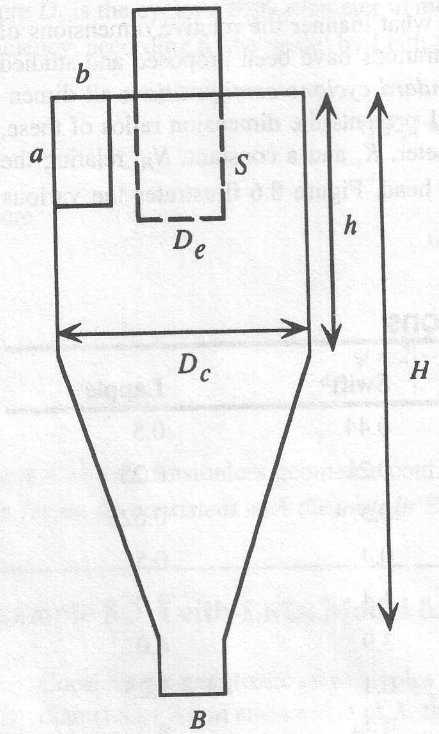

Figure 8.6 Dimensions of a standard cyclone

Equation (8.22) can be rewritten in terms of the cyclone body diameter and the gas volumetric flow rate.

$$\Delta P = \frac{N_H \rho_f Q^2}{2 K_a^2 K_b^2 D_c^4} \tag{8.23}$$

where

$K_a = a/D_c$

$K_b = b/D_c$

It is evident from Eq. (8.23) that the pressure drop is extremely sensitive to the cyclone body diameter, increasing rapidly as the device becomes smaller. Notice in Table 8.1 that, for a given set of operating conditions and body diameter, Swift standard configuration is more efficient (higher value of K), but results in a higher pressure drop (higher value of N_H). Lapple configuration, with a relatively high pressure drop, is not nearly as efficient as the other two.

Example 8.4 Pressure Drop through Cyclone of Standard Configuration

(a) Estimate the pressure drop for the conditions of Example 8.3. According to the value of K in that example, the cyclone proportions correspond to Stairmand configuration.
(b) If the cyclone body diameter decreases to 1.0 m, estimate the removal efficiency for 30-μm particles, and the corresponding pressure drop.

Solution

a) From Table 8.1, for Stairmand configuration, $N_H = 6.4$, $K_a = 0.5$, $K_b = 0.2$. The ideal gas law gives the fluid density, $\rho_f = 1.186$ kg/m^3. The cyclone body diameter, D_c, is 2.0 m, the gas volumetric flow rate is 5 m^3/s. From Eq. (8.23), $\Delta P = 593$ Pa.
(b) For a body diameter of 1.0 m, and Stairmand standard configuration, $m = 0.665$, $M = 0.6$, $\Psi = 2{,}491$, $\eta(30\ \mu m) = 0.992$. Because the body diameter is twice as small as that of the cyclone of part a, the pressure drop is 16 times higher. Therefore, $\Delta P = 9{,}500$ Pa.

Comments

Usually, the pressure drop is the limiting factor in the design of cyclones. To maintain the pressure drop within acceptable levels, the removal efficiency for small particles must remain relatively low.

8.4 SIZE DISTRIBUTION OF PENETRATING PARTICLES

Your objectives in studying this section are to

1. Develop an equation relating the cumulative mass distribution function of the particles penetrating a cyclone to the inlet conditions and the device grade efficiency function.
2. Estimate the resulting integrals with Gauss-Legendre quadrature formulas.
3. Estimate MMD and σ_g for the aerosol that penetrates the cyclone.

Cyclones are frequently used as precleaners for more sophisticated particulate control devices. The size distribution characteristics of the aerosol population entering the cyclone are known, but they must be calculated for those particles penetrating the device.

The size distribution function of the particles penetrating the cyclone, by cumulative mass fraction less than size, $Go(D_p)$, may be determined by a material balance taken over all particles finer than a given size D_p as shown in Figure 8.7. The mass flow rate of particles finer than $D_p$entering the cyclone is given by $Qc_iG_i(D_p)$, where

Q = volumetric flow rate of the gas

c_i = mass of total particulate matter per unit volume of entering gas

$G_i\,(D_p)$ = cumulative mass fraction of particles finer than D_p at the entrance

The mass flow rate of particles finer than D_p penetrating the cyclone is $Qc_iPt_MG_o(D_p)$, where

Pt_M = overall penetration = 1 - η_M

$G_o(D_p)$ = cumulative mass fraction of particles finer than D_p at the outlet

The mass flow rate of particles finer than D_p in the collected dust is given by

$$Qc_i\int_0^{D_p}\eta(D_p')n_{mi}(D_p')d(D_p')$$

where

$n_{mi}(D_p')d(D_p')$ = the mass fraction of particles with diameters around D_p' at the inlet of the cyclone

For a log-normal size distribution at the cyclone inlet,

$$n_{mi}(D_p') = \frac{1}{\sqrt{2\pi}\,D_p'\ln\sigma_{gi}}\exp\left[-\frac{(\ln D_p' - \ln \mathrm{MMD}_i)^2}{2(\ln\sigma_{gi})^2}\right] \tag{8.24}$$

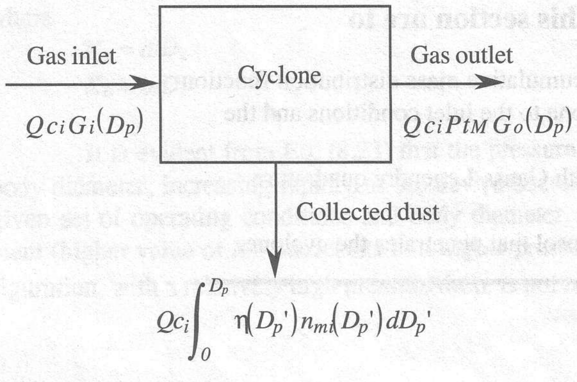

Figure 8.7 Material balance on particles finer than D_p

where MMD_i and σ_{gi} are characteristic of the inlet aerosol population.

Assuming that the size of the individual particles do not change as they flow through the cyclone, a material balance yields

$$Qc_i Pt_M G_o(D_p) + Qc_i \int_0^{D_p} \eta(D_p')n_{mi}(D_p')dD_p' = Qc_i G_i(D_p) \tag{8.25}$$

Simplifying and rearranging Eq. (8.25) becomes

$$G_o(D_p) = \frac{G_i(D_p) - \int_0^{D_p} \eta n_{mi} d(D_p')}{Pt_M} \tag{8.26}$$

Substituting in Eq. (8.26) the definition of $G_i(D_p)$,

$$G_o(D_p) = \frac{\int_0^{D_p} (1-\eta)n_{mi} dD_p'}{Pt_M} \tag{8.27}$$

Define the grade penetration, $Pt(D_p') = 1 - \eta(D_p')$. Then, Eq. (8.27) becomes

$$G_o(D_p) = \frac{\int_0^{D_p} Pt(D_p')n_{mi}(D_p')dD_p'}{Pt_M} \tag{8.28}$$

For a log-normal distribution, the integral in Eq. (8.28) can not be evaluated in closed form. Define a new variable, x, such that

$$x = \frac{2D_p'}{D_p} - 1 \tag{8.29}$$

Then,

$$\int_0^{D_p} Pt(D_p')\, n_{mi}(D_p')\, dD_p' = \frac{D_p}{2}\int_{-1}^{+1} Pt(x)\, n_{mi}(x)\, dx \qquad (8.30)$$

The integral on the right hand side of Eq. (8.30) can be approximated numerically with a Gauss-Legendre quadrature formula:

$$\int_{-1}^{+1} Pt(x)\, n_{mi}(x)\, dx \cong \sum_{i=1}^{N} w_i Pt(x_i)\, n_{mi}(x_i) \qquad (8.31)$$

where w_i and x_i are the weight factors and roots of the Nth degree Legendre polynomial. Equation (8.28) becomes

$$G_o(D_p) \cong \frac{\dfrac{D_p}{2}\displaystyle\sum_{i=1}^{N} w_i Pt(x_i)\, n_{mi}(x_i)}{Pt_M} \qquad (8.32)$$

The overall penetration in Eq. (8.32) can be estimated with a Gauss-Hermite quadrature formula (as illustrated in Chapter 7) given MMD_i, σ_{gi}, and the grade efficiency equation for the cyclone. The following example illustrates the computational scheme to characterize the particle size distribution function at the cyclone gas outlet.

Example 8.5 Size Distribution Function of Particles Penetrating a Cyclone

Consider Example 8.3. The aerosol population entering the cyclone is log-normally distributed with MMD = 8.0 μm and $\sigma_g = 2.5$. Estimate the overall removal efficiency for the device. Calculate the cumulative mass fraction of particles penetrating the cyclone for various particle sizes. Determine if these cumulative results follow a log-normal distribution, and, if so, estimate the corresponding MMD and σ_g values.

Solution

The grade efficiency function for the cyclone at the operating conditions of Example 8.3 is, from Eq. (8.20),

$$\eta\,(D_p') = 1 - \exp\,(-\,1{,}041\; D_p'^{\,0.577}) \qquad (8.20\text{ a})$$

Follow the computational scheme outlined in Example 7.4 to estimate the overall removal efficiency through application of Gauss-Hermite quadrature formula. The result, based on a 16-point formula, is $\eta_M = 0.686$. Therefore, the overall penetration is $Pt_M = 0.314$.

The procedure to characterize the particle size distribution penetrating the cyclone is as follows:

- Choose N, the number of quadrature points for Gauss-Legendre formula.
- Obtain the values of the roots, x_i, and weight factors, w_i, of the corresponding Nth degree Legendre polynomial, either from a mathematical table or from the computer subprogram GAULEG already provided (Section 4.3.1).
- Choose a value of D_p.
- For each of the roots calculate the corresponding D_p' from Eq. (8.29).
- From Eq. (8.20 a) calculate $Pt\,(D_p') = 1 - \eta\,(D_p')$.
- From Eq. (8.24) calculate $n_{mi}\,(D_p')$.
- From Eq. (8.32) calculate $G_o\,(D_p)$.
- Choose other values of D_p and repeat the calculations.
- Plot G_o versus D_p on log-probability paper.
- If a reasonable straight line results, estimate MMD and σ_g at the cyclone outlet.

The following table summarizes the results of the outlet cumulative mass fraction calculations with a 16-point Gauss-Legendre quadrature formula.

D_p (μm)	$G_o\,(D_p)$
1.0	0.0274
2.0	0.1355
4.0	0.3958
5.0	0.5037
7.0	0.6651
10.0	0.809
11.0	0.8404
12.0	0.8659
13.0	0.8868
15.0	0.9184

Figure 8.8 is a log-probability plot of the tabulated results. It shows that the cumulative mass fraction distribution of the particles penetrating the cyclone is log-normal, with MMD = 4.96 μm and $\sigma_g = 2.22$.

Comments

The net effects of the cyclone are to remove 68.6% of all the particulate mass, to displace the size distribution toward finer particles, and to reduce the spread in particle sizes exhibited by the original distribution.

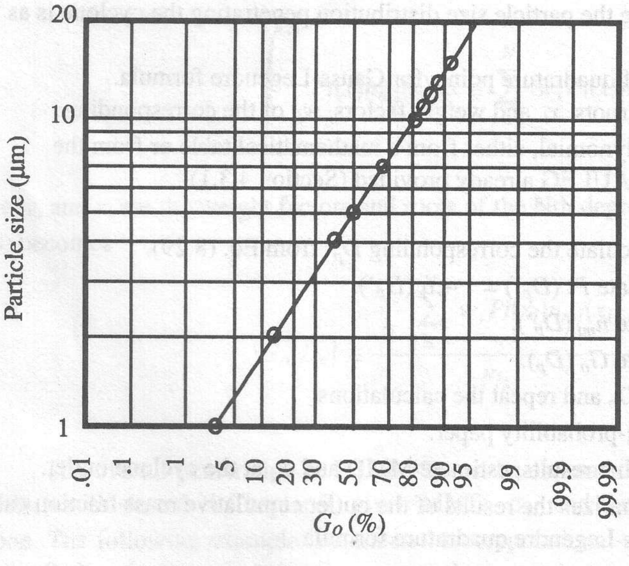

Figure 8.8 Size distribution of penetrating particles in Example 8.5

8.5 MULTIPLE CYCLONES

Your objectives in studying this section are to

1. Understand the advantage of multiple small-size cyclones operating in parallel—as compared to a single cyclone—for high particulate removal efficiency.
2. Calculate the overall removal efficiency and gas pressure drop for a multiple cyclone system.

The centrifugal force—the driving force for particulate removal in cyclones—is proportional to the square of the tangential velocity and inversely proportional to the radius of curvature of the gas trajectory. Therefore, the efficiency of a cyclone increases as the diameter of the device is reduced. To achieve higher efficiencies dictates the use of smaller cyclones. However, the pressure drop through the cyclone increases rapidly as the tangential velocity increases. A way to maintain high efficiencies with a moderate pressure drop is to use a large number of small cyclones placed in parallel. If N_c is the number of cyclones in parallel in a multiple cyclone system, the gas volumetric flow rate through each one is Q/N_c. Equation (8.20) for the grade efficiency of the system can be rewritten as (see Problem 8.4):

$$\eta(D_p) = 1 - \exp\left\{-2\left[\frac{KQ\tau}{MN_cD_c^3}\right]^{\frac{M}{2}}\right\} \tag{8.33}$$

Equation (8.23) for the pressure drop through the multiple cyclone system becomes:

$$\Delta P = \frac{N_H \rho_f Q^2}{2K_a^2 K_b^2 N_c^2 D_c^4} \tag{8.34}$$

Equation (8.34) suggests that the pressure drop decreases rapidly as the number of cyclones increases. The following example shows that a multiple cyclone system can achieve a high removal efficiency for small particles at a much lower pressure drop than a single cyclone of similar efficiency.

Example 8.6 Collection Efficiency and Pressure Drop of Multiple Cyclone System

A paper mill in Louisiana operates a power plant that produces steam for the paper making process, and enough electricity to run a city of 200,000. The plant has four boilers—two recovery boilers that burn "black liquor" from the pulp mill, and two power boilers that burn bark, coal, oil, and gas. The power boilers have a flue gas cleaning system that consists of a multiple cyclone precleaner, followed by an electrostatic precipitator. The flue gas flow rate is 165 m^3/s, measured at the actual conditions of 450 K and 1 atm. The average particle density is 1,600 kg/m^3. The precleaner consists of 2 units in parallel, each with 450 Stairmand cyclones with body diameter of 0.25 m. Assuming that the flue gases behave as air,

(a) Estimate the removal efficiency of the multiple cyclone system for 10-μm particles and the resulting pressure drop.
(b) If a single Stairmand cyclone removes 10-μm particles with the same efficiency as the multiple cyclone system of part a, estimate the corresponding pressure drop.

Solution

(a) The total number of cyclones, N_c, is 900; $D_c = 0.25$ m. From Eq. (8.19), $m = 0.485$, $M = 0.674$. Calculate the characteristic or relaxation time for 10-μm particles. The viscosity of air at 450 K is 2.48×10^{-5} kg/m-s, the Cunningham correction factor is approximately 1.0. Therefore, $\tau = 3.58 \times 10^{-4}$ s. From Eq. (8.33), η (10 μm) = 0.952 (95.2%). The pressure drop is from Eq. (8.34): $\Delta P = 2.16$ kPa.
(b) Calculate, by trial-and-error, the body diameter of a single Stairmand cyclone for 95.2% removal of 10-μm particles. The answer is $D_c = 2.37$ m. The pressure drop is from Eq. (8.23): $\Delta P = 216.8$ kPa.

Comments

This example illustrates the advantages of multiple cyclone systems over single cyclones as high efficiency precleaners. Many combinations of number of cyclones and body diameter result in the desired efficiency. The optimum combination is obtained from a cost analysis.

8.6 COST ANALYSIS FOR CYCLONES

Your objectives in studying this section are to

1. Estimate the initial and annual cost for single and multiple cyclones.
2. Design a multiple cyclone system to minimize its total annual cost.

Cyclones are very inexpensive, having capital costs at least an order of magnitude less than final control devices such as baghouses and electrostatic precipitators. Because of their simplicity, the only significant operating expense is the cost of electricity to overcome the pressure drop through the device.

8.6.1 Costs of Single Cyclones

Vatavuk (1990) gives a correlation to estimate the equipment cost of single cyclones (updated to June 1990) for a range of inlet duct areas. The cost includes a carbon steel cyclone, support stand, a fan and motor, and a hopper for collecting captured dust. The correlation is:

$$EC = 57{,}800\,[ab]^{0.903} \tag{8.35}$$

where

EC = equipment cost, in \$ of June 1990
a and b are the inlet height and width, respectively
Equation (8.35) is valid in the range $0.020 \leq ab,\ \text{m}^2 \leq 0.4$

Installation costs and ductwork connections are often more expensive than the cyclone itself. The total capital investment is about twice the purchased equipment cost.

Example 8.7 TCI and TAC for Cyclone

Consider the cyclone of Examples 8.3 and 8.4a. Estimate the total capital investment and annual cost if the device operates 8,000 h/yr. Assume a useful life of 10 yr and a minimum attractive rate of return on investment of 15%. The mechanical efficiency of the motor-fan system is 65%, and the cost of electricity is \$0.08/kW-h.

Solution

The cyclone is of Stairmand standard configuration with a body diameter of 2 m. The inlet area is $ab = K_a K_b D_c^2 = (0.5)(0.2)(2.0)^2 = 0.4\ \text{m}^2$. The equipment cost is from Eq. (8.35), EC = \$25,300. Add 8% to EC to account for freight and taxes, and multiply by 2.0 to estimate the total capital investment: TCI = \$54,650.

The only two significant expenses in the total annual cost calculation are the capital recovery cost and the cost of electricity. For $i = 0.15$ and $n = 10$ yr, the capital recovery factor is CRF = 0.20/yr. Therefore, the capital recovery cost is CRC = \$10,930/yr. For a volumetric flow rate of the gases of 5 m^3/s and a pressure drop of 0.593 kPa, the power to operate the fan is (5)(0.593)/(0.65) = 4.56 kW. The annual cost of electricity is (4.56)(8,000)(0.08) = \$2,920/yr. TAC = 10,930 + 2,920 = \$13,850/yr.

8.6.2 Costs of Multiple Cyclones

The following correlation for the equipment cost of multiple cyclone systems is based on suggestions by Crawford (1976) and cost data presented by Cooper and Alley (1986), and Peters and Timmerhaus (1991):

$$EC = 7{,}000 N_c\, ab + 72 N_c \tag{8.36}$$

where

EC = equipment cost, in \$ of June 1990
N_c = number of cyclones.

Equation (8.36) is valid in the range $1.0 \le N_c ab,\ \text{m}^2 \le 6.0$

Example 8.8 TCI and TAC of a Multiple Cyclone System

Estimate the total capital investment and annual cost for the multiple cyclone system of Example 8.6a. Assume that the conditions are similar to those of Example 8.7.

Solution

For a total of 900 Stairmand cyclones with body diameter of 0.25 m, the total inlet area is $N_c ab = N_c K_a K_b D_c^2 = (900)(0.5)(0.2)(0.25)^2 = 5.625\ \text{m}^2$. The equipment cost is from Eq. (8.36): EC = \$104,200. TCI = (2)(1.08)(104,200) = \$225,000.

For a capital recovery factor of 0.2/yr, the capital recovery cost is \$45,000/yr For a pressure drop of 2.16 kPa and a gas flow rate of 165 m³/s, the power to push the gas through the system is (2.16)(165)/(0.65) = 548 kW. The annual cost of electricity is (548)(8,000)(0.08) = \$351,000/yr. TAC = \$396,000/yr.

8.6.3 Optimization of Multiple Cyclone System Design

It is possible to estimate the most economical design for a multiple cyclone system, given the required collection efficiency. The total annual cost for the system is

$$\text{TAC} = K_1 N_c ab + K_2 W t + K_3 N_c \tag{8.37}$$

where

K_1 = the capital recovery factor times the installed cost of a cyclone of unit inlet area, \$/yr-m^2

K_2 = the cost of electric energy, in \$/kW-h

W = electric power, in kW

t = the number of hours the unit operates per year

K_3 = the capital recovery factor times the portion of the installed cost of the system that is proportional to the number of cyclones, in \$/yr.

Equation (8.37) can be rewritten as

$$\text{TAC} = K_1 N_c K_a K_b D_c^2 + \frac{K_2 t N_H \rho_f Q^3}{2E K_a^2 K_b^2 N_c^2 D_c^4} + K_3 N_c \tag{8.38}$$

where E = mechanical efficiency of the motor-blower system.

Equation (8.33), which gives the grade efficiency of the multiple cyclone system, may be written as

$$N_c D_c^3 = \frac{K Q \tau}{M\left[-\frac{1}{2}\ln(1-\eta)\right]^{2/M}} \tag{8.39}$$

The only term on the right hand side of Eq. (8.39) that depends on the system design is M. However, it is only a weak function of the cyclone body diameter and may be assumed constant without introducing significant errors in the following analysis. For a given set of operating conditions, then, the combination $N_c D_c^3$ is fixed. Equation (3.38) can be written in terms of ND3 = $N_c D_c^3$.

$$\text{TAC} = \frac{K_1 K_a K_b (\text{ND3})}{D_c} + \frac{K_2 t N_H \rho_f Q^3 D_c^2}{2E K_a^2 K_b^2 (\text{ND3})^2} + \frac{K_3 (\text{ND3})}{D_c^3} \tag{8.40}$$

Define

$K_1' = K_1 K_a K_b\,(\text{ND3})$,

$$K_2' = \frac{K_2 t N_H \rho_f Q^3}{2E K_a^2 K_b^2 (\text{ND3})^2}$$

$K_3' = K_3(\text{ND3})$

Then,

$$\text{TAC} = \frac{K_1'}{D_c} + K_2' D_c^2 + \frac{K_3'}{D_c^3} \tag{8.41}$$

To minimize TAC, take the derivative of Eq. (8.41) with respect to D_c and set it equal to zero.

$$\left[\frac{\partial \text{TAC}}{\partial D_c}\right]_{\text{ND3}} = -\frac{K_1'}{D_c^2} + 2K_2' D_c - \frac{3K_3'}{D_c^4} = 0 \tag{8.42}$$

Rearranging Eq. (8.42),

$$D_c^5 - \frac{K_1'}{2K_2'} D_c^2 - \frac{3K_3'}{2K_2'} = 0 \tag{8.43}$$

The optimum cyclone body diameter is the real root of Eq. (8.43).

Example 8.9 Optimal Design of Multiple Cyclone System

Determine the number and body diameter of Stairmand cyclones that minimize the TAC in Example 8.8. Your design should achieve the same efficiency for 10-μm particles, namely, 95.2%.

Solution

Assume $M = 0.7$. From Eq. (8.39), ND3 = 14.1 m³. The capital recovery factor is 0.20/yr. Based on Eq. (8.36), $K_1 = (7{,}000)(1.08)(2)(0.2) = 3{,}024$ \$/yr-m². According to their definitions, $K_1' = (3{,}024)(0.5)(0.2)(14.1) = 4{,}265$ \$-m/yr, $K_2' = (0.08)(8{,}000)(6.4)(0.785)(165)^3/[(2)(0.65)(0.5)^2(0.2)^2(14.1)^2(1{,}000)] = 5.62 \times 10^6$ \$/yr-m², $K_3' = (72)(1.08)(2)(0.2)(14.1) = 438.5$ \$-m³/yr. Equation (8.43) becomes

$$D_c^5 - 0.000378\, D_c^2 - 0.0001167 = 0$$

The real root of this polynomial is $D_c = 0.166$ m. The number of cyclones is $N_c =$

$14.1/(0.166)^3 = 3{,}080$. Check the value of M assumed. From Eq. (8.19), $m = 0.45$, $M = 0.69$, which is sufficiently close to the assumed value. Therefore, the optimal design is 3,080 cyclones with body diameter of 0.166 m. Equation (8.41) gives the total annual cost for this design, TAC = \$276,000/yr. That is \$120,000/yr lower than the TAC for Example 8.8!

Example 8.10 Optimal Design Based on Overall Efficiency

The aerosol population entering the multiple cyclone system of Example 8.9 is log-normally distributed with MMD = 4.0 μm and $\sigma_g = 2.5$. Determine the optimum design that achieves an overall particulate removal efficiency of 70%. Characterize the aerosol population penetrating the system.

Solution

The efficiency specified is not a grade efficiency, but an overall efficiency. The relation between ND3 and η_M is more complex than Eq. (8.39). The solution involves trial and error. The computational scheme is as follows:

- Assume a value of ND3.
- Calculate K_1', K_2', and K_3'.
- Solve Eq. (8.43) for D_c ; calculate M and $N_c = \text{ND3}/D_c^3$.
- Apply Gauss-Hermite quadrature formula to estimate the resulting overall efficiency.
- If the calculated efficiency differs from the specified efficiency by less than a specified tolerance, the design is optimal; characterize the penetrating aerosol according to the scheme presented in Example 8.5.
- Otherwise, assume a new value of ND3 and repeat the procedure.

The computer program MLTCYC implements this scheme. This program calls the subprograms RTSAFE (see Example 2.9), GAULEG (see Section 4.3.1), and HERMIT (see Example 7.4) The input data are defined as follows:

PENR = specified overall penetration, fraction
MMD = mass median diameter of initial aerosol, mm
SIGG = σ_g, dimensionless

Q = volumetric flow rate, m^3/s
LAMBDA = mean free path, μm
TEMP = temperature, K

DENP = particle density, kg/m^3
VIS = gas viscosity, kg/m-s
K1P = $K_1K_aK_b$
K2P = $K_2tN_H\rho_fQ^3/(2EK_a^2K_b^2)$
K3P = K_3

The optimal design for an overall efficiency of 70% is N_c = 1,310 cyclones, D_c = 0.30 m. The total annual cost is TAC = $155,700. The penetrating aerosol is log-normally distributed with MMD = 2.39 μm and σ_g = 2.16.

```
      BLOCK DATA
       REAL PENR, MMD, SIGG, Q, LAMBDA, TEMP, DENP, VIS, K1P, K2P,K3P,K
       COMMON /BLOCK1/ PENR, MMD, SIGG, Q, LAMBDA, TEMP, DENP, VIS,
      * K1P, K2P, K3P,K1PP, K2PP, K3PP, K
       DATA PENR/0.30/, MMD/4.00/, SIGG/2.50/, Q/165./, LAMBDA/0.1500/,
      * TEMP/450.0/, DENP/1600.0/, VIS/2.48E-5/, K1P/302.40/,
      * K2P/1.11E9/, K3P/31.10/, K/551.3/
       END

      PROGRAM MLTCYC
       REAL PENR, MMD, SIGG, K, Q, LAMBDA, TEMP, DENP
       REAL VIS, K1P, K2P, K3P, X1, X2, XACC, K1PP, K2PP, K3PP
       REAL CA, CB, DIAM, ND3, DPI, RTSAFE, RTBIS, SUMA
       REAL W, Y, G, D, TAU, ENE, PEN, Q0
       INTEGER N, NC
       LOGICAL SUCCES
       EXTERNAL FUNCD, FUNCP,FUN
       COMMON /BLOCK1/ PENR, MMD, SIGG, Q, LAMBDA, TEMP, DENP, VIS,
      * K1P, K2P, K3P,K1PP,K2PP,K3PP,K
       COMMON DIAM
       PARAMETER ( N = 10, FACTOR=1.6, NTRY = 50)
       DIMENSION Y(N), W(N)
       TAU1=(MMD*1.0E-6)**2*DENP/(18.*VIS)
       X1=1.43*TAU1*Q*K/(-0.5*LOG(PENR))**2.86
       X2=X1/SIGG
       F1 = FUNCP(X1)
       F2 = FUNCP(X2)
       SUCCES = .TRUE.
       DO 1 J =1, NTRY
         IF(F1*F2 .LT. 0) GO TO 2
         IF(ABS(F1) .LT. ABS(F2)) THEN
            X1 = X1+FACTOR*(X1-X2)
            F1=FUNCP(X1)
```

```
      ELSE
         X2=X2+FACTOR*(X2-X1)
         F2=FUNCP(X2)
      ENDIF
1   CONTINUE
    SUCCES = .FALSE.
2   IF (SUCCES) THEN
      XACC=1.0E-3
      ND3=RTBIS(FUNCP,X1,X2,XACC)
      PRINT *, 'ND3 = ', ND3
         NC=INT(ND3/(DIAM**3))
         DIAM=(ND3/NC)**(1./3.)
         TAC = K1PP/DIAM +K2PP*DIAM**2 + K3PP/DIAM**3
         PRINT *, ' NUMBER OF CYCLONES = ', NC
         PRINT *, ' DIAMETER OF EACH CYCLONE = ', DIAM
         PRINT *, ' TOTAL ANNUAL COST = ' , TAC
         PAUSE 'PRESS THE ENTER KEY TO CONTINUE'
5        PRINT *, 'ENTER PARTICLE SIZE, ENTER 0.0 TO TERMINATE
         READ *, DPI
         IF (DPI .NE. 0.0) THEN
           Y1=0.0
           CALL GAULEG (Y1,DPI,Y,W,N)
           ENE=1.-(1.-0.67*DIAM**0.14)*(TEMP/283.)**0.3
           SUMA=0.0
           DO 10 I=1,N
             D=Y(I)
             IF (0.55*D/LAMBDA .GT. 80.) THEN
               CUN = 1. + 2.*LAMBDA/D
             ELSE
               CUN=1.+(2.*LAMBDA/D)*(1.257+0.4*EXP(-0.55*D/
     *         LAMBDA))
             ENDIF
             TAU=CUN*(1.0E-6*D)**2*DENP/(18.*VIS)
             ARGUM=2.*((K*Q*TAU*(ENE+1))/ND3)**(1./
     *         (2.*ENE+2.))
             IF (ARGUM .GT. 80.0) THEN
               PEN = 0.0
             ELSE
               PEN = EXP(-ARGUM)
             ENDIF
             Q0=(1./(D*SQRT(2.*3.1416)*LOG(SIGG)))*EXP(-(
     *         LOG(D/MMD)/(SQRT(2.)*LOG(SIGG)))**2)
             SUMA=SUMA+W(I)*PEN*Q0
10         CONTINUE
           G=SUMA/PENR
```

```
          PRINT *, 'MASS CUMULATIVE FRACTION= ', G
          PRINT *, 'FOR PARTICLE SIZE = ', DPI
          GO TO 5
        END IF
        GO TO 15
      END IF
      PRINT *, ' FAILURE IN BRACKETING OPTIMUM DIAMETER'
      GO TO 15
15    STOP
      END

      FUNCTION FUNCP(ND3)
      REAL K1P,K2P,K3P,K1PP,K2PP,K3PP,ND3,K,MMD, LAMBDA, NMD
      LOGICAL SUCCES
      EXTERNAL FUNCD, FUN
      COMMON /BLOCK1/ PENR, MMD, SIGG, Q, LAMBDA, TEMP, DENP, VIS,
     * K1P, K2P, K3P,K1PP, K2PP, K3PP,K
      PARAMETER ( N = 10)
      COMMON DIAM
      DIMENSION X(N), A(N)
        K1PP=K1P*ND3
        K2PP=K2P/ND3**2
        K3PP=K3P*ND3
        CA=K1PP/(2.*K2PP)
        CB=(3.*K3PP)/(2.*K2PP)
      X1=0.0
      X2=1.0
      XACC=1.0E-3
      CALL ZBRAC1(FUN,X1,X2,SUCCES,CA,CB)
      IF (SUCCES) THEN
        DIAM=RTSAFE(FUNCD,X1,X2,XACC,CA,CB)
        EPS=1.0E-6
        CALL HERMIT(N,X,A,EPS)
        ENE=1.-(1.-0.67*DIAM**0.14)*(TEMP/283.)**0.3
        NMD = EXP(LOG(MMD)-3*(LOG(SIGG))**2)
        SUMA=0.0
        DO 10 I=1,N
          D=EXP(SQRT(2.)*LOG(SIGG)*X(I)+LOG(NMD))
          IF (0.55*D/LAMBDA .GT. 80.0) THEN
            CUN = 1. + 2.*LAMBDA/D
          ELSE
            CUN=1.+(2.*LAMBDA/D)*(1.257+0.4*EXP(-0.55*D/
     *            LAMBDA))
          ENDIF
          TAU=CUN*(1.0E-6*D)**2*DENP/(18.*VIS)
          ARGUM=(2.*((K*Q*TAU*(ENE+1))/ND3)**(1./
```

```
     *          (2.*ENE+2.)))
          IF (ARGUM .GT. 80.0) THEN
            PEN = 0.0
          ELSE
            PEN=EXP(-ARGUM)
          ENDIF
          SUMA=SUMA+A(I)*PEN*EXP(3.*SQRT(2.)*LOG(SIGG)*X(I))
 10     CONTINUE
        PENC=SUMA/(SQRT(3.1416)*EXP(4.5*(LOG(SIGG))**2))
        ABC=PENC-PENR
        FUNCP=ABC
      ENDIF
      END

      SUBROUTINE FUNCD(X,FN,DF,CA,CB)
      FN=X**5-CA*X**2-CB
      DF=5.*X**4-2.*CA*X
      RETURN
      END

      FUNCTION FUN(X,CA,CB)
      FUN=X**5-CA*X**2-CB
      RETURN
      END

    FUNCTION RTBIS(FUNCP,X1,X2,XACC)
      © 1986 by Numerical Recipes Software. Reprinted with permission from Numerical
      Recipes: The Art of Scientific Computing, Cambridge University Press, New York (1986).
      PARAMETER (JMAX=40)
      FMID=FUNCP(X2)
      F=FUNCP(X1)
      IF(F*FMID.GE.0.) PAUSE 'Root must be bracketed for bisection.'
      IF(F.LT.0.)THEN
        RTBIS=X1
        DX=X2-X1
      ELSE
        RTBIS=X2
        DX=X1-X2
      ENDIF
      DO 11 J=1,JMAX
        DX=DX*.5
        XMID=RTBIS+DX
        FMID=FUNCP(XMID)
        IF(FMID.LE.0.)RTBIS=XMID
        IF(ABS(DX).LT.XACC .OR. FMID.EQ.0.) RETURN
```

```
11  CONTINUE
    PAUSE 'too many bisections'
    END
C
    SUBROUTINE ZBRAC1(FUN,X1,X2,SUCCES,CA,CB)
    © 1986 by Numerical Recipes Software. Reprinted with permission from Numerical
    Recipes: The Art of Scientific Computing, Cambridge University Press, New York (1986).
    EXTERNAL FUN
    PARAMETER (FACTOR=1.6,NTRY=50)
    LOGICAL SUCCES
    IF(X1.EQ.X2)PAUSE 'You have to guess an initial range'
    F1=FUN(X1,CA,CB)
    F2=FUN(X2,CA,CB)
    SUCCES=.TRUE.
    DO 11 J=1,NTRY
      IF(F1*F2.LT.0.)RETURN
      IF(ABS(F1).LT.ABS(F2))THEN
        X1=X1+FACTOR*(X1-X2)
        F1=FUN(X1,CA,CB)
      ELSE
        X2=X2+FACTOR*(X2-X1)
        F2=FUN(X2,CA,CB)
      ENDIF
11  CONTINUE
    SUCCES=.FALSE.
    RETURN
    END
```

8.7 CONCLUSION

Cyclones are very simple devices for particulate removal. They are effective to remove relatively big particles from waste gas streams. The pressure drop as the gas flows through the cyclone is the main factor limiting the removal efficiency achieved. Multiple cyclone systems exhibit high removal efficiencies at moderate pressure drops. Careful selection of the number and size of cyclones operating in parallel in a multiple cyclone system can result in significant annual savings. When these devices are used as pre-cleaners for more sophisticated particulate control equipment, such as electrostatic precipitators or fabric filters, the size distribution of the aerosol penetrating the cyclonic device must be characterized. The next chapter shows how the design of an electrostatic precipitator depends on the entering aerosol population characteristics.

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PROBLEMS

The problems at the end of each chapter have been grouped into four classes (designated by a superscript after the problem number)

Class a: Illustrates direct numerical application of the formulas in the text.
Class b: Requires elementary analysis of physical situations, based on the subject material in the chapter.
Class c: Requires somewhat more mature analysis.
Class d: Requires computer solution.

8.1^{a}. Ideal, laminar cyclonic flow

A stream of 15 m^3/s of air at 298 K and 1 atm flows in laminar cyclonic flow through a duct where the radii are 0.5 m and 1.0 m, and the height is 2.0 m. What angle of turn is necessary to collect 20-μm particles with perfect efficiency? The particle density is 2,000 kg/m^3.

Answer: 14.35 rad

8.2^{a}. Ideal, turbulent cyclonic flow

Repeat Problem 8.1 assuming turbulent flow. What angle of turn is necessary to collect 20-μm particles with 99% efficiency?

Answer: 88 rad

8.3^{b}. Collection efficiency for a cyclone of standard proportions

A cyclone with a body diameter of 1.0 m and with Stairmand standard proportions processes air at 298 K and 1 atm, which carries particles with a density of 1,000 kg/m^3.

The gas velocity in the entrance duct is 20.0 m/s. The number of turns that the gas makes while flowing through a cyclone of standard proportions, N_t, is given by

$$N_t = \frac{H + h}{2a}$$

and $\theta_f = 2\pi N_t$ rad. Estimate the collection efficiency for 10-μm particles assuming

(a) Ideal, laminar cyclonic flow

Answer: 56%

b) Ideal, turbulent cyclonic flow

Answer: 38.2%

c) Leith-Licht semiempirical model applies

Answer: 81.3%

8.4b. Alternative form of Leith-Licht model

Show that, for a multiple cyclone system, Eq. (8.20) can be rewritten as

$$\eta(D_p) = 1 - \exp\left\{-2\left[\frac{KQ\tau}{MN_cD_c^3}\right]^{\frac{M}{2}}\right\}$$

where

N_c = the number of cyclones

τ = relaxation time for a particle of diameter D_p, which is given by

$$\tau = \frac{D_p^2 \rho_p C_c}{18\mu}$$

8.5^b. Design of a cyclone of standard proportions

Design a cyclone to remove 80% of particles of 20 μm diameter and density 1,500 kg/m³ from a stream of 20 m³/s of air at 298 K and 1 atm. Determine suitable values for the major dimensions of the cyclone, assuming Swift standard configuration has been chosen.

Answer: H = *18.25 m*

8.6[b]. Design of a cyclone of standard proportions

Repeat Problem 8.5 for Lapple standard configuration.

Answer: h = *7.71 m*

8.7[d]. Overall efficiency of a cyclone of standard proportions

Estimate the overall efficiency for the conditions of Problem 8.5 if the particles are log-normally distributed with MMD = 15 μm and $\sigma_g = 2.0$. Approximate the integral with a 16-point Gauss-Hermite quadrature formula.

Answer: 73.8%

8.8[a]. Pressure drop through a cyclone

Estimate the pressure drop and the power to overcome it if the mechanical efficiency of the motor-blower system is 60%

(a) For Problem 8.5

Answer: 17.8 kW

(b) For Problem 8.6

Answer: 18.3 kW

8.9[d]. Pressure drop and overall efficiency

A stream of air at 298 K and 1 atm flows at the rate of 10 m³/s and carries with it particles with a density of 2000 kg/m3. The particles are log-normally distributed with MMD = 10 μm and $\sigma_g = 2.5$. You must design a cyclone of Swift standard configuration to serve as a precleaner for this stream. A blower is available with a motor rated at 20 kW and a mechanical efficiency of 65%. Incorporating this blower in your design, specify the cyclone body diameter and the overall efficiency achievable.

Answer: 76.1%

8.10[b]. Saltation effect

Equations (8.20) and (8.21) suggest that as the size of a cyclone decreases, which means higher inlet velocities, the grade efficiency will continue to increase and approach 100% in the limit. However, it is known that this is not so. There is a limit to the inlet velocity above which further increase results in a decrease in collection efficiency. This is due to the reentrainment of particles by the saltation effect described by Kalen and Zenz (Kalen, B. and Zenz, F.A. *A.I.Ch.E. Sympos. Ser.*, **70**:388, 1974). Licht (1980) proposed the following empirical correlation to estimate the inlet velocity, v_M, that results in the maximum cyclone collection efficiency, just before the saltation effect becomes important:

$$v_M = 3{,}025 \frac{\mu \rho_p}{\rho_f^2} \frac{K_b^{1.2}}{1 - K_b} D_c^{0.201}$$

Consider the multiple cyclone system of Example 8.6. Estimate v_M for those conditions, and compare it to the actual inlet velocity.

Answer: v_M *= 26.7 m/s*

8.11[d]. Overall efficiency of a multiple cyclone system.

The aerosol population entering the multiple cyclone system of Example 8.6 is log-normally distributed with MMD = 4.0 μm and $\sigma_g = 2.5$. The particulate loading at the entrance of the system is 0.028 kg/m³. Estimate the rate at which the collected dust must be removed from the system, in kg/d. Neglect the Cunningham correction factor.

Answer: 308,600 kg/d

8.12[c]. Effects of hopper evacuation on the performance of multiple cyclone systems

Tucker et al. (*JAPCA* **39**:1614, 1989) showed that the collection efficiency of multiple cyclone systems increases when a small amount of the gas flow (typically less than 15%) is withdrawn from the dust hopper. This hopper evacuation flow then goes through a small baghouse before joining the remainder of the system exhaust flow and going out the stack. They demonstrated the concept experimentally in a system containing 10 cyclones of 0.25-m body diameter and 3.61-m overall body height. The air flow rate was 52 m³/min at 298 K and 101.3 kPa. The particulate matter was spherical glass beads with a density of 2,200 kg/m³. The size distribution function was approximately log-normal with MMD = 12 μm and σ_g = 2.0. With no hopper evacuation, the measured overall efficiency was 89.5%. With 14% hopper evacuation, the overall efficiency increased to 95%. The authors suggest that the improved performance with hopper evacuation is due to a higher gas velocity and a more uniform gas distribution through the individual cyclones.

(a) Estimate the value of the geometric configuration parameter, *K*, for the cyclones in the experimental work described and compare it to the Swift and Stairmand standard configurations.

(b) If the improved performance was due solely to the effect of higher gas velocities, estimate the percent increase in gas velocity to explain the observed efficiency of 95%.

Answer: 190%

8.13[a]. Cost of a single cyclone

Consider the cyclone of Problem 8.9. Calculate the total capital investment and total annual cost. The useful life of the cyclone is 5 yr and the minimum attractive return on investment is 20%/yr. The cyclone operates 8,000 h/yr; the cost of electricity is $0.06/kW-h.

Answer: TAC = $37,800

8.14[b]. Optimization of multiple cyclone system design

Repeat Example 8.9, but for Swift standard configuration. Calculate the number of cyclones and body diameter that minimize the total annual cost.

Answer: TAC = $312,000/yr

8.15[b] Effect of the cost of electricity on multiple cyclone system design

Repeat Example 8.9, but for a cost of electricity of $0.04/kW-h. Calculate the number of cyclones and body diameter that minimize the TAC.

Answer: D_c = *0.192 m*

8.16[d]. Optimization of multiple cyclone system based on overall efficiency

Repeat Example 8.10 for cyclones of Swift standard configuration. Calculate the number and size of cyclones that minimize TAC. Estimate TAC, and MMD and σ_g for the aerosol penetrating the system.

Answer: TAC = $175,800

8.17[d] Multiple cyclone system for a Portland cement kiln

The cement dust from a Portland cement kiln is log-normally distributed with MMD = 12 μm and $\sigma_g = 3.08$, having a density of 1,500 kg/m³ (Licht 1980). The waste gas from the kiln flows at the rate of 377 m³/min, at 394.4 K and 101.3 kPa, and its properties are similar to those of air. The particulate loading of the waste gas is 0.023 kg/m³. To satisfy the corresponding NSPS, 99.8% of the particulate matter in the gas must be removed. A multiple cyclone system, followed by a fabric filter or an ESP is being considered for this purpose.

Design the multiple cyclone system for overall efficiencies of 80%, 85%, and 90%. Specify, for each case, which standard configuration to use, the size and number of cyclones, TCI, and TAC. Characterize the size distribution of the penetrating aerosol, and calculate the particulate loading at the outlet. The following data apply:

- Useful life, 10 yr; no salvage value
- Minimum attractive return on investment, 20%
- 8,400 h/yr of operation
- Cost of electricity, $0.08/kW-h