7

Fundamentals of Particulate Emissions Control

7.1 INTRODUCTION

Particulates constitute a major class of air pollutants. Particles have a diversity of shapes and sizes; they can be either liquid droplets or dry dusts, with a wide variety of physical and chemical properties. They are emitted from many sources including both combustion and non-combustion industrial processes. In addition, primary gaseous emissions may react in the atmosphere to form secondary species that nucleate to form particles or condense on preexisting ones. An important class of industrial gas-cleaning processes remove particles from exhaust gas streams, and such processes are the subject of the following chapters. This chapter presents information about certain characteristics of particles and particulate behavior in fluids, with particular emphasis on those that are relevant to the engineering task of separating and removing particles from a stream of gas.

7.2 CHARACTERISTICS OF PARTICLES

Your objectives in studying this section are to

1. Understand the importance of an aerosol size distribution.
2. Characterize an aerosol size distribution with data from a cascade impactor.
3. Develop and apply a log-normal size distribution function.

An aerosol is a suspension of small particles in air or another gas. Important aerosol characteristics include size, size distribution, shape, density, stickiness, corrosivity, reactivity, and toxicity. From the viewpoint of air pollution, the most important of these is the particle size distri bution. The most common aerosols cover a wide range of sizes—from 0.001 μm to 100 μm. As mentioned in Chapter 1, the effects of aerosols on human health and visibility are strongly size-dependent, with particles in the range of 0.1 to 1.0 μm being the worst.

In addition to average particle concentration per unit atmospheric volume, it is important to note the size distribution by particle count and by mass. Such distributions for a typical atmospheric particulate sample are shown in Table 7.1. From data in the last two entries, particles in the 0 to 1-μm range constitute only 3% by mass. However, the number of particles in that range is overwhelming compared with the rest of the sample. Particles of this size range are capable of entering the lungs. *From a health standpoint, it is not so much a question of lowering the overall atmospheric dust loading in an urban area but of decreasing the heavy particulate count in the smaller size range.*

Table 7.1 Particle Distribution of a Typical Atmospheric Sample

Size range (μm)	Average size (μm)	Particle count[a]	Mass percent
10-30	20	1	27
5-10	7.5	112	53
3-5	4	167	12
1-3	2	555	5
0.5-1	0.75	4,215	2
0-0.5	0.25	56,900	1

[a] Count of other sizes relative to count of 20-μm size.
Source: Wark and Warner (1981).

As can be expected from such a wide range of sizes, one type of particulate collection device might be better suited than others for a specific aerosol. Furthermore, the collection efficiency of these devices depends on particle size, with bigger particles usually removed more efficiently, as shown on Figure 7.1. Thus, to calculate the overall collection efficiency of a device, it is imperative to have good information on the size distribution of the particles.

A good device to obtain this information is a cascade impactor. It separates and sizes suspended particles in a manner similar to the way that sieves separate and size samples of sand. Air with particles is drawn through a series of stages that consist of slots and impaction plates as shown on Figure 7.2. Each successive stage has narrower slots and closer plates so that each successive stage captures increasingly smaller particles. The masses of particles collected on all stages are then used to determine the size distribution of the aerosol. It must be emphasized that a cascade impactor sizes particles according to their aerodynamic diameters rather than their physical size. The aerodynamic diameter is a measure of the reaction of a particle to inertial forces.

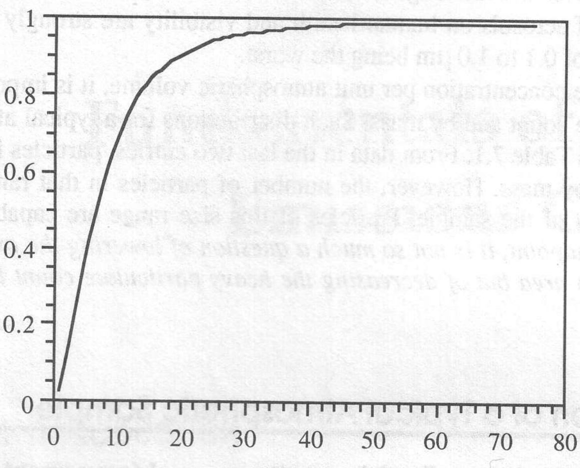

Figure 7.1 Collection efficiency versus particle size for a typical mechanical collector

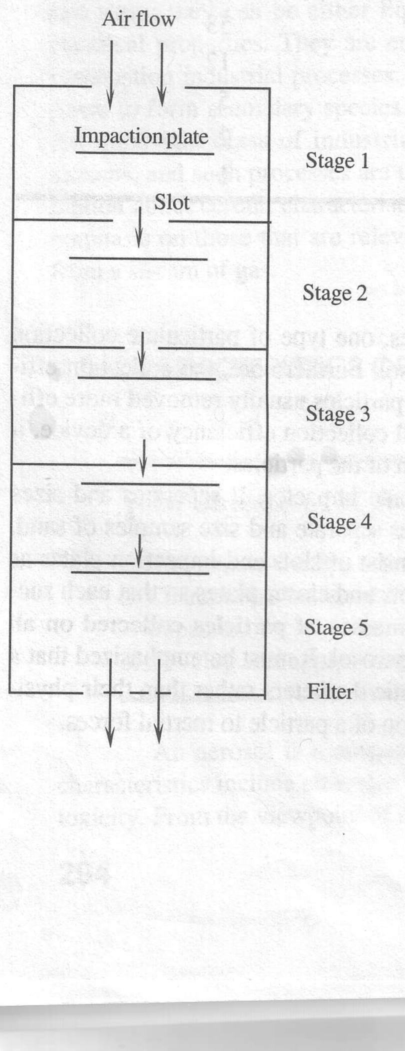

Figure 7.2 Schematic diagram of a cascade impactor

Because the ultimate objective of our analysis is to remove particles from a flowing stream of gas, the aerodynamic diameter is more relevant than the actual dimensions of the particle.

An aerosol population can be treated as if its size variation is essentially continuous. The normalized size distribution function, $n(D_p)$, can be defined as follows:

$n(D_p)\ dD_p$ = fraction of the total number of particles per unit volume of air having diameters in the range D_p to $D_p + dD_p$

Integrating over all particle sizes:

$$1.0 = \int_0^{\infty} n(D_p)\, dD_p \tag{7.1}$$

The units of $n(D_p)$ are μm^{-1}. The normalized distribution of particle mass with respect to particle size is defined as follows:

$n_m(D_p)dD_p$ = mass-fraction of particles having diameters in the range D_p to $D_p + dD_p$

The cumulative frequency distribution function, $F(D_p)$, is the fraction of the number of particles with diameters smaller or equal to D_p. Therefore,

$$F(D_p) = \int_0^{D_p} n(D_p')\, dD_p' \tag{7.2}$$

The cumulative mass distribution function, $G(D_p)$, is the mass-fraction of the particles with diameters smaller or equal than D_p. Then,

$$G(D_p) = \int_0^{D_p} n_m(D_p')\, dD_p' \tag{7.3}$$

Because particle sizes in an aerosol population typically vary over several orders of magnitude, it is often convenient to express the size distribution in terms of the natural logarithm of the diameter, $\ln D_p$ In a particular incremental particle size range D_p to $D_p + dD_p$ the fraction of particles is a certain quantity, and that quantity is the same regardless of how the size distribution function is expressed. Thus,

$$n(D_p)dD_p = n(\ln D_p)\, d(\ln D_p) \tag{7.4}$$

Because $d(\ln D_p) = dD_p/D_p$

$$D_p n(D_p) = n(\ln D_p) \tag{7.5}$$

The next question that arises is: What functions are commonly used to represent particle size distributions? A popular function for this purpose is the log-normal distribution. If a quantity u is normally distributed, the probability function for u obeys the Gaussian distribution:

$$n(u) = \frac{1}{\sqrt{2\pi}\,\sigma_u} \exp\left[-\frac{(u - u_m)^2}{2\sigma_u^2}\right] \tag{7.6}$$

where

u_m is the mean value of the distribution

σ_u is the standard deviation

A quantity that is log-normally distributed has its logarithm governed by a normal distribution. If the quantity of interest is particle diameter D_p, then saying that an aerosol population is log-normally distributed means that $u = \ln D_p$ satisfies Eq. (7.6):

$$n(\ln D_p) = \frac{1}{\sqrt{2\pi}\,\ln \sigma_g} \exp\left[-\frac{(\ln D_p - \ln D_{pm})^2}{2(\ln \sigma_g)^2}\right] \tag{7.7}$$

The physical significance of the parameters D_{pm} and σ_g will be discussed shortly. It is more convenient to express the size distribution function in terms of D_p rather than $\ln D_p$. Combining Eqs. (7.5) and (7.7):

$$n(D_p) = \frac{1}{\sqrt{2\pi}\,D_p \ln\sigma_g} \exp\left[-\frac{(\ln D_p - \ln D_{pm})^2}{2(\ln\sigma_g)^2}\right] \tag{7.8}$$

For a normally distributed quantity, the cumulative frequency distribution function, $F(u)$, is

$$F(u) = \frac{1}{\sqrt{2\pi}\,\sigma_u} \int_{-\infty}^{u} \exp\left[-\frac{(u' - u_m)^2}{2\sigma_u^2}\right] du' \tag{7.9}$$

To evaluate this integral, let $\eta = (u' - u_m)/2^{1/2}\,\sigma_u$, then,

$$F(u) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{(u-u_m)/\sqrt{2}\,\sigma_u} \exp\left(-\eta^2\right) d\eta \tag{7.10}$$

Integrating in terms of the error function, erf η (see Problem 6.1),

$$F(u) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{u - u_m}{\sqrt{2}\,\sigma_u}\right)\right] \tag{7.11}$$

For the log-normal distribution, $u = \ln D_p$, so Eq. (7.11) can be expressed as

$$F(D_p) = \frac{1}{2} + \frac{1}{2}\,\text{erf}\left[\frac{\ln(D_p/D_{pm})}{\sqrt{2}\ln\sigma_g}\right] \tag{7.12}$$

It is evident from Eq. (7.12) that $F(D_{pm}) = 0.5$. Thus D_{pm} is the number median diameter (NMD) defined as the diameter for which exactly one-half the particles are smaller and one-half are larger. To understand the significance of σ_g, consider that diameter $D_{p\sigma}$ for which $\sigma_g = D_{p\sigma}/D_{pm}$. At that diameter

$$F(D_{p\sigma}) = \frac{1}{2} + \frac{1}{2}\,\text{erf}\left(\frac{1}{\sqrt{2}}\right) = 0.841 \tag{7.13}$$

Thus, σ_g is the ratio of the diameter below which 84.1% of the particles lie to the number median diameter. $D_{p\sigma}$ is one standard deviation from the median, so σ_g is called the *geometric standard deviation*.

It can be shown (Seinfeld and Flagan 1988) that if the number size distribution function is log-normal, then the mass size distribution function is also log-normal with the same geometric standard deviation and the *mass median diameter* (MMD) given by

$$\ln \text{MMD} = \ln \text{NMD} + 3\left(\ln \sigma_g\right)^2 \tag{7.14}$$

Therefore,

$$G(D_p) = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left[\frac{\ln(D_p/\text{MMD})}{\sqrt{2} \ln \sigma_g}\right] \tag{7.15}$$

The log-normal distribution has the useful property that when the cumulative distribution function, either F or G, is plotted against the logarithm of particle diameter on a special graph paper with one axis scaled according to the error function, so-called *log-probability paper*, a straight line results. Such a plot with actual data from an aerosol population obtained with a cascade impactor serves two purposes: (1) to determine if the log-normal model fits the data, and (2) if so, to estimate the parameters MMD and σ_g.

Example 7.1 Analysis of data from a cascade impactor

The following data were obtained from a cascade impactor run on a sample from an aerosol population (Cooper and Alley 1986):

Size range (μm)	Mass (mg)
0–2	4.5
2–5	179.5
5–9	368
9–15	276
15–25	73.5
> 25	18.5

Show that a log-normal distribution fits the data, and estimate the corresponding values of MMD, NMD, and σ_g.

Solution

Prepare a table of particle size versus cumulative mass fraction, G, less than the stated size, as follows:

D_p (μm)	G (%)
2.0	0.5
5.0	20.0
9.0	60.0
15.0	90.0
25.0	98.0

A plot of these data is presented in Figure 7.3, using log-probability scales. The resulting straight line is evidence that a log-normal distribution is an adequate model for the size distribution function.

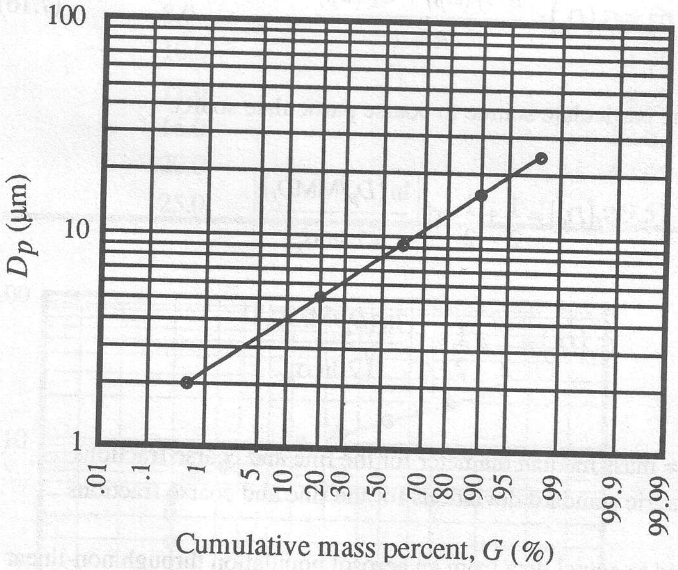

Figure 7.3 Log-normal distribution, data of Example 7.1

From Figure 7.3, MMD = 8.0 μm, $D_{p\sigma}$ = 14 μm. Therefore, σ_g = 14/8 = 1.75. From Eq. (7.14), NMD = 3.13 μm.

Comments

Although there is no completely satisfactory theoretical explanation for it, many investigators have reported that the distribution of several quantities related to environmental pollution, such as particle size, ambient air quality data, indoor radon measurements, stream water quality data, phosphorus in lakes, radio nuclides in soil, trace metals in human tissue, lung function reaction to ozone, and others often appear log-normal (Larsen et al. 1991; Ott 1990).

Example 7.2 The log-bimodal size distribution function

It has been observed that frequently the log-normal size distribution becomes less accurate to describe the end of the distribution representing fine particulate. Kerr (1989) suggested to overcome this difficulty by breaking the size distribution into a fine and a coarse log-normal distributions—a log-bimodal distribution. This is a five-parameter model given by:

$$G(D_p) = \frac{R\,G_f(D_p) + G_c(D_p)}{R+1} \tag{7.16}$$

where

R = mass ratio of fine particulate source to coarse particulate source

$$G_f(D_p) = \frac{1}{2} + \frac{1}{2}\,\text{erf}\left[\frac{\ln(D_p/\text{MMD}_f)}{\sqrt{2}\,\ln\sigma_{gf}}\right]$$

$$G_c(D_p) = \frac{1}{2} + \frac{1}{2}\,\text{erf}\left[\frac{\ln(D_p/\text{MMD}_c)}{\sqrt{2}\,\ln\sigma_{gc}}\right]$$

MMD_f and MMD_c = mass median diameter for the fine and coarse fractions

σ_{gf} and σ_{gc} = geometric standard deviations for the fine and coarse fractions

This model can be fitted to actual data from an aerosol population through non-linear regression techniques. Software packages for performing this task are plentiful. Obviously, the validity of this approach should be tested by calculating some criteria, such as the standard error of the estimate, that tests the goodness of fit. Consider the data on Table 7.2 characterizing an aerosol population. Figure 7.4 shows these data plotted on log-probability scales. It is evident from it that the distribution is not log-normal. Using a nonlinear regression program a log-bimodal distribution is fitted to the data. The best estimate of the parameters are: $R = 0.01023$, $\text{MMD}_f = 0.5028$ μm, $\text{MMD}_c = 11.29$ μm, $\sigma_{gf} = 1.202$, and $\sigma_{gc} = 1.353$. The standard error of the estimate is 0.42%. The solid line on Figure 7.4 corresponds to the size distribution predicted by the log-bimodal model whereas the circles illustrate the actual experimental data. The model is remarkably good.

Table 7.2 Log-Bimodal Size Distribution Data

Particle size (μm)	$G \times 100$
0.3	0.00252
0.5	0.4950
0.7	0.9579
1.0	0.9901
2.0	1.012
3.0	1.035
4.0	1.039
5.0	1.343
6.0	2.808
7.0	6.636
8.0	13.59
10.0	34.99
12.0	58.28
15.0	82.67
20.0	97.05
25.0	99.57

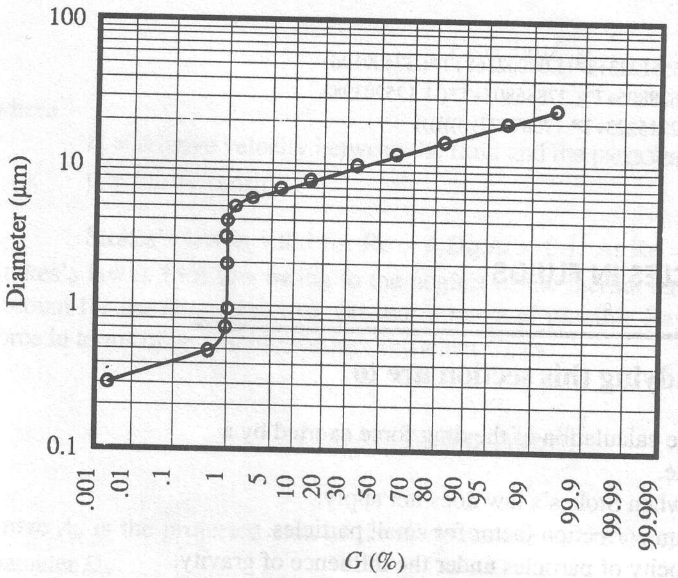

Figure 7.4 Log-bimodal particle size distribution of Example 7.2

Comments

Flagan and Seinfeld (1988) observed that particulate size distributions in the flue gases of pulverized-coal combustion systems exhibit two distinct peaks, one in the submicron-size range, and one in the 3- to 50-μm range. According to them, submicron ash constitutes less than 2% of the total fly ash mass. Ash residue particles that remain when the carbon burns out account for the large-diameter fraction, whereas ash volatilization followed by nucleation and coagulation into very small particles accounts for the fine fraction.

The following computer program, based on a very compact and efficient subroutine presented by Press et al. (1989), estimates the error function with a fractional error everywhere less than 10^{-7}.

```
PROGRAM ERFUNC
PRINT *, ' ENTER VALUE OF X '
READ *, X
IF (X . GT. 0) THEN
ERF = 1.- ERFCC(X)
ELSE
ERF = ERFCC(X) -1.
END IF
PRINT *, ' THE VALUE OF ERF(X) IS ', ERF
END
FUNCTION ERFCC(X)
Z=ABS(X)
T=1./(1.+0.5*Z)
ERFCC=T*EXP(-Z*Z-1.26551223+T*(1.00002368+T*(.37409196+
*  T*(.09678418+T*(-.18628806+T*(.27886807+T*(-1.13520398+
*  T*(1.48851587+T*(-.82215223+T*.17087277)))))))))
IF (X.LT.0.) ERFCC=2.-ERFCC
RETURN
```

7.3 DYNAMICS OF PARTICLES IN FLUIDS

Your objectives in studying this section are to

1. Apply Stokes's law to the calculation of the drag force exerted by a fluid on a moving particle.
2. Estimate the drag force when Stokes's law does not apply.
3. Calculate the Cunningham correction factor for small particles.
4. Estimate the settling velocity of particles under the influence of gravity.

Particles are often separated from a fluid as part of a pollution control system. In air pollution control, particles can be removed by gravity settlers, centrifugal collectors, fabric filters, electrostatic precipitators, or wet scrubbers. In all of these devices, particles are separated from the surrounding fluid by the application of one or more external forces. These forces—gravitational, inertial, centrifugal, and electrostatic—cause the particles to accelerate away from the direction of the mean fluid flow. The particles must then be collected and removed from the system to prevent ultimate reentrainment into the fluid. Thus, design and operation of particulate control equipment require a basic understanding of the dynamics of particles in fluids.

7.3.1 Drag Force

A good place to start to study the dynamical behavior of aerosol particles in a fluid is to consider the drag force exerted on a particle as it moves in a fluid. To calculate this force, the equations of fluid motion must be solved to obtain the velocity and pressure fields around the particle, a formidable task. These equations can be solved analytically only at very low velocities, when viscous forces dominate inertial forces. The type of flow that results is called *creeping flow* or low-Reynolds number flow.

The solution of the equations of motion for the velocity and pressure distribution around a sphere in creeping flow was first obtained by Stokes. The drag force, which is the net force exerted by the fluid on the particle in the direction of flow, can be calculated once the velocity and pressure fields are known. The result, known as *Stokes's law*, is (Bird et al. 1960):

$$F_D = 3\pi\mu D_p u_r \tag{7.17}$$

where

u_r = relative velocity between the fluid and the particle
μ = fluid viscosity

Stokes's law is valid for $\mathrm{Re} = u_r D_p \rho/\mu < 0.1$. At Re = 1.0, the drag force predicted by Stokes's law is 13% low owing to the neglect of the inertial terms in the equation of motion. To account for the drag force over the entire range of possible Reynolds numbers, express the drag force in terms of an empirical drag coefficient C_D as

$$F_D = C_D A_p \rho \frac{u_r^2}{2} \tag{7.18}$$

where A_p is the projected area of the body normal to the flow. Thus, for a spherical particle of diameter D_p

$$F_D = \frac{\pi}{4} C_D \rho D_p^2 \frac{u_r^2}{2} \tag{7.19}$$

Table 7.3 summarizes some of the correlations available for the drag coefficient as a function of the Reynolds number.

Table 7.3 Correlations for Drag Coefficient of Spherical Particles

Range of Reynolds Number	Correlation for C_D
Re < 0.1 (Stokes's law)	$\frac{24}{Re}$
$0.1 < Re < 2$	$\frac{24}{Re}\left(1+\frac{3}{16}Re+\frac{9}{160}Re^2 \ln 2Re\right)$
$2 < Re < 500$	$\frac{24}{Re}\left(1+0.15 Re^{0.687}\right)$
$500 < Re < 2 \times 10^5$	0.44

Source: Flagan and Seinfeld 1988.

7.3.2 Noncontinuum Effects

Aerosol particles are small. The particle size is often comparable to the distances that gas molecules travel between collisions with other gas molecules. Consequently, the basic continuum transport equations must be modified to account for non continuum effects. The Knudsen number, $Kn = 2\lambda_g/D_p$, where λ_g is the mean free path of the gas, is the key dimensionless number in this respect.

The mean free path of a gas can be calculated from the kinetic theory of gases as

$$\lambda_g = \frac{0.1145\ \mu}{P\sqrt{\frac{M}{T}}} \tag{7.20}$$

where P is the gas pressure in kPa, and M is the gas molecular weight. For example, for air at 298 K and 1 atm the mean free path is 6.51×10^{-8} m = 0.0651 μm. Stokes's law derives from the equations of continuum fluid mechanics. When the particle diameter approaches the same order as the mean free path of the suspending gas molecules, the resisting force offered by the fluid is

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Table 7.
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Table 7.4 Cu

D_p (μm)
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7.3.3 Gravita

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smaller than that predicted by Stokes's law. To account for this effect that becomes important as D_p becomes smaller, a slip factor, C_c, also called Cunningham correction factor, is introduced into Stokes's law:

$$F_D = \frac{3\pi\mu u_r D_p}{C_c} \tag{7.21}$$

where an empirical correlation for C_c was developed, based on experiments performed by Millikan between 1909 and 1923, as (Allen and Raabe 1982)

$$C_c = 1 + \mathrm{Kn}\left[1.257 + 0.40\exp\left(-\frac{1.10}{\mathrm{Kn}}\right)\right] \tag{7.22}$$

Table 7.4 shows the value of the Cunningham correction factor for particles in air at 1 atm and 298 K.

Table 7.4 Cunningham Correction Factor for Air at 1 atm and 298 K

D_p (μm)	Knudsen number (Kn)	Cunningham factor (C_c)
0.01	13.02	22.7
0.05	2.60	5.06
0.10	1.30	2.91
0.50	0.26	1.337
1.00	0.13	1.168
5.00	0.026	1.034
10.00	0.013	1.017

7.3.3 Gravitational Settling

For a relative motion to exist between a fluid and a freely suspended particle, at least one external force must exist. Considering an external force, F_e, which is opposed by the drag force, Newton's second law of motion for a particle of mass mp can be written as

$$F_e - F_D = m_p \frac{du_r}{dt} \tag{7.23}$$

For a spherical particle in the Stokes's region, Eq. (7.21) can be substituted into Eq.(7.23) to yield

$$\frac{du_r}{dt} + \frac{18\mu}{\rho_p C_c D_p^2} u_r = \frac{F_e}{m_p} \tag{7.24}$$

Equation (7.24) can be rewritten as

$$\frac{du_r}{dt} + \frac{u_r}{\tau} = \frac{F_e}{m_p} \tag{7.25}$$

where τ is a characteristic time associated with the motion of the particle given by:

$$\tau = \frac{D_p^2 \rho_p C_c}{18\mu} \tag{7.26}$$

Equation (7.25) is the basic differential equation governing the motion of a particle in a fluid when Stokes's law applies. Consider, for example, the resultant motion when gravity is the only external force (the buoyancy force can be neglected when the fluid is a gas). Equation (7.25) becomes

$$\frac{du_r}{dt} + \frac{u_r}{\tau} = g \tag{7.27}$$

where g is the gravitational constant. If the initial relative velocity is zero, the solution to Eq. (7.27) is

$$u_r = \tau g \left[1 - \exp\left(-\frac{t}{\tau} \right) \right] \tag{7.28}$$

For $t \gg \tau$, the particle attains a constant velocity, called its terminal settling velocity, u_t,

$$u_t = \tau\, g = \frac{D_p^2 \rho_p C_c g}{18\mu} \tag{7.29}$$

Figure 7.5 illustrates the transient behavior of a particle settling under the influence of gravity. After four characteristic times, the particle's velocity is virtually equal to its terminal velocity.

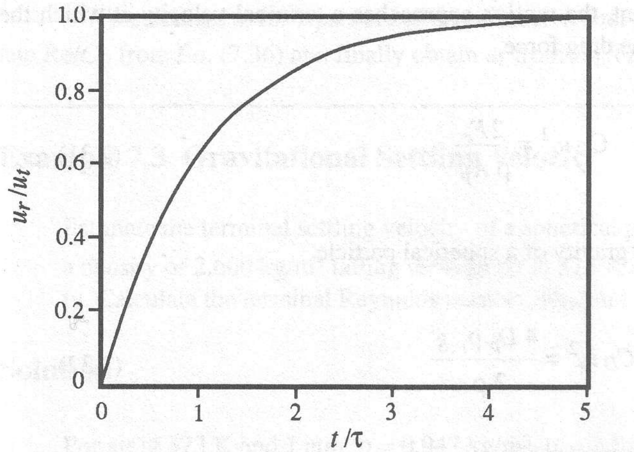

Figure 7.5 Dimensionless particle velocity versus dimensionless time

The transient portion of a particle's settling time is usually ignored. Table 7.5 shows the terminal velocity and the characteristic time for several particles in the size range of interest in air pollution control applications. Because τ is so small, it is justified to assume that the terminal velocity is attained almost instantaneously.

Table 7.5 Gravitational Settling of Unit Density Spheres in Air At 298 K and 1 atm

D_p (μm)	Characteristic time (τ, s)	Terminal velocity (m/s)
0.1	8.8×10^{-8}	8.6×10^{-7}
0.5	1.0×10^{-6}	1.0×10^{-5}
1.0	3.6×10^{-6}	3.5×10^{-5}
5.0	7.9×10^{-5}	7.8×10^{-4}
10.0	3.1×10^{-4}	3.1×10^{-3}

For a particle larger than 10 to 20 μm settling at its terminal velocity, the Reynolds number is too high for the Stokes's regime analysis to be valid. The drag coefficient is a useful way to represent the drag force on a particle over the entire range of Reynolds number. Newton's second law of motion can be rewritten in terms of the drag coefficient:

$$m_p \frac{du_r}{dt} = F_e - \frac{1}{2} C_D \rho A_p u_r^2 \tag{7.30}$$

If the external force is constant, the motion approaches a terminal velocity at which the external force is exactly balanced by the drag force,

$$C_D u_t^2 = \frac{2F_e}{\rho A_p} \qquad (7.31)$$

For terminal settling owing to gravity of a spherical particle,

$$C_D u_t^2 = \frac{4 D_p \rho_p g}{3 \rho} \qquad (7.32)$$

Because C_D depends on u_t through Re, this equation can not be solved explicitly for u_t. Instead, Eq. (7.32) must be solved for u_t by trial and error or through the following procedure. Define a new dimensionless number, the Galileo number.

$$\mathrm{Ga} = C_D \mathrm{Re}^2 = C_D u_t^2 \left(\frac{D_p \rho}{\mu}\right)^2 \qquad (7.33)$$

Substituting Eq. (7.32) in Eq. (7.33),

$$\mathrm{Ga} = \frac{4 D_p^3 \rho \rho_p g}{3 \mu^2} \qquad (7.34)$$

Another useful relation between C_D and Re is:

$$\frac{\mathrm{Re}}{C_D} = \frac{\mathrm{Re}^3}{\mathrm{Ga}} = \frac{3 \rho^2 u_t^3}{4 g \rho_p \mu} \qquad (7.35)$$

The following correlation due to Koch can be used to relate Re/C_D to Ga (Licht 1980):

$$\ln \sqrt[3]{\frac{\mathrm{Re}}{C_D}} = -3.194 + 2.153 \ln \mathrm{Ga}^{1/3} - 0.238 \left(\ln \mathrm{Ga}^{1/3}\right)^2 + 0.01068 \left(\ln \mathrm{Ga}^{1/3}\right)^3 \qquad (7.36)$$

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7.3.4 Collection

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To calculate u_t for a particle of any diameter, first calculate the value of Ga. Then, calculate Re/C_D from Eq. (7.36) and finally obtain u_t from Eq. (7.35).

Example 7.3 Gravitational Settling Velocity

Estimate the terminal settling velocity of a spherical particle with a diameter of 100 μm and a density of 2,600 kg/m³ falling through air at 373 K and 1 atm under the influence of gravity. Calculate the terminal Reynolds number, Re_t, and the terminal drag force on the particle.

Solution

For air at 373 K and 1 atm, $\rho = 0.947$ kg/m³, $\mu = 2.1 \times 10^{-5}$ kg/m-s. From Eq. (7.34), $Ga^{1/3} = 4.174$. Equation (7.36) yields $(Re/C_D)^{1/3} = 0.564$. From Eq. (7.35), $u_t = 0.522$ m / s. Calculate $Re_t = u_t D_p \rho/\mu = 2.35$. There are two ways to calculate the terminal drag force. The easy way is to notice that when the particle attains constant velocity, the drag force is perfectly balanced by the external force, gravity in this case. Therefore, the drag force is equal to the weight of the particle: $F_D = m_p g = 1.33 \times 10^{-8}$ N. Remember that this is true only at the terminal velocity. A more general approach to calculate the drag force is through Eq. (7.19). Calculate the drag coefficient with the corresponding correlation from Table 7.2:

$C_D = 24/Re\left(1 + 0.15\, Re^{0.687}\right) = 12.97$. Substituting in Eq. (7.19), $F_D = 1.32 \times 10^{-8}$ N. There is a slight difference between the two results due to the use of Eq. (7.36), an approximate correlation, as part of the procedure to calculate the terminal velocity.

7.3.4 Collection Of Particles By Impaction, Interception, and Diffusion

When a flowing fluid approaches a stationary object such as a fabric filter thread, a large water droplet , or a metal plate, the fluid flow streamlines will diverge around that object. Because of their inertia, particles in the fluid will tend to continue in their original direction. If the particles have enough inertia and are located close enough to the stationary object, they will collide and be collected by it.

Impaction occurs when the center of mass of a particle that is diverging from the fluid streamlines strikes a stationary object. *Interception* occurs when the particle's center of mass closely misses the object, but, because of its finite size, the particle strikes the object. Collection of particles by *diffusion* occurs when small particles (which are subject to random motion about the mean path) diffuse toward the object while passing near it. Once striking the object by any of these means, particles are collected only if there are short-range forces strong enough to hold them to the surface.

A simple means to explain impaction is with the concept of stopping distance. If a sphere in the Stokes's regime is projected with an initial velocity u_0 into a motionless fluid, its velocity as a function of time (ignoring all but the drag force) is

$$u_r = u_0 e^{-t/\tau} \tag{7.37}$$

The total distance traveled by the particle before it comes to rest is

$$x_{stop} = \int_0^\infty u_r \, dt = u_0 \tau \tag{7.38}$$

If the particle stops before striking the object, it can be swept around the object by the altered fluid flow. Because τ is very small, x_{stop} is also small. For example, if a 1.0-μm particle with unit density is projected at 10 m/s into air, it will travel only 36 μm.

An impaction parameter, N_t, can be defined as the ratio of the stopping distance of a particle (based on the upstream fluid velocity) to the diameter of the stationary object, d_0, or:

$$N_t = \frac{x_{stop}}{d_0} \tag{7.39}$$

If N_t is large, most of the particles will strike the object, otherwise, most will follow the fluid flow around it.

7.4 EFFECTIVENESS OF COLLECTION

Your objectives in studying this section are to

1. Define fractional efficiency, overall efficiency based on particle number, overall efficiency based on particle mass, and penetration.
2. Develop a relation between overall mass collection efficiency and fractional efficiency for a log-normal aerosol population.
3. Apply Gauss-Hermite quadrature formulas to the evaluation of overall mass collection efficiencies.
4. Estimate the fractional and overall collection efficiencies of settling chambers operating in the turbulent flow regime.
5. Calculate the overall collection efficiency of two or more particulate collection devices operating in series.

The success of a particulate collection system may be expressed either in terms of the amount of aerosol removed from the air stream, or the amount permitted to remain in it. The collection or removal efficiency of a device may be defined in various ways. For instance, the *fractional or grade efficiency* $\eta\,(D_p)$ is defined as:

$$\eta(D_p) = 1 - \frac{\text{number of particles of diameter } D_p \text{ out}}{\text{number of particles of diameter } D_p \text{ in}} \tag{7.40}$$

This efficiency can be expressed in terms of the particle size distribution functions at the inlet and outlet sides of the device:

$$\eta(D_p) = \frac{n_{in}(D_p)\,dD_p - n_{out}(D_p)\,dD_p}{n_{in}(D_p)\,dD_p} \tag{7.41}$$

The overall efficiency based on particle number η_N is defined as

$$\eta_N = 1 - \frac{\text{number of particles out}}{\text{number of particles in}} \tag{7.42}$$

In terms of the particle size distribution functions, the overall efficiency is

$$\eta_N = 1 - \frac{\int_0^\infty n_{out}(D_p)\,dD_p}{\int_0^\infty n_{in}(D_p)\,dD_p} \tag{7.43}$$

Combining Eqs. (7.41) and (7.43):

$$\eta_N = \frac{\int_0^\infty \eta(D_p)\,n_{in}(D_p)\,dD_p}{\int_0^\infty n_{in}(D_p)\,dD_p} \tag{7.44}$$

The overall efficiency based on particle mass η_M is defined as

$$\eta_M = 1 - \frac{\text{mass of particles out}}{\text{mass of particles in}} \tag{7.45}$$

For spherical particles of uniform density:

$$\eta_M = \frac{\displaystyle\int_0^\infty \eta(D_p)\, D_p^3\, n_{in}(D_p)\, dD_p}{\displaystyle\int_0^\infty n_{in}(D_p)\, D_p^3\, dD_p} \tag{7.46}$$

The overall collection efficiency by mass is usually the easiest to measure experimentally. The inlet and outlet streams may be sampled by a collection device, such as a filter, that collects virtually all of the particles.

The collection efficiency is frequently expressed in terms of *penetration*. The penetration is based on the amount emitted rather than on the amount collected; based on particle mass, it is just $Pt_M = 1 - \eta_M$.

The fractional efficiency $\eta(D_p)$ is, for most collectors, a unique single-valued function for a particular set of operating conditions. It depends on such parameters as the nature and design dimensions of the collector, and the rate of flow and particulate loading of the gas stream. The following chapters will develop the fractional efficiency function for the most common devices used for particulate removal from gaseous streams.

For a log-normal particle size distribution, Eq. (7.46) can be written as

$$\eta_M = \frac{\displaystyle\int_{-\infty}^\infty \eta(u)\, e^{3u}\, n_{in}(u)\, du}{\displaystyle\int_{-\infty}^\infty n_{in}(u)\, e^{3u}\, du} \tag{7.47}$$

$$\eta_M = \frac{\displaystyle\int_{-\infty}^{\infty} \eta(u)\, e^{3u}\, e^{-(u-u_m)^2/2\sigma_u^2}\, du}{\displaystyle\int_{-\infty}^{\infty} e^{3u}\, e^{-(u-u_m)^2/2\sigma_u^2}\, du} \tag{7.48}$$

where u = ln D_p, u_m = ln NMD and $\sigma_u = \ln \sigma_g$. The denominator of Eq. (7.48) is easily evaluated (see Problem 7.7), leading to:

$$\eta_M = \frac{\displaystyle\int_{-\infty}^{\infty} \eta(u)\, e^{3u}\, e^{-(u-u_m)^2/2\sigma_u^2}\, du}{\sqrt{2\pi}\, \sigma_u\, e^{3u_m}\, e^{9\sigma_u^2/2}} \tag{7.49}$$

Equation (7.49) can be further simplified (see Problem 7.12):

$$\eta_M = \frac{\displaystyle\int_{-\infty}^{\infty} \eta(u)\, e^{-(u-\bar{u})^2/2\sigma_u^2}\, du}{\sqrt{2\pi}\, \sigma_u} \tag{7.50}$$

where $\bar{u}$ = ln MMD. Define a new variable v such that:

$$v = \frac{u-\bar{u}}{\sqrt{2}\, \sigma_u} \tag{7.51}$$

In terms of this variable, Eq. (7.50) becomes:

$$\eta_M = \frac{\displaystyle\int_{-\infty}^{\infty} \eta(v) e^{-v^2}\, dv}{\sqrt{\pi}} \tag{7.52}$$

For a given fractional efficiency function, the integral of Eq. (7.52) can be approximated numerically using the Gauss-Hermite quadrature method (Carnahan et al. 1969). Kerr (1981,

1989) and Benítez (1988) have used this technique where the log-normal distribution applies. The integral becomes:

$$\int_{-\infty}^{\infty} \eta(v)e^{-v^2}dv \cong \sum_{i=1}^{N} w_i\eta(v_i) \tag{7.53}$$

where w_i and v_i are the weight factors and roots of the Nth degree Hermite polynomial (Abramowitz and Stegun 1972). As a general rule, the accuracy of the numerical integration increases with increasing polynomial degree. Benítez (1988) suggested that an 8-point formula was adequate for preliminary design purposes.

Example 7.4 Overall Mass Collection Efficiency Calculations

The fractional efficiency function of a particulate removal device is given by:

$$\eta(D_p) = 1 - \exp\left(-0.000651D_p^2\right) \tag{7.54}$$

where D_p is in microns. The device processes a log-normally distributed aerosol with a MMD of 50 μm and σ_g of 2.5. Estimate the overall mass collection efficiency. Use an 8-point Gauss-Hermite quadrature formula to estimate the integral.

Solution

The procedure to estimate the overall mass collection efficiency is as follows:

- Choose the number of quadrature points to use, N.
- Obtain the values of the roots, v_i, and weight factors, w_i, of the corresponding Hermite polynomial either from a mathematical table or from a computer program provided subsequently.
- For each of the roots, calculate the corresponding u_i from Eq. (7.51).
- Calculate $D_{pi} = \exp u_i$.
- Calculate η (D_{pi}) from Eq. (7.54).
- Calculate $w_i\,\eta$ (D_{pi}) for $i = 1, 2, ..., N$.
- Calculate η_M from Eqs. (7.52) and (7.53).

The following table summarizes the calculations for an aerosol population with MMD = 50 μm and $\sigma_g = 2.5$.

vi
–2.93063
–1.98165
–1.15719
–0.38119
0.38119
1.15719
1.98165
2.93063

$\eta_M = 1.$

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vi	D_{pi} (μm)	η (D_{pi})	*wi*	*wi* η (D_{pi})
−2.93063	1.12	0.0008	0.00020	0
−1.98165	3.83	0.0095	0.01708	0.00016
−1.15719	11.16	0.0779	0.207802	0.01619
−0.38119	30.51	0.4545	0.66115	0.30050
0.38119	81.94	0.9870	0.66115	0.65260
1.15719	223.97	1.0000	0.207802	0.20780
1.98165	651.89	1.0000	0.01708	0.01708
2.93063	2,246.10	1.0000	0.00020	0.00020
				Σ = 1.19453.

$\eta_M = 1.19453/\pi^{1/2} = 0.6739$ (67.39%)

Comments

Notice the symmetry of the roots and weight functions of the Hermite polynomials. This characteristic simplifies computer implementation of the method. The following is a computer subroutine to calculate the roots and weight factors of the Nth degree Hermite polynomial.

```
      SUBROUTINE HERMIT(NN,X,A,EPS)
C     CALCULATES THE ZEROES X(I) OF THE NN-TH ORDER
C     HERMITE POLYNOMIAL. ALSO CALCULATES THE CORRESPONDING
C     WEIGHT FACTOR A(I) FOR GAUSS-HERMITE QUADRATURE
C
      DIMENSION X(50), A(50)
      FN = NN
      N1 = NN-1
      N2 = (NN+1)/2
      CC = 1.7724538509*GAMMA(FN)/(2.**N1)
      S = (2.*FN+1.)**.16667
      DO 11 I =1, N2
       IF (I .EQ. 1) THEN
        XT = S**3 - 1.85575/S
       ELSE IF (I .EQ. 2) THEN
          XT = XT- 1.14*FN**.426/XT
       ELSE IF (I .EQ. 3) THEN
          XT = 1.86*XT-0.86*X(1)
```

```
      ELSE IF (I .EQ. 4) THEN
         XT = 1.91*XT-.91*X(2)
      ELSE
         XT = 2.*XT-X(I-2)
      END IF
      CALL HROOT(XT,NN,DPN,PN1,EPS)
      X(I) = XT
      A(I) = CC/DPN/PN1
      NI = NN-I+1
      X(NI) = -XT
      A(NI) = A(I)
 11   CONTINUE
      RETURN
      END
      FUNCTION GAM(Y)
C
      GAM = (((((((0.035868343*Y- .193527818)*Y + .482199394)*Y-
     1 .756704078)*Y + .918206857)*Y - .897056937)*Y + .988205891)*Y
     2 -.577191652)*Y + 1.0
      RETURN
      END
      FUNCTION GAMMA(X)
C     COMPUTES THE GAMMA FUNCTION OF X FOR X BETWEEN 0 AND 35.
C
      Z = X
      IF (Z .LE. 0.0 .OR. Z .GT. 35.) THEN
         GAMMA = 0.
      ELSE IF ( Z .EQ.1.) THEN
         GAMMA = 1.
      ELSE IF (Z .LT. 1.0) THEN
         GAMMA = GAM(Z)/Z
      ELSE IF (Z .GT. 1.0) THEN
         ZA = 1.
 10      Z = Z -1.
         IF ( Z .EQ. 1.) THEN
            GAMMA = ZA
         ELSE IF (Z .GT. 1.0) THEN
            ZA = ZA*Z
            GO TO 10
         ELSE IF (Z .LT. 1.0) THEN
            GAMMA = ZA*GAM(Z)
         END IF
      END IF
      RETURN
      END
      SUBROUTINE HROOT(X,NN,DPN,PN1,EPS)
```

```
C     IMPROVES THE APPROXIMATE ROOT X
C     DPN = DERIVATIVE OF H(N) AT X
C     PN1 = VALUE OF H(N-1) AT X
C
      ITER = 0
 1    ITER = ITER + 1
      CALL HRECUR(P,DP,PN1,X,NN)
      D = P/DP
      X = X- D
      IF( ABS(D) .GT. EPS .AND. ITER .LT. 10) THEN
        GO TO 1
      ELSE
        DPN = DP
      END IF
      RETURN
      END
      SUBROUTINE HRECUR(PN,DPN,PN1,X,NN)
      P1 = 1.
      P = X
      DP1 = 0.
      DP = 1.
      DO 1 J =2, NN
        FJ = J
        FJ2 = (FJ-1.)/2.
        Q = X*P-FJ2*P1
        DQ = X*DP + P - FJ2*DP1
        P1 = P
        P = Q
        DP1 = DP
        DP = DQ
 1    CONTINUE
      PN = P
      DPN = DP
      PN1 = P1
      RETURN
      END
```

Example 7.5 Overall Mass Collection Efficiency of Settling Chamber

The settling chamber is perhaps the simplest of all air pollution control devices. Its main usefulness lies in serving as a preliminary screening device for a more efficient control system. Where the mass of the larger particles is huge, the settling chamber can remove much of the mass of the particulate population which would otherwise choke up the other control device, impairing its operation or requiring too frequent cleaning.

The use of several trays improves the collection efficiency of a settling chamber since the particles have a much shorter distance to travel before reaching the bottom of the passage between trays. Figure 7.6 shows a settling chamber in which trays are provided.

The flow in a rectangular channel, such as the ones in a settling chamber with trays, is turbulent if the Reynolds number, $Re_c > 4{,}000$ (McCabe and Smith, 1976). For the situation depicted in Figure 7.6, the Reynolds number is (Crawford 1976)

$$Re_c = \frac{2Q\rho}{\mu(N_{tr}W + H)} \qquad (7.55)$$

where N_{tr} is the number of trays, including the bottom surface of the chamber. For turbulent flow, the fractional efficiency for particulate collection by a settling chamber is given by (see Problem 7.13):

$$\eta(D_p) = 1 - \exp\left[-\frac{N_{tr}LWu_t}{Q}\right] \qquad (7.56)$$

where u_t is the terminal velocity of a particle of diameter D_p. Notice that $N_{tr}LW = A_c$ where A_c is the total area available for particle collection. Therefore, Eq. (7.56) can be rewritten as

$$\eta(D_p) = 1 - exp\left[-\frac{A_c u_t}{Q}\right] \qquad (7.57)$$

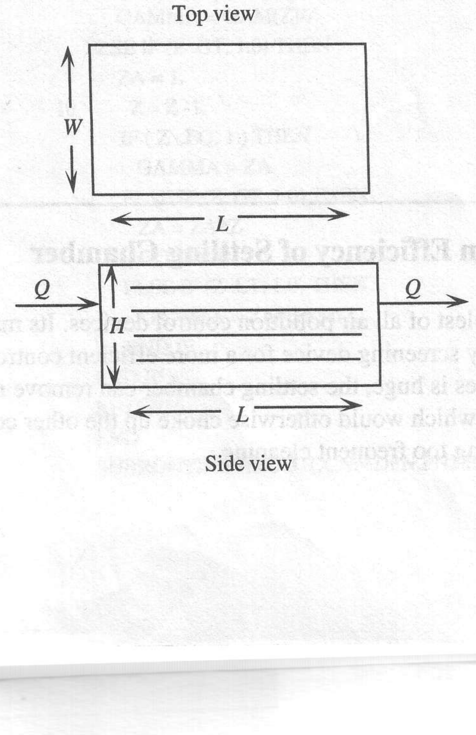

Figure 7.6 Schematic diagram of a settling chamber with trays

Consider a 5-m long settling chamber with 10 square trays. The device processes 10 m³/s of air at 298 K and 101.3 kPa, carrying a log-normal aerosol with MMD = 25 μm and $\sigma_g = 2.0$. The density of the particles is 2,000 kg/m³. The total height of the chamber is 3 m. Estimate the overall mass collection efficiency of the chamber.

Solution

The first step in the solution is to determine if the flow through the chamber is turbulent. For air at 298 K and 101.3 kPa, $\rho = 1.185$ kg/m³ and $\mu = 1.84 \times 10^{-5}$ kg/m-s. The chamber dimensions are: $L = W = 5$ m., $H = 3$ m. For $N_{tr} = 10$, Eq. (7.55) yields $\text{Re}_c = 24{,}300$ which means fully developed turbulent flow. Calculate $A_c = (10)\,(5)2 = 250$ m². Equation (7.57) becomes

$$\eta(D_p) = 1 - \exp\left[-25\,u_t\right] \tag{7.58}$$

To illustrate the concept, choose a 4-point quadrature formula to relate the fractional and overall collection efficiencies. The terminal velocities in Eq. (7.58) must be carefully calculated because Stokes's law is not valid at the higher end of the particle size range covered. At the lower end of the range, the Cunningham correction factor must be included in the calculations.

v_i	D_{pi} (μm)	u_t (m/s)	Re_t	$\eta\,(D_{pi})$	w_i	$w_i\eta(D_{pi})$
−1.65068	5	0.0015	0.0002	0.0368	0.08131	0.0030
−0.52465	15	0.0118	0.0113	0.2548	0.80491	0.2051
0.52465	41.8	0.1020	0.275	0.9220	0.80491	0.7421
1.65068	126	0.6220	5.05	1.0000	0.08131	0.0813
						$\Sigma = 1.0315$

$\eta_M = 1.0315/\sqrt{\pi} = 0.582\ (58.2\%)$

Frequently, a particulate collection system consists of two or more devices operating in series. The overall collection efficiency is not simply the sum nor the product of the efficiencies of each device. Each device's efficiency is based on the mass loading of particles entering that device, but the overall system efficiency is based on the total mass collected as a fraction of the total mass entering the first device. It can be easily shown that the overall penetration of such a system, Pt_o, is simply the product of the penetrations of all of the individual devices (Cooper and Alley 1986).

$$Pt_o = \prod_{i=1}^{n} Pt_i \qquad (7.59)$$

The overall collection efficiency of the system is $\eta_M = 1 - Pt_o$.

Example 7.6 Overall Collection Efficiency with Two Control Devices in Series

A particulate control system consists of a settling chamber with an overall mass collection efficiency of 65%, followed by an electrostatic precipitator with an overall mass collection efficiency of 95%. Calculate the overall collection efficiency for the system.

Solution

Calculate the penetrations of the individual devices: $Pt_1 = 1 - 0.65 = 0.35$, $Pt_2 = 1 - 0.95 = 0.05$. From Eq. (7.59), $Pt_o = (0.35)\,(0.05) = 0.0175$. Therefore, the overall efficiency for the system is $\eta_M = 0.9825$ (98.25%)

7.5 CONCLUSION

Removal of particulate matter from exhaust gases is a very important engineering task because particles constitute a major class of air pollutants. The most important characteristic of an aerosol population is its size distribution function. Not only are the deleterious effects of particulates dependent on their size, but the nature and design of the pollution control device appropriate for a given aerosol are highly dependent on the characteristics of the size distribution function. Most aerosols of interest in air pollution control are log-normally distributed, which is fortunate because such a function is easily characterized in terms of only two parameters.

Most particulate collection devices depend on an external force to impart on the particles a velocity component that is normal to the direction of the gas flow. Settling chambers depend on gravity for that purpose. The following chapters explore the mechanisms through which the most popular particulate control devices operate, and derive the corresponding fractional efficiency equations. Sizing and costing of particulate control equipment are covered in detail.

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PROBLEMS

The problems at the end of each chapter have been grouped into four classes (designated by a superscript after the problem number).

Class a: Illustrates direct numerical application of the formulas in the text.
Class b: Requires elementary analysis of physical situations, based on the subject material in the chapter.
Class c: Requires somewhat more mature analysis.
Class d: Requires computer solution.

7.1^a. Analysis of data from a cascade impactor

The following data were obtained when a sample of an aerosol population was analyzed with a cascade impactor:

Size range (μm)	Mass (mg)
0-4	25
4-8	125
8-16	100
16-30	75
30-50	30
> 50	5

Determine whether a log-normal distribution fits this data and, if so, estimate MMD, NMD, and σ_g.

Answer: MMD = 10.8 μm

7.2[b]. Log-normal distribution

For the aerosol population of Problem 7.1, calculate the following:

(a) Mass-fraction of particles with diameters between 5 and 10 μm.

Answer: 32.3%

(b) Fraction of the total number of particles with diameters between 1 and 5 μm.

Answer: 74.6%

7.3[b]. Modal diameter of a log-normal distribution

An important parameter of a size distribution function is the modal diameter, D_{pMO}, defined as the diameter at which the greatest number of particles is clustered. This diameter is located at the maximum point of the curve for $n(D_p)$. Show that, for a log-normal distribution,

$$D_{pMO} = \text{NMD} \exp\left[-(\ln \sigma_g)^2\right]$$

Estimate the modal diameter for the aerosol population of Problem 7.1.

Answer: 1.58 μm

7.4[d]. Log-bimodal size distribution

The following data were obtained with a cascade impactor

D_p (µm)	G (%)
0.5	5.4
1.0	14.4
2.0	23.4
4.0	37.0
8.0	55.1
10.0	61.4
15.0	72.7
20.0	79.6
25.0	84.6
50.0	94.8

Fit a log-bimodal distribution function to these data and comment on the goodness of fit of the model.

7.5[b] Log-bimodal size distribution

An aerosol population results from the combination of particulate matter from two distinct sources. The coarser source is log-normally distributed with MMD = 10 µm and σ_g =3.0. The finer source is also log-normally distributed with σ_g = 2.5, but unknown MMD. Twenty-three percent of all the mass originates at the fine particle source. It was found experimentally that the combined, cumulative mass fraction up to 3 µm was 30.8%. Estimate the value of MMD for the fine source.

7.6[a]. Urban aerosols

The size distribution of urban aerosols containing photochemical smog are usually bimodal. The "fine particle" mode—less than 2µm—contains from one-third to two-thirds of the total mass, with the remainder in the "coarse particle" mode. The fine particles are produced by photochemical atmospheric reactions and the coagulation of combustion products. The coarse particles are mainly of mechanical origin.

The aerosol over Pasadena, California, was sampled on September 3, 1969 under light to moderate smog conditions. The MMD of the fine particle mode was 0.3 µm with a σ_g of 2.05. The corresponding parameters for the coarse particles were 8.0 µm and 2.3, respectively (Hinds 1982). The cumulative mass fraction up to 1.0 µm was 55%.

Estimate what fraction of the total atmospheric aerosol was of photochemical origin.

Answer: 57.3%

7.7[b]. Mass concentration of a log-normal aerosol population

An aerosol with a log-normal size distribution has a NMD of 0.3 μm and a σ_g of 1.5. If the number concentration is 10^6 particles/cm^3, what is the mass concentration? Particles may be assumed spherical with a density of 4,500 kg/m^3. The following identity may be useful:

$$\int_{-\infty}^{\infty} e^{ru} e^{-(u-\bar{u})^2/2\sigma_u^2}\, du = \sqrt{2\pi}\, \sigma_u e^{r\bar{u}} e^{(r^2\sigma_u^2/2)}$$

Answer: 133 μg/m^3

7.8[a]. Terminal gravitational settling velocity

(a) Estimate the terminal gravitational settling velocity of a unit-density, 200-μm diameter sphere in air at 298 K and 1 atm.

Answer: 0.68 m/s

(b) For a 0.15-μm diameter spherical particle (ρ_p = 2,500 kg/m^3) determine the Cunningham correction factor and the terminal settling velocity in air at 298 K and 1 atm.

Answer: 3.64 × 10^{-6} m/s

7.9[a]. Dynamic shape factors

A correction factor called the dynamic shape factor, χ, is applied to Stokes's law to account for the effect of shape on particle motion. Stokes's law for irregular particles becomes

$$F_D = \frac{3\pi\mu u_r D_p \chi}{C_c}$$

where D_p is the diameter of a spherical particle with the same volume as the irregularly shaped particle. The following table gives dynamic shape factors for particles of various shapes.

Shape	Dynamic shape factor
Sphere	1.00
Cube (*L/D* = 4)	1.08
axis horizontal	1.32
axis vertical	1.07
Bituminous coal dust	1.05-1.11
Quartz	1.36
Sand	1.57
Talc	2.04

Source: Davies, C. N. *J. Aerosol Sci.*, **10**:477 (1979).

An old industrial hygiene rule of thumb is that a 10-μm silica particle settles in atmospheric air at a rate of 1 cm/s. What is the true settling velocity of such a particle? The specific gravity of silica is 2.6. Use the dynamic shape factor for quartz.

Answer: 0.566 cm/s

7.10^b. Gravitational settling velocity

Atmospheric air is dried by bubbling it through concentrated sulfuric acid (ρ_p = 1,840 kg/m^3).The acid container is a 0.1-m diameter, 2-m long tube which holds 1.5 L of acid. The air flow rate is 10 L/min. When the bubbles burst at the liquid surface, they form droplets. What is the largest droplet that can be carried out of this system?

Answer: 19.5 μm

7.11^b.Terminal velocity for electrically charged particles

When a particle possessing an electrical charge q_p enters a region where an electric field of strength E_c is also present, an electrostatic force F will act on the particle. The magnitude of this force is given by $F = q_pE_c$, where F is in newtons, q_p in coulombs, and E_c in volts/m. Estimate the terminal velocity in air at 298 K and 1 atm of a 1.0-μm diameter particle with a charge of 0.3×10^{-15} coulombs under the influence of an electric field of 10^5 V/m.

Answer: 0.2 m/s

7.12[b]. Overall efficiency based on particle mass for log-normal function

Derive Eq. (7.50), which relates overall mass efficiency to fractional efficiency for a log-normal distribution, beginning with Eqs. (7.49) and (7.14).

7.13[c]. Turbulent flow in settling chambers

Derive Eq. (7.56) for the fractional efficiency of particulate collection by a settling chamber operating in the turbulent flow regime. Assume that there is a laminar layer adjacent to the bottom surface of the passage into which turbulent eddies do not penetrate, so that any particle that crosses into this layer is captured shortly. In the remainder of the flow passage, the eddying motion owing to turbulence will cause a uniform distribution of particles of all sizes. The vertical component of the velocity of the particles in the laminar layer is the corresponding terminal settling velocity. Horizontally, the particles move at the average velocity of the gas through the passage.

7.14[d]. Gauss-Hermite quadrature for overall efficiency estimation

Write a computer program to implement the method outlined in Example 7.4 to estimate the overall mass collection efficiency for a device with a given fractional collection efficiency equation when it operates on a log-normally distributed aerosol population. The user should be able to specify the fractional efficiency function, the parameters of the log-normal distribution, and the number of quadrature points up to $N = 50$. Test your program with the information on Example 7.4. Increase the number of points to 16 and observe the effect on the overall efficiency predicted.

7.15[d]. Overall efficiency of a settling chamber

Write a computer program to estimate the overall mass collection efficiency of a settling chamber operating in the turbulent flow regime. Assume that the aerosol is log-normally distributed. Estimate the integral with an N-point Gauss-Hermite quadrature formula. Test your program with the information on Example 7.5. Increase the number of points to 8 and observe the effect on the overall efficiency predicted.

7.16[b]. Overall penetration of a log-bimodal aerosol population

Kerr (1989) showed that the overall mass penetration of a log-bimodal aerosol population through a particulate control device can be estimated by:

$$Pt_M \cong \frac{R\sum_{i=1}^{N} w_i Pt(v_{fi}) + \sum_{i=1}^{N} w_i Pt(v_{ci})}{\sqrt{\pi}\,(R+1)}$$

where v_{ci} and v_{fi} are as defined by Eq. (7.51) for the coarse and fine fractions respectively. Consider a particulate control device with a fractional penetration function given by

$$Pt(D_p) = \exp(-0.066174 - 78{,}371 D_p)$$

where D_p is in meters. The aerosol is log-bimodal with $MMD_f = 0.5028$ μm, $MMD_c = 11.29$ μm, $\sigma_{gf} = 1.202$, $\sigma_{gc} = 1.353$, and $R = 0.01023$. Estimate the overall mass collection efficiency of the device based on a 5-point quadrature formula.

Answer: 61%

7.17[b] Particulate matter deposition in the alveolar region

Table 7.6 shows the fraction of inhaled particles deposited in the alveolar region for nose breathing at a rate of 14 L/min as a function of particle size D_p. Estimate how much mass deposits in a person's alveolar region daily owing to breathing local air which contains 150 mg/m^3 of a log-normally distributed aerosol with MMD = 2.72 μm and $\sigma_g = 1.649$.

Answer: 0.54 mg/d

Table 7.6 Fractional Deposition of Particles in Alveolar Region

D_p (μm)	Fraction	D_p (μm)	Fraction
0.10	0.20	1.2	0.21
0.20	0.15	1.3	0.22
0.30	0.12	1.4	0.23
0.40	0.12	1.5	0.25
0.50	0.12	2.0	0.28
0.60	0.12	3.0	0.26
0.70	0.15	4.0	0.19
0.80	0.18	5.0	0.11
0.9	0.20	6.0	0.03
1.0	0.20	7.0	0.0
1.1	0.20	8.0	0.0

Source:Hattis et al. *JAPCA* **37**:1060 (1987).

7.18[a]. Particulate control devices in series

Particulate removal efficiency on a certain gas stream must be 98.5% to satisfy emission standards. If a 60%-efficient cyclone precleaner is used with a wet scrubber, what is the required efficiency of the scrubber?

Answer: 96.2%

7.19[c]. Design of a settling chamber

Design a settling chamber to serve as a precleaner for an electrostatic precipitator (ESP). The removal efficiency for the system must be at least 98%. The ESP efficiency is 96.7% for a gas flow rate of 10 m^3/s of air at 298 K and 1 atm. The aerosol entering the settling chamber is log-normally distributed with MMD = 15.0 μm and $\sigma_g = 2.5$, and a particle density of 2,000 kg/m^3. Because of floor space limitations, the dimensions of the trays in the settling chamber cannot exceed 4 m. The tray spacing must be 0.3 m. Calculate the chamber dimensions and the number of trays required.

7.20[b]. Optimal design of a settling chamber

Crawford (1976) showed that, when there are no space limitations, the optimal design of a settling chamber operating in the turbulent flow regime is given by:

$$L = W = \sqrt{\frac{A_c}{N_{tr}}}$$

$$N_{tr} = \frac{A_c^{1/3}}{(2\Delta H)^{2/3}}$$

where A_c is the total collection area and ΔH is the tray spacing. Calculate the optimal dimensions and number of trays of a settling chamber to collect 50-μm particles with 90% efficiency. The gas flow rate is 25 m³/s of air at 298 K and 1 atm. The particle density is 2,000 kg/m³. The tray spacing is 0.3 m.

Answer: L = *5.53 m*