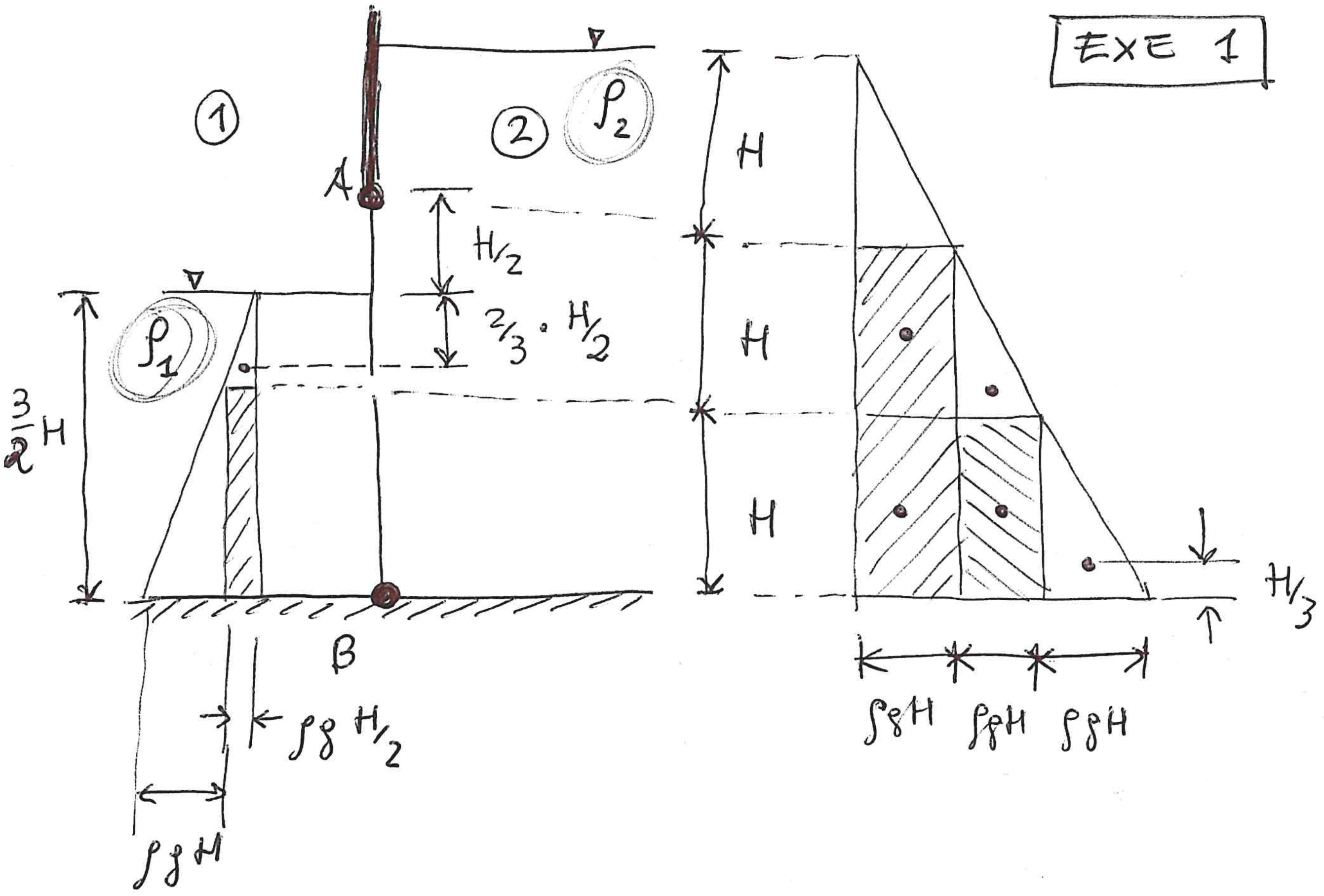


PARATIA SUPERIORE :

Momento prodotto dal liquido sul lato 1 :

$$M_{①}^{up} = \left(\rho_1 g\,\frac{H}{2}\cdot\frac{H}{2}\cdot b\right)W \quad \text{con} \quad b = \frac{H}{2}+\frac{H}{3} = \frac{5}{6}H$$

$$= \rho_1 g W H^3 \cdot \frac{5}{24}$$

Momento prodotto dal liquido sul lato 2 :

$$M_{②}^{up} = \Big(\underbrace{\rho_2 g H\cdot H\cdot\frac{H}{2}}_{\text{Area rettangolo superiore}} + \underbrace{\rho_2 g H\cdot\frac{H}{2}\cdot\frac{2}{3}H}_{\text{Area triangolo superiore}}\Big)W$$

$$= \rho_2 g W H^3 \cdot \frac{5}{6} \left[> M_{①}^{up} \text{ se } \rho_1 = \rho_2 = \rho \,!\right)$$

Momento netto agente sulla paratia superiore:

$$\Delta M = M_{2}^{up} - M_{1}^{up} = 5\rho g W H^3\left(\frac{1}{6} - \frac{1}{24}\right) \qquad \underline{\rho_1 = \rho_2 = \rho}$$

$$= \frac{5}{8}\rho g W H^3 \quad (\text{rotaz. senso } \underline{\text{ORARIO}})$$

Affinché il momento netto sia nullo, invece, il rapporto tra le densità dei due liquidi dovrà essere tale da avere:

$$M_{1}^{up} = M_{2}^{up}$$

$$\frac{5}{24}\rho_1 \cancel{g W H^3} = \frac{5}{6}\rho_2 \cancel{g W H^3}$$

$$\boxed{\rho_1/\rho_2 = \frac{5}{6}\cdot\frac{24}{5} = 4}$$

PARATIA INFERIORE:

$$M_{1}^{down} = \left(\rho_1 g \frac{H}{2}\cdot H\cdot\frac{H}{2} + \rho_1 g H\cdot\frac{H}{2}\cdot\frac{H}{3}\right)W$$

$$= \rho_1 g W \frac{H^3}{4} + \rho_1 g W \frac{H^3}{6} = \rho_1 g W H^3 \cdot \frac{5}{12}$$

$$M_{2}^{down} = \left(2\rho_2 g H\cdot H\cdot\frac{H}{2} + \rho_2 g H\cdot\frac{H}{2}\cdot\frac{H}{3}\right)\cdot W =$$

$$M_{②}^{down} = \rho_2 g W H^3 + \rho_2 g W \frac{H^3}{6} = \frac{7}{6} \rho_2 g W H^3$$

Momento netto agente sulla paratia inferiore

se $\rho_1 = \rho_2 = \rho$:

$$\Delta M = M_{②}^{down} - M_{①}^{down} = \frac{7}{6} \rho g W H^3 - \frac{5}{12} \rho g W H^3$$

$$= \frac{3}{4} \rho g W H^3$$ (rotaz. senso ANTIORARIO)

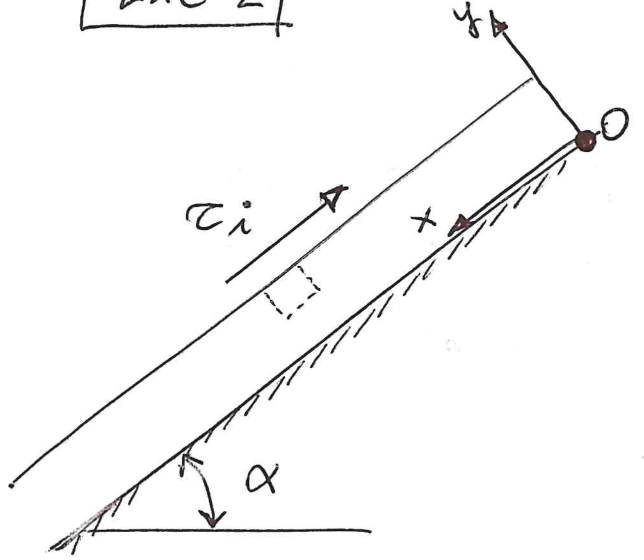
Affinché il momento netto sia nullo:

$$M_{①}^{down} = M_{②}^{down} \Rightarrow \frac{5}{12} \rho_1 \cancel{g W H^3} = \frac{7}{6} \rho_2 \cancel{g W H^3}$$

$$\boxed{\rho_1 / \rho_2 = \frac{7}{6} \cdot \frac{12}{5} = 14/5}$$

1

EXE 2



$v_x \neq 0$; $v_y = v_z = 0$

$\frac{\partial \cdot}{\partial z} = 0$; $\frac{\partial \cdot}{\partial t} = 0$

$\frac{\partial p}{\partial x} = -\rho g \operatorname{sen}\alpha$

[5%] com eq.

$$v_x(y) = \frac{1}{2\mu}(-\rho g \operatorname{sen}\alpha)\, y^2 + C_1 y + C_2$$

- $v_x(y=0) = 0 \Rightarrow C_2 = 0$
- $\tau_{yx}(y=\delta) = -\tau_i \Rightarrow \mu \left.\frac{\partial v_x}{\partial y}\right|_{y=\delta} = -\tau_i$

$$\left.\frac{\partial v_x}{\partial y}\right|_{y=\delta} = \left.\left[\frac{1}{\mu}(-\rho g \operatorname{sen}\alpha)\, y + C_1\right]\right|_{y=\delta} = -\frac{\rho g \delta \operatorname{sen}\alpha}{\mu} + C_1$$

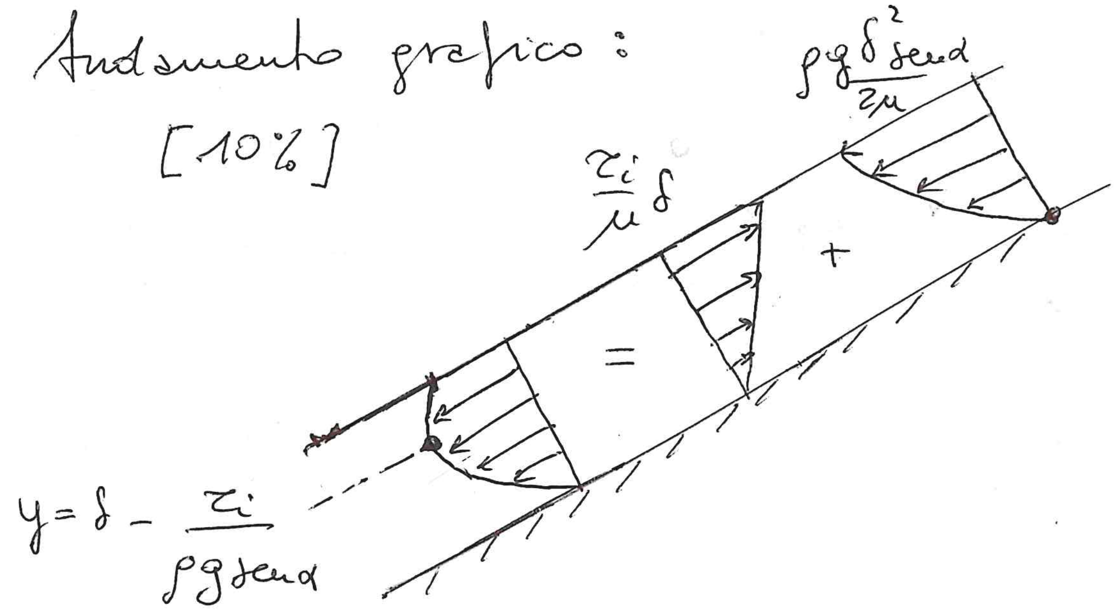
$$C_1 = \frac{1}{\mu}\left(-\tau_i + \rho g \delta \operatorname{sen}\alpha\right)$$

[10%]

$$\boxed{v_x(y) = \frac{1}{2\mu}(-\rho g \operatorname{sen}\alpha)(y^2 - 2\delta \cdot y) - \frac{\tau_i}{\mu} \cdot y}$$

Andamento grafico:

[10%]



$$\tau_{yx} = \mu \frac{\partial v_x}{\partial y} = \mu \left[\frac{1}{2\mu}(-\rho g \operatorname{sen}\alpha)(2y - 2\delta) - \frac{\tau_i}{\mu} \right]$$

$$= \rho g \operatorname{sen}\alpha\,(\delta - y) - \tau_i$$

Il taglio si annulla per $\boxed{y = \delta - \frac{\tau_i}{\rho g \operatorname{sen}\alpha}}$

$$Q = \int_0^w \int_0^\delta v_x(y)\,dy\,dz = \left(\frac{\rho g \delta^3 \operatorname{sen}\alpha}{3\mu} - \frac{\tau_i}{2\mu}\delta^2 \right) W$$

$$\frac{d(Q/W)}{d\delta} = 0 \Rightarrow \frac{\rho g \delta^2 \operatorname{sen}\alpha}{\mu} - \frac{\tau_i}{\mu}\delta = 0$$

Soluz. $\delta = 0$ (NA) e $\boxed{\delta = \tau_i / \rho g \operatorname{sen}\alpha}$ Acc

2] Per un tale valore di δ, la portata trasferita verso il basso risulta:

$$\boxed{\frac{Q}{W} = \frac{\rho g \operatorname{sen}\alpha}{3\mu} \cdot \left(\frac{\tau_i}{\rho g \operatorname{sen}\alpha}\right)^3 - \frac{\tau_i}{2\mu}\left(\frac{\tau_i}{\rho g \operatorname{sen}\alpha}\right)^2}$$

$$= \frac{1}{3} \cdot \frac{\tau_i^3}{\mu(\rho g \operatorname{sen}\alpha)^2} - \frac{1}{2}\frac{\tau_i^3}{\mu(\rho g \operatorname{sen}\alpha)^2}$$

$$= \boxed{-\frac{1}{6} \cdot \frac{\tau_i^3}{\mu(\rho g \operatorname{sen}\alpha)^2}}$$

[10%] $\begin{cases} \text{Calcolo } \delta + \\ \text{Calcolo } Q_{max} \end{cases}$

SVOLGIMENTO CON SISTEMA DI RIFERIMENTO SULL'INTERFACCIA:

- $v_x(y=\delta) = 0$
- $\tau_{yx}(y=0) = +\tau_i \Rightarrow \mu \left.\frac{\partial v_x}{\partial y}\right|_{y=0} = \tau_i$

$$\mu \left.\frac{\partial v_x}{\partial y}\right|_{y=0} = \mu \cdot C_1 \Longrightarrow \boxed{C_1 = \frac{\tau_i}{\mu}}$$

Dall'altra c.c.:

$$0 = \frac{1}{2\mu}(-\rho g \operatorname{sen}\alpha)\delta^2 + C_1 \cdot \delta + C_2$$

$$C_2 = -\frac{1}{2\mu}(-\rho g \operatorname{sen}\alpha)\delta^2 + \frac{\tau_i \cdot \delta}{\mu}$$

$$\Downarrow$$

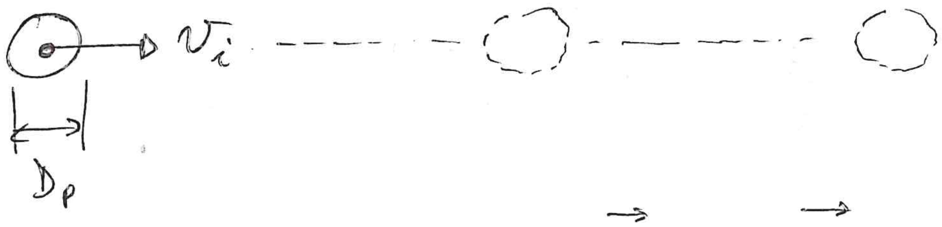
$$\boxed{v_x(y) = \underbrace{\frac{1}{2\mu}(-\rho g \operatorname{sen}\alpha)(y^2 - \delta^2)}_{\substack{\text{POISEUILLE} \\ (\geq 0)}} + \underbrace{\frac{\tau_i}{\mu}(y - \delta)}_{\substack{\text{TAGLIO} \\ (\leq 0)}}}$$

Il disegno è lo stesso di prima.

$$Q = \int_0^w \int_0^\delta v_x(y)\, dy\, dz = \frac{\rho g \delta^3 \operatorname{sen}\alpha}{3\mu} - \frac{\tau_i \delta^2}{2\mu}$$

ovvero, come era ovvio, anche la portata è esprimibile tramite la stessa equazione di prima, in quanto Q non dipende dalla particolare posizione dell'origine del rif.

Quindi anche $Q_{max} = \frac{1}{6} \cdot \frac{\tau_i^3}{\mu (\rho g \operatorname{sen}\alpha)^2}$!



Moto orizzontale : $\vec{F}_I = \vec{F}_D + \cancel{\vec{F}_p + \vec{F}_{g,ell}}$ 0 in x

$$m_p \frac{d\vec{v}_p}{dt} = \frac{1}{2} C_D \rho_f A_p (\vec{u}_f - \vec{v}_p)|\vec{u}_f - \vec{v}_p|$$

Lungo x : $$\frac{dv_{p,x}}{dt} = -\frac{v_{p,x}}{\tau_p} \qquad (u_{f,x} = 0)$$

$$\boxed{v_{p,x}(t) = v_i \cdot e^{-t/\tau_p}}$$

$$\frac{dx_p}{dt} = v_{p,x}(t) \longrightarrow \int_{x_0=0}^{x_p(t)} dx_p = \int_{0}^{t} v_{p,x}(t)\, dt$$

$$\boxed{x_p(t) = v_i \tau_p \left(1 - e^{-t/\tau_p}\right)}$$

Stopping distance : $v_{p,x} = 0$ per $t \to \infty$

$$x_p(t \to \infty) = v_i \tau_p = x_p^{max}$$

Quando $x_p(t) = \frac{x_p^{max}}{4} = \frac{v_i \tau_p}{4}$ abbiamo:

$\longrightarrow$

$$\frac{\cancel{v_i \tau_p}}{4} = \cancel{v_i \tau_p}\left(1 - e^{-t/\tau_p}\right)$$

$$e^{-t/\tau_p} = \frac{3}{4} \Rightarrow \boxed{v_p = \frac{3}{4} v_i}$$

CALCOLO $v_{p,x}(t)$ CON MASSA VARIABILE:

$$\frac{dm_p}{dt} = -C = -K\pi D_p^2 \quad ; \quad \frac{dm_p}{dt} = \rho_p \frac{\pi D_p^2}{2}\frac{dD_p}{dt}$$

$$\frac{dD_p}{dt} = -\frac{2K}{\rho_p}$$

$$D_p(t) = D_i - \frac{2k}{\rho_p}\cdot t \qquad dt = -\frac{\rho_p}{2k} dD_p$$

Il bilancio di forze diventa:

$$\frac{d\vec{v}_p}{dt} + \frac{\vec{v}_p}{m_p}\frac{dm_p}{dt} = -\frac{\vec{v}_p}{\tau_p} \Rightarrow \frac{dv_{p,x}}{dt} + \frac{v_{p,x}}{m_p}\cdot\frac{dm_p}{dt} = -\frac{v_{p,x}}{\tau_p}$$

$$\frac{dv_{p,x}}{dt} = \left(-\frac{1}{\tau_p} - \frac{1}{m_p}\cdot\frac{dm_p}{dt}\right)v_{p,x}$$

$$= \left(-\frac{18\mu}{\rho_p}\cdot\frac{1}{D_p^2} - \frac{6}{\rho_p \pi D_p^{\cancel{3}}}\left(-K\pi \cancel{D_p^2}\right)\right)v_{p,x}$$

$$= \left(\underbrace{\frac{6K}{\rho_p}}_{C_1}\cdot\frac{1}{D_p} - \underbrace{\frac{18\mu}{\rho_p}}_{C_2}\cdot\frac{1}{D_p^2}\right)v_{p,x}$$

$$\int_{v_i}^{v_{p,x}(t)} \frac{dv_{p,x}}{v_{p,x}} = \left(C_1 \cdot \frac{1}{D_p} - C_2 \cdot \frac{1}{D_p^2} \right) \left(- \frac{\rho_p}{2k} \, dD_p \right)$$

$$= \int_{D_i}^{D_p(t)} \left(-3 \cdot \frac{1}{D_p} + \frac{9\mu}{k} \cdot \frac{1}{D_p^2} \right) dD_p$$

$$\ln \left[\frac{v_{p,x}(t)}{v_i} \right] = -3 \ln \left[\frac{D_p(t)}{D_i} \right] + \frac{9\mu}{k} \left(- D_p^{-1} \right) \Big|_{D_i}^{D_p(t)}$$

$$= \ln \left(\frac{D_i}{D_p(t)} \right)^3 + \frac{9\mu}{k} \left(\frac{1}{D_i} - \frac{1}{D_p(t)} \right)$$

$$\boxed{v_{p,x}(t) = v_i \cdot \left(\frac{D_i}{D_p(t)} \right)^3 \cdot e^{\frac{9\mu}{k} \left(\frac{1}{D_i} - \frac{1}{D_p(t)} \right)}}$$