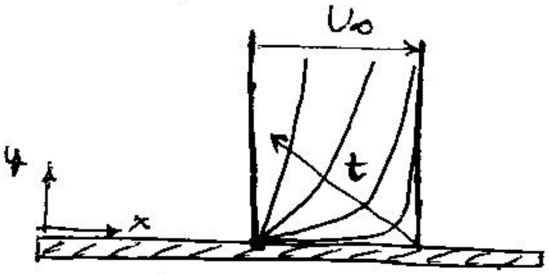
FLUIDODINAMICA

A.A. 2004/2005

SECONDA PROVETTA (25 NOV. 2005)

$\boxed{\text{EXE } 1}$

1.1) Lastra piana istantaneamente fermata



$v_x = v_x(y, t)$

$v_y = v_z = 0$

Continuità : $\dfrac{\partial v_x}{\partial x} = 0$ (profilo completamente sviluppato)

NS_x : $\rho \dfrac{\partial v_x}{\partial t} = \mu \dfrac{\partial^2 v_x}{\partial y^2}$

Infatti : $\rho\left(\dfrac{\partial v_x}{\partial t} + \cancel{v_x \dfrac{\partial v_x}{\partial x}}^{0} + \cancel{v_y \dfrac{\partial v_x}{\partial y}}^{0} + \cancel{v_z \dfrac{\partial v_z}{\partial z}}^{0}\right) = -\cancel{\dfrac{\partial P}{\partial x}}^{0} + \mu\left(\cancel{\dfrac{\partial^2 v_x}{\partial x^2}}^{0} + \dfrac{\partial^2 v_x}{\partial y^2} + \cancel{\dfrac{\partial^2 v_x}{\partial z^2}}^{0}\right)$

da Bernoulli

NS_y : $0 = -\dfrac{\partial P}{\partial y}$

NS_z : $0 = -\dfrac{\partial P}{\partial z}$

1.2) Calcolo profilo di velocità.

→ Ipotizzo la similitudine del profilo di velocità :

$$\frac{v_x}{U_\infty} = f(\eta) \qquad \text{con} \quad \eta = \frac{y}{\delta(t)} = \frac{y}{2\sqrt{\nu t}}$$

$$\frac{\partial v_x}{\partial t} = \underbrace{U_\infty \frac{\partial f(\eta)}{\partial \eta} \cdot \frac{\partial \eta}{\partial t}}_{\frac{\partial f(\eta)}{\partial t}} = U_\infty f' \cdot \underbrace{\frac{y}{2\sqrt{\nu}}\left(-\frac{1}{2} t^{-3/2}\right)}_{\frac{y}{2\sqrt{\nu t}} \cdot \frac{1}{t}} = -U_\infty f' \eta \frac{1}{2t}$$

$$\frac{\partial v_x}{\partial y} = U_\infty \frac{\partial f(\eta)}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = U_\infty f' \cdot \frac{1}{2\sqrt{\nu t}}$$

$$\frac{\partial^2 v_x}{\partial y^2} = \frac{\partial}{\partial y}\left(U_\infty f' \frac{1}{2\sqrt{\nu t}} \right) = U_\infty \frac{1}{2\sqrt{\nu t}} \cdot \frac{\partial f'}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = \frac{U_\infty f''}{4\nu t}$$

La NS_x si riscrive come:

$$-\rho \cancel{U_\infty} f' \eta \cdot \frac{1}{2\cancel{t}} = \mu \frac{\cancel{U_\infty} f''}{4\nu \cancel{t}} \; ; -\frac{1}{2} f' \cdot \eta = \frac{\cancel{\mu}}{\cancel{\rho}} \frac{f''}{4\cancel{\nu}}$$

$$\boxed{2\eta f' + f'' = 0} \qquad (1)$$

La (1) si integra con le seguenti c.c.

i) $v_x(y=0, t \le 0) = U_\infty$

ii) $v_x(y=0, t>0) = 0 \quad \Longrightarrow \quad f(\eta = 0) = 0$

iii) $v_x(y \to \infty, t) = U_\infty \quad \Longrightarrow \quad f(\eta \to \infty) = 1$

La soluzione della (1) è:

$$f(\eta) = C_1 \int_0^{\eta} e^{-\eta^2} d\eta + C_2$$

Dalle c.c. ricavo C_1 e C_2:

ii) $f(\eta = 0) = 0 \quad \Rightarrow \quad C_2 = 0$

iii) $f(\eta \to \infty) = 1 \quad \Rightarrow \quad 1 = C_1 \int_0^{\infty} e^{-\eta^2} d\eta$

$$C_1 = \frac{1}{\int_0^{\infty} e^{-\eta^2} d\eta} = \frac{2}{\sqrt{\pi}}$$

Quindi:

$$f(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta$$

$$\Downarrow$$

$$\boxed{v_x(y,t) = U_\infty \cdot \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta} \quad (2)$$

1.3) Calcolo del taglio:

$$\tau_{xy} = \mu \left(\frac{\partial v_x}{\partial y} + \cancel{\frac{\partial v_y}{\partial x}}^{\,0} \right) = \mu \frac{\partial}{\partial y} \left(U_\infty \cdot \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta \right) =$$

$$= \frac{2 \mu U_\infty}{\sqrt{\pi}} \underbrace{\left(\frac{\partial}{\partial \eta} \int_0^{\eta} e^{-\eta^2} d\eta \right)}_{e^{-\eta^2}} \cdot \underbrace{\frac{\partial \eta}{\partial y}}_{\frac{1}{2\sqrt{\nu t}}} = \frac{\cancel{2} \mu U_\infty}{\sqrt{\pi}} \cdot \frac{1}{\cancel{2}\sqrt{\nu t}} e^{-\eta^2}$$

Alla parete :

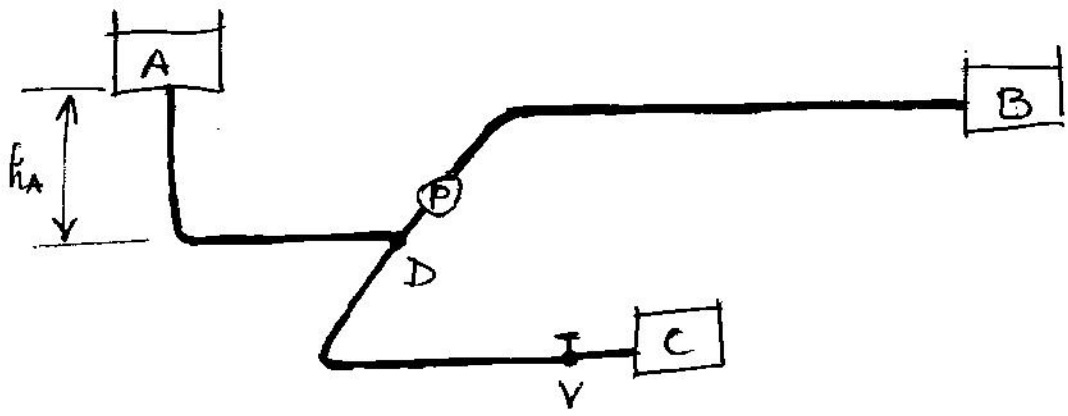
$$\tau_{xy}\Big|_{y=0} = \frac{\mu U_\infty}{\sqrt{\pi \nu t}} e^{-\eta^2}\Big|_{y=0}$$

$$\Downarrow$$

$$\boxed{\tau_{xy}\Big|_{y=0} = \frac{\mu U_\infty}{\sqrt{\pi \nu t}}}$$

SFORZO DI TAGLIO ALLA PARETE

EXE 2



$h_A = 10$ m

$Q = 20 \cdot 10^{-3}\ \frac{m^3}{s}$

$\rho = 10^3\ kg/m^3$

$\mu = 10^{-3}$ Pa·s

$D = 0{,}2$ m

$L_{AD} = 5000$ m $\quad L_{DB} = 5000$ m $\quad L_D = 2500$ m

2.1 Calcolo di P_D

$$B_{AD}: \cancel{\frac{1}{2}v_A^2}^{\,0} + g h_A + \frac{P_A}{\rho} + \cancel{W_s}^{\,0} = \frac{1}{2}v_{AD}^2 + g h_D + \frac{P_D}{\rho} + l v_{AD}$$

$$P_D = P_{atm} + \rho g (h_A - h_D) - \frac{1}{2}\rho v_{AD}^2 - 2\rho f_{AD} v_{AD}^2 \frac{L_{AD}}{D} \quad (1)$$

Ora :

$$v_{AD} = \frac{4Q}{\pi D^2} = 0{,}637 \ m/s$$

$$\hookrightarrow f_{AD} = 0{,}079 \ Re_{AD}^{-0{,}25} = 4{,}18 \cdot 10^{-3}$$

con $Re_{AD} = 127320$

Si trova immediatamente : $\boxed{p_D \cong 1{,}145 \cdot 10^5 \ Pa}$

1.2) Potenza della pompa ed altezze cinetiche K_v perse nella valvola V

$$B_{DB} : \frac{1}{2} v_{DB}^2 + \frac{P_D}{\rho} + \cancel{g h_D} + W_s = \frac{1}{2} \cancel{v_B^2}^{0} + \frac{P_B}{\rho} + \cancel{g h_B} + l v_{DB}$$

$h_D \simeq h_B$

$$W_s = \frac{P_B - P_D}{\rho} - \frac{1}{2} v_{DB}^2 + 2 f_{DB} v_{DB}^2 \frac{L_{DB}}{D} \quad (2)$$

Ora, poiché $Q_{DB} = Q_{DC} = \frac{Q}{2} = 10^{-2} \ m^3/s$ si ha :

$$v_{DB} = v_{DC} = \frac{4 Q_{DB}}{\pi D^2} = 0{,}3185 \ m/s$$

$$\hookrightarrow f_{DB} = 0{,}079 \ Re_{DB}^{-0{,}25} = 4{,}97 \cdot 10^{-3} \quad (Re_{DB} \cong 63660)$$

Dunque, dalla (2) : $W_s = 12{,}03 \ m^2/s^2$

$$\boxed{Pot = \rho Q \cdot W_s = 120{,}4 \ W}$$

$$B_{DC} : \frac{1}{2} v_{DC}^2 + \frac{P_D}{\rho} + \cancel{g h_D} = \cancel{\frac{1}{2} v_C^2}^{\,0} + \frac{P_C}{\rho} + \cancel{g h_C} + l_{v_{DC}}$$

$$l_{v_{DC}} = \frac{P_D - P_{atm}}{\rho} + \frac{1}{2} v_{DC}^2 \qquad (3)$$

Nel tratto DC abbiamo sia perdite concentrate che distribuite :

$$l_{v_{DC}} = \frac{1}{2} K_v v_{DC}^2 + 2 f_{DC} v_{DC}^2 \frac{L_{DC}}{D} \qquad (4)$$

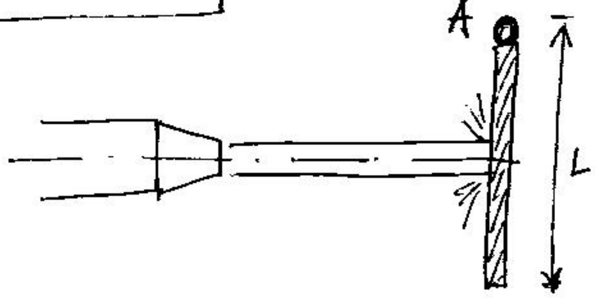
INCOGNITA!

Sostituendo la (4) nella (3) ed esplicitando per K_v :

$$K_v = \left[\left(\frac{P_D - P_{atm}}{\rho} \right) + \frac{1}{2} v_{DC}^2 - 2 f_{DC} v_{DC}^2 \frac{L_{DC}}{D} \right] \cdot \frac{2}{v_{DC}^2}$$

$$= 11,18 \implies \boxed{K_v = 11,18}$$

EXE 3



$Q = 2 \cdot 10^{-3}\ m^3/s$

$d = 10^{-2}\ m$

$L = 0,5\ m$

3.1 Calcolo della forza resistente R

Applicando l' eq. di conservazione della q.d.m. al →

volume di controllo indicato in figura si ottiene:

$$\text{direz. } x) \quad 0 = \dot{m}\,(v_1 - \cancel{v_2}^{\,0}) + \cancel{P_1 A_1} - \cancel{P_2 A_2} - F_x + \cancel{\int_{z_1}^{z_2} \rho g_x A\, dz}^{\,0}$$

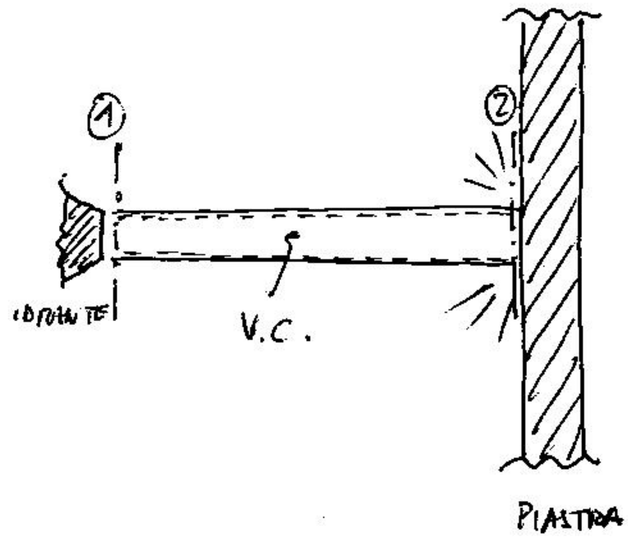
essendo:

$\dot{m} = \rho Q = 10^3 \cdot 2 \cdot 10^{-3} = 2 \ kg/s$

$P_1 = P_2 = P_{atm}$

$A_1 = A_2 = \dfrac{\pi d^2}{4}$

$v_2 = 0 \quad , \quad v_1 = \dfrac{4Q}{\pi d^2}$



Pertanto risulta: $F_x = \dfrac{4\rho Q^2}{\pi d^2}$

Per mantenere la lastra verticale, deve valere la seguente condizione di equilibrio alla rotazione rispetto ad A:

$$\underbrace{F_x \cdot \frac{L}{2}}_{\text{braccio}} = \underbrace{R \cdot L}_{\text{braccio}} \quad \rightarrow \quad R = \frac{F_x}{2} = \frac{2\rho Q^2}{\pi d^2} = \frac{2 \cdot 10^3 \cdot (2 \cdot 10^{-3})^2}{\pi \cdot (10^{-2})^2}$$

$$= \frac{80}{\pi}\,[N] \cong \underline{\underline{25{,}46\ N}}$$

Controllo dimensionale: $\dfrac{kg}{m^3} \cdot \dfrac{m^6}{s^2} \cdot \dfrac{1}{m^2} = kg\,\dfrac{m}{s^2}$ ok!

3.2 Calcolo della massa della piastra: se la forza resistente R non agisce la piastra ruota fino a raggiungere una posizione di equilibrio stabile in cui i momenti prodotti dalla forza trasmessa dal

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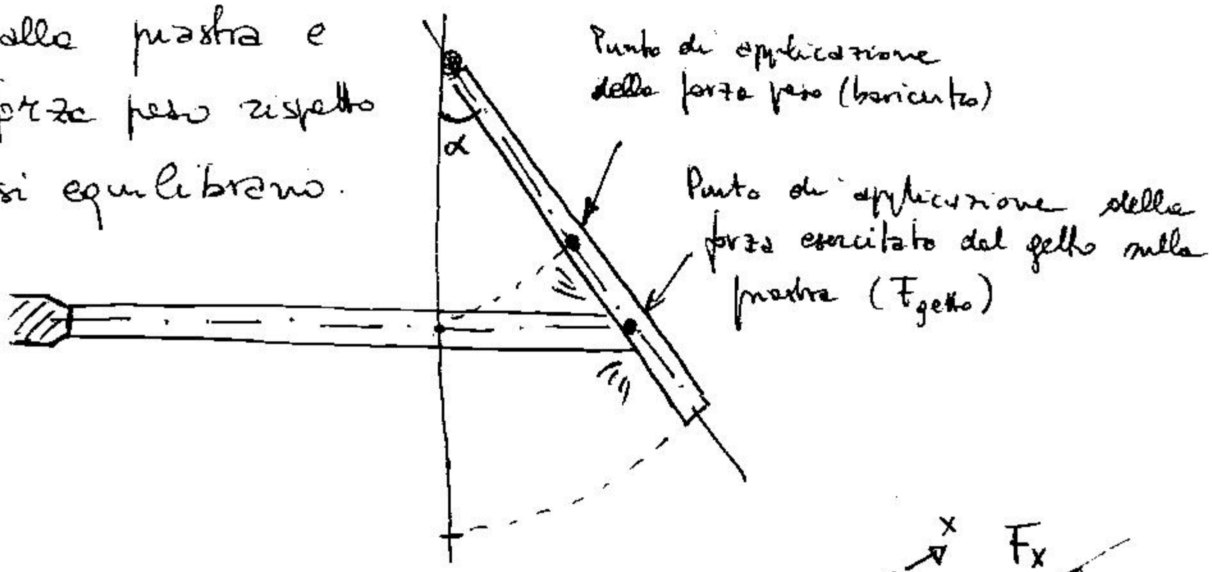
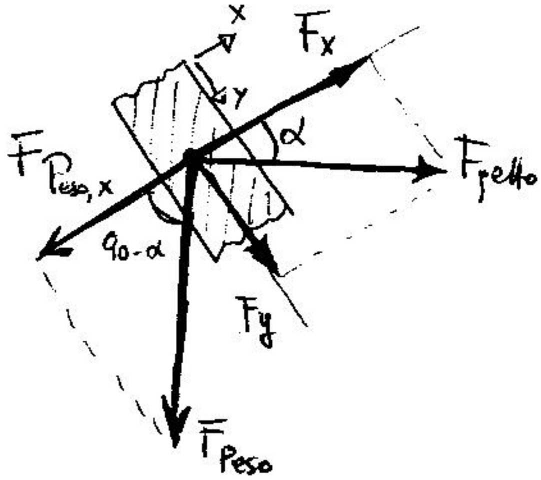
getto alla piastra e della forza peso rispetto ad A si equilibrano.





$$F_x = w \cdot v_1 \cos\alpha = \frac{4\rho Q^2}{\pi d^2}\cos\alpha$$

$$F_{Peso,x} = mg\sin\alpha$$

- <u>Opzione A</u>: si considera che i punti di applicazione delle due forze siano circa coincidenti

$$F_x \cdot \cancel{\frac{L}{2}} = F_{Peso,x} \cdot \cancel{\frac{L}{2}} \Rightarrow mg\sin\alpha = \frac{4\rho Q^2}{\pi d^2}\cos\alpha$$

$$\boxed{m = \frac{4\rho Q^2}{\pi d^2 g} \cdot \frac{1}{tg\alpha}}$$

* La differenza nel valore di m trovato con le due diverse opzioni e' $\sim 0,92$ kg

$$m = \frac{4 \cdot 10^3 (2\cdot 10^{-3})^2}{\pi (10^{-2})^2 \cdot 9{,}81} \cdot \frac{1}{tg(20°)} = 14{,}27 \text{ Kg}$$

- <u>Opzione B</u>: si considera che i punti di applicazione delle due forze non coincidano

$$F_x \cdot \left(\cancel{\frac{L}{2}} \cdot \frac{1}{\cos\alpha}\right) = F_{P,x} \cdot \cancel{\frac{L}{2}} \Rightarrow \boxed{m = \frac{4\rho Q^2}{\pi d^2 g} \cdot \frac{1}{\sin\alpha}} \Rightarrow m \simeq 15{,}19 \text{ kg}^{*}$$