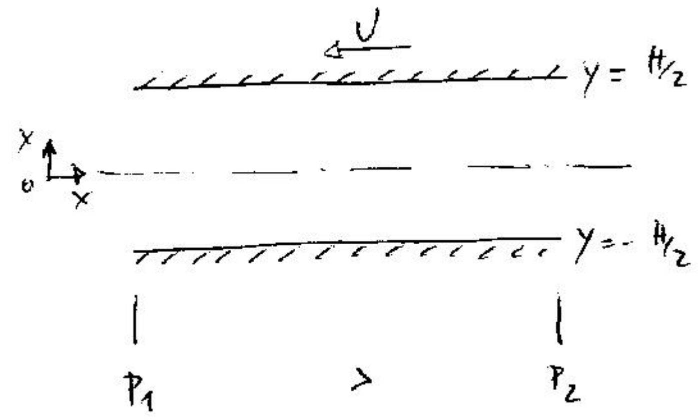
$\boxed{\text{EXE 1}}$



Hp: moto stazionario $\left(\frac{\partial}{\partial t}=0\right)$

moto bidimensionale $\left(\frac{\partial}{\partial z}=0\right)$

moto unidirezionale $(v_x = v_x(y)$

$\hookrightarrow v_y = v_z = 0$

Eq. semplificate : CONTINUITA' $\frac{\partial v_x}{\partial x}+\cancel{\frac{\partial v_y}{\partial y}}+\cancel{\frac{\partial v_z}{\partial z}}=0 \rightarrow \boxed{\frac{\partial v_x}{\partial x}=0}$

FLUSSO COMPLETAMENTE SVILUPPATO

$$NS_x \quad \rho\left(\cancel{\frac{\partial v_x}{\partial t}}+\cancel{v_x\frac{\partial v_x}{\partial x}}+\cancel{v_y\frac{\partial v_x}{\partial y}}+\cancel{v_z\frac{\partial v_x}{\partial z}}\right)=$$

$$=-\frac{\partial P}{\partial x}+\mu\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}}+\frac{\partial^2 v_x}{\partial y^2}+\cancel{\frac{\partial^2 v_x}{\partial z^2}}\right)$$

$$\boxed{0=-\frac{\partial P}{\partial x}+\mu\frac{\partial^2 v_x}{\partial y^2}} \quad (1)$$

$$NS_y \quad 0=-\frac{\partial P}{\partial y}$$

Integrando la (1) : $\frac{\partial^2 v_x}{\partial y^2}=\frac{1}{\mu}\left(\frac{\Delta P}{L}\right) \xrightarrow{\int^{\#1}} \frac{\partial v_x}{\partial y}=\frac{1}{\mu}\left(\frac{\Delta P}{L}\right)y+C_1$

$$\xrightarrow{\int^{\#2}} v_x(y)=\frac{1}{2\mu}\left(\frac{\Delta P}{L}\right)y^2+C_1 y+C_2 \quad (*)$$

C.C. #1 : $v_x(y=H/2)=-U \rightarrow -U=\frac{1}{2\mu}\left(\frac{\Delta P}{L}\right)\frac{H^2}{4}+C_1\frac{H}{2}+C_2$ (2)

C.C. #2 : $v_x(y=-H/2)=0 \Rightarrow 0=\frac{1}{2\mu}\left(\frac{\Delta P}{L}\right)\frac{H^2}{4}-C_1\frac{H}{2}+C_2$ (3)

Sottraendo la (3) alla (2) si ottiene : $-U=2\left(C_1\frac{H}{2}\right)$

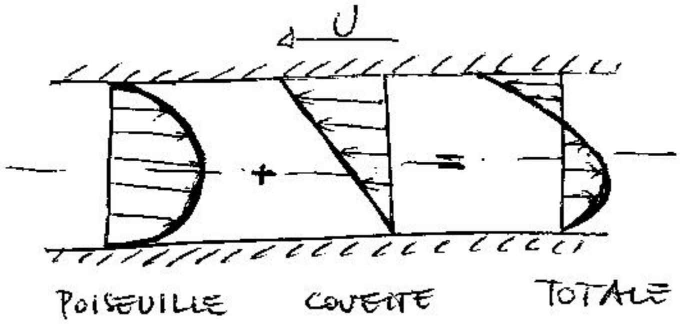
ovvero : $\boxed{C_1=-U/H}$ (4)

Sostituendo la (4) nella (2):

$$-U = \frac{1}{2\mu}\left(\frac{\Delta P}{L}\right)\frac{H^2}{4} - \frac{U}{2} + C_2 \Rightarrow \boxed{C_2 = -\frac{1}{2\mu}\left(\frac{\Delta P}{L}\right)\frac{H^2}{4} - \frac{U}{2}} \quad (5)$$

Sostituendo la (4) e la (5) nella (*):

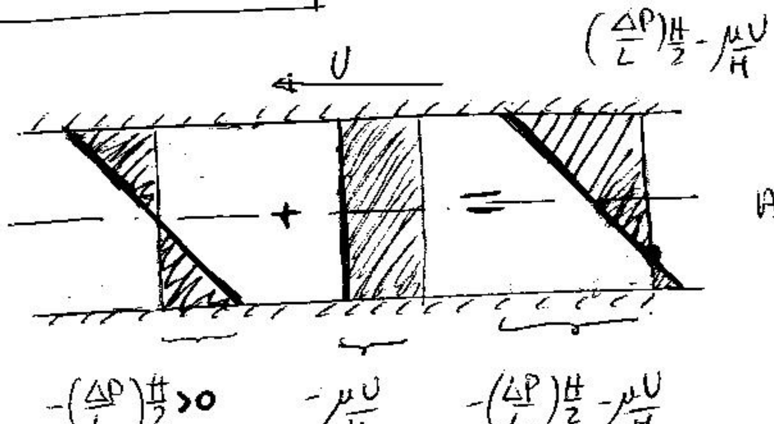
$$\boxed{v_x(y) = \frac{1}{2\mu}\left(\frac{\Delta P}{L}\right)\left(y^2 - \frac{H^2}{4}\right) - \frac{U}{H}y - \frac{U}{2}}$$ PROFILO DI VELOCITA'



ANDAMENTO QUALITATIVO

$$\tau_{xy} = \mu \frac{\partial v_x}{\partial y} = \cancel{\mu}\frac{1}{2\cancel{\mu}}\left(\frac{\Delta P}{L}\right)(2y) - \mu\frac{U}{H} = \left(\frac{\Delta P}{L}\right)y - \mu\frac{U}{H}$$

$$\boxed{\tau_{xy} = \left(\frac{\Delta P}{L}\right)y - \mu\frac{U}{H}}$$ PROFILO DEL TAGLIO



ANDAMENTO QUALITATIVO

$$\frac{Q}{W} = \int_{-\frac{H}{2}}^{\frac{H}{2}} v_x(y)\,dy = \frac{1}{2\mu}\left(\frac{\Delta P}{L}\right)\left(\frac{y^3}{3} - \frac{H^2}{4}y\right)\Bigg|_{-H/2}^{H/2} - \frac{U}{H}\,\frac{y^2}{2}\Bigg|_{-H/2}^{H/2} - \frac{U}{2}y\Bigg|_{-H/2}^{H/2} =$$

$$\frac{Q}{W} = \frac{1}{2\mu}\left(\frac{\Delta P}{L}\right)\underbrace{\left(\frac{H^3}{24} - \frac{H^3}{8} + \frac{H^3}{24} - \frac{H^3}{8}\right)}_{\frac{H^3}{12} - \frac{H^3}{4} = -\frac{H^3}{6}} - \frac{U}{H}\cancel{\left(\frac{H^2}{8} - \frac{H^2}{8}\right)} - \frac{U}{2}\underbrace{\left(\frac{H}{2} + \frac{H}{2}\right)}_{H}$$

$$\frac{Q}{W} = \underbrace{-\frac{1}{12\mu}\left(\frac{\Delta P}{L}\right)H^3}_{>0} - \underbrace{\frac{UH}{2}}_{<0} \Rightarrow \frac{Q}{W} = 0 \; ; \quad U = -\frac{1}{12\mu}\left(\frac{\Delta P}{L}\right)H^3 \cdot \frac{2}{H}$$

$$\boxed{U = -\frac{1}{6\mu}\left(\frac{\Delta P}{L}\right)H^2}$$

$$\frac{Q}{W} = 0 \quad \text{per} \quad \boxed{U = \frac{1}{6\mu}\left(-\frac{\Delta P}{L}\right)H^2} \quad (6)$$

Calcolo della forza F: $\quad F = \tau_{xy}\Big|_{y=H/2} \cdot (LW)$

$$\tau_{xy}\Big|_{y=H/2} = \left(\frac{\Delta P}{L}\right)y\Big|_{y=H/2} - \frac{\mu U}{H} = \left(\frac{\Delta P}{L}\right)\cdot\frac{H}{2} - \frac{\mu U}{H} \Rightarrow \boxed{F = \left(\frac{\Delta P}{L}\right)\frac{HLW}{2} - \frac{\mu U}{H}}$$

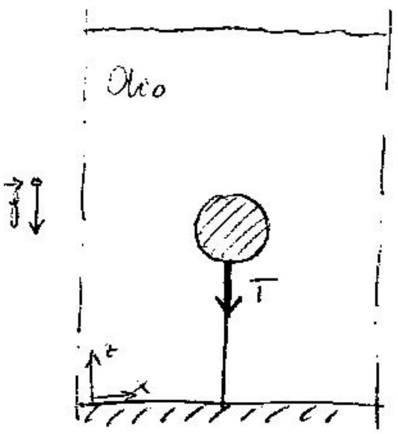
Poiché vale la (6): $F = \left(\frac{\Delta P}{L}\right)\frac{HLW}{2} - \frac{\cancel{\mu}LW}{H}\cdot\frac{1}{6\cancel{\mu}}\left(-\frac{\Delta P}{L}\right)H^2$

$$= \left(\frac{\Delta P}{L}\right)\frac{HLW}{2} + \left(\frac{\Delta P}{L}\right)\frac{HLW}{6}$$

$$= \left(\frac{\Delta P}{L}\right)\frac{3HLW + HLW}{6}$$

Pertanto: $\boxed{F = \frac{2}{3}\left(\frac{\Delta P}{L}\right)HLW}$ FORZA NECESSARIA PER TRASCINARE LA PARETE SUP.

ESE 2



$V = 10\ dm^3 = 10^{-2}\ m^3$

$\rho_{olio} = 800\ kg/m^3$

$\mu_{olio} = 0.1\ Pa \cdot s$

$T = 50\ N$

1) Calcolo di ρ:

All'equilibrio: $\boxed{F_p + T = F_{gall}}$ EQUILIBRIO STATICO

$$\rho g V + T = \rho_{olio} g V \rightarrow \rho = \rho_{olio} - \frac{T}{gV}$$

$$= \rho_{olio} - \frac{T \cdot 6}{g \pi D_p^3}$$

$$= 800 - \frac{50}{10 \cdot 10^{-2}} = 300\ kg/m^3$$

$\boxed{\rho = 300\ kg/m^3}$ DENSITA' DELLA SFERA

2) Calcolo di v_{term}:

In condizioni stazionarie: $\cancel{\vec{F}_I}^{\,0} = \vec{F}_D + \vec{F}_{Peso} + \vec{F}_{gall}$

$$0 = \frac{1}{2} \rho_{olio} C_D \frac{\pi D_p^2}{4} (\vec{u} - \vec{v}_p) |\vec{u} - \vec{v}_p| + \rho \vec{g} V - \rho_{olio} \vec{g} V \qquad (1)$$

Il moto di risalita avviene in direzione verticale, quindi l'eq. vettoriale (1) ammette un'unica componente scalare

$$0 = \frac{1}{2} \rho_{olio} C_D \frac{\pi D_p^2}{4} (\cancel{u}^{\,0} - v_p) |\cancel{u}^{\,0} - v_p| - \rho g V + \rho_{olio} g V$$

$$0 = \frac{1}{2} \rho_{olio} (0.44) \cdot \frac{\pi D_p^2}{4} (-v_p) |-v_p| + (\rho_{olio} - \rho) g V$$

$$0 = \frac{1}{8}\rho_{olio}(0.44)\pi D_p^2(-v_p^2) + (\rho_{olio} - \rho)gV$$

$$v_p = \sqrt{\frac{(\rho_{olio} - \rho)gV}{\frac{1}{8}\rho_{olio}(0.44)\pi D_p^2}} = \sqrt{\left(1 - \frac{\rho}{\rho_{olio}}\right)g\frac{\frac{\cancel{\pi} D_p^{\cancel{3}}}{6}}{\frac{1}{8}(0.44)\cancel{\pi}\cancel{D_p^2}}}$$

$$= \sqrt{\frac{8}{6}\left(1 - \frac{\rho}{\rho_{olio}}\right)\frac{gD_p}{0.44}}$$

$$v_p = \sqrt{\frac{4}{3}\left(1 - \frac{\rho}{\rho_{olio}}\right)\frac{gD_p}{0.44}} = \sqrt{\frac{4}{3}\left(1 - \frac{300}{800}\right)\frac{10 \cdot 0{,}2673}{0.44}} = 2{,}25\ m/s$$

con $D_p = \left(\frac{6V}{\pi}\right)^{1/3} = \left(\frac{6 \cdot 10^{-2}}{\pi}\right)^{1/3} = 0.2673\ m$. Pertanto $\boxed{v_{term} = 2{,}25\ m/s}$

Verifica: $Re_p = \frac{\rho_{olio}|u - v_p|D_p}{\mu_{olio}} = \frac{800 \cdot 2{,}25 \cdot 0.2673}{0.1} \simeq 4800 > 10^3$

3) Calcolo della forza di attrito:

$$F_{D,max} = F_D\Big|_{v_p = v_{term}} = \frac{1}{2}\rho_{olio}C_D\frac{\pi D_p^2}{4}v_{term}^2 = \frac{1}{2}800 \cdot 0{,}44\left(\pi\frac{0.2673^2}{4}\right)2{,}$$

$\boxed{F_{D,max} \simeq 50\ N}$ Valore massimo della F_{DRAG}

4) Calcolo della frazione di volume immerso:

Volume immerso : V_{imm}

Volume emerso : $V - V_{imm}$

All'equilibrio (senza fune): $\vec{F}_{peso} + \vec{F}_{gall} = \vec{0}$

$$\rho g V = \rho_{olio} g V_{imm} \Rightarrow V_{imm} = (\rho/\rho_{olio})V = \frac{3}{8}V \rightarrow \boxed{\frac{V_{imm}}{V} = \frac{3}{8}}$$

$\boxed{\text{Exe 3}}$

$$T(z) = T_0 - \alpha z$$

$$dp = -\rho(z) g\, dz \implies dp = -\frac{Mg}{R}\left[p(z)/T(z)\right] dz \implies \frac{dp}{p(z)} = -\frac{Mg}{R}\frac{dz}{T(z)}$$

$$\frac{p(z)}{\rho(z)} = \frac{R}{M} T(z) \implies \rho(z) = \frac{M}{R}\cdot\frac{p(z)}{T(z)}$$

$$\ln p(z)/p_0 = -\frac{Mg}{R}\ln\left(\frac{T_0-\alpha z}{T_0}\right)\cdot\left(-\frac{1}{\alpha}\right) = \frac{Mg}{R\alpha}\ln\left(\frac{T_0-\alpha z}{T_0}\right)$$

$$p(z) = p_0\left(\frac{T_0-\alpha z}{T_0}\right)^{Mg/R\alpha} = p_0\left(\frac{T_0-\alpha z}{T_0}\right)^{\gamma} \qquad \gamma = \frac{Mg}{R\alpha}$$

Quindi: $$\rho(z) = \frac{M}{R}\cdot p_0\left(\frac{T_0-\alpha z}{T_0}\right)^{\gamma}\cdot\frac{1}{T_0-\alpha z}\cdot\frac{T_0}{T_0}$$

$$= \frac{Mp_0}{RT_0}\left(\frac{T_0-\alpha z}{T_0}\right)^{\gamma-1} = \rho_0\left(\frac{T_0-\alpha z}{T_0}\right)^{\gamma-1}$$

$$\boxed{p(z) = p_0\left(\frac{T_0-\alpha z}{T_0}\right)^{\gamma}} \quad (1) \qquad\qquad \boxed{\rho(z) = \rho_0\left(\frac{T_0-\alpha z}{T_0}\right)^{\gamma-1}} \quad (2)$$

Dalla (1): $p(z^*) = p_0/2 \rightarrow \left(\frac{T_0-\alpha z^*}{T_0}\right)^{\gamma} = \frac{1}{2}$

$T_0 - \alpha z^* = T_0\cdot\frac{1}{2^{1/\gamma}} \rightarrow \boxed{z^* = \frac{T_0}{\alpha}\left(1-2^{-1/\gamma}\right)}$ Quota alla quale $p(z^*) = p_0/2$

Sostituendo nella (2):

$$\rho(z^*) = \rho_0\left\{\frac{T_0 - \left[\frac{T_0}{\alpha}\left(1-2^{-1/\gamma}\right)\right]}{T_0}\right\}^{\gamma-1} = \rho_0\left(\cancel{1}-\cancel{1}+2^{-1/\gamma}\right)^{\gamma-1}$$

$$= \rho_0\, 2^{1/\gamma-1} \implies \boxed{\rho(z^*) = \rho_0\cdot 2^{1/\gamma-1}} \quad e \quad \boxed{T(z^*) = T_0\cdot 2^{-1/\gamma}}$$