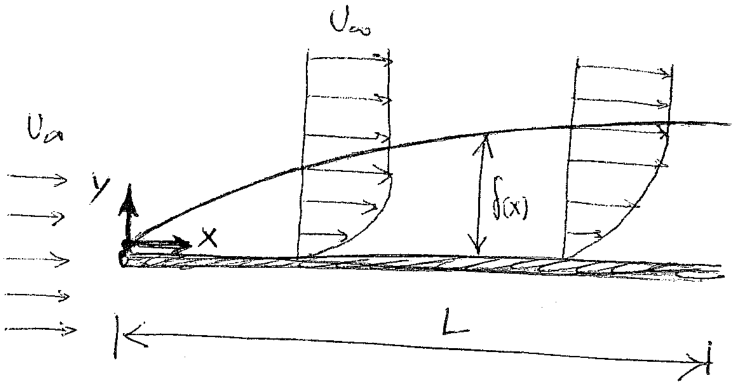
EQUAZIONI DELLO STRATO LIMITE



$$\tilde{v}_x = \frac{v_x}{U_\infty} \qquad \tilde{x} = \frac{x}{L}$$

$$\tilde{v}_y = \frac{v_y}{V} \qquad \tilde{y} = \frac{y}{\delta(x)} \rightarrow \delta \ll L$$

$$+ \quad \tilde{p} = \frac{p}{\pi}$$

CONTINUITÀ : $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \rightarrow \frac{U_\infty}{L}\frac{\partial \tilde{v}_x}{\partial \tilde{x}} + \frac{V}{\delta}\frac{\partial \tilde{v}_y}{\partial \tilde{y}} = 0$

$$\frac{U_\infty}{L}\cdot\frac{\delta}{V}\frac{\partial \tilde{v}_x}{\partial \tilde{x}} + \frac{\partial \tilde{v}_y}{\partial \tilde{y}} = 0$$

Dev'essere $\frac{U_\infty}{L}\cdot\frac{\delta}{V} \sim O(1) \Rightarrow \boxed{V = U_\infty \cdot \frac{\delta}{L}} \Rightarrow V \ll U_\infty$

Pertanto : $\boxed{\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0}$ EQ. CONTINUITÀ

NS_x (FLUSSO 2D STAZIONARIO) : $\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right)$

$$\rho\left(\frac{U_\infty^2}{L}\tilde{v}_x\frac{\partial \tilde{v}_x}{\partial \tilde{x}} + \underbrace{\frac{U_\infty V}{\delta}}\tilde{v}_y\frac{\partial \tilde{v}_x}{\partial \tilde{y}}\right) = -\frac{\pi}{L}\frac{\partial \tilde{p}}{\partial \tilde{x}} + \mu\left(\frac{U_\infty}{L^2}\frac{\partial^2 \tilde{v}_x}{\partial \tilde{x}^2} + \frac{U_\infty}{\delta^2}\frac{\partial^2 \tilde{v}_x}{\partial \tilde{y}^2}\right)$$

$$\left(\frac{U_\infty}{\delta}\right)\cdot\frac{U_\infty \delta}{L} = \frac{U_\infty^2}{L}$$

Dividiamo tutto per $\frac{\mu U_\infty}{\delta^2}$:

$$\rho\frac{U_\infty^2}{L}\cdot\frac{\delta^2}{\mu U_\infty}\left(\tilde{v}_x\frac{\partial \tilde{v}_x}{\partial \tilde{x}} + \tilde{v}_y\frac{\partial \tilde{v}_x}{\partial \tilde{y}}\right) = -\frac{\pi}{L}\cdot\frac{\delta^2}{\mu U_\infty}\frac{\partial \tilde{p}}{\partial \tilde{x}} + \left(\cancel{\overset{\ll 1 \text{ trasc.}}{\frac{\delta^2}{L^2}\frac{\partial^2 \tilde{v}_x}{\partial \tilde{x}^2}}} + \frac{\partial^2 \tilde{v}_x}{\partial \tilde{y}^2}\right)$$

Valutiamo i singoli coefficienti:

$$\frac{\rho U_\infty^2}{L}\cdot\frac{\delta^2}{\mu V_\infty}=\frac{\rho U_\infty \delta}{\mu}\cdot\frac{\delta}{L}=Re\cdot\frac{\delta}{L}$$

$$\Rightarrow\ Re\cdot\frac{\delta}{L}\left(\tilde{v}_x\frac{\partial\tilde{v}_x}{\partial\tilde{x}}+\tilde{v}_y\frac{\partial\tilde{v}_x}{\partial\tilde{y}}\right)=-\frac{\partial\tilde{p}}{\partial\tilde{x}}\cdot\frac{\Pi\,\delta^2}{\mu U_\infty L}+\frac{\partial^2\tilde{v}_x}{\partial\tilde{y}^2}$$

Deve risultare: $Re\cdot\frac{\delta}{L}\sim O(1) \Longrightarrow \boxed{\delta(x)=\sqrt{\frac{\mu L}{\rho V_\infty}}}$

$Re \gg 1$

$\frac{\delta}{L} \ll 1$

$$\frac{\Pi\,\delta^2}{\mu U_\infty L}\sim O(1) \Longrightarrow \boxed{\Pi=\frac{\mu U_\infty L}{\delta^2}\overset{*}{=}\rho U_\infty^2}$$

$$*\ \frac{\mu U_\infty L}{\left(\frac{\mu L}{\rho U_\infty}\right)}=\rho U_\infty^2$$

L'eq. di NS_x per lo strato limite è:

$$\boxed{\rho\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\cdot\frac{\partial^2 v_x}{\partial y^2}}\quad \text{eq. } NS_x$$

NS_y (FLUSSO 2D STAZIONARIO): $\rho\left(v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}\right)$

$$\rho\left(\underbrace{\frac{U_\infty V}{L}}_{\frac{U_\infty}{L}\cdot\frac{U_\infty\delta}{L}=\frac{U_\infty^2\delta}{L^2}}\tilde{v}_x\frac{\partial\tilde{v}_y}{\partial\tilde{x}}+\underbrace{\frac{V^2}{\delta}}_{\frac{U_\infty^2\delta^2}{L^2}\cdot\frac{1}{\delta}=\frac{U_\infty^2\delta}{L^2}}\tilde{v}_y\frac{\partial\tilde{v}_y}{\partial\tilde{y}}\right)=-\underbrace{\frac{\Pi}{\delta}}_{\frac{\rho U_\infty^2}{\delta}}\frac{\partial\tilde{p}}{\partial\tilde{y}}+\mu\left(\frac{V}{L^2}\frac{\partial^2\tilde{v}_y}{\partial\tilde{x}^2}+\frac{V}{\delta^2}\frac{\partial^2\tilde{v}_y}{\partial\tilde{y}^2}\right)$$

Dividiamo tutto per $\frac{\mu V}{\delta^2}\equiv\frac{\mu}{\delta^2}\frac{U_\infty\delta}{L}=\frac{\mu U_\infty}{\delta\cdot L}$

$$\underbrace{\frac{\rho U_\infty^2 \delta}{L^2}\cdot\frac{\delta L}{\mu U_\infty}}\left(\tilde v_x \frac{\partial \tilde v_y}{\partial \tilde x} + \tilde v_y \frac{\partial \tilde v_y}{\partial \tilde y}\right) = -\frac{\partial \tilde p}{\partial \tilde y}\cdot \underbrace{\frac{\rho U_\infty^2}{\delta}\cdot\frac{\delta L}{\mu U_\infty}} + \left(\cancel{\frac{\delta^2}{L^2}\frac{\partial^2 \tilde v_y}{\partial \tilde x^2}}^{\text{trasc.}} + \frac{\partial^2 \tilde v_y}{\partial \tilde y^2}\right)$$

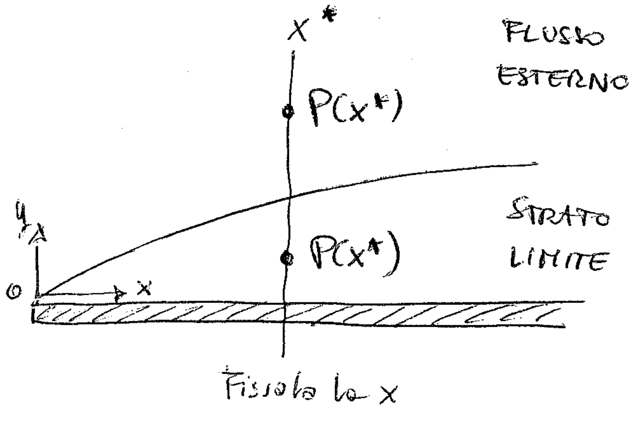
L3

$$\underbrace{\frac{\rho U_\infty \delta}{\mu}\cdot\frac{\delta}{L}}_{Re\cdot\frac{\delta}{L}\sim O(1)} \qquad\qquad \underbrace{\frac{\rho U_\infty L}{\mu}}_{\frac{\rho U_\infty \delta}{\mu}\cdot\frac{L}{\delta} = Re\cdot\frac{L}{\delta}}$$

$$\underbrace{Re\cdot\frac{\delta}{L}}_{O(1)}\cancel{\left(\tilde v_x \frac{\partial \tilde v_y}{\partial \tilde x} + \tilde v_y \frac{\partial \tilde v_y}{\partial \tilde y}\right)}^{\text{trasc.}} = -\frac{\partial \tilde p}{\partial \tilde y}\cdot \underbrace{Re\frac{L}{\delta}}_{\gg 1} + \cancel{\frac{\partial^2 \tilde v_y}{\partial \tilde y^2}}^{\text{trasc.}}$$

$$0 = -\frac{\partial \tilde p}{\partial \tilde y} \Longrightarrow \boxed{0 = -\frac{\partial p}{\partial y}} \quad \text{EQ. } NS_y$$

Dalla NS_y: $p = p(x)$ ma NON dipende da y !!



Applicando Bernoulli lungo una linea di flusso esterna allo strato limite:

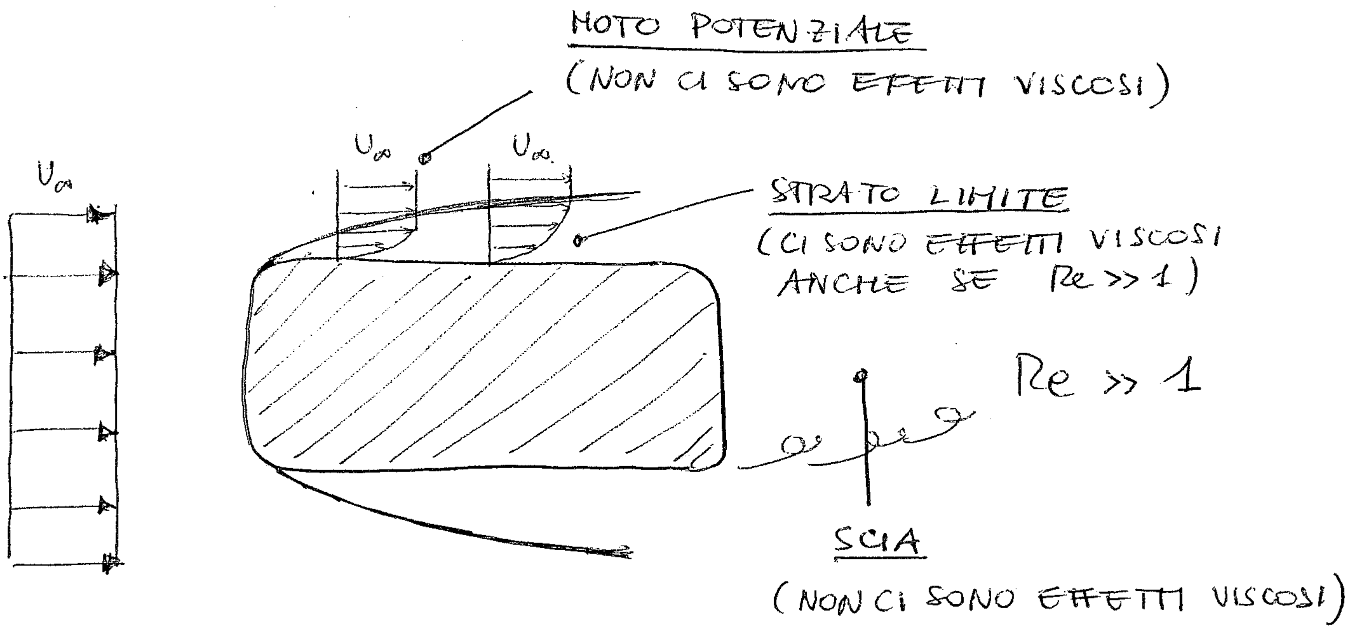
$$p + \frac{1}{2}\rho U_\infty^2 = \text{COST.}$$

$$\Downarrow$$

$$\frac{\partial p}{\partial x} = \frac{dp}{dx} = -\rho U_\infty \underbrace{\frac{dU_\infty}{dx}}_{\neq 0 \text{ se } U_\infty = U_\infty(x)}$$

La NS_x diventa pertanto:

$$\boxed{\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{v_x}{\partial y}\right) = \rho U_\infty \frac{dU_\infty}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2}}$$

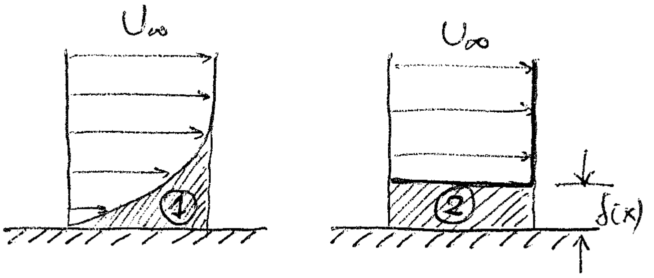


VORTICITA' → NULLA per MOTO POTENZIALE

VORTICITA' → NON NULLA per STRATO LIMITE e SCIA

DEFINIZIONI PER $\delta(x)$:

- $\delta(x)$ tale che $v_x(x,y) = 0,99\, U_\infty$
- $\delta(x) = \int_0^\infty \left[1 - \frac{v_x(x,y)}{U_\infty}\right] dy$ SPESSORE DI SPOSTAMENTO



$\delta(x)$ tale che l'area 1 è uguale all'area 2