Vorlesung 10

Turbulence

Phenomena & Modeling

Small Recap:

We examined the different terms of the balance equations to identify one single scaling parameter ($Re \equiv$ the Reynolds number) and have the possibility to neglect some terms of the balance equations:

(2)

Continuity : $\frac{\partial v_i}{\partial x_i} = 0$

N-S eqs : $\rho\left[\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j}\right] = -\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_j^2}$

Simplifications examined.

[1] Creeping flow $Re = \rho \frac{VD}{\mu} \to 0$

Inertia forces ~~vanishing~~ independent of the geometry

[2] Lubrication Approximation:

Inertia forces negligible But with the help of a ~~single~~ particular geometry

[3] Potential Flow $Re \to \infty$

Viscous forces Truly negligible

4 Boundary Layer Theory.

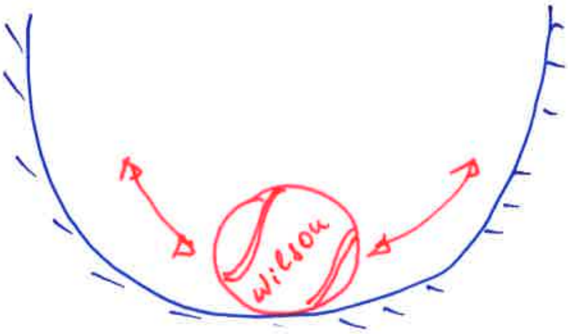
All Terms are To be considered:
pressure may be estimated by The potential flow Theory applied in The outer flow region.

Look To The N-S equations from another viewpoint

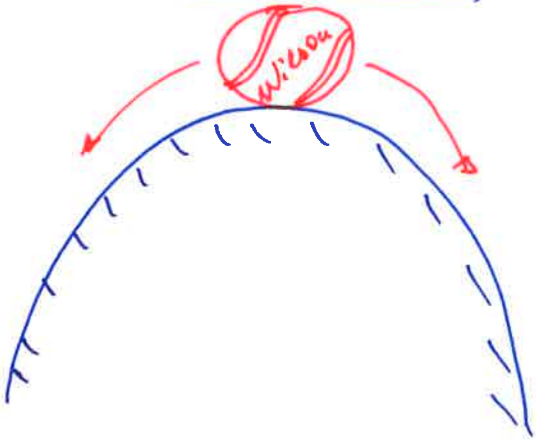
$$\underbrace{\rho\left(\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right)}_{\text{Inertial Term}}=\underbrace{-\frac{\partial P}{\partial x_i}}_{\text{Pressure Term}}+\underbrace{\mu\frac{\partial^2 v_i}{\partial x_j^2}}_{\text{Viscous Term (dissipat.)}}$$

The pressure Term feeds energy into The flow system and inertial Terms increase (so There is acceleration) BUT The viscous Terms ~~brakes~~ damps strongly with The viscous damping.

Stable System



Unstable System

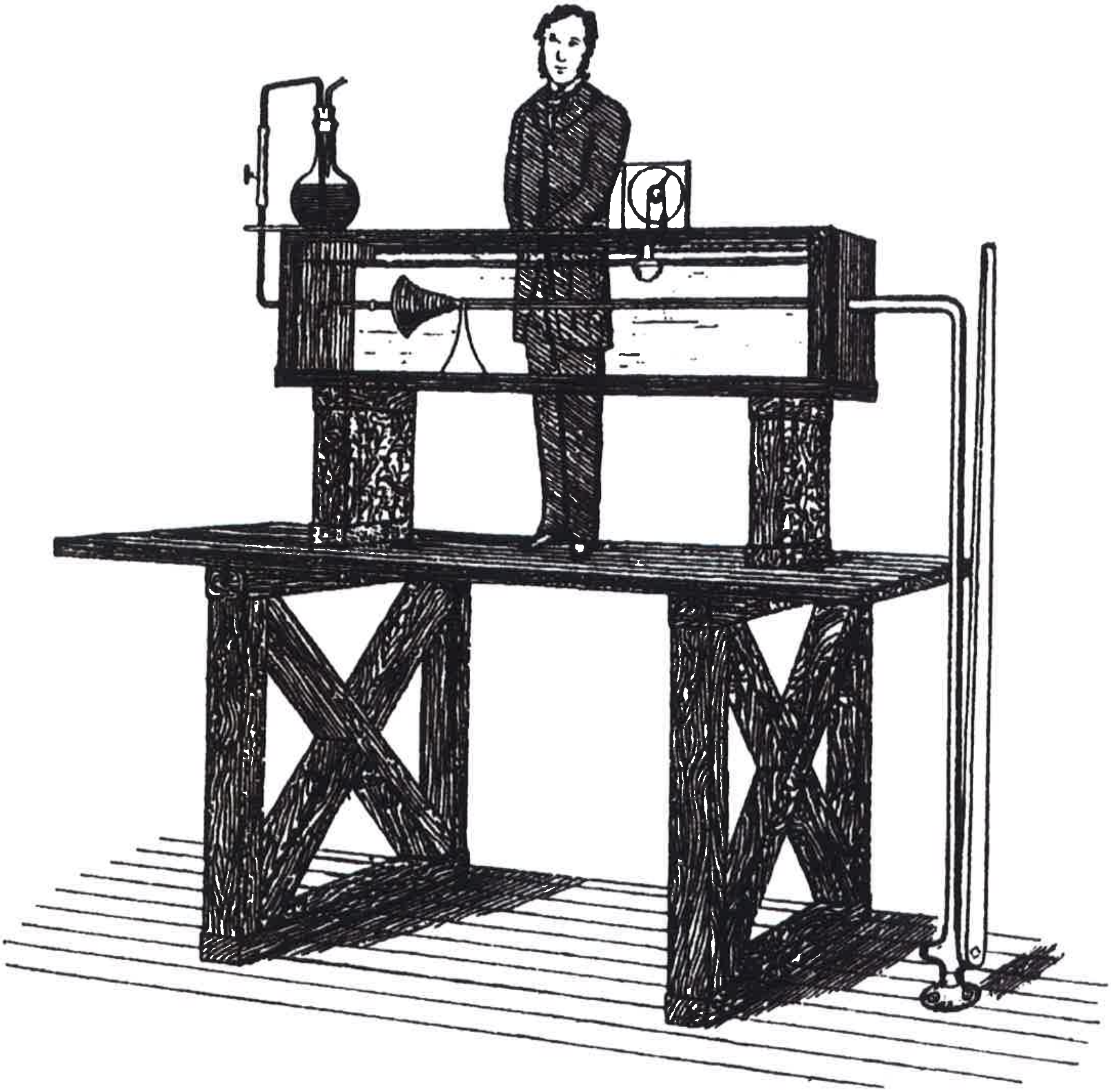


The Navier Stokes equations describe a stable system if the inertial terms are not too big compared with the viscous terms. If they become too large, then equations describe an unstable system.

If the system is stable then it is stable for any amplitude of the perturbation.

If the system is unstable it is unstable even for minimal amplitude of the perturbation.

Artist Concept of Reynolds's flow visualization experiment (Manchester, U.K, 1880s)



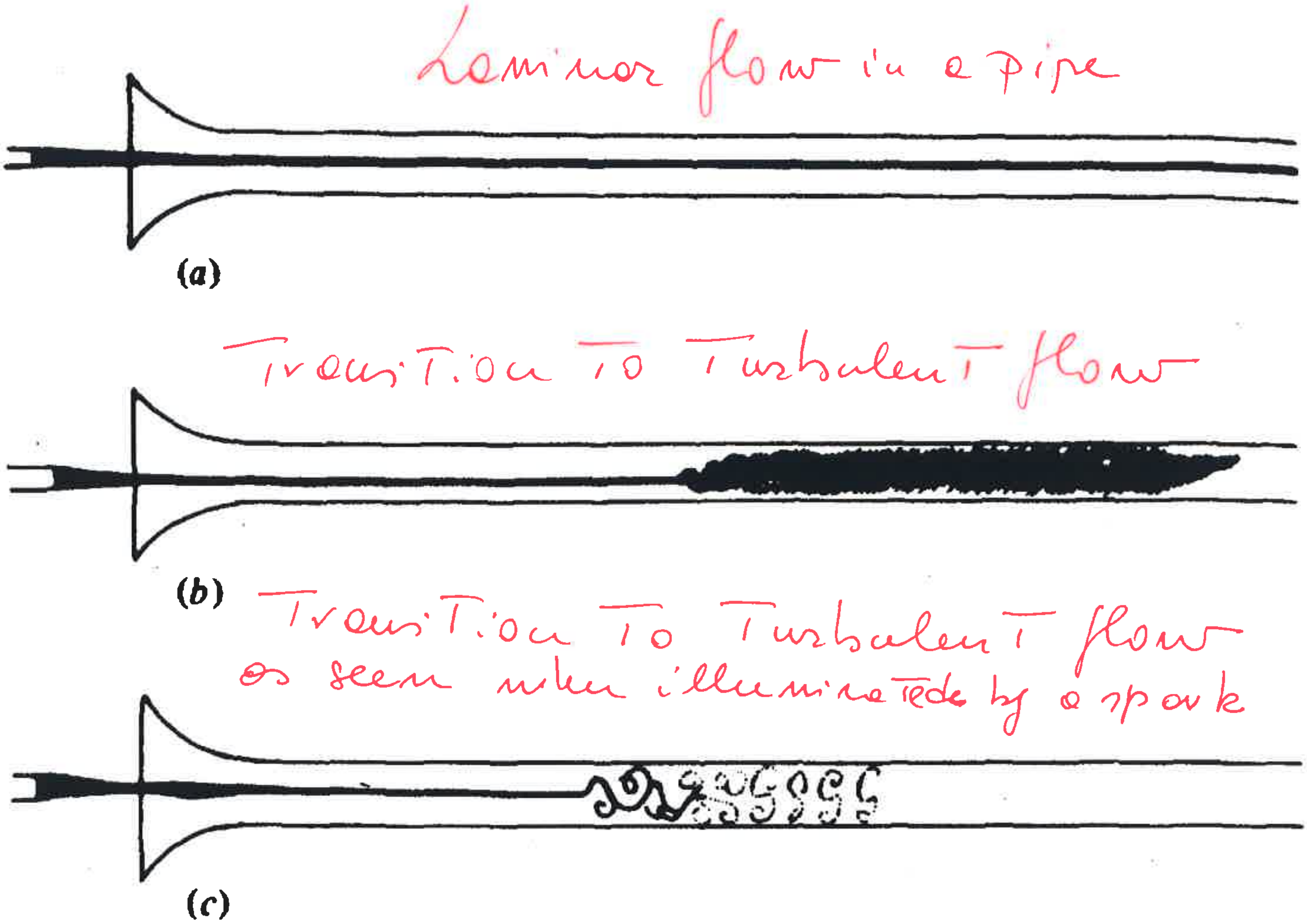
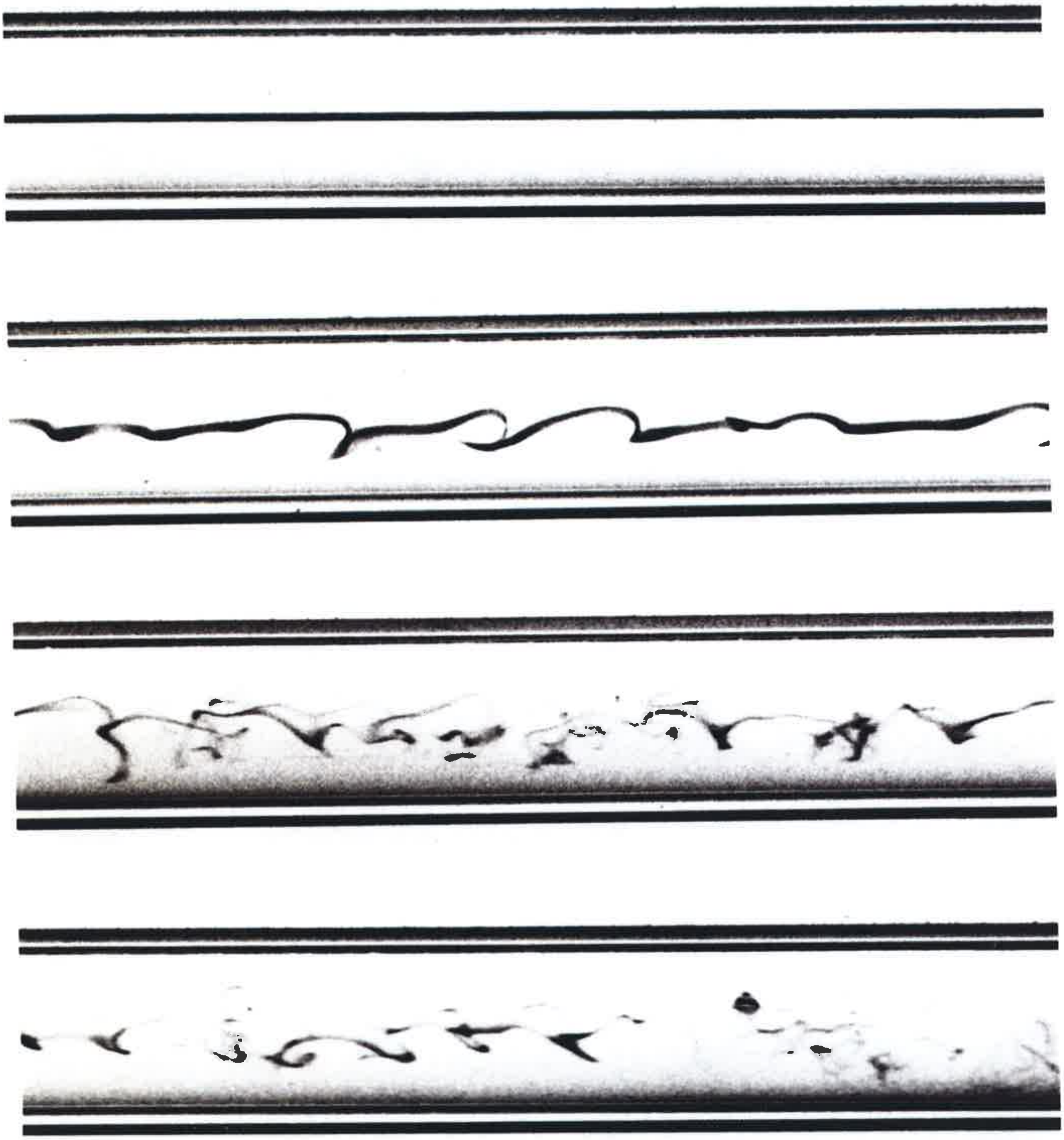


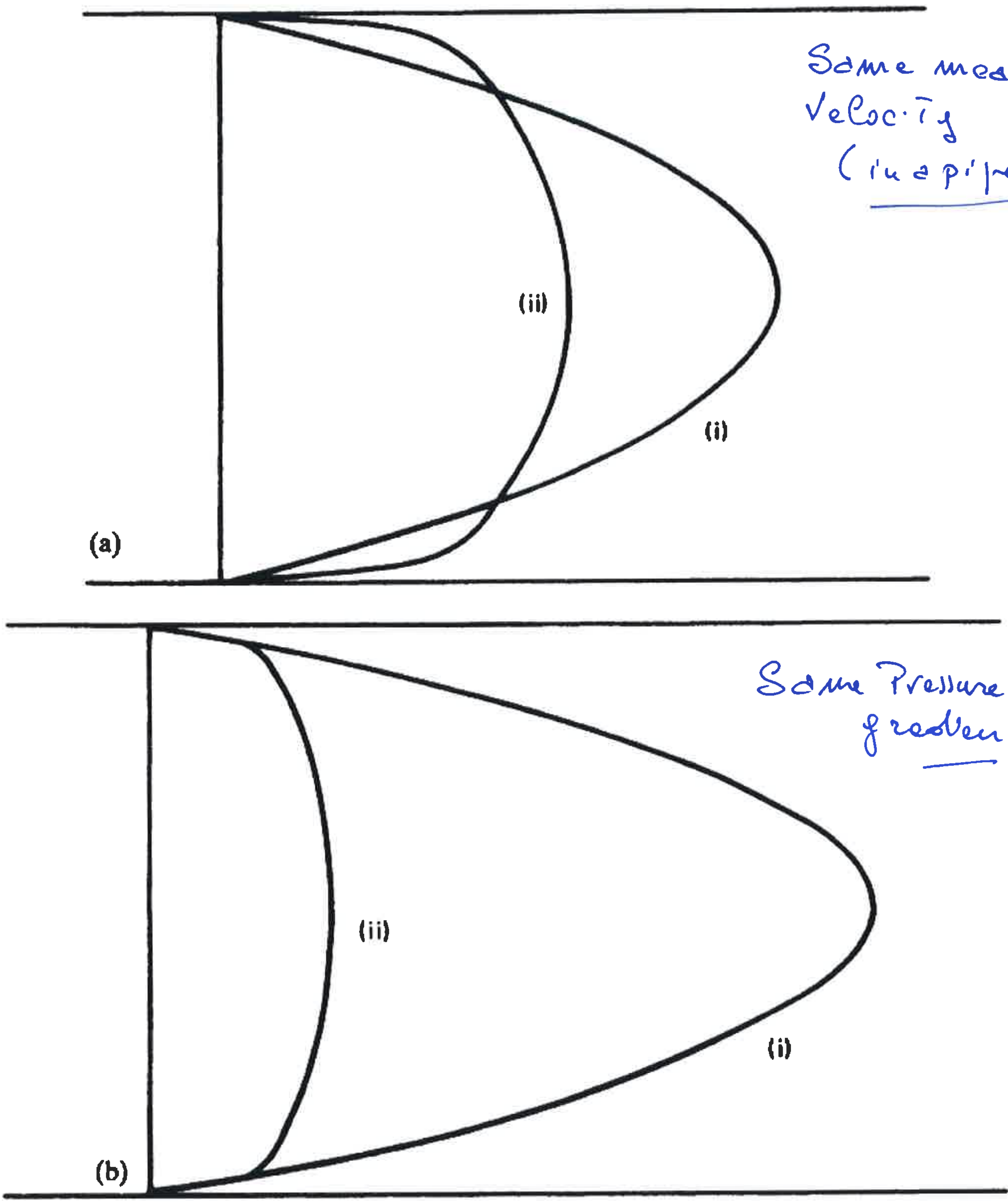
Fig. 9.2. Reynolds's drawings of the flow in his dye experiment.

Repetition of the Reynolds' dye experiment



103. Repetition of Reynolds' dye experiment. Osborne Reynolds' celebrated 1883 investigation of stability of flow in a tube was documented by sketches rather than photography. However the original apparatus has survived at the University of Manchester. Using it a century later, N. H. Johannesen and C. Lowe have taken this sequence of photographs. In laminar flow a filament of colored water introduced at a bell-shaped entry extends undisturbed the whole length of the glass tube. Transition is seen in the second of the photographs as the speed is increased; and the last two photographs show fully turbulent flow. Modern traffic in the streets of Manchester made the critical Reynolds number lower than the value 13,000 found by Reynolds.

Macroscopic Consequences of Turbulence

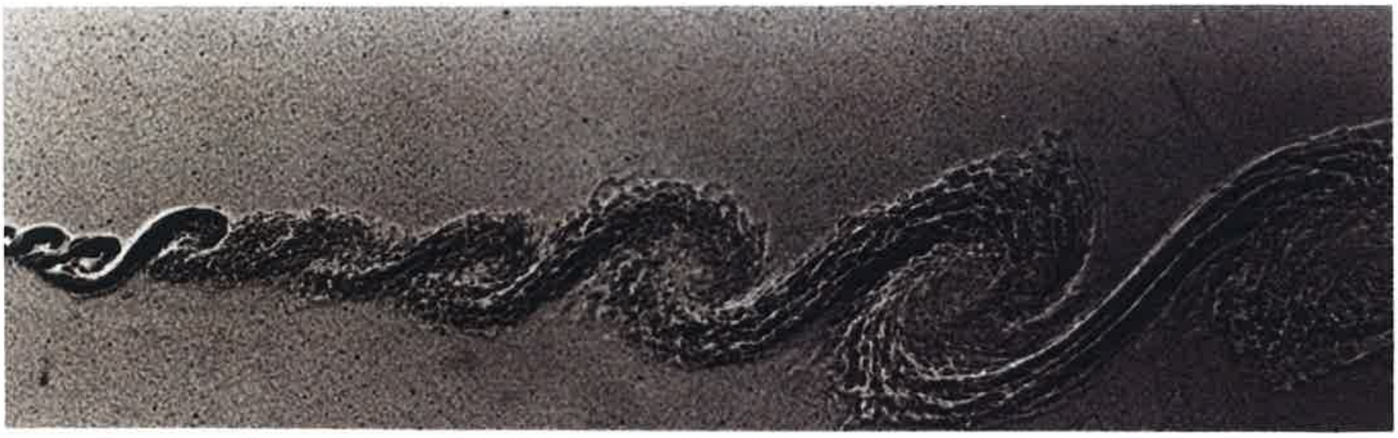


Same mean velocity (in a pipe)

Same Pressure gradient

FIG. 21.14 Comparison of (i) laminar and (ii) turbulent velocity profiles in a pipe for (a) the same mean velocity, and (b) the same pressure gradient. (The diagrams correspond to a turbulent flow Reynolds number of about 4000; for higher Re the contrast is more marked, cf. Fig. 2.11.)

Large Scale & Small scale structures in Turbulence



176. Large-scale structure in a turbulent mixing layer. Nitrogen above flowing at 1000 cm/s mixes with a helium-argon mixture below at the same density flowing at 380 cm/s under a pressure of 4 atmospheres. Spark shadow photography shows simultaneous edge and plan views, demonstrating the spanwise organization of the large eddies. The streamwise streaks in the plan view (of which half the span is shown) correspond to a system of secondary vortex pairs oriented in the streamwise direction. Their spacing at the downstream side of the layer is larger than near the beginning. *Photograph by J. H. Konrad, Ph.D. thesis, Calif. Inst. of Tech., 1976.*

177. Coherent structure at higher Reynolds number. This flow is as above but at twice the pressure. Doubling the Reynolds number has produced more small-scale structure without significantly altering the large-scale structure. *M. R. Rebollo, Ph.D. thesis, Calif. Inst. of Tech., 1976; Brown & Roshko 1974*

Increasing the Reynolds number produces smaller structures.

Turbulent flow Similarity

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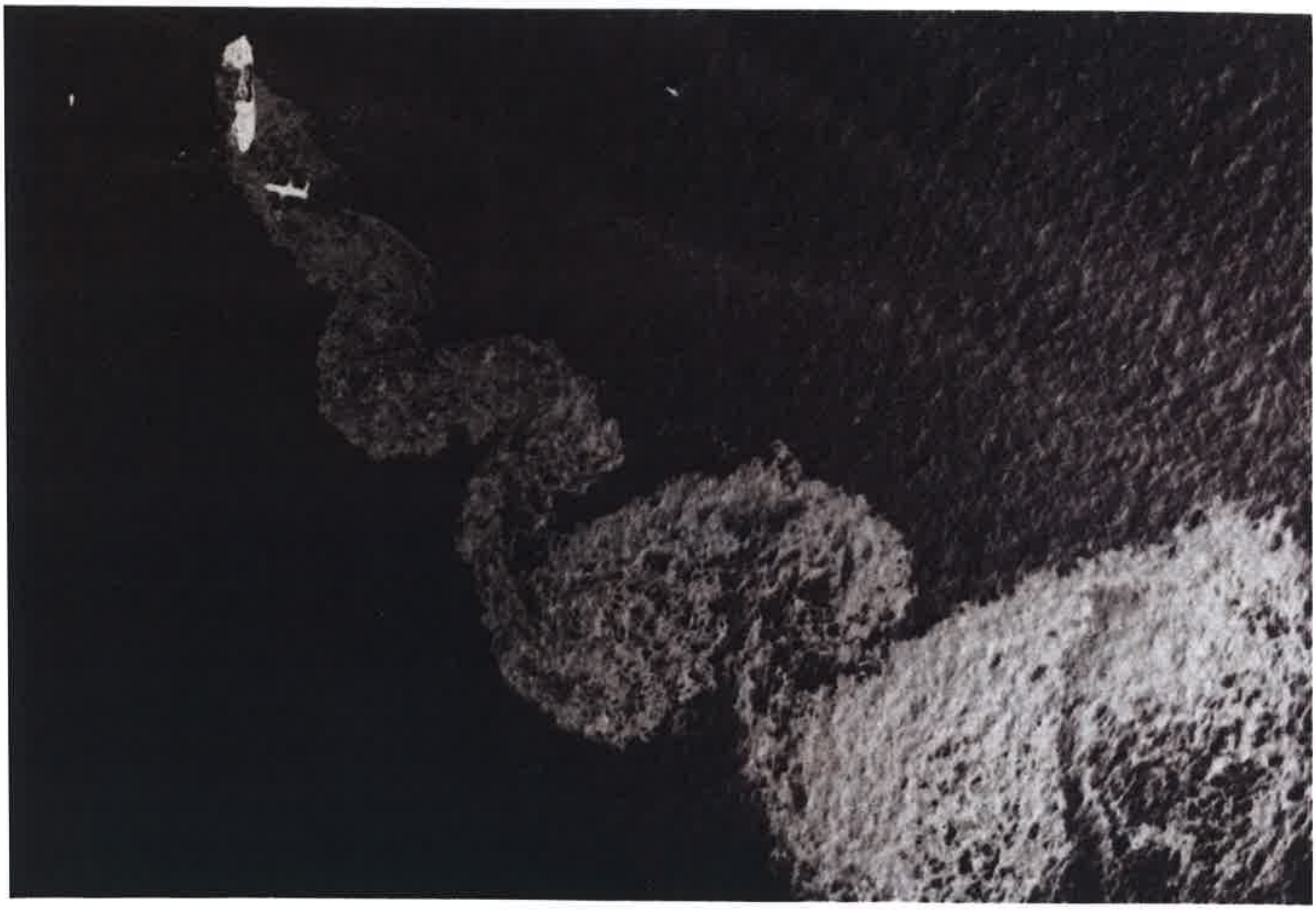
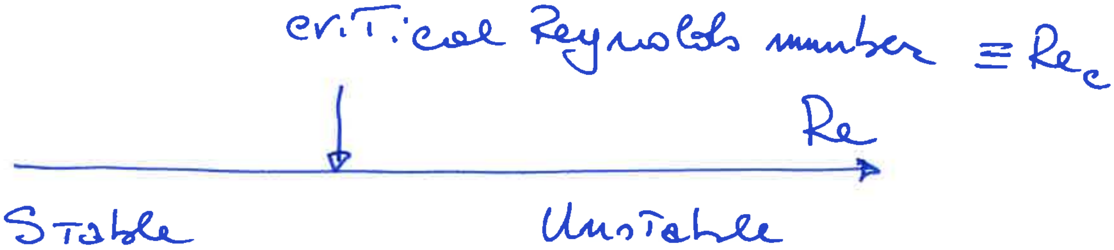
172. Wake of an inclined flat plate. The wake behind a plate at 45° angle of attack is turbulent at a Reynolds number of 4300. Aluminum flakes suspended in water show its characteristic sinuous form. *Cantwell 1981. Reproduced, with permission, from the* Annual Review of Fluid Mechanics, *Volume 13. © 1981 by Annual Reviews Inc.*



173. Wake of a grounded tankship. The tanker *Argo Merchant* went aground on the Nantucket shoals in 1976. Leaking crude oil shows that she happened to be inclined at about 45° to the current. Although the Reynolds number is approximately 10^7, the wake pattern is remarkably similar to that in the photograph at the top of the page. *NASA photograph, courtesy of O. M. Griffin, Naval Research Laboratory.*

From the Reynolds' experiment we see that:

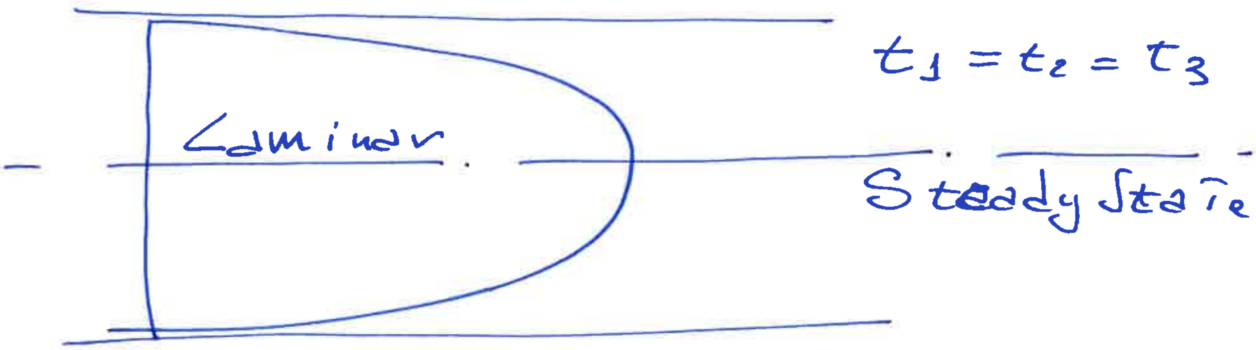


Below the Re_c perturbations are damped.
Above the Re_c perturbations are amplified.

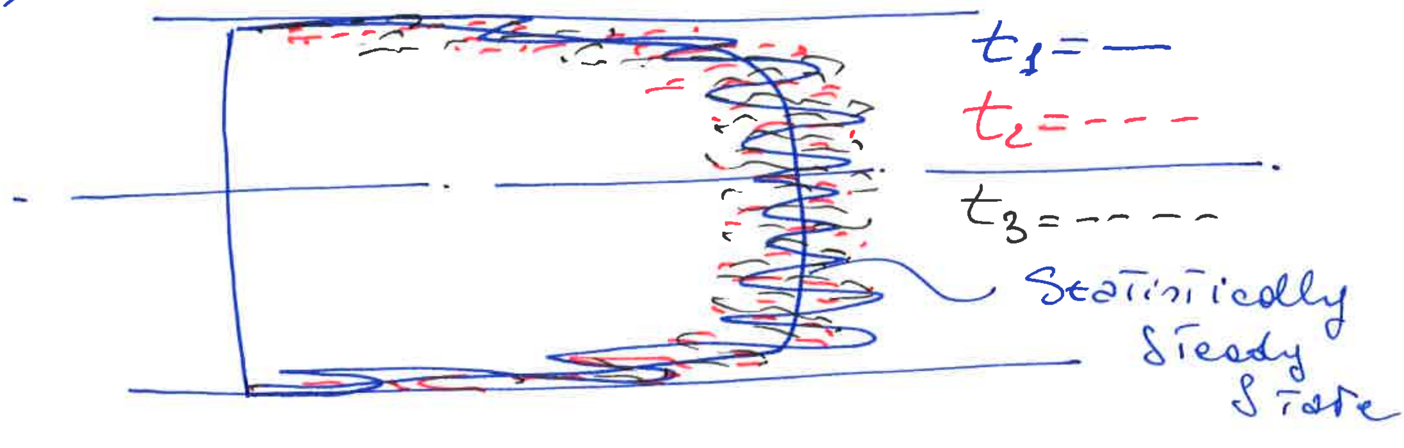
[Examples of the boxes]

Suppose we have a channel flow, a Poiseuille flow (driven by a pressure gradient).

1)



2)



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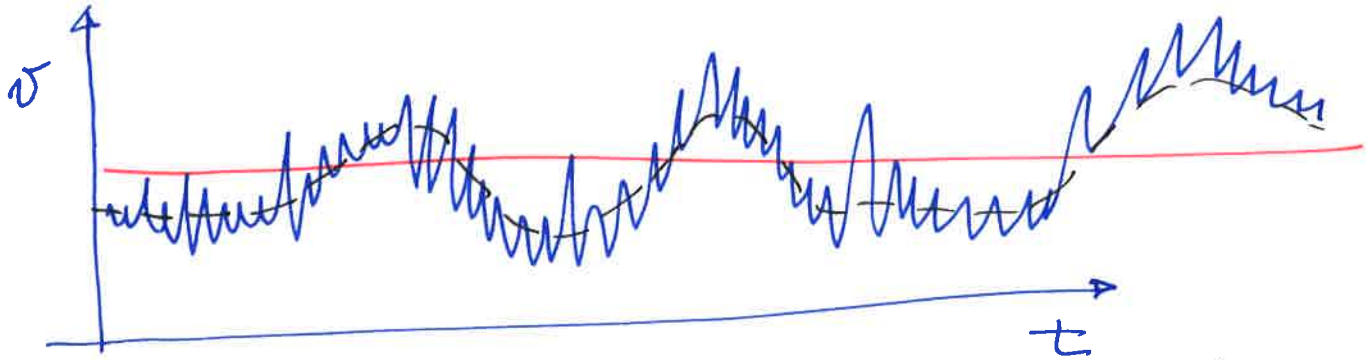
In fact, we are not really interested in the instantaneous behavior of the quantities, velocity, pressure etc. We are rather interested in average quantities. A velocity probe in our channel flow will yield:



– – – – – – average over short period

——————— average over long period

In the discrete space, the average of a generic variable ξ_i is:

$$\bar{\xi} = \frac{1}{N} \sum_{i=1}^{N} \xi_i$$

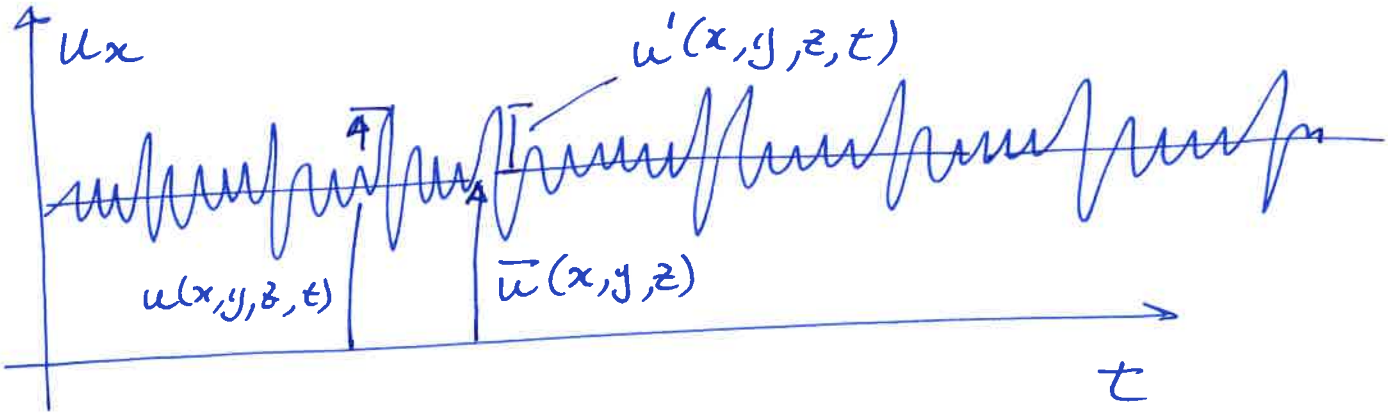
and in the continuous space:

$$\bar{\xi}(t) = \frac{1}{2T} \int_{t-T}^{t+T} \xi(t) dt$$

with $T \to \infty$

In a channel with constant pressure drop we have for the average velocity:

$$\bar{u}_{\substack{x\\y\\z}}(\cancel{t}) = \bar{u}_{\substack{x\\y\\z}} = \lim_{t\to\infty} \frac{1}{2T}\int_{-T}^{T} u_{\substack{x\\y\\z}}(x,y,z,t)\,dt$$



So we can decompose our velocity as:

$$u(x,y,z,t) = \bar{u}(x,y,z) + u'(x,y,z,t)$$

with

$\bar{u}$ = Average velocity NOT depending on time

u' = fluctuating velocity

u = instantaneous velocity.

In general applications we are not interested in the fluctuating part of the variables, we are rather interested in their average value. In the same way reasoned Reynolds who proposed the following procedure:

1. We decompose the variables into average and fluctuating part and we substitute them into the balance equations.

2. We average the equations over time so to obtain time independent equations with ~~ees~~ most (hopefully all) terms ~~#~~ fluctuating which are eliminated.

3. We can solve these equations to obtain the average variables.

The Balance equations are:

$$\frac{\partial v_i}{\partial x_i} = 0$$

$$\rho\left[\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j}\right] = -\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_j^2}$$

and the variables $v_i = \overline{v_i} + v_i'$; $P = \overline{P} + P'$

We start with the continuity equation:

$$\frac{\partial}{\partial x_i}\left(\overline{v_i} + v_i'\right) = \frac{\partial \overline{v_i}}{\partial x_i} + \frac{\partial v_i'}{\partial x_i} = 0$$

Applying the time average to the equation we have:

$$\overline{\frac{\partial \overline{v_i}}{\partial x_i}} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{\partial \overline{v_i}}{\partial x_i}\, dt = \frac{\partial \overline{v_i}}{\partial x_i}$$

$$\overline{\frac{\partial v_i'}{\partial x_i}} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{\partial v_i'}{\partial x_i}\, dt =$$

x and t are independent.

$$= \lim_{T \to \infty} \frac{1}{2T} \frac{\partial}{\partial x_i} \underbrace{\int_{-T}^{T} v_i'\, dt}_{=0 \text{ by definition}} = 0$$

The continuity equation becomes:

$$\boxed{\frac{\partial \bar{v}_i}{\partial x_i} = 0}$$

Going back to the non-averaged equation we have:

$$\underbrace{\frac{\partial \bar{v}_x}{\partial x} + \frac{\partial \bar{v}_y}{\partial y} + \frac{\partial \bar{v}_z}{\partial z}}_{=0} + \underbrace{\frac{\partial v'_x}{\partial x} + \frac{\partial v'_y}{\partial y} + \frac{\partial v'_z}{\partial z}}_{=0} = 0$$

So we have that BOTH the average continuity and the fluctuating continuity are zero.

Now we apply the same procedure to the N/S equations: We substitute:

$$\rho\left[\frac{\partial(\bar{v}_i+v_i')}{\partial t}+(\bar{v}_j+v_j')\frac{\partial(\bar{v}_i+v_i')}{\partial x_j}\right]=-\frac{\partial}{\partial x_i}(\bar{P}+P')+$$

$$+\mu\frac{\partial^2}{\partial x_j^2}(\bar{v}_i+v_i')$$

Then we average:

$$\rho\left[\overline{\frac{\partial}{\partial t}(\bar{v}_i+v_i')}+\overline{(\bar{v}_j+v_j')\frac{\partial}{\partial x_j}(\bar{v}_i+v_i')}\right]=-\overline{\frac{\partial}{\partial x_i}(\bar{P}+P_i')}+$$

$$+\mu\overline{\frac{\partial^2}{\partial x_j^2}(\bar{v}_i+v_i')}$$

$$\rho\left[\overline{\frac{\partial}{\partial t}\bar{v}_i}+\overline{\frac{\partial}{\partial t}v_i'}+\overline{\bar{v}_j\frac{\partial\bar{v}_i}{\partial x_j}}+\overline{\bar{v}_j\frac{\partial v_i'}{\partial x_j}}+\overline{v_j'\frac{\partial\bar{v}_i}{\partial x_j}}+\right.$$

$$\left.+\overline{v_j'\frac{\partial v_i'}{\partial x_j}}\right]=-\overline{\frac{\partial\bar{P}}{\partial x_i}}-\overline{\frac{\partial P'}{\partial x_i}}+\mu\overline{\frac{\partial^2\bar{v}_i}{\partial x_j^2}}+\mu\overline{\frac{\partial^2 v_i'}{\partial x_j^2}}$$

- Time and space are independent
- The average of an average is the average.

... and then :

- $\overline{\dfrac{\partial \overline{v_i}}{\partial t}} = 0$

- $\overline{\dfrac{\partial v_i'}{\partial t}} = 0$ $\qquad$ $\overline{\dfrac{\partial p'}{\partial x_i}} = 0$

- $\overline{\overline{v_j}\dfrac{\partial \overline{v_i}}{\partial x_j}} = \overline{v_j}\dfrac{\partial \overline{v_i}}{\partial x_j}$

- $\overline{\overline{v_j}\dfrac{\partial v_i'}{\partial x_j}} = 0$ and $\overline{v_j'\dfrac{\partial \overline{v_i}}{\partial x_j}} = 0$

- $\overline{v_j'\dfrac{\partial v_i'}{\partial x_j}} = \overline{\dfrac{\partial v_j' v_i'}{\partial x_j}} - \overline{v_i'\dfrac{\partial v_j'}{\partial x_j}}$

  ↳ continuity of the fluctuating part and $= \emptyset$!

- $\overline{\dfrac{\partial^2 v_i'}{\partial x_j^2}} = 0$

Substituting into the equation, we have

$$\boxed{\rho\left[\overline{v_j}\frac{\partial \overline{v_i}}{\partial x_j}\right] = -\frac{\partial \overline{P}}{\partial x_i} + \mu\frac{\partial^2 \overline{v_i}}{\partial x_j^2} - \rho\frac{\partial \overline{v_j' v_i'}}{\partial x_j}}$$

So we have an average equation with averaged variables BUT we have an extra term.

$$\Rightarrow \rho \overline{v_j} \frac{\partial \overline{v_i}}{\partial x_j} = -\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j} \mu \frac{\partial \overline{v_i}}{\partial x_j} - \frac{\partial}{\partial x_j} \rho \overline{v_j' v_i'}$$

$$\boxed{\rho \overline{v_j} \frac{\partial \overline{v_i}}{\partial x_j} = -\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu \frac{\partial \overline{v_i}}{\partial x_j} - \rho \overline{v_j' v_i'} \right]}$$

with $\mu \frac{\partial \overline{v_i}}{\partial x_j} = \tau_{ij}$

and $-\rho \overline{v_j' v_i'}$ = Reynolds stresses.

The Reynolds stress is a tensor defined as $\overline{\tau}^R_{ij} = -\rho \overline{v_i' v_j'}$

We have been able to derive an average equation in which only average variables appear. However, we have now the problem of getting acquainted with the new character: $\overline{\tau}^R_{ij}$.

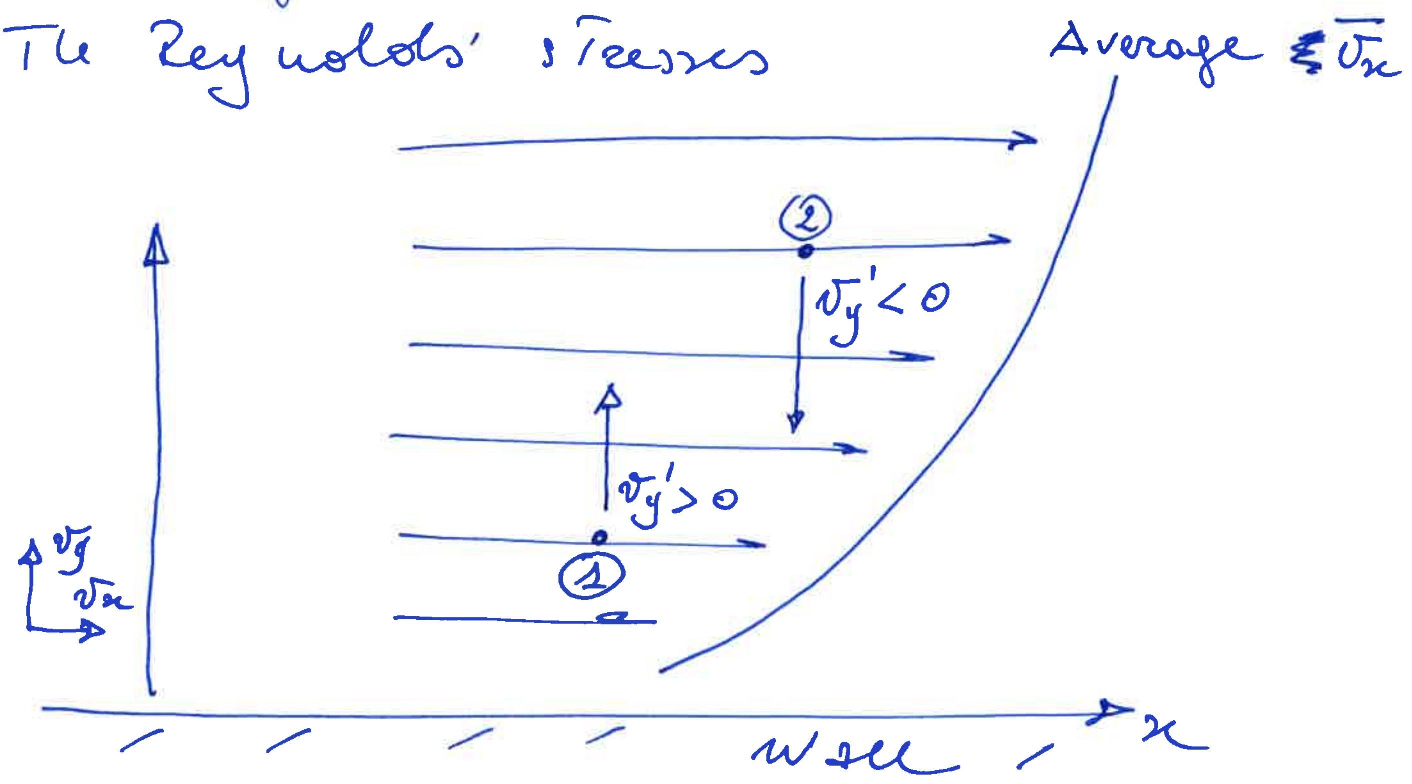
As the equation is now cast, it seems that we have more damping for the same pressure gradient.

And if it were true, that would explain the velocity profiles we have just seen: in turbulent flow for the same pressure gradient we are able to transfer less flowrate.

Reynolds' stresses

$$\overline{\tau}^{R}_{ij} = -\rho\,\overline{v_i' v_j'} \neq 0$$ Only if the two variables, v_i' and v_j' are NOT independent of each other.

We observe the situation of the turbulent channel flow to examine the behavior of the Reynolds' stresses



1. The particle 1 has a positive velocity fluctuation (average $\bar{v}_y = 0$) in the y direction and moves in a region where all particles have a larger velocity in the x direction. It is likely (we speak of statistical fluctuations) that the instantaneous velocity in x will be lower than the mean $v_x' < 0$

$$\Rightarrow \quad v_y' > 0 \quad \text{and} \quad v_x' < 0 \quad \Rightarrow \quad v_x' v_y' < 0$$

2. Particle 2 has a negative velocity fluctuation in y and has likely a positive velocity fluctuation in x.

$$\Rightarrow \quad v_y' < 0 \quad \text{and} \quad v_x' > 0 \quad \Rightarrow \quad v_x' v_y' < 0$$

<u>Then</u>

$$\bar{\tau}_{yx} = \mu \frac{\partial \bar{v}_x}{\partial y} - \rho \overline{v_x' v_y'} > \mu \frac{\partial \bar{v}_x}{\partial y}$$