Drag on a Slender Body.

- Last Time we examined the application of Potential Flow Theory to compute the Drag on a Cylinder — The reasoning was the following:

- Potential Flow ⟹ only inertial forces are important.

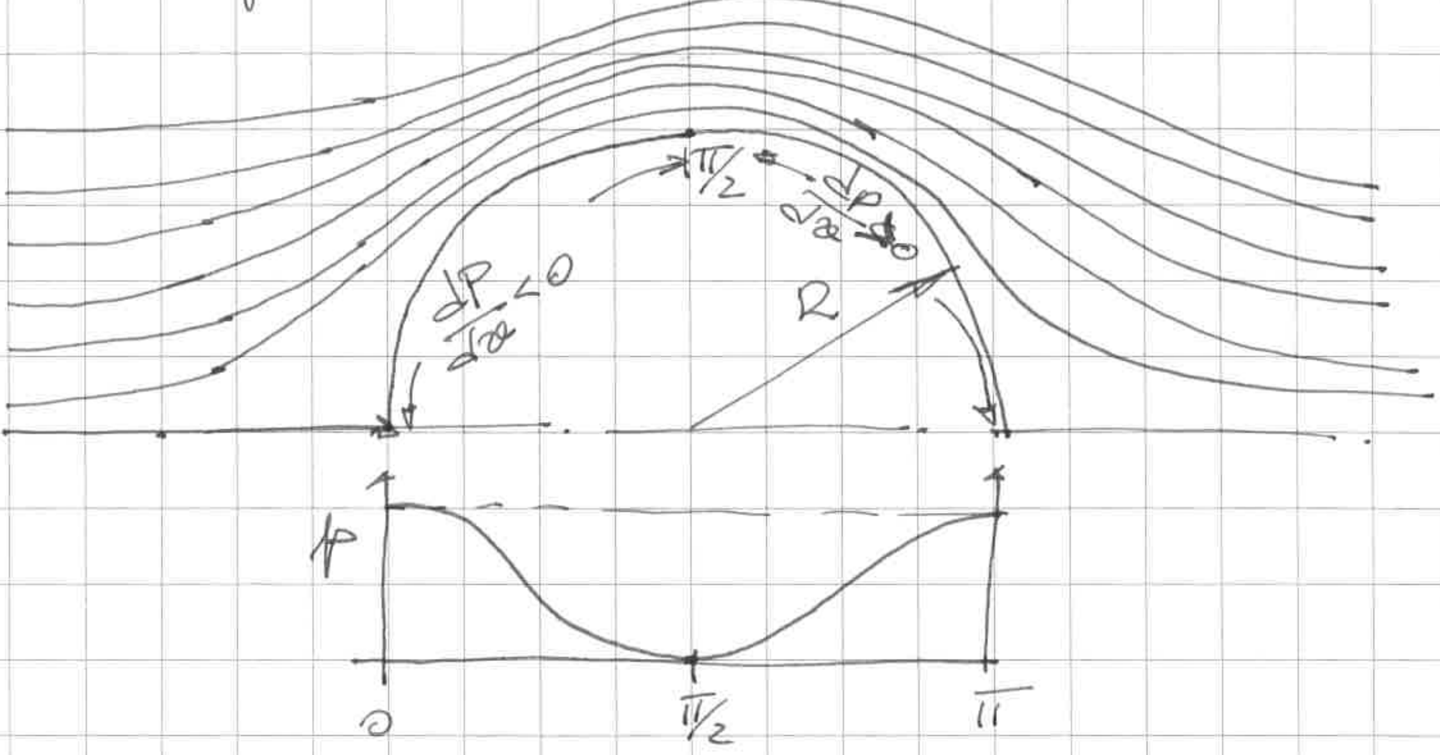
- Drag on a Body is due to

    a Viscous Contribution
    an ~~Inertial~~ Pressure Contribution *

→ If the Re is very high, only inertial forces are important; ~~and~~ pressure contribution should be balanced by inertial forces and viscous contribution should be negligible

* Define a Blunt Body and a Slender Body with the example of hands out of the windows —

For the flow about a circular cylinder Potential Flow Theory predicts:



There is No Pressure DRAG!.

PARADOX: An assertion that is essentially self-contradictory though based on a valid deduction from acceptable premises

Our model for the flow physics (i.e. Potential Flow) was not sophisticate enough for the question!

The reason is the boundary condition: near the body the flow has a No slip condition (Reality) - Therefore the local Reynolds number is small and precisely where we want ~~our~~ model to work (near the boundary) viscous forces become comparable to inertial forces. ~~Our models~~
Consequence: our model is not slightly off, it is totally wrong!
[when it rains, it pours]*
Ein Unglück kommt selten allein

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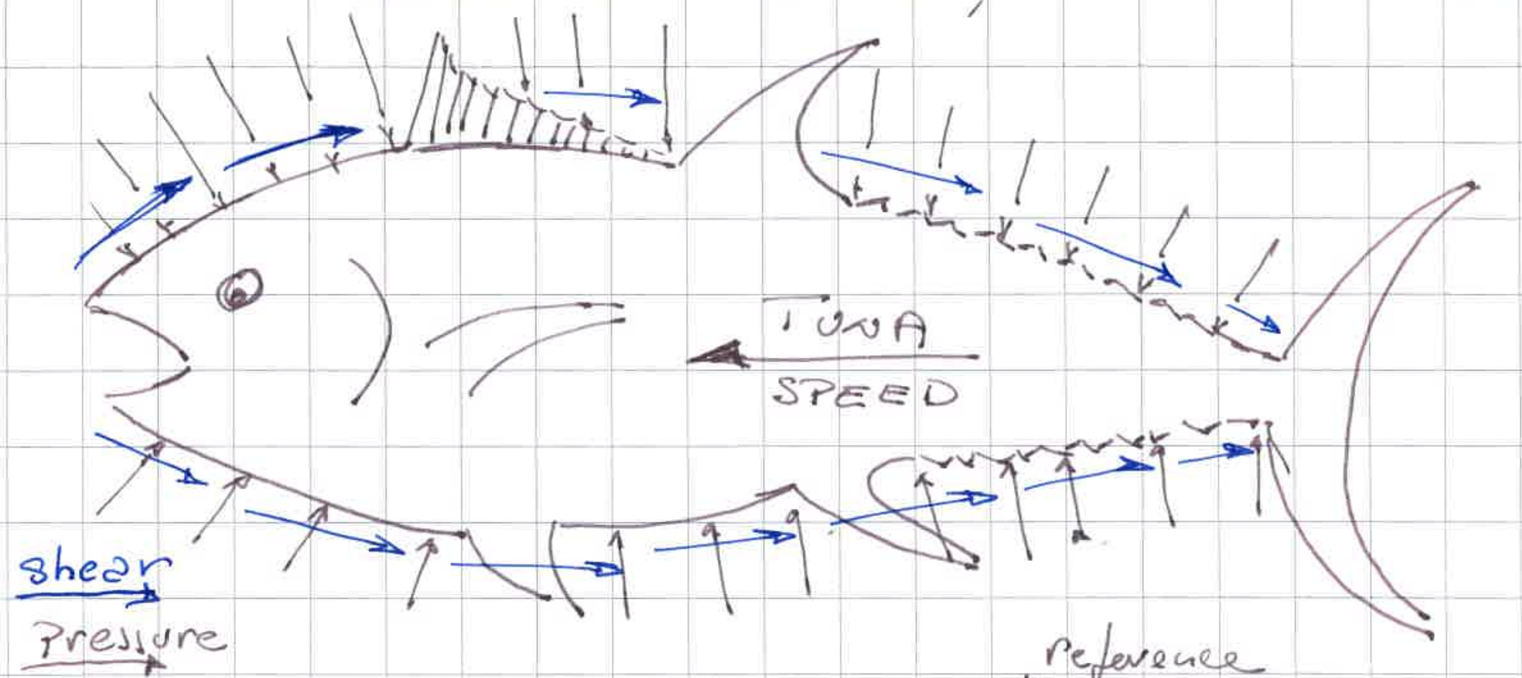
What is the Total DRAG made of?

The drag on a body immersed in a fluid is given by Two contributions:

PRESSURE contribution, FORM DRAG

VISCOUS contribution, SKIN FRICTION



reference

$$\text{PRESSURE FORCE} = \int_S (p - p_\infty)\, \vec{n}\, \hat{i}\, dA$$

$$\text{Viscous Force} = \int_S \tau_w\, \vec{t}\, \hat{i}\, dA$$

Our Topic is a <u>SLENDER BODY</u>

PRESSURE FORCE NEGLIGIBLE SLENDER
streamlined

VISCOUS FORCE NEGLIGIBLE BLUFF
Blunt

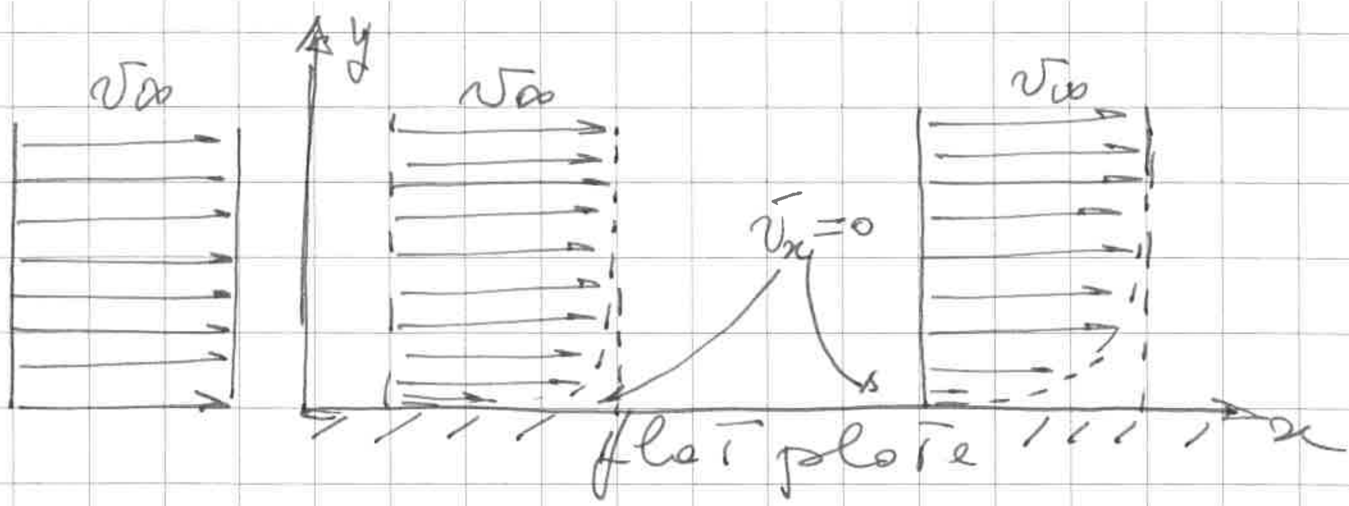
[driving no hands on wheel example]

PLATE 23 Van Dyke gives a nice representation.

Ideal representation of a slender body is ~~a~~ plate n. 29 Van Dyke.

In this case, a flat plate at zero incidence, the slenderest body produces a wake (a flow region where some momentum is missing) ~~as~~ purely due to viscous effects.

It is indeed the effect of the No SLIP Boundary condition.



Streamwise (v_x) velocity goes from zero ($\emptyset$) to v_∞ from the plate surface outward, a velocity gradient develops.

$$\frac{d v_x}{d y} \neq 0$$ and a

corresponding shear stress

$$\tau_w = \mu \frac{d v_x}{d y} \neq 0$$

Which is what we want to find.

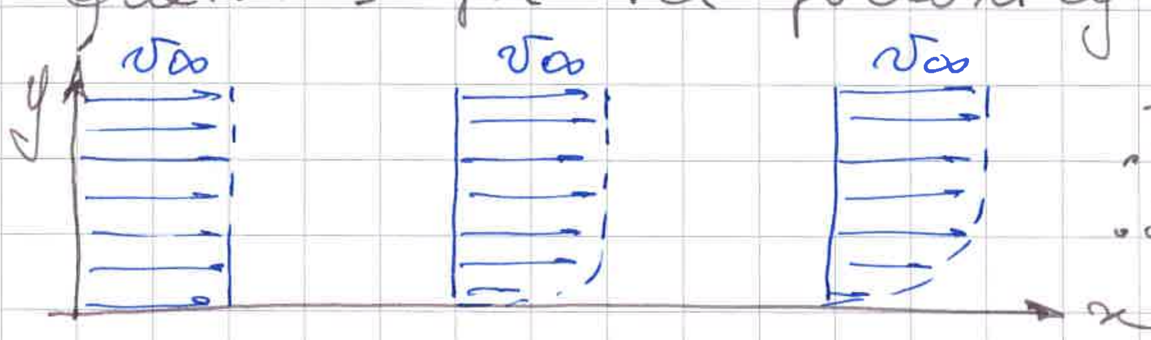
Now we know the concept of a Slender Body and why it produces a force on the fluid passing by - We also know the name of this force Skin friction

How can we have an estimate for this Drag?
A new concept is required

The BOUNDARY LAYER.

"The Boundary Layer is the region near the body where inertial terms and viscous terms in the equation of fluid motion have similar importance"

We will apply an order of magnitude analysis to the Navier Stokes equations for the following scenario.



. Two D.
. Steady

For a dimensional analysis we need SCALES: Start with observation. The plate is long (L) and the boundary layer occupies a relatively thin region (δ).

The velocity along x scales with v_∞

Is there a velocity along y? How is it big?

From the continuity $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$ we see that not only it exists but it varies with y. We call it V. How big is that?

A dimensionless form of continuity

$$\frac{v_\infty}{L}\frac{\partial \tilde{v}_x}{\partial \tilde{x}} + \frac{V}{\delta}\frac{\partial \tilde{v}_y}{\partial \tilde{y}} = 0$$ requires

$$V \simeq \frac{\delta}{L} v_\infty$$

We know the scale of v_y, but we do not know how big δ is and how it grows.

We need the Navier-Stokes equations along x and a scale for the pressure Π which we do not know yet.

$$NS_x)\quad \rho\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}\right)$$

making it dimensionless with
v_∞, V, L, δ and Π with $\frac{V}{v_\infty}\ll 1$; $\frac{\delta}{L}\ll 1$

$$\rho\frac{\delta^2 v_\infty}{L\mu}\left(\tilde{v}_x\frac{\partial \tilde{v}_x}{\partial \tilde{x}}+\tilde{v}_y\frac{\partial \tilde{v}_x}{\partial \tilde{y}}\right)=-\frac{\Pi\delta^2}{\mu v_\infty L}\frac{\partial \tilde{P}}{\partial \tilde{x}}+\left[\left(\frac{\delta}{L}\right)^2\frac{\partial^2 \tilde{v}_x}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{v}_x}{\partial \tilde{y}^2}\right]$$

Inertial and viscous terms have to be the same order, therefore

$$\rho\frac{\delta^2 v_\infty}{L\mu}\simeq 1$$

$$\Rightarrow\quad \delta\simeq\sqrt{\frac{\mu L}{\rho v_\infty}}$$

and

$$\delta\simeq\sqrt{\frac{\mu L\cdot L}{\rho v_\infty L}}=L\cdot Re_L^{-1/2}$$

So now we know that the boundary layer grows with the square root of L.

In addition, we get for free a modified and useful version of the Reynolds number based on the length of the boundary layer.

Finally since pressure terms in the Navier-Stokes equations should scale as the others, we have a scale for Π

$$\Pi = \mu \frac{v_\infty L}{\delta^2} = \rho v_\infty^2$$

So now we know all scales and we can write simplified equations for the Boundary Layer.

Solution will be done in the next lecture.

We can anticipate that, for the case of a laminar boundary layer

The thickness $\delta_L = \dfrac{4.64\ x}{\sqrt{Re_x}}$

The shear stress $\tau = 0.332 \sqrt{\mu \rho \dfrac{v_\infty^3}{x}}$

The total drag $\int_S \tau_w \hat{t} \hat{i}\, dA = \frac{1}{2} \rho v_\infty^2 A \cdot C_F$

where C_F is the skin friction coefficient. and is

$$C_F = \frac{0.664}{\sqrt{Re_x}}$$