PROCEDURA DI ADIMENSIONALIZZAZIONE

DELLE EQ. DI TRASPORTO REYNOLDS - AVERAGED

1. TRASPORTO DI k : $0 = \frac{\partial}{\partial y}\left(\frac{C_\mu}{\sigma_k} \cdot \frac{k^2}{\varepsilon} \frac{\partial k}{\partial y}\right) + C_\mu \frac{k^2}{\varepsilon}\left(\frac{\partial \bar{u}}{\partial y}\right)^2 - \varepsilon$

Poniamo:

$y^+ = y \frac{u_\tau}{\nu} \Rightarrow y = y^+ \frac{\nu}{u_\tau}$

$\bar{u}^+ = \bar{u}/u_\tau \Rightarrow \bar{u} = \bar{u}^+ \cdot u_\tau$

$k^+ = k/u_\tau^2 \Rightarrow k = k^+ \cdot u_\tau^2$

$\varepsilon^+ = \varepsilon \frac{\nu}{u_\tau^4} \rightarrow \varepsilon = \varepsilon^+ \frac{u_\tau^4}{\nu}$

Avremo:

$$0 = \frac{u_\tau}{\nu}\frac{\partial}{\partial y^+}\left[\frac{C_\mu}{\sigma_k} \cdot \frac{k^{+2}}{\varepsilon^+} \cdot \frac{u_\tau^4}{\left(\frac{u_\tau^4}{\nu}\right)} \cdot \left(\frac{u_\tau^3}{\nu}\frac{\partial k^+}{\partial y^+}\right)\right] + C_\mu \frac{k^{+2}}{\varepsilon^+} \cdot \frac{u_\tau^4}{\left(\frac{u_\tau^4}{\nu}\right)}\left(\frac{\partial \bar{u}^+}{\partial y^+}\right)^2 \cdot \frac{u_\tau^2}{\left(\frac{\nu}{u_\tau}\right)^2} - \frac{\varepsilon^+ u_\tau^4}{\nu}$$

$$0 = \frac{u_\tau}{\nu}\frac{\partial}{\partial y^+}\left(\frac{C_\mu}{\sigma_k} \cdot \frac{k^{+2}}{\varepsilon^+}\frac{\partial k^+}{\partial y^+} \cdot u_\tau^3\right) + C_\mu \frac{k^{+2}}{\varepsilon^+}\left(\frac{\partial \bar{u}^+}{\partial y^+}\right)^2 \cdot \frac{u_\tau^4}{\nu} - \frac{\varepsilon^+ u_\tau^4}{\nu}$$

$$0 = \cancel{\frac{u_\tau^4}{\nu}}\frac{\partial}{\partial y^+}\left(\frac{C_\mu}{\sigma_k} \cdot \frac{k^{+2}}{\varepsilon^+}\frac{\partial k^+}{\partial y^+}\right) + C_\mu \frac{k^{+2}}{\varepsilon^+}\left(\frac{\partial \bar{u}^+}{\partial y^+}\right)^2 \cdot \cancel{\frac{u_\tau^4}{\nu}} - \varepsilon^+ \cancel{\frac{u_\tau^4}{\nu}}$$

$$0 = \frac{\partial}{\partial y^+}\left(\frac{C_\mu}{\sigma_k} \cdot \frac{k^{+2}}{\varepsilon^+} \cdot \frac{\partial k^+}{\partial y^+}\right) + C_\mu \frac{k^{+2}}{\varepsilon^+}\left(\frac{\partial \bar{u}^+}{\partial y^+}\right)^2 - \varepsilon^+ \quad \text{C.V.D.}$$

2. TRASPORTO DI ε : $0 = \frac{\partial}{\partial y}\left(\frac{C_\mu}{\sigma_\varepsilon} \cdot \frac{k^2}{\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + C_\mu C_{\varepsilon 1} k \left(\frac{\partial \bar{u}}{\partial y}\right)^2 - C_{\varepsilon 2}\frac{\varepsilon^2}{k}$

$$0 = \frac{u_\tau}{\nu}\frac{\partial}{\partial y^+}\left[\frac{C_\mu}{\sigma_\varepsilon} \cdot \frac{k^{+2}}{\varepsilon^+} \cdot \frac{u_\tau^4}{\left(\frac{u_\tau^4}{\nu}\right)} \cdot \frac{\partial \varepsilon^+}{\partial y^+}\left(\frac{u_\tau^4}{\nu} \cdot \frac{u_\tau}{\nu}\right)\right] + C_\mu C_{\varepsilon 1} k^+ \cdot u_\tau^2 \left(\frac{\partial \bar{u}^+}{\partial y^+}\right)^2 \cdot \frac{u_\tau^2}{\left(\frac{\nu}{u_\tau}\right)^2}$$

$$- C_{\varepsilon 2}\frac{\varepsilon^{+2}}{k^+} \cdot \frac{u_\tau^8}{\nu^2} \cdot \frac{1}{u_\tau^2}$$

$\Rightarrow$

$$0 = \frac{u_\tau}{\nu}\frac{\partial}{\partial y^+}\left[\frac{C_\mu}{\sigma_\varepsilon}\cdot\frac{k^{+2}}{\varepsilon^+}\frac{\partial \varepsilon^+}{\partial y^+}\cdot\frac{u_\tau^5}{\nu}\right] + C_\mu C_{\varepsilon 1}\, k^+\left(\frac{\partial \bar{u}^+}{\partial y^+}\right)^2\cdot\frac{u_\tau^6}{\nu^2} - C_{\varepsilon 2}\frac{\varepsilon^{+2}}{k^+}\cdot\frac{u_\tau^6}{\nu^2}$$

- $$0 = \frac{\partial}{\partial y^+}\left[\frac{C_\mu}{\sigma_\varepsilon}\cdot\frac{k^{+2}}{\varepsilon^+}\cdot\frac{\partial \varepsilon^+}{\partial y^+}\right] + C_\mu C_{\varepsilon 1}\, k^+\left(\frac{\partial \bar{u}^+}{\partial y^+}\right)^2 - C_{\varepsilon 2}\frac{\varepsilon^{+2}}{k^+} \quad \text{c.v.d.}$$

3. EQUAZ. DI NS : $0 = -\frac{\partial \bar{p}}{\partial x} + \mu\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial}{\partial y}\left(\rho C_\mu \frac{k^2}{\varepsilon}\frac{\partial \bar{u}}{\partial y}\right)$

Poniamo :

$\bar{p}^+ = \bar{p}/\pi \Rightarrow \bar{p} = \bar{p}^+\cdot\bar{\pi}$ con $\bar{\pi}$ = press. caratteristica

Avremo :

$$0 = -\frac{\pi}{\left(\frac{\nu}{u_\tau}\right)}\frac{\partial \bar{p}^+}{\partial x^+} + \mu\cdot\frac{u_\tau}{\left(\frac{\nu}{u_\tau}\right)^2}\frac{\partial^2 \bar{u}^+}{\partial y^{+2}} + \frac{u_\tau}{\nu}\frac{\partial}{\partial y^+}\left(\rho C_\mu \frac{k^{+2}}{\varepsilon^+}\cdot\frac{u_\tau^4}{\left(\frac{u_\tau^4}{\nu}\right)}\cdot\frac{\partial \bar{u}^+}{\partial y^+}\cdot\frac{u_\tau}{\left(\frac{\nu}{u_\tau}\right)}\right)$$

$$0 = -\frac{\pi u_\tau}{\nu}\frac{\partial \bar{p}^+}{\partial x^+} + \mu\frac{u_\tau^3}{\nu^2}\frac{\partial^2 \bar{u}^+}{\partial y^{+2}} + \frac{u_\tau}{\nu}\frac{\partial}{\partial y^+}\left(\rho C_\mu \frac{k^{+2}}{\varepsilon^+}\cdot\frac{\partial \bar{u}^+}{\partial y^+}\cdot u_\tau^2\right)$$

$$0 = -\frac{\pi u_\tau}{\nu}\frac{\partial \bar{p}^+}{\partial x^+} + \mu\frac{u_\tau^3}{\nu^2}\frac{\partial^2 \bar{u}^+}{\partial y^{+2}} + \frac{u_\tau^3}{\nu}\frac{\partial}{\partial y^+}\left(\rho C_\mu \frac{k^{+2}}{\varepsilon^+}\cdot\frac{\partial \bar{u}^+}{\partial y^+}\right) \qquad \rho = \frac{\mu}{\nu}$$

$$0 = -\frac{\pi u_\tau}{\nu}\frac{\partial \bar{p}^+}{\partial x^+} + \mu\frac{u_\tau^3}{\nu^2}\frac{\partial^2 \bar{u}^+}{\partial y^{+2}} + \mu\frac{u_\tau^3}{\nu^2}\frac{\partial}{\partial y^+}\left(C_\mu \frac{k^{+2}}{\varepsilon^+}\cdot\frac{\partial \bar{u}^+}{\partial y^+}\right)$$

Ponendo $\bar{\pi} = \rho u_\tau^2 = \mu\frac{u_\tau^2}{\nu}$ abbiamo $\frac{\pi u_\tau}{\nu} = \mu\frac{u_\tau^3}{\nu^2}$

Semplificando il termine comune (e costante) $\mu\frac{u_\tau^3}{\nu^2}$ troviamo:

- $$\boxed{0 = -\frac{\partial \bar{p}^+}{\partial x^+} + \frac{\partial^2 \bar{u}^+}{\partial y^{+2}} + \frac{\partial}{\partial y^+}\left(C_\mu \frac{k^{+2}}{\varepsilon^+}\frac{\partial \bar{u}^+}{\partial y^+}\right)}$$

Poniamo ora : $z_5 = \bar{u}$, $z_6 = \frac{\partial \bar{u}^+}{\partial y^+} = \frac{\partial z_5}{\partial y^+}$

Avremo :

$$0 = -\frac{\partial \bar{p}^+}{\partial x^+} + \underbrace{\frac{\partial}{\partial y^+}\left(\frac{\partial \bar{u}^+}{\partial y^+}\right)}_{\frac{\partial z_6}{\partial y^+}} + C_\mu \underbrace{\frac{\partial}{\partial y^+}\left(\frac{k^{+2}}{\varepsilon^+}\cdot\frac{\partial \bar{u}^+}{\partial y^+}\right)}_{2\frac{k^+}{\varepsilon^+}\cdot\frac{\partial k^+}{\partial y^+}\cdot\frac{\partial \bar{u}^+}{\partial y^+} - \left(\frac{k^+}{\varepsilon^+}\right)^2 \frac{\partial \varepsilon^+}{\partial y^+}\frac{\partial \bar{u}^+}{\partial y^+} + \frac{k^{+2}}{\varepsilon^+}\underbrace{\frac{\partial}{\partial y^+}\left(\frac{\partial \bar{u}^+}{\partial y^+}\right)}_{\frac{\partial z_6}{\partial y^+}}}$$

$$0 = -\frac{\partial \bar{p}^+}{\partial x^+} + \underline{\frac{\partial z_6}{\partial y^+}} + \left[2\,\frac{z_1 z_3}{z_2}\cdot z_6 - \left(\frac{z_1}{z_2}\right)^2 z_4 z_6 + \underline{\frac{z_1^2}{z_2}\cdot\frac{\partial z_6}{\partial y^+}}\right] C_\mu$$

$$\frac{\partial z_6}{\partial y^+}\left(1 + C_\mu \frac{z_1^2}{z_2}\right) = \frac{\partial \bar{p}^+}{\partial x^+} - C_\mu z_6\left[\frac{2 z_1 z_3}{z_2} - \left(\frac{z_1}{z_2}\right)^2 z_4\right]$$

$$\boxed{\frac{\partial z_6}{\partial y^+} = \left[\frac{\partial \bar{p}^+}{\partial x^+} - C_\mu z_6\left(\frac{2 z_1 z_3}{z_2} - z_4\frac{z_1^2}{z_2^2}\right)\right]\cdot\frac{1}{\left(1 + C_\mu \frac{z_1^2}{z_2}\right)}}$$

~~EQ.~~ DA INTEGRARE PER OTTENERE z_6 OVVERO $\frac{\partial u^+}{\partial y^+} = f(y^+)$

Il <u>SISTEMA DI ODE</u> da risolvere è pertanto il seguente:

$$\begin{cases}
\dfrac{\partial z_1}{\partial y^+} = z_3 & (z_1 = k) \\[2ex]
\dfrac{\partial z_2}{\partial y^+} = z_4 & (z_2 = \varepsilon) \\[2ex]
\dfrac{\partial z_5}{\partial y^+} = z_6 & (z_5 = u) \\[2ex]
\dfrac{\partial z_3}{\partial y^+} = -\dfrac{2 z_3^2}{z_1} + \dfrac{z_3 z_4}{z_2} - \sigma_k \cdot z_6^2 + \dfrac{\sigma_k}{C_\mu}\dfrac{z_2^2}{z_1^2} \\[2ex]
\dfrac{\partial z_4}{\partial y^+} = -\dfrac{2 z_3 z_4}{z_1} + \dfrac{z_4^2}{z_2} - C_{\varepsilon 1}\sigma_\varepsilon z_6^2 \dfrac{z_2}{z_1} + \dfrac{C_{\varepsilon 2}\sigma_\varepsilon}{C_\mu}\left(\dfrac{z_2}{z_1}\right)^3 \\[2ex]
\dfrac{\partial z_6}{\partial y^+} = \left[\dfrac{\partial \bar{p}^+}{\partial x^+} - C_\mu z_6\left(\dfrac{2 z_1 z_3}{z_2} - \dfrac{z_1^2 z_4}{z_2^2}\right)\right]\cdot\dfrac{1}{\left(1 + C_\mu \dfrac{z_1^2}{z_2}\right)}
\end{cases}$$

Soluzione = $\underbrace{\text{ode45}}$ (argomenti)

calcola la soluzione di una ODE del tipo $y' = f(t, y)$ nell'intervallo $tspan \equiv [t_0 ; t_f]$ con condizione iniziale y_0

Argomenti (in input):

ode45($\underbrace{\text{odefun}}_{\text{ODE da risolvere}}$, tspan, y0)

AD ESEMPIO, PER RISOLVERE IL SISTEMA :

$$\frac{\partial X_1}{\partial z^+} = X_3$$

$$\frac{\partial X_2}{\partial z^+} = X_4$$

$$\frac{\partial X_3}{\partial z^+} = \dots$$

$$\frac{\partial X_4}{\partial z^+} = \dots$$

$\Rightarrow$

- soluzione = ode45(@solve4ode, [150 0], ...

  nome dato a odefun

con ziniz precedentemente definito:

- ziniz = [0.7   0.006   0   0]

  - 0.7 → valore di K @ $z = H/2$
  - 0.006 → valore di ε_K @ $z = H/2$
  - 0 → valore di $\partial K / \partial z^+$ @ $z = H/2$
  - 0 → valore di $\frac{\partial \varepsilon_K}{\partial z^+}$ @ $z = H/2$

Si definisce poi:

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function dz = solve4ode(t,z)
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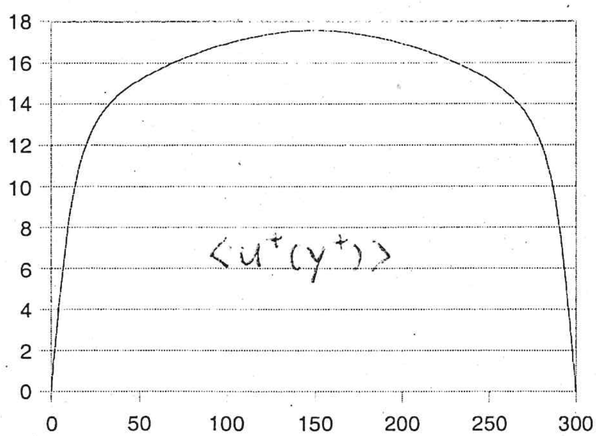
che serve per definire il sistema di 4 ode:

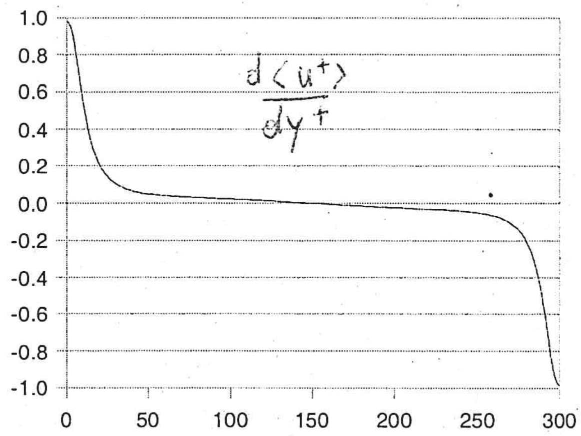
$dz(1) = z(3);$ ← $\frac{\partial x_1}{\partial z^+} = x_3$

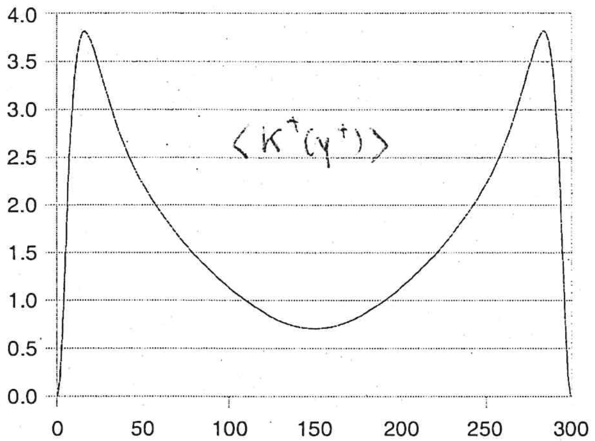
$dz(2) = z(4);$ ← $\frac{\partial x_2}{\partial z^+} = x_4$

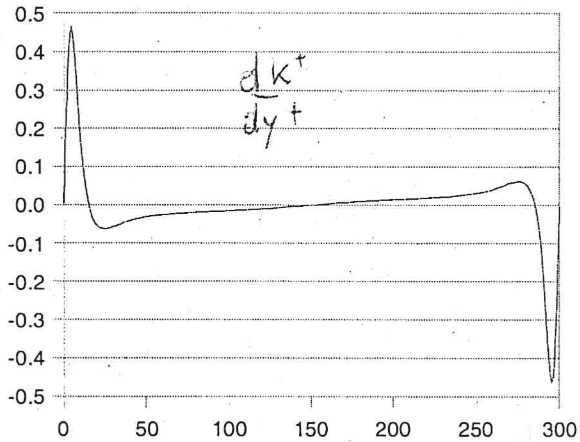
$dz(3) = \ldots;$ ← $\frac{\partial x_3}{\partial z^+} = \ldots$

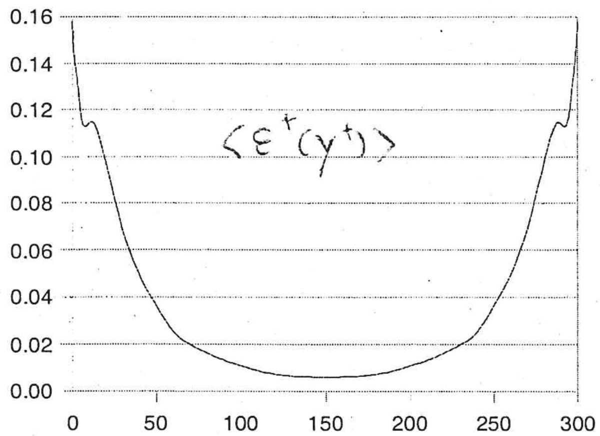
$dz(4) = \ldots;$ ← $\frac{\partial x_4}{\partial z^+} = \ldots$

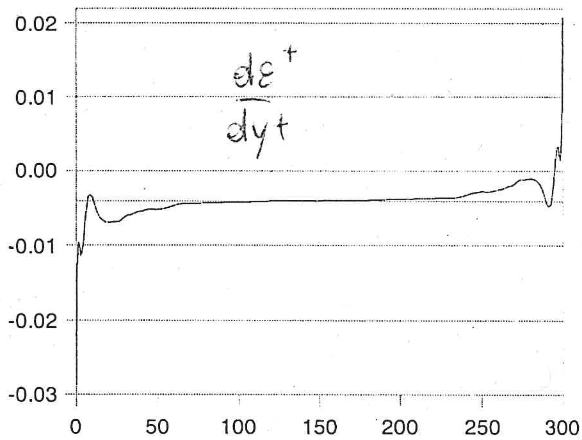

⟨u⁺(y⁺)⟩
18
16
14
12
10
8
6
4
2
0
0 50 100 150 200 250 300

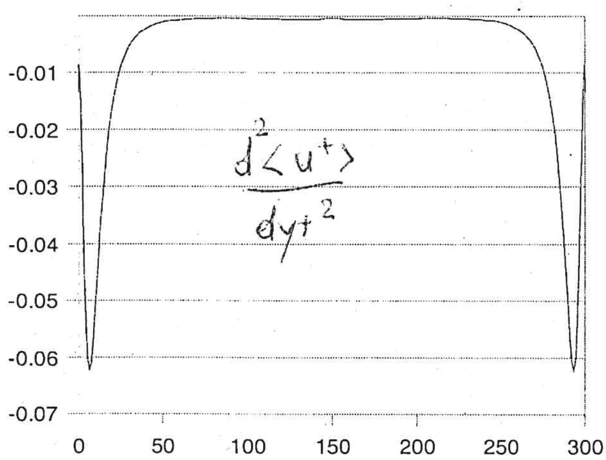

d⟨u⁺⟩/dy⁺
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0 50 100 150 200 250 300

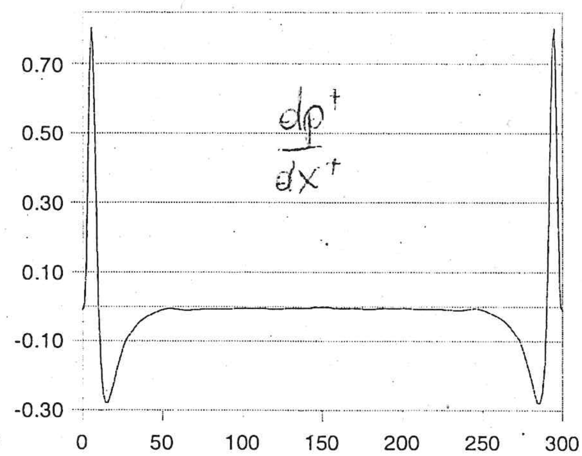

⟨K⁺(y⁺)⟩
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0 50 100 150 200 250 300


dK⁺/dy⁺
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
0 50 100 150 200 250 300


⟨ε⁺(y⁺)⟩
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0 50 100 150 200 250 300


dε⁺/dy⁺
0.02
0.01
0.00
-0.01
-0.02
-0.03
0 50 100 150 200 250 300


d²⟨u⁺⟩/dy⁺²
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
0 50 100 150 200 250 300


dp⁺/dx⁺
0.70
0.50
0.30
0.10
-0.10
-0.30
0 50 100 150 200 250 300