Natural convection in a square cavity

In this work I treat a natural convection problem (namely, the velocity field is induced by a temperature difference) in a square domain .
Consider a compartment with a square cross section. The fluid (water) in the compartment is assumed to be initially motionless and at a temperature of 21.5 °C. The left side wall is cooled at a temperature of 20.0 °C while the right side wall is heated at 23 °C, so that the compartment feels the temperature difference Δt=3°C.
The top and bottom side are thermally insulated, which means:

$$\frac{\partial T}{\partial n} = 0$$

The solution of the problem is reached by numerically solving the Navier-Sokes equations (which describe the velocity field) and the energy equation (which describes the thermal field). In natural convection problems, these equation are fully coupled (the temperature field induces fluid motion which in turn drive the temperature).
Once the Navier-Stokes equations are written in terms of stream function and vorticity, the fundamental equations become:

$$-\omega = \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right)$$

$$\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + Gr \frac{\partial T}{\partial x}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\Pr} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

The dimensionless groups used in the above equations are defined as:

$$\Pr = \frac{\nu}{\lambda}$$

$$Gr = \frac{g \beta \Delta T L^3}{\nu^2}$$

If we define the Rayleigh number as

$$Gr = \frac{g \beta \Delta T L^3}{\lambda \nu} = Gr \Pr$$

the problem can be solved once the Rayleigh and Prandtl number are chosen.

Simulations

The following results are obtained numerically solving the fundamental equations listed above. These equations are discretized in a 2D Cartesian domain by means of finite difference method at Prandtl number $Pr=1$ and Rayleigh number $Ra=10^4$.

Stream Function

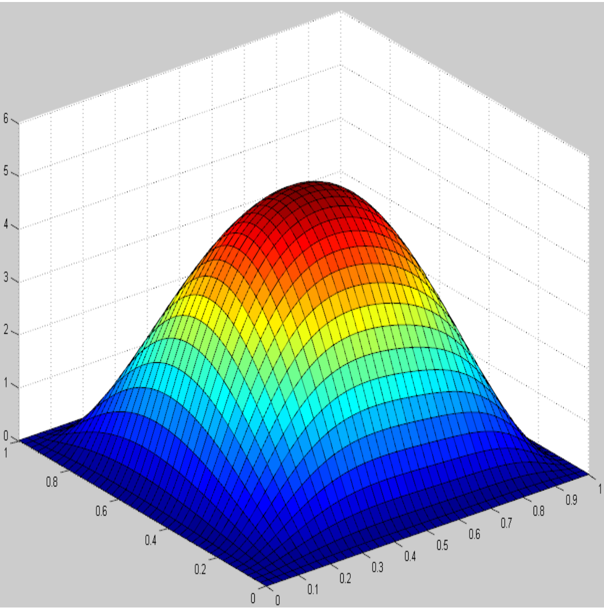

Figura 1: Stream function, 3D surface

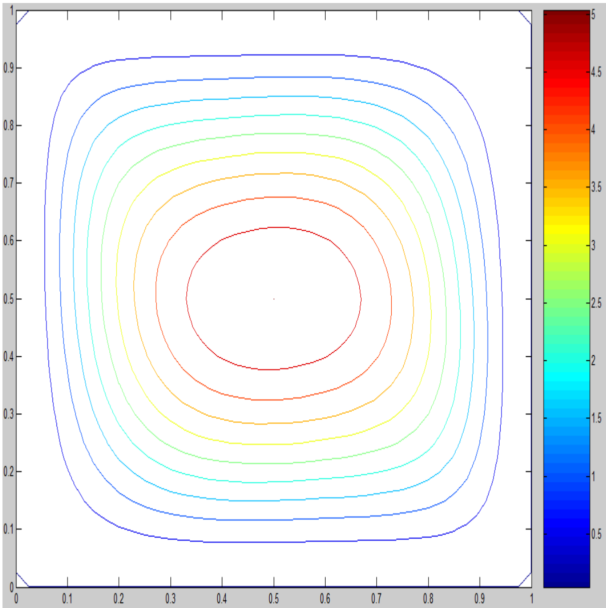

Figura 2: Stream function, isocontours

Velocity Field

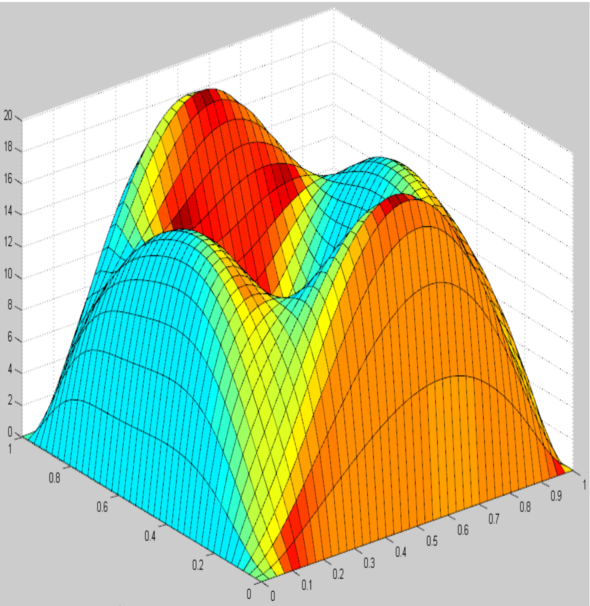

Figure 3: Magnitude of velocity field

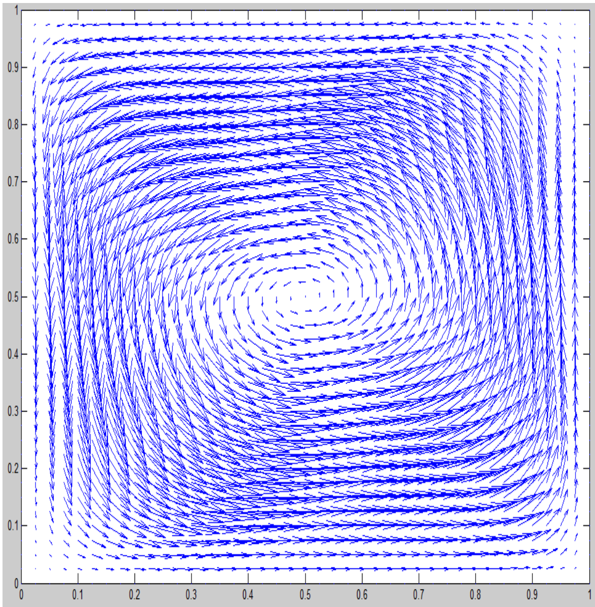

Figure 4: Velocity field, vector visualization

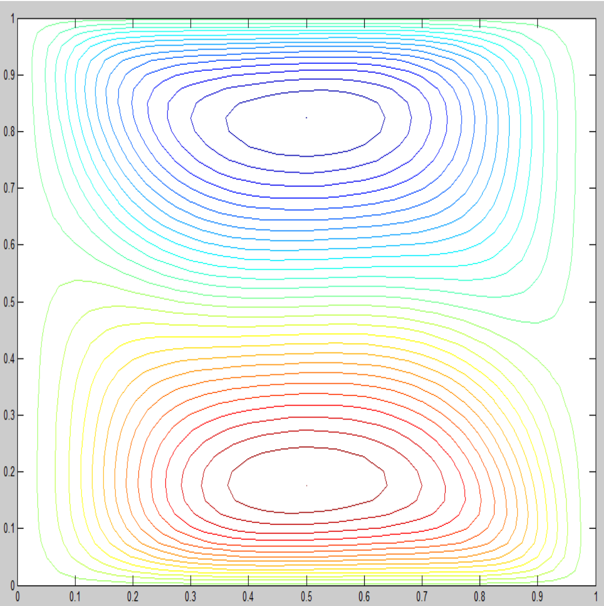

Figure 5: Magnitude of u-velocity, isocontours

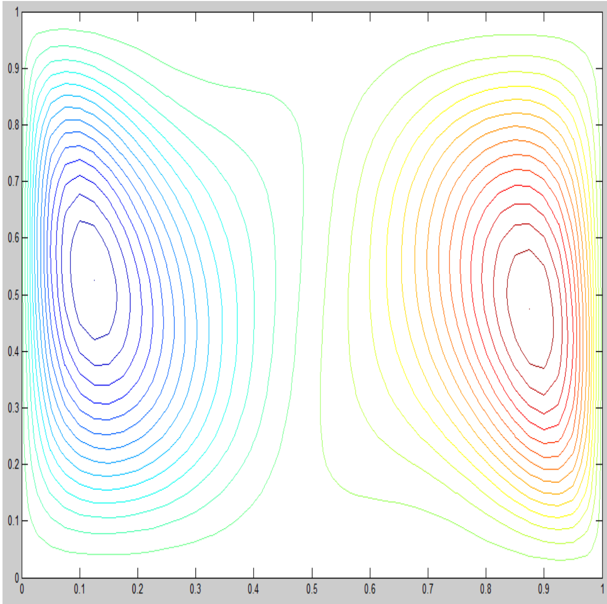

Figure 6: magnitude of v-velocity, isocontours

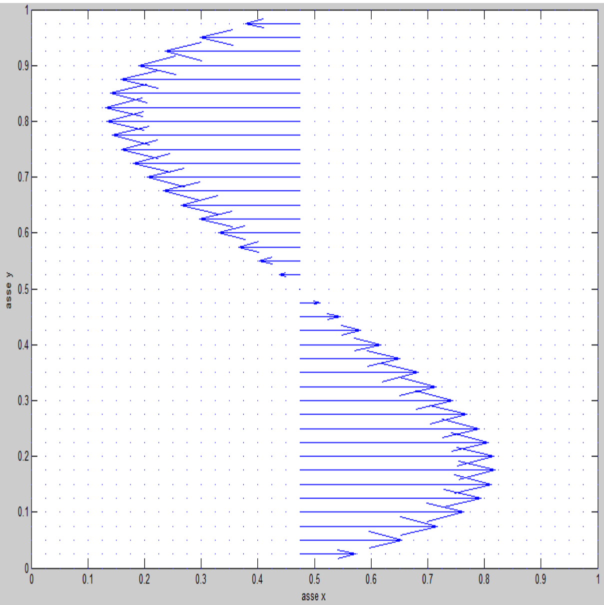

Figure 7: u-velocity in the middle section

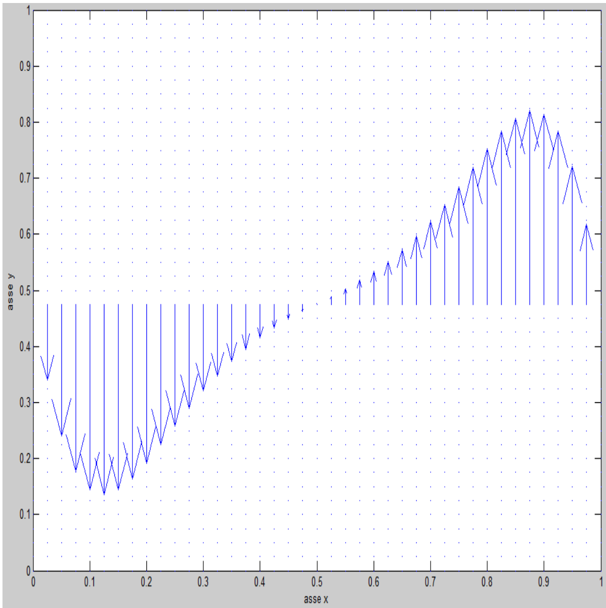

Figure 8: v-velocity in the middle section

Temperature Field

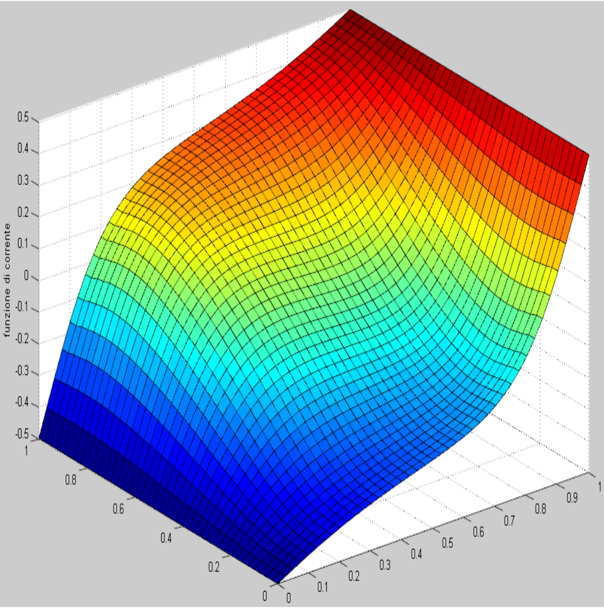

Figure 9: Temperature field, 3D surface

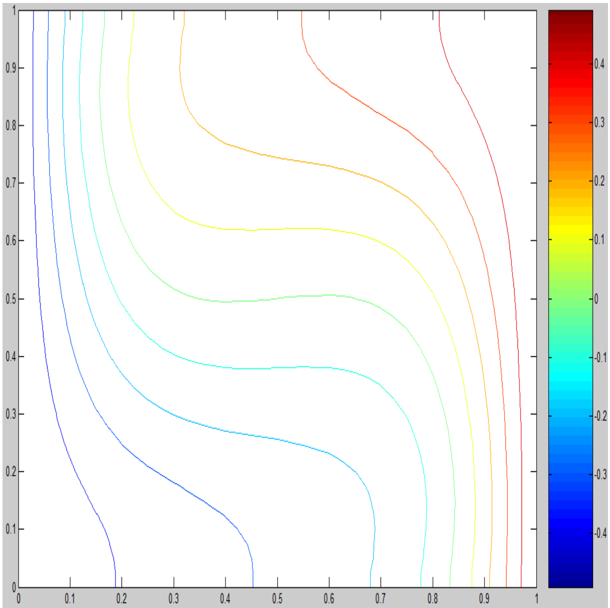

Figure 10: Temperature field, isocontours

Vorticity

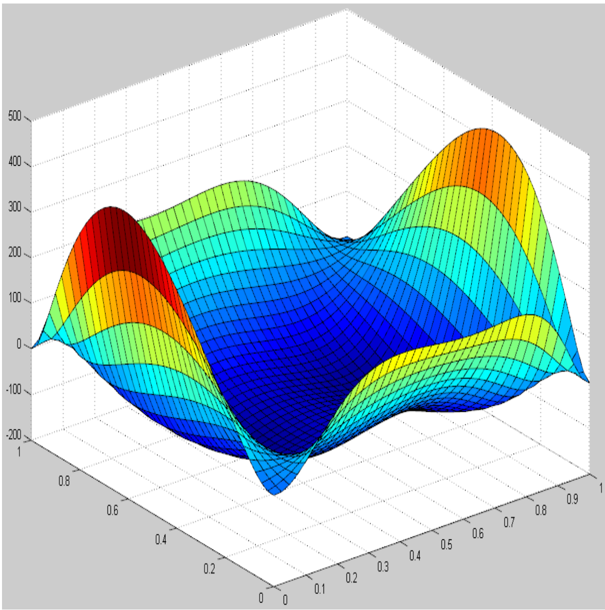

Figure 11: Vorticity field, 3D surface