

Stochastic models for turbulent particle dispersion in general inhomogeneous flows

A. Dehbi Paul Scherrer Institut Switzerland

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Outline

- > Background. Why stochastic models?
- > Discrete random walk model (DRW). Shortcomings
- > Continuous random walk (CRW) based on the Langevin equation
 - Standard Langevin equation
 - > Non-dimensional Langevin equation
- Sample results of the CRW model
 - Isothermal flows
 - Flows with thermal gradients (active thermophoresis)
- Concluding remarks



Particles-turbulence: applications

- Particle-turbulence interactions play a crucial role in wide range of applications
 - Atmospheric dispersion of pollutants
 - Sediment transport in rivers
 - > Drug delivery in human airways
 - Combustion
 - Fouling in compressor and turbine blades
 - Chemical pulping
 - > Nuclear fission products transport
- "Turbulence has a strong influence on plankton contact rate, which is a crucial parameter for plankton ecology". ③

Recent paper in J. Marine Systems



Background

> CFD method increasingly successful in prediction of turbulent flows in general geometries

> Particle dispersion in CFD codes predicted using:

- > Eulerian two-fluid methods
 - Particles regarded as continuous phase with own averaged equations (mass, momentum, etc)
 - > Better suited for denser suspensions when particle-particle interactions important
 - > Main challenges: defining interphasial exchange terms, boundary conditions
- Lagrangian particle tracking (LPT)
 - > One solves first for the continuous phase (Eulerian)
 - Then: one follows paths of a "large" sample of particles by integration of Newton's 2nd Law



Lagrangian methods: Pros & Cons

≻Pros:

- > Rigorous and intuitive inclusion of all relevant forces on particle (e.g. drag, gravity, thermophoretic force, etc)
- > Rigorous and intuitive treatment of boundary conditions
- > More appropriate for dispersed flows, with low particle loading

≻Cons:

Computational expense: Necessary to track a large number of particles until stationary statistics are achieved



Background

- > CFD with LPT successful in predicting laminar flows
- In turbulent flows, DNS and LES coupled to LPT offer most rigorous way of treating particle dispersion in Euler/Lagrange frameworks. However:
 - Very time consuming
 - > Difficult (sometimes impossible) to apply in general geometries
 - > Want quick answers with "good enough" accuracy using today's CFD codes
- In past, CFD-LPT treatment in turbulent flows has showed unsatisfactory accuracy due to:
 - Inappropriate modeling of turbulence seen by particles
 - > Rather rough assumptions e.g. turbulence isotropic in whole domain
- > Recent advances in stochastic models and coupling to CFD codes offer hope for a good compromise between accuracy and computer expense



Particle-Turbulence interactions in LPT

Supposing drag is the only significant force on the particle. The particle path is extracted from:

$$\frac{dU_{p}}{dt} = F_{D}(U - U_{p}), \quad F_{D} = \frac{18\mu}{\rho_{p}d_{p}^{2}}C_{D}\frac{\text{Re}}{24}, \qquad \text{Re} = \frac{\rho_{p}d_{p}|U - U_{p}}{\mu}$$

> A major issue in Lagrangian particle tracking: modeling fluid turbulence.



- > RANS turbulent models in CFD produce *averaged* fluid field quantities
- > How to extract instantaneous fields from averaged fields? **Stochastic models**



Random Walk Models: preview

> Premise:

- A random walk model consisting of a large number of statistically independent steps is suitable to represent the chaotic nature of turbulent diffusion
- The mean flow equations solved analytically/numerically (CFD-RANS)
- > Turbulence modeled with a random walk model
 - Discrete Random Walk
 - Continuous Random Walk





Discrete Random Walk (DRW) Model

- > Also known as Eddy Interaction Model (EIM). Due to Gosman et al., 1983
- > Particle interacts with turbulence in "Discrete Random Walks"
 - > Particle is "trapped" by an eddy during an "eddy lifetime"

$$\tau_e = 2 \cdot \tau_L = 2 \cdot C_L \frac{k}{\varepsilon}$$

- > During the lifetime of the eddy:
 - > The mean fluid velocities seen by the particle are those of the fluid
 - The fluctuating fluid components are randomly distributed Gaussian variables whose rms value are equal and deduced from the turbulent kinetic energy k:

$$\sqrt{u^{'2}} = \sqrt{v^{'2}} = \sqrt{w^{'2}} = \sqrt{2k/3}$$

- > The instantaneous fluid velocity seen by a particle is: $u_i = \lambda_i \sqrt{u_i^2}$
- $> \lambda$'s are Gaussian random variables with 0 mean and standard deviation 1



Discrete Random Walk (DRW) Model

- > Integrate trajectory until eddy life is over
- > When the eddy lifetime is over, generate another eddy with random rms of velocity
- > The particle trajectory is determined by the Lagrangian tracking

$$\frac{d\mathbf{x}_{p}}{dt} = U_{p}$$

$$m_{p} \frac{dU_{p}}{dt} = \mathbf{f} = \mathbf{f}_{Drag} + \mathbf{f}_{Gravity} + \mathbf{f}_{thermophoresis} + \mathbf{f}_{lift} + \dots$$

- > In 3D: trajectory obtained by integrating 6 coupled ODE's
- > Tacking is continued until particle is hits the wall or leaves domain



Typical Discrete Random Walk Trajectory



Fig. 1. Typical particle trajectory in the boundary layer of turbulent pipe flow: $\tau_p^+ = 0.02$; initial velocity $u_x^+ = 18$, $v_x^+ = 12$.



Shortcomings of original DRW model

- Many practical flows can be approximated as having isotropic turbulence in the *bulk*
- > However: turbulence is very anisotropic in boundary layers
- In presence of walls, particle deposition dictated by phenomena in boundary layer
- > Thus: Original DRW prediction of deposition is poor even in simple geometries (always strong over-prediction of deposition)
- > Better treatment of boundary layer effects is required because of:
 - > Anisotropy
 - Different time scales



Turbulent velocity scales in boundary layer





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Improvement of DRW: boundary layer model

Keep default model as is as long as particle in the bulk (y⁺ >100)
 If particle in boundary layer (y⁺ < 100) introduce rms values of gas velocities obtained from curve fits of DNS data in channel flow:

$$\sigma_{1} \equiv \sqrt{u'^{2}} = \frac{0.40 \cdot y^{+}}{1 + 0.0239(y^{+})^{1.496}} \cdot u^{*} \qquad \text{(streamwise direction)}$$

$$\sigma_{2} \equiv \sqrt{v'^{2}} = \frac{0.0116 \cdot (y^{+})^{2}}{1 + 0.203 \cdot y^{+} + 0.00140(y^{+})^{2.421}} \cdot u^{*} \qquad \text{(normal direction to wall)}$$

$$\sigma_{3} \equiv \sqrt{w'^{2}} = \frac{0.19 \cdot y^{+}}{1 + 0.0361(y^{+})^{1.322}} \cdot u^{*} \qquad \text{(spanwise direction)}$$

$$\text{ith the friction velocity} \qquad u^{*} = \sqrt{\frac{\tau_{w}}{\rho}}$$



Times scales

> Lagrangian time scale of fluid particle defined as:

$$R_{u_i}(\tau) = \frac{u_i(t)u_i(t+\tau)}{u_i(t)u_i(t)}$$

$$\tau_L = \int_0^\infty R_{u_i}(\tau) d\tau$$

Computed from LPT-DNS of fluid particles & ensemble averaging

 $\tau_{\text{L}} :$ typical time before $\,$ particle loses memory of its history. Velocities are

-Correlated in time intervals $O(\tau_L)$

-Uncorrelated for greater time intervals



Times scales





Times scales, fits

DNS computed scales reasonably approximated by wall function fits given by Kallio & Reeks (1989)

$$\tau_L^+ = 7.122 + 0.5731 \cdot y^+ - 0.00129 \cdot y^{+2} \qquad \text{for } 5.0 < y^+ < 100$$

$$\tau_L^+ = 10.0 \qquad \text{for } y^+ <= 5.0$$

> with the Lagrangian time scale τ_L obtained from

$$\tau_L^+ = \tau_L \cdot \frac{(u^*)^2}{v}$$



Results: Liu deposition in pipe experiments ('74)



> Unphysical deposition is significantly reduced compared to original model



Shortcomings of DRW Model

- > Still suffers from inherent deficiencies:
 - Modeled turbulence too synthetic
 - In limit of massless particles, DRW still predicts some concentration build-up near the wall ("spurious drift"), with as a result:
 - > Non-vanishing deposition velocity in the tracer limit
 - > Over-prediction of particle deposition when external forces are present (e.g. thermophoresis)
 - > A good dispersion model should obey the "well-mixed criterion" (Thompson 1987) i.e.:
 - If initially well mixed, tracer particles should remained well mixed in the domain as time evolves



Beyond DRW: CRW

- Continuous random walk (CRW) offers a more physically sound way of modeling particle dispersion
- > Fluid velocity seen by particles continuously fluctuates with time
- > Original Langevin equation (ca. 1910) used by Langevin to model Brownian velocity fluctuations
- > The stochastic Langevin equation applied for homogeneous turbulence (Obukhof 1959)



Langevin equation in homogeneous turbulence

> A spherical particle moves in a Eulerian flow domain according to:

$$\frac{dU_p}{dt} = F_{Drag}(U - U_p) \qquad \qquad F_{Drag} = \frac{18\mu}{\rho_p d_p^2} C_D \frac{\text{Re}}{24}$$

- > In turbulent flows, the carrier gas velocity: $U = \overline{U} + u$
- > Mean velocity U from CFD. How to model the fluctuating velocity u?

> The Langevin equation tries to mimic turbulence:





Radial fluid velocity v seen by particle



Classical Langevin equation: a few words

Langevin equation has intuitively the right physics

➢Produces velocity fluctuations which are "credible"

≻However:

- > Equation is a postulate i.e. is not derived from first principles
- Only comparison with experiments will allow us to conclude to its usefulness or lack thereof
- Does not obey, in its original format, the "well-mixed criterion" (Thompson, 1987). It leads to non-physical accumulation of small particles in regions of high kinetic energy (in laminar sublayer). Luckily one can correct for this.



Corrections for inhomogeneous turbulence

Sampling from rms of velocity values introduces "spurious drift", i.e. unphysical migration of small, fluid-like particles from bulk to walls

➤Correction: start with acceleration of fluid particle:

$$a_i = U_j \frac{\partial U_i}{\partial x_j}$$

>Write velocity as mean + fluctuation: $U_i = \overline{U_i} + u_i$

Plugging 2nd equation in first above, and averaging in time, while using continuity, one gets after algebra:





Tracer limit corrections

>Physics dictates what terms are dominant in the turbulent acceleration

>Example: DNS statistics are used to close the drift correction in boundary layers:

$$\overline{a_i}' = \overline{u_j \frac{\partial u_i}{\partial x_j}}$$

➢ Finally, the correction velocity in inhomogeneous turbulence:

$$\overline{\Delta u_i} = \overline{a_i} \, \Delta t = \Delta t \cdot u_j \, \frac{\partial u_i}{\partial x_j}$$

➢With correction, "spurious drift" and deposition of tracer particles significantly reduced. Periodic pipe flow of Re=10000





Tracer limit corrections, pipe flow, Re=10000





Correction for arbitrary inertia

>Inertia particles "sees" different fluid turbulence than would a fluid particle

>Bocksell & Loth (2006) have extended the drift correction to inertial particles with arbitrary Stokes number *Stk* (measure of particle relaxation vs flow scales)

≻The correction is given by:

$$\overline{\Delta u_i} = \frac{1}{1 + Stk} \cdot \left(\overline{\Delta u_i}\right)_{fluid} \qquad Stk = \frac{\tau_p}{\tau_L}$$

Expression has correct limits:

- Very low inertia particles (Stk=0) have correction of fluid particles
- > Very high inertia particles (Stk $\rightarrow \infty$) have no correction. Particle motion and turbulence increasingly decoupled
- Expression is a significant finding



Classical Langevin with corrections: assessment



>Not accurate enough in strongly inhomogeneous flows such as boundary layers



Non-dimensional Langevin equation for boundary layers

- In recent years: many improvements to Langevin equation to tackle inhomogeneous turbulence (e.g. pipe). Transported quantity in inhomogeneous turbulence is
 No longer *u* but *u/o*
- > One writes the so-called non-dimensional Langevin equation in boundary layer:



- > Requires Eulerian statistics from DNS databases. Readily available.
- Can couple CFD mean flow with Langevin fluctuating flow to predict more accurately particle motion in general flows



Non-dimensional Langevin equation outside boundary layer

> Outside boundary layer in bulk: turbulence roughly isotropic:

$$\sigma = \sigma_1 = \sigma_2 = \sigma_3 = \sqrt{\frac{2}{3} \cdot k}$$

It can be shown (my paper Int. J. Multiphase Flows, 2008) that the drift correction in the bulk takes the form:

$$A_{i}dt = \delta(\frac{u_{i}}{\sigma_{i}}) = u_{j}\frac{\partial(\frac{u_{i}}{\sigma_{i}})}{\partial x_{j}} \cdot \frac{dt}{1 + Stk} \cong \frac{1}{3\sigma}\frac{\partial k}{\partial x_{i}} \cdot \frac{dt}{1 + Stk}$$

> CFD codes solve for k, so drift correction readily computed in CFD



Non-dimensional Langevin equation in & outside boundary layer





Langevin equation in inhomogeneous media

Langevin equations:



- > Time scales τ_i in boundary layer: roughly equal in all directions (DNS findings by Bocksell & Loth, 2006):
- $\begin{aligned} \tau_L^+ &= 10 & y^+ \leq 5 \\ \tau_L^+ &= 7.122 + 0.5731 \cdot y^+ 0.00129 \cdot y^{+2} & 5 \leq y^+ \leq 100 \end{aligned} \qquad \qquad \tau_L = \frac{2}{C_o} \cdot \frac{k}{\varepsilon}, \qquad C_o = 14 \end{aligned}$



Algorithm of CFD model implementation

- Note: need to know "local" coordinate system at any particle position
 - Requires knowing location of "closest" wall to particle at any time.
 - Computation done once at post process. Not trivial, especially in complex geometry.
 - Shuttling between local and computational coordinate systems at every ∆t





Benchmarking model: deposition in turbulent flows

>Benchmarking of the model in isothermal flow

- Particle dispersion data from recent DNS computations (2007)
- > Deposition: Comparison with particle deposition data in:
 - > 2D: pipe flow (Liu-Agarwal correlation)
 - ≻ 3D flow
 - > 90° bend (Pui correlation)
 - > Mouth-throat geometry (Stahlhofen data fit, Grgic et al. data)

>Benchmarking of the model with active thermophoresis

- > TUBA tests (Dumaz, 1993)
- > Tsai tests (2004)



Comparison with DNS database statistics

Extensive DNS database for particle dispersion statistics assembled by Marchioli, Soldati et al. (IJMF, 2007).



Fig. 1. Particle-laden turbulent gas flow in a flat channel: computational domain.



Comparison with DNS database statistics

- > Re_{τ} =150, Re_{h} =2100. Periodic boundary condition in 2 directions
- > 6 classes of particles, with τ^+ =0.2,1, 5, 15, 25, 125
- > Database spans $\Delta t^+=1200$, i.e. about 10 channel transit times
- > Statistics:
 - > Particle concentration profiles at two times, $t^+=675,1125$
 - > Mean and rms of axial and normal velocities between t+=742 & t+= 1192
- Investigation studies effects of: drag, lift, gravity
- > Here we compare against results with drag only with particles with $\tau^+=0.2, 25, 125$
- > Boundary conditions: particles reflect elastically on impact with wall



Concentrations, tracer particles τ **+=0.2**





Concentrations, mid-inertia $\tau^+=15$





Concentrations, heavy particle τ^+ =125





Normal mean velocity and rms, tracer particle $\tau^+=0.2$





Normal mean velocity and rms, mid-inertia τ^+ =25





Normal mean velocity and rms, heavy particle τ^+ =125





Conclusions from comparison with DNS

- Model predictions of particle dispersion surprisingly good
 - Concentration
 - Velocity profiles (deposition rates)
- rms values of velocity slightly larger. Due to assumption of Gaussin distribution for the turbulent fluctuations.
- Every term in non-dimensional Langevin equation counts e.g. not including the Stokes correction factor





Typical heat transfer correlation graph: $\Delta = \pm 30\%$



Fig.7 Effect of grid spacer type, Type-A/Type-B and SS, on film boiling heat transfer



Deposition in pipe flow: experimental data. $\Delta = \pm 100-1000\%$!





Pause: Why particle deposition so uncertain?

> Standard way of measuring particle deposition rates:

- > Assume particle profiles are fully developed after a few 10's of L/D's
- > Draw a sample from somewhere in the bulk, and filter it.
- Assume concentration profile is flat because "turbulence mixes up things" (counterpart to temperature/velocity profiles in turbulent flows)
- > Recent DNS show procedure above is seriously flawed:
 - Preferential concentration in boundary layer. Assuming fully mixed profiles in sampling may induce large errors!
 - Very long times needed for particles to reach fully developed profiles, several 1000's of L/D! Get different deposition rates depending on where deposition is measured.
- > Recent measurements confirm phenomena of preferential concentration
- > Turbulence actually de-mixes particles!



Particle flow over plate. Tests Wang '07





Particle concentration. Diameter= 60 μ m





Particle concentration. Diameter= 200 μm





Synthetic turbulence. Particles demixing





Model assessment vs pipe flow data, low turbulence (Re =10000)





Model assessment vs pipe flow data, high turbulence (Re=50000)





Model assessment: deposition in 90° bend flow





Model assessment in 3D flows: deposition in mouth-throat geometry (MTG)





Deposition in mouth-throat (Finlay et al.)

> Research by Prof. Finlay's group Uni. of Alberta

- Deposition of DEHS particles obtained by
 - > Gravimetry
 - Gamma scintigraphy





Quality assurance of MTG CFD computations

- > Before using dispersion model, need to have confidence that the computed flow field is free of user-induced errors
- > Best practice guidelines (BPG) followed to ensure in particular:
 - > Grid-independence of results
 - Required grid resolution in the boundary layer
- > For mouth-thoat geometry
 - > Reynolds Stress Model (RSM, 7 equations) used
 - > RSM considered the "best" CFD turbulence model for general flows



Best Practice Guidelines for mouth-throat simulation

Hybrid mesh: hex in boundary layer & tet elsewhere
Fine enough to ensure y+ order 1 in wall adjacent cells (
3 grid levels

Second order accuracy



Sample velocity contours





Flow in throat section





Close-up view of flow in throat





Sample velocity profiles









Particle deposition in mouth-throat geometry



Percent deposited, 90 l/min

Particle	Data by	CRW	Mean flow
diameter	Grgic et al.	Model	tracking
μm			
3.0	33 ± 5	23.4	4.0
5.0	68 ± 3	59.5	17.2
6.5	78 ± 3	80.1	33.0

Percent deposited, 30 l/min

Particle diameter	Data by Grgic et al.	CRW Model	Mean flow tracking
μm			
3.0	2 ± 2	6.4	4.4
5.0	11 ± 3	11.8	4.1
6.5	32 ± 3	21.6	5.8



Particle dispersion in presence of thermophoresis

- > Thermophoresis: Force that drives particles from hot to cold regions of fluid
- > A spherical particle moves in a Eulerian flow domain according to:

$$\frac{dU_p}{dt} = F_{Drag}(U - U_p) + F_{Thermo} \qquad F_{Drag} = \frac{18\mu}{\rho_p d_p^2} C_D \frac{\text{Re}}{24} \qquad F_{Thermo} = C \frac{1}{T} \nabla T$$



Thermophoresis: Tests by Tsai (2004)





Thermophoresis: Tests by Tsai (2004)





Thermophoresis: TUBA tests (Dumaz, 1993)





Thermophoresis: TUBA tests (Dumaz, 1993)





Turbulence and thermophoresis for tracer particles

- > In isothermal flows, small inertia (tracer) particles
 - > Don't deposit
 - > Tend to remain fully mixed
- > If thermophoresis acts on them: particles go towards the wall <u>but</u>
 - They do have a chance to reflect back to the bulk because they respond very quickly to random turbulence bursts (unlike high inertia particles)
- > Hence: tracer particles that go to the wall will *not* all deposit there
- > Therefore: turbulence actually reduces thermophoretic deposition of very low inertia particles
- > This explains why if one ignores radial fluctuating fluid velocities, the model will over-estimate thermophoretic deposition



Conclusions

- > Non-dimensional Langevin based CRW model offers the best hope for accurate predictions of practical CFD-based particle dispersion
- > Model relies heavily on DNS statistics
- > Hence DNS research is of great importance to help produce better dispersion models
- > Best chance of success in predicting dilute particle dispersion in turbulence flows with CFD:
 - Accurate mean flow
 - Good stochastic model
- > Further benchmarking still necessary, but goal within reach