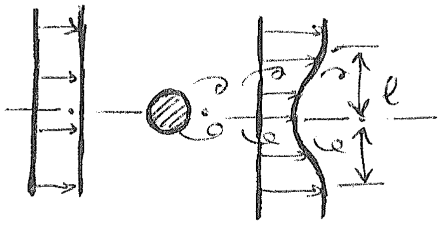
science and oceanography.

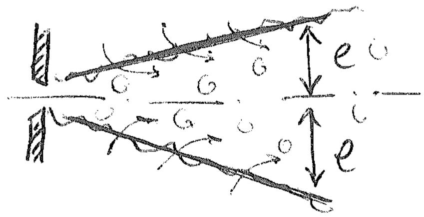
To properly close the model, one must provide the expression for l_m: such expression depends on the type of turbulent flow and, in general, is space dependent.

For instance:

- <u>Free shear flow</u> : $l_m = l$
  where l is the half-width of the shear layer

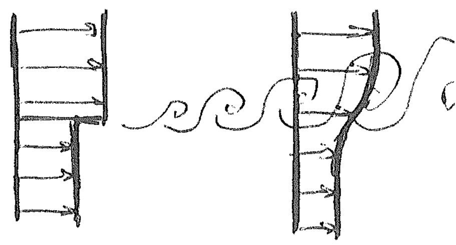


- <u>Plane jet</u> : $l_m = 0{,}09\, l$

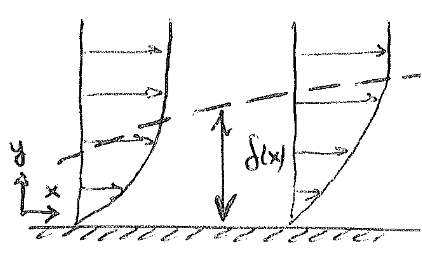


- <u>Circular jet</u> : $l_m = 0{,}075\, l$

- <u>Mixing layer</u> : $l_m = 0{,}07\, l$



- <u>Boundary layer</u> : $l_m = K \cdot y$
  with $K = 0.41$ (Von Karman constant)



• Pipe flow : $l_m = \frac{D}{2}\left[0{,}14 - 0{,}08\left(1-\frac{y}{D/2}\right)^2 - 0{,}06\left(1-\frac{y}{D/2}\right)^4\right]$

where D is the pipe diameter (expression derived by Nikuradse).

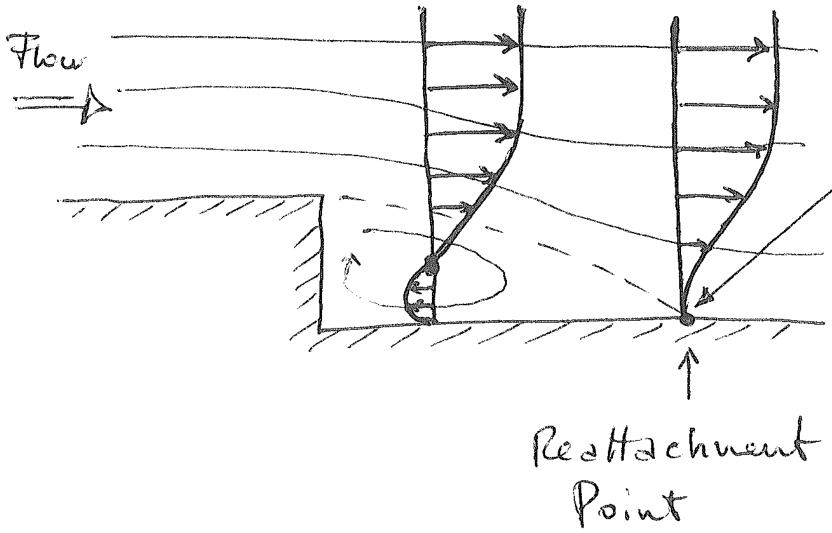
Shortcomings of the mixing length hypothesis:

From its definition ($\mu^{TURB} = \rho l_m^2 \left|\frac{\partial v_x}{\partial y}\right|$), it must be $\mu^{TURB} = 0$ when $\frac{\partial v_x}{\partial y} = 0$.

However, it has been observed experimentally that this is not always true (for instance, at the center of a pipe flow, where $\partial v_x/\partial y$ vanishes but $\mu^{TURB} = 0{,}8\, \mu^{TURB}_{max}$!).

Example of failure of the mixing length theory:

Backward-facing step



Experiments show that maximum heat flux occurs here, where $\frac{\partial v_x}{\partial y} = 0$ and therefore:

$$\gamma_T = \nu^{TURB} = l_m^2 \left|\frac{\partial v_x}{\partial y}\right| = 0$$

Heat diffusivity

SCALES OF TURBULENCE

Turbulent flows are highly unsteady flows, where the main velocity field is superimposed by (practically) random velocity fluctuations.

The major problem making turbulent flows difficult to examine (both numerically and experimentally) is the broad range of length and time scales that appear in a turbulent flow. A widely-used assessment quantifying the ratio between the largest flow scale L and the smallest flow scale η_k (estimated based on the theory developed by Kolmogorov) is given as:

$$\frac{L}{\eta_k} \sim Re_L^{3/4}$$

where $Re_L = \frac{U_L \cdot L}{\nu}$ is the flow Reynolds number based on L and on a characteristic flow velocity U_L that can be associated to L.

This means that scale separation (namely the difference in length scales appearing in the turbulent flow comprised between L and η_k)

is of order $\mathcal{O}(10^3)$ for $Re_L \sim 10^4$.

For instance, in a turbulent pipe flow at $Re_L \sim 10^5$ with pipe diameter $D = 50$ mm and mean fluid velocity of ~ 2 m/s, the largest flow scale (namely the largest eddies) have size approx equal to 25 mm ($L \sim D/2$) whereas the smallest flow scale (namely the smallest eddies) have size approx 25 μm!

Usually, the largest flow scale is determined (or limited) by the specific geometry of the flow. Therefore it may easily change depending on the specific flow instance, and has no universality.

The smallest flow scales are more universal in nature, and tend to share common characteristics among different flow instances. This has made it possible to develop a theory (Kolmogorov theory) that is capable of predicting the characteristic length, time and velocity scales of the smallest eddies that can appear in a turbulent flow.

These are given as :

- $\eta_K = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$ KOLMOGOROV LENGTH SCALE [m]
- $\tau_K = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$ KOLMOGOROV TIME SCALE [s]
- $v_K = (\nu \cdot \varepsilon)^{1/4}$ KOLMOGOROV VELOCITY SCALE $\left[\frac{m}{s}\right]$

where ν is the kinematic fluid viscosity [m^2/s] and ε is the energy dissipation rate [m^2/s^3], namely the rate at which the energy of the flow is dissipated as heat due to viscosity.

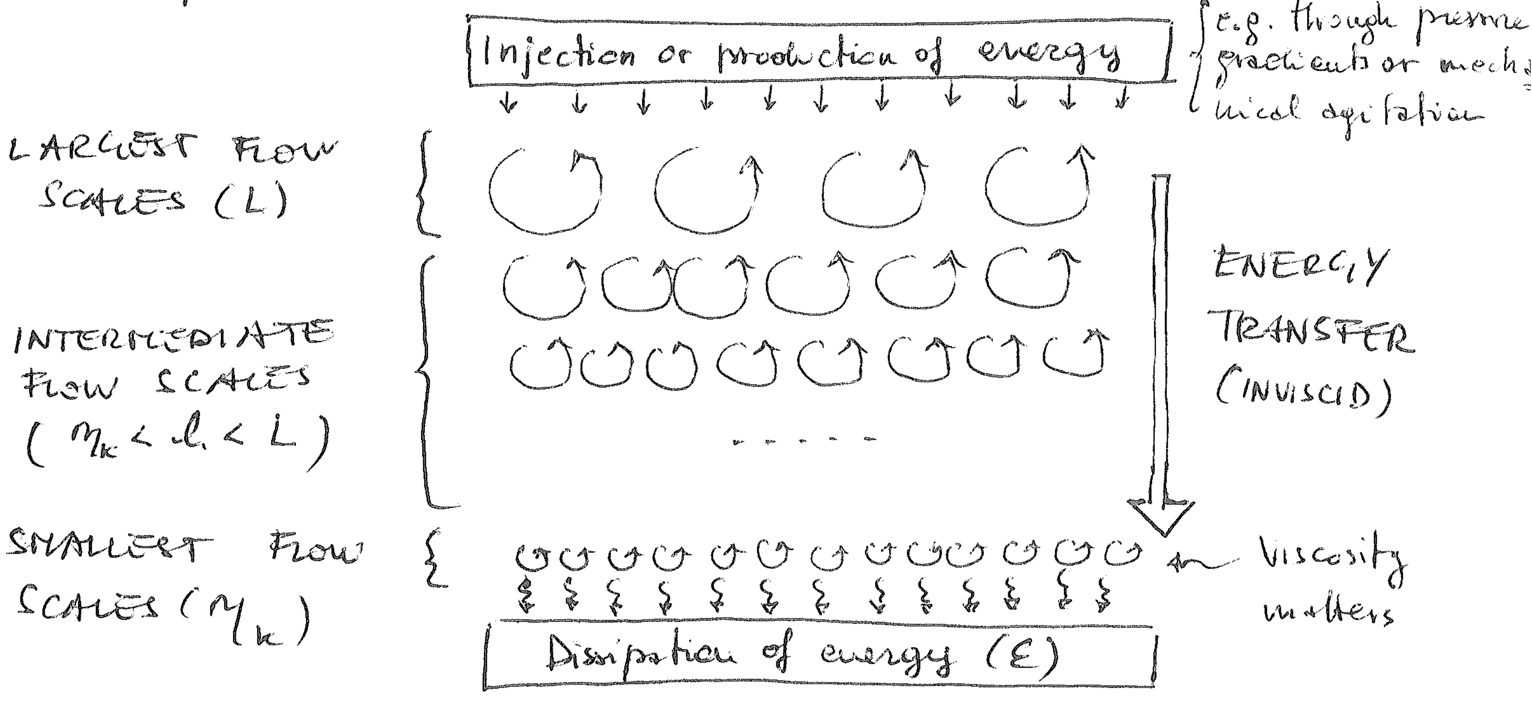
Note that the Kolmogorov scales can be combined to form a Kolmogorov Reynolds number :

$$Re_K = \frac{v_K \cdot \eta_K}{\nu} = \frac{(\nu \cdot \varepsilon)^{1/4} \cdot (\nu^3/\varepsilon)^{1/4}}{\nu} = 1$$

In wind-driven oceanic turbulence, for instance, field measurements show that ε has typical values in the range $10^{-3} \div 1$ cm^2/s^3, η_K in the range $0.3 \div 2$ mm, v_K in the range $0.5 \div 3$ mm/s.

The concept of energy dissipation is linked to the phenomenological picture behind Kolmogorov's theory. According to this picture, the largest eddies contain the energy injected into the flow (either at the boundaries or by the initial flow condition) but sooner or later become unstable due to turbulence and start transferring energy to smaller eddies. These smaller eddies, in turn, repeat the process and transfer energy to even smaller scales (ENERGY CASCADE).

As long as the eddies are sufficiently large for viscosity effects to be negligible, then the energy transfer process is inviscid and no energy is dissipated. However, the process cannot continue forever because eventually an eddy size (the Kolmogorov scale) is reached for which viscosity cannot be neglected: At this point energy is dissipated.



To better explain the physical meaning of energy dissipation rate (without providing a formal mathematical definition), consider a situation in which energy is fed into the flow at scale L. The Kolmogorov theory assumes that the eddies of size L become unstable within a time of the order of their TURNOVER TIME T_L: After this time, eddies of size L transfer an amount Δu_L^2 of energy (per unit mass) to the smaller eddies.

This allows us to define $T_L = \dfrac{L}{\Delta u_L}$ where Δu_L is the velocity difference between the eddies which must exist to trigger energy transfer.

The energy dissipation rate can be qualitatively defined as:

$$\varepsilon = \frac{\Delta u_L^2}{T} = \frac{\Delta u_L^3}{L}$$

Amount of energy per unit mass transferred to the small scales

Time required to transfer this energy

As apparent from this definition, the energy dissipation rate has to be the same for all eddy sizes, so even for the intermediate-size ones: $\varepsilon \simeq \dfrac{\Delta u_\ell^3}{\ell}$!