CONVECTIVE INSTABILITY OF A HORIZONTAL FLUID LAYER

In the Rayleigh-Benard problem, a horizontal fluid layer is heated from below and natural convection sets in between the horizontal plane surfaces that bound the layer provided that the temperature difference across the layer is above a certain critical value. In such problem, convection is driven by buoyancy and the instability of the fluid layer arises as a consequence of gradients of a single entity (temperature)

A generalization of the Rayleigh-Benard problem is given by the case of density gradients established as a consequence of gradients of two independent quantities, such as temperature and a solute, or two solutes.

A classical example is the case of vertical density gradients established in oceans or seas by gradients of temperature and salinity that provide opposing contributions to the density

gradient itself. For instance, evaporation can make water more salty near the upper surface, such that salinity increases in moving vertically upward. At the same time, the fluid is warmed up by heating near the upper surface (because of radiant heating), producing an increase of temperature in moving vertically upward.

In this case, density should increase (decrease) in moving vertically upward due to salinity (temperature) and the overall gradient depends on which of these two opposing contributions prevails.

Another common system is one containing two solutes, like sugar and salt.

In any case, the physical problem can be represented by a stationary fluid layer with a vertical density profile, in which instabilities can occur even if the overall density decreases upward (a situation that would be judged, intuitively, as stable since one would conclude that the heavier fluid is below the lighter fluid).

Indeed, the system is stable only if the vertical density profile (with density decreasing upward) is produced by a single entity, but can be unstable if the density profile is produced by two independent entities.

To trigger such instability two conditions must be verified:

1. the density profile for one of the two entities must be unstable (e.g. salinity increasing in moving vertically upward) even though the overall density profile is apparently stable (e.g. density decreases in moving vertically upward).

2. the diffusivity for transport of the two entities must be significantly different

When these conditions are met, the so called DOUBLE-DIFFUSIVE CONVECTION sets in.

Two different regimes of double-diffusive convection can be identified:

1. FINGER REGIME : in this case, warm and salty water overlays cooler and fresher water, with

an overall density profile that has the density decreasing as one goes from the bottom to the top of the water layer.

The contribution to the overall density profile (which should correspond to a stable situation) produced by the salinity gradient is unstable (salty water overlays fresh water!), but the contribution produced by the temperature gradient is stable and stronger.

Nevertheless, an instability leading to the spontaneous onset of convection may occur due to the difference in the thermal and mass diffusivities (the ratio being equal to the LEWIS NUMBER).

To understand why such convective instability can occur, consider a fluid element that is displaced upward by some perturbation.

The fluid element will quickly equilibrate thermally with its new surroundings but will still have less salt (namely, the fluid element will still be lighter that its new surroundings) due to the fact that thermal diffusivity is

characterized by a faster time scale compared
to mass diffusivity (which is characterized by
a slower time scale).

As a result, there is a net buoyancy force that
makes the fluid element move further upwards:
The system will spontaneously undergo convection
and become unstable, with a resultant motion
given by cellular convection (not oscillatory in
time).

> NOTE: $Le = \dfrac{\alpha}{D} = \dfrac{\text{thermal diffusivity}}{\text{mass diffusivity}} = \dfrac{Sc}{Pr}$

2. <u>DIFFUSIVE REGIME</u> : in this case, cold and
fresh water overlays warmer and saltier water.
The contribution of the salinity gradient to the
overall density gradient is stable, whereas the
contribution of the temperature gradient is unstable.
The unstable contribution is, therefore, associated
with the entity that has the larger diffusivity
(temperature), which is exactly the opposite of
what happens in the finger regime (where the
unstable contribution to the overall density
gradient is associated to the entity, salt, that

diffuses more slowly).

Considering, again, a fluid element displaced upward by some perturbation, it is observed that the element loses heat to the surroundings but retains its original salinity (more or less), thus being heavier than the surroundings. This produces a net buoyancy force that drives the fluid element back towards its initial position.

If the fluid element is a little cooler than its surroundings (this may happen because heat transport occurs at a fast, yet finite rate), then the "restoring" force driving the element back may be larger than the force we would have with the salinity gradient alone : if the restoring force is indeed larger, then the fluid element can overshoot its initial position (of neutral equilibrium) and may start an oscillatory motion. If the magnitude of the oscillations grows in time, then the system is (clearly) unstable.

It is apparent that, in both the regimes, the driving

mechanism for convective instability is the difference in the diffusive transport rate (quantified by the diffusion coefficients) of the two entities that control the density distribution.

MARANGONI INSTABILITY

Another type of convective instability that can be observed for a horizontal fluid layer is the well-known MARANGONI INSTABILITY.

This type of instability can be observed whenever there is at least one of the surfaces of the fluid layer that is an interface, and the physical mechanism driving the instability is associated with the existence of gradients of the SURFACE TENSION, σ [N/m].

Surface tension is the energy required to increase the surface area of a liquid by an amount equal to a unit of area. The increase is due to inter-molecular forces, namely attractive or repulsive forces between molecules.
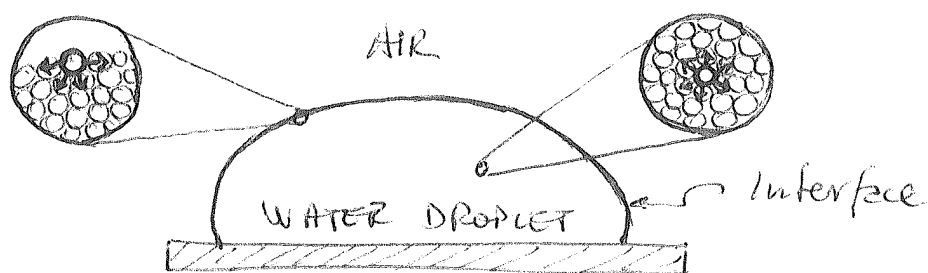
Inter-molecular forces can be of two types: (1) short-

range forces, acting when molecules are separated by 3 Å or less (~ 10^{-8} cm); and (2) long-range forces, acting when the separation distance is larger than 3 Å.

Short-range (long-range) forces tend to be repulsive (attractive).

Long-range forces are also known as <u>Van der Waals</u> forces and are responsible for surface tension, as well as other physical properties of fluids like viscosity and friction.
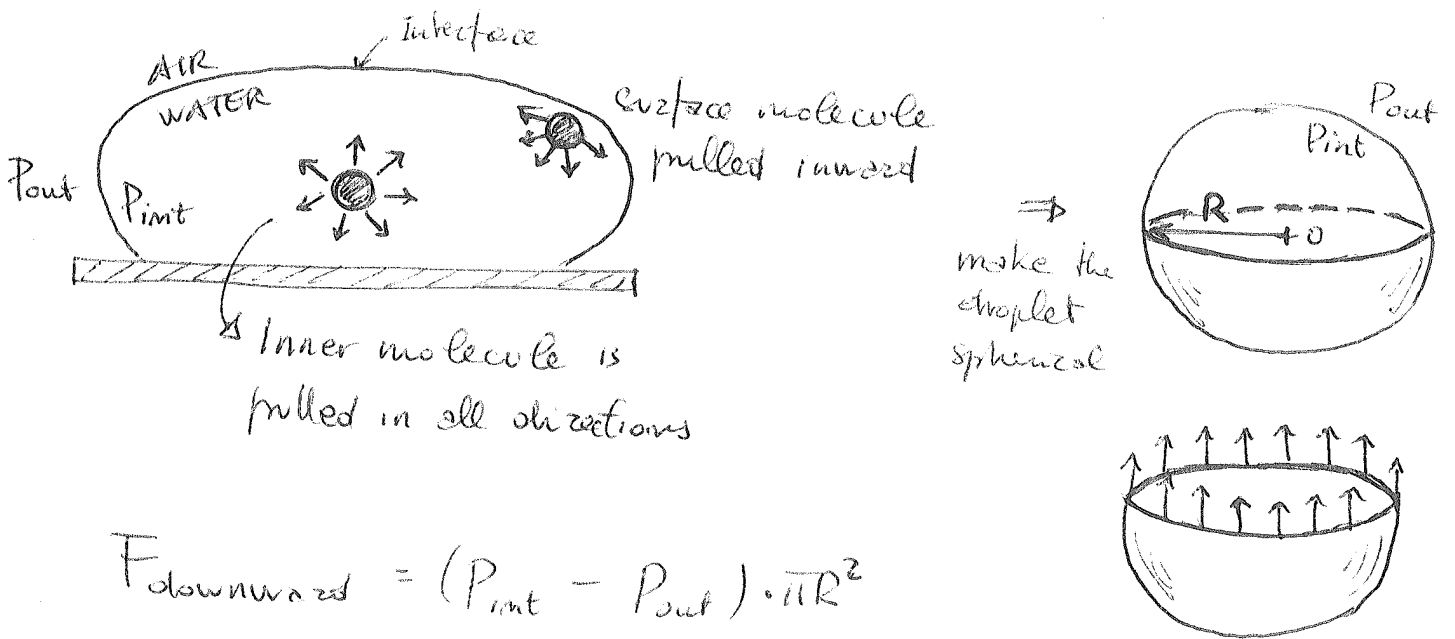
Since inter-molecular forces vary depending on the nature of the liquid (e.g. water vs gasoline) or the solute in the liquid (e.g. surfactants, like soap or detergents), surface tension properties also vary depending on the liquid.

Molecules at the interface experience a net attraction to other molecules inside the droplet: It is this attraction that holds the interface together. In contrast, molecules in the interior of the droplet experience uniform attractive forces.

The unbalanced attraction of molecules at the interface tends to pull molecules back into the interior, leaving the minimum number of molecules on the interface (which is thus minimized in terms of area). Therefore, energy must be spent to increase the surface area of the interface because a larger interface contains more molecules.

Surface tension of a water droplet:

$$F_{downward} = (P_{int} - P_{out}) \cdot \pi R^2$$
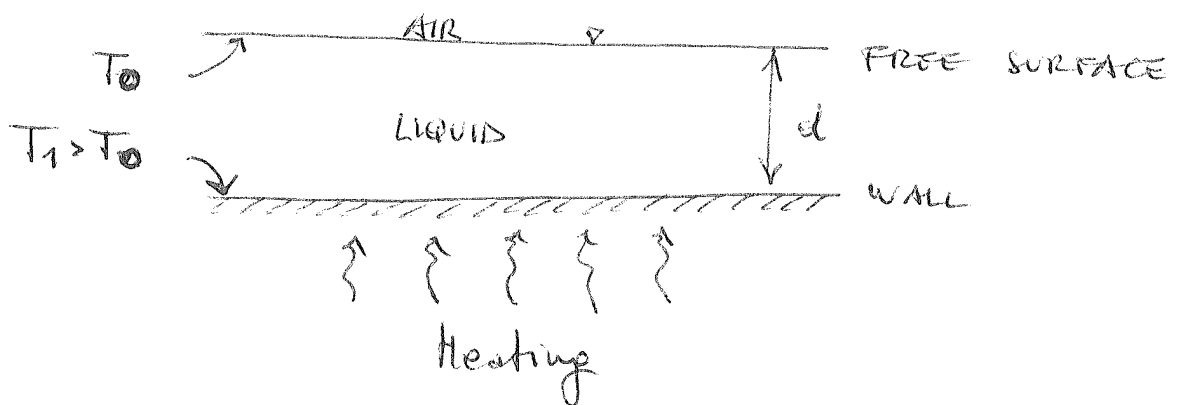
$$F_{upward} = \gamma \cdot 2\pi R$$

$$\Delta F = F_{upward} - F_{downward} = 0 \implies \boxed{\gamma = \frac{(P_{int} - P_{out})R}{2}}$$

Note: for a bubble $\gamma = \dfrac{(P_{int} - P_{out})R}{4}$ because the bubble actually has an interface made of two surfaces
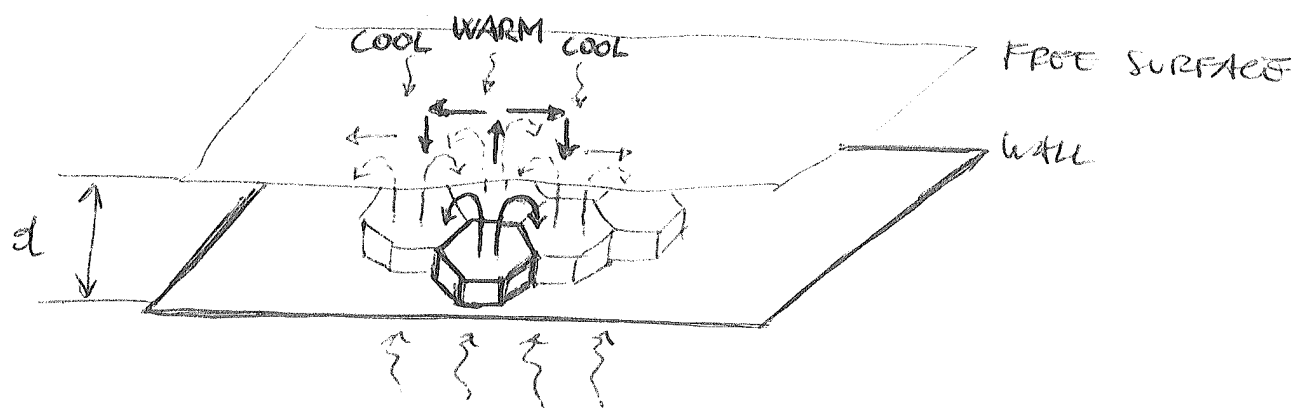
Going back to the Marangoni instability, surface tension gradients can be generated by temperature gradients.

The simplest situation in which this can be observed is that of a fluid layer comprised between a horizontal solid wall and a free surface, heated from below:

This configuration is very similar to the one already considered to study the Rayleigh-Benard instability (which appears because the difference in the temperatures induces a density gradient).

In the case of Marangoni instability, the difference in the temperature induces a surface tension gradient, which can of course add to the density gradient.

If the gradient is strong enough to induce instability, its effect is to eject the liquid from the

heated region towards the free surface, which is colder. Because of mass conservation, cold liquid will be pushed towards the wall, giving rise to a system of cells where the liquid moves upward in the center of each cell and moves downward at the periphery of each cell:

As mentioned, the surface tension gradient must be high enough to trigger the instability and form the cells. The dimensionless number that determines the threshold for the onset of Marangoni instability is:

$$Ma = \frac{\frac{d\sigma}{dT} \cdot \Delta T \cdot d}{\mu \cdot \chi}$$

MARANGONI NUMBER

where $d\sigma/dT$ is the variation rate of surface tension with temperature, $\Delta T = T_1 - T_0$ is the
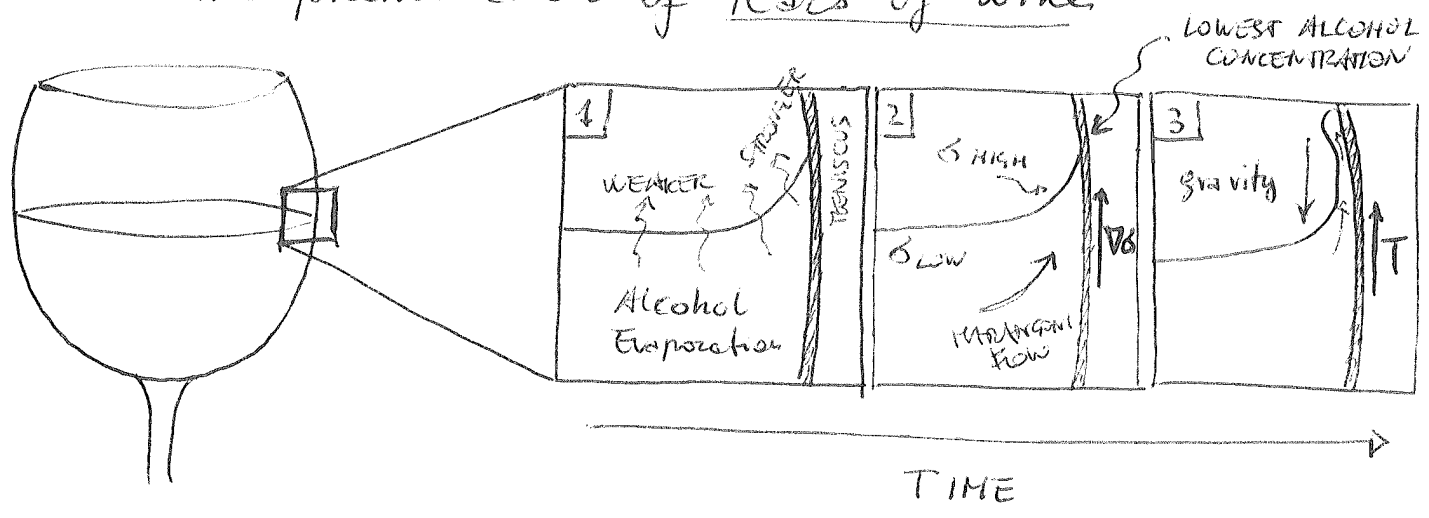
temperature difference between the wall and the
free surface and α is thermal diffusivity.

For the configuration considered here (see schematic
at page 10), the critical value of Marangoni number
to develop surface tension - driven instabilities
is $Ma_{crit} = 80$.

Physical meaning of the Marangoni number:

$$Ma = \frac{surface\ tension\ forces}{viscous\ forces}$$

NOTE : The phenomenon of tears of wine.

Tears of wine form because the transport rate of alcohol
from the interior of the wine to the flat surface is
different from the transport rate of alcohol from
the interior to the meniscus; This generates a
gradient in alcohol concentration that, in turn,
generates surface tension and a

The surface tension gradient $\nabla\sigma$ generates a tangential stress T that, initially, is stronger than gravitational force thus generating a net upward force that pulls the wine up the wall of the glass (wine is in fact accelerated up the wall of the glass). Through this process, droplets of wine start to form and grow at the upper end of the meniscus.

Eventually, when the droplets reach a critical "size", gravitational force can balance the tangential stress and the droplet cannot remain attached to the upper end of the meniscus and (when gravity exceeds stress) slip back to the rest of the glass but with a depleted alcohol content. This process continues until the alcohol is completely evaporated.