

# Heat Transfer

## 9.1. INTRODUCTION

In the majority of chemical processes heat is either given out or absorbed, and fluids must often be either heated or cooled in a wide range of plant, such as furnaces, evaporators, distillation units, dryers, and reaction vessels where one of the major problems is that of transferring heat at the desired rate. In addition, it may be necessary to prevent the loss of heat from a hot vessel or pipe system. The control of the flow of heat at the desired rate forms one of the most important areas of chemical engineering. Provided that a temperature difference exists between two parts of a system, heat transfer will take place in one or more of three different ways.

*Conduction.* In a solid, the flow of heat by conduction is the result of the transfer of vibrational energy from one molecule to another, and in fluids it occurs in addition as a result of the transfer of kinetic energy. Heat transfer by conduction may also arise from the movement of free electrons, a process which is particularly important with metals and accounts for their high thermal conductivities.

*Convection.* Heat transfer by convection arises from the mixing of elements of fluid. If this mixing occurs as a result of density differences as, for example, when a pool of liquid is heated from below, the process is known as *natural convection*. If the mixing results from eddy movement in the fluid, for example when a fluid flows through a pipe heated on the outside, it is called *forced convection*. It is important to note that convection requires mixing of fluid elements, and is not governed by temperature difference alone as is the case in conduction and radiation.

*Radiation.* All materials radiate thermal energy in the form of electromagnetic waves. When this radiation falls on a second body it may be partially reflected, transmitted, or absorbed. It is only the fraction that is absorbed that appears as heat in the body.

## 9.2. BASIC CONSIDERATIONS

### 9.2.1. Individual and overall coefficients of heat transfer

In many of the applications of heat transfer in process plants, one or more of the mechanisms of heat transfer may be involved. In the majority of heat exchangers heat passes through a series of different intervening layers before reaching the second fluid (Figure 9.1). These layers may be of different thicknesses and of different thermal conductivities. The problem of transferring heat to crude oil in the primary furnace before it enters the first distillation column may be considered as an example. The heat from the flames passes by radiation and convection to the pipes in the furnace, by conduction through the

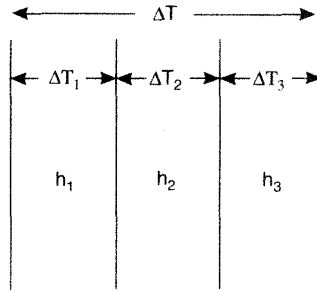


Figure 9.1. Heat transfer through a composite wall

pipe walls, and by forced convection from the inside of the pipe to the oil. Here all three modes of transfer are involved. After prolonged usage, solid deposits may form on both the inner and outer walls of the pipes, and these will then contribute additional resistance to the transfer of heat. The simplest form of equation which represents this heat transfer operation may be written as:

$$Q = UA\Delta T \quad (9.1)$$

where  $Q$  is the heat transferred per unit time,  $A$  the area available for the flow of heat,  $\Delta T$  the difference in temperature between the flame and the boiling oil, and  $U$  is known as the overall heat transfer coefficient ( $\text{W/m}^2 \text{K}$  in SI units).

At first sight, equation 9.1 implies that the relationship between  $Q$  and  $\Delta T$  is linear. Whereas this is approximately so over limited ranges of temperature difference for which  $U$  is nearly constant, in practice  $U$  may well be influenced both by the temperature difference and by the absolute value of the temperatures.

If it is required to know the area needed for the transfer of heat at a specified rate, the temperature difference  $\Delta T$ , and the value of the overall heat-transfer coefficient must be known. Thus the calculation of the value of  $U$  is a key requirement in any design problem in which heating or cooling is involved. A large part of the study of heat transfer is therefore devoted to the evaluation of this coefficient.

The value of the coefficient will depend on the mechanism by which heat is transferred, on the fluid dynamics of both the heated and the cooled fluids, on the properties of the materials through which the heat must pass, and on the geometry of the fluid paths. In solids, heat is normally transferred by conduction; some materials such as metals have a high thermal conductivity, whilst others such as ceramics have a low conductivity. Transparent solids like glass also transmit radiant energy particularly in the visible part of the spectrum.

Liquids also transmit heat readily by conduction, though circulating currents are frequently set up and the resulting convective transfer may be considerably greater than the transfer by conduction. Many liquids also transmit radiant energy. Gases are poor conductors of heat and circulating currents are difficult to suppress; convection is therefore much more important than conduction in a gas. Radiant energy is transmitted with only limited absorption in gases and, of course, without any absorption *in vacuo*. Radiation is the only mode of heat transfer which does not require the presence of an intervening medium.

If the heat is being transmitted through a number of media in series, the overall heat transfer coefficient may be broken down into individual coefficients  $h$  each relating to a single medium. This is as shown in Figure 9.1. It is assumed that there is good contact between each pair of elements so that the temperature is the same on the two sides of each junction.

If heat is being transferred through three media, each of area  $A$ , and individual coefficients for each of the media are  $h_1$ ,  $h_2$ , and  $h_3$ , and the corresponding temperature changes are  $\Delta T_1$ ,  $\Delta T_2$ , and  $\Delta T_3$  then, provided that there is no accumulation of heat in the media, the heat transfer rate  $Q$  will be the same through each. Three equations, analogous to equation 9.1 can therefore be written:

$$\left. \begin{aligned} Q &= h_1 A \Delta T_1 \\ Q &= h_2 A \Delta T_2 \\ Q &= h_3 A \Delta T_3 \end{aligned} \right\} \quad (9.2)$$

Rearranging: 
$$\Delta T_1 = \frac{Q}{A} \frac{1}{h_1}$$

$$\Delta T_2 = \frac{Q}{A} \frac{1}{h_2}$$

$$\Delta T_3 = \frac{Q}{A} \frac{1}{h_3}$$

Adding: 
$$\Delta T_1 + \Delta T_2 + \Delta T_3 = \frac{Q}{A} \left( \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} \right) \quad (9.3)$$

Noting that  $(\Delta T_1 + \Delta T_2 + \Delta T_3) =$  total temperature difference  $\Delta T$ :

then: 
$$\Delta T = \frac{Q}{A} \left( \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} \right) \quad (9.4)$$

From equation 9.1: 
$$\Delta T = \frac{Q}{A} \frac{1}{U} \quad (9.5)$$

Comparing equations 9.4 and 9.5:

$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} \quad (9.6)$$

The reciprocals of the heat transfer coefficients are resistances, and equation 9.6 therefore illustrates that the resistances are additive.

In some cases, particularly for the radial flow of heat through a thick pipe wall or cylinder, the area for heat transfer is a function of position. Thus the area for transfer applicable to each of the three media could differ and may be  $A_1$ ,  $A_2$  and  $A_3$ . Equation 9.3 then becomes:

$$\Delta T_1 + \Delta T_2 + \Delta T_3 = Q \left( \frac{1}{h_1 A_1} + \frac{1}{h_2 A_2} + \frac{1}{h_3 A_3} \right) \quad (9.7)$$

Equation 9.7 must then be written in terms of one of the area terms  $A_1$ ,  $A_2$ , and  $A_3$ , or sometimes in terms of a mean area. Since  $Q$  and  $\Delta T$  must be independent of the particular

area considered, the value of  $U$  will vary according to which area is used as the basis. Thus equation 9.7 may be written, for example:

$$Q = U_1 A_1 \Delta T \quad \text{or} \quad \Delta T = \frac{Q}{U_1 A_1}$$

This will then give  $U_1$  as:

$$\frac{1}{U_1} = \frac{1}{h_1} + \frac{A_1}{A_2} \left( \frac{1}{h_2} \right) + \frac{A_1}{A_3} \left( \frac{1}{h_3} \right) \quad (9.8)$$

In this analysis it is assumed that the heat flowing per unit time through each of the media is the same.

Now that the overall coefficient  $U$  has been broken down into its component parts, each of the individual coefficients  $h_1$ ,  $h_2$ , and  $h_3$  must be evaluated. This can be done from a knowledge of the nature of the heat transfer process in each of the media. A study will therefore be made of how these individual coefficients can be calculated for conduction, convection, and radiation.

### 9.2.2. Mean temperature difference

Where heat is being transferred from one fluid to a second fluid through the wall of a vessel and the temperature is the same throughout the bulk of each of the fluids, there is no difficulty in specifying the overall temperature difference  $\Delta T$ . Frequently, however, each fluid is flowing through a heat exchanger such as a pipe or a series of pipes in parallel, and its temperature changes as it flows, and consequently the temperature difference is continuously changing. If the two fluids are flowing in the same direction (*co-current flow*), the temperatures of the two streams progressively approach one another as shown in Figure 9.2. In these circumstances the outlet temperature of the heating fluid must always be higher than that of the cooling fluid. If the fluids are flowing in opposite directions (*countercurrent flow*), the temperature difference will show less variation throughout the heat exchanger as shown in Figure 9.3. In this case it is possible for the cooling liquid to leave at a higher temperature than the heating liquid, and one of the great advantages of

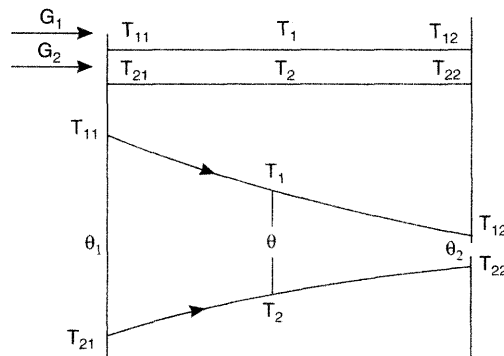


Figure 9.2. Mean temperature difference for co-current flow

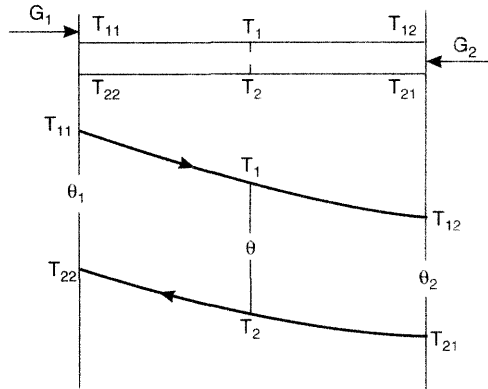


Figure 9.3. Mean temperature difference for countercurrent flow

countercurrent flow is that it is possible to extract a higher proportion of the heat content of the heating fluid. The calculation of the appropriate value of the temperature difference for co-current and for countercurrent flow is now considered. It is assumed that the overall heat transfer coefficient  $U$  remains constant throughout the heat exchanger.

It is necessary to find the average value of the temperature difference  $\theta_m$  to be used in the general equation:

$$Q = UA\theta_m \quad (\text{equation 9.1})$$

Figure 9.3 shows the temperature conditions for the fluids flowing in opposite directions, a condition known as countercurrent flow.

The outside stream specific heat  $C_{p1}$  and mass flow rate  $G_1$  falls in temperature from  $T_{11}$  to  $T_{12}$ .

The inside stream specific heat  $C_{p2}$  and mass flow rate  $G_2$  rises in temperature from  $T_{21}$  to  $T_{22}$ .

Over a small element of area  $dA$  where the temperatures of the streams are  $T_1$  and  $T_2$ . The temperature difference:

$$\theta = T_1 - T_2$$

$$\therefore d\theta = dT_1 - dT_2$$

$$\text{Heat given out by the hot stream} = dQ = -G_1 C_{p1} dT_1$$

$$\text{Heat taken up by the cold stream} = dQ = G_2 C_{p2} dT_2$$

$$\therefore d\theta = -\frac{dQ}{G_1 C_{p1}} - \frac{dQ}{G_2 C_{p2}} = -dQ \left( \frac{G_1 C_{p1} + G_2 C_{p2}}{G_1 C_{p1} \times G_2 C_{p2}} \right) = -\psi dQ \quad (\text{say})$$

$$\therefore \theta_1 - \theta_2 = \psi Q$$

$$\text{Over this element:} \quad U dA\theta = dQ$$

$$\therefore U dA\theta = -\frac{d\theta}{\psi}$$

If  $U$  may be taken as constant:

$$-\psi U \int_0^A dA = \int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta}$$

$$\therefore -\psi UA = -\ln \frac{\theta_1}{\theta_2}$$

From the definition of  $\theta_m$ ,  $Q = UA\theta_m$ .

$$\therefore \theta_1 - \theta_2 = \psi Q = \psi UA\theta_m = \ln \frac{\theta_1}{\theta_2} (\theta_m)$$

and:

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} \quad (9.9)$$

where  $\theta_m$  is known as the *logarithmic mean temperature difference*.

UNDERWOOD<sup>(1)</sup> proposed the following approximation for the logarithmic mean temperature difference:

$$(\theta_m)^{1/3} = \frac{1}{2}(\theta_1^{1/3} + \theta_2^{1/3}) \quad (9.10)$$

and, for example, when  $\theta_1 = 1$  K and  $\theta_2 = 100$  K,  $\theta_m$  is 22.4 K compared with a logarithmic mean of 21.5 K. When  $\theta_1 = 10$  K and  $\theta_2 = 100$  K, both the approximation and the logarithmic mean values coincide at 39 K.

If the two fluids flow in the same direction on each side of a tube, co-current flow is taking place and the general shape of the temperature profile along the tube is as shown in Figure 9.2. A similar analysis will show that this gives the same expression for  $\theta_m$ , the logarithmic mean temperature difference. For the same terminal temperatures it is important to note that the value of  $\theta_m$  for countercurrent flow is appreciably greater than the value for co-current flow. This is seen from the temperature profiles, where with co-current flow the cold fluid cannot be heated to a higher temperature than the exit temperature of the hot fluid as illustrated in Example 9.1.

### Example 9.1

A heat exchanger is required to cool 20 kg/s of water from 360 K to 340 K by means of 25 kg/s water entering at 300 K. If the overall coefficient of heat transfer is constant at 2 kW/m<sup>2</sup>K, calculate the surface area required in (a) a countercurrent concentric tube exchanger, and (b) a co-current flow concentric tube exchanger.

### Solution

Heat load:  $Q = 20 \times 4.18(360 - 340) = 1672$  kW

The cooling water outlet temperature is given by:

$$1672 = 25 \times 4.18(\theta_2 - 300) \quad \text{or} \quad \theta_2 = 316 \text{ K}$$

(a) *Counterflow*

In equation 9.9:

$$\theta_m = \frac{44 - 40}{\ln(44/40)} = 41.9 \text{ K}$$

Heat transfer area:

$$A = \frac{Q}{U\theta_m}$$

$$\begin{aligned}
 &= \frac{1672}{2 \times 41.9} \\
 &= \underline{\underline{19.95 \text{ m}^2}}
 \end{aligned}$$

(b) *Co-current flow*

In equation 9.9:

$$\theta_m = \frac{60 - 24}{\ln(60/24)} = 39.3 \text{ K}$$

Heat transfer area:

$$\begin{aligned}
 A &= \frac{1672}{2 \times 39.3} \\
 &= \underline{\underline{21.27 \text{ m}^2}}
 \end{aligned}$$

It may be noted that using Underwood's approximation (equation 9.10), the calculated values for the mean temperature driving forces are 41.9 K and 39.3 K for counter- and co-current flow respectively, which agree exactly with the logarithmic mean values.

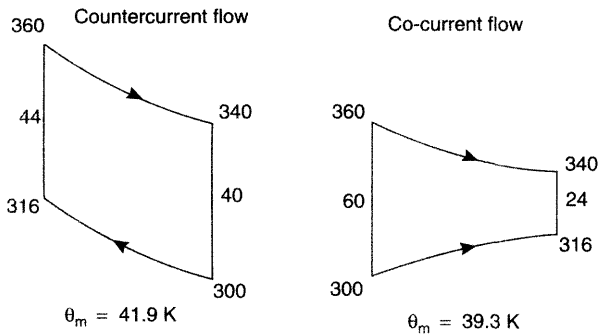


Figure 9.4. Data for Example 9.1

## 9.3. HEAT TRANSFER BY CONDUCTION

### 9.3.1. Conduction through a plane wall

This important mechanism of heat transfer is now considered in more detail for the flow of heat through a plane wall of thickness  $x$  as shown in Figure 9.5.

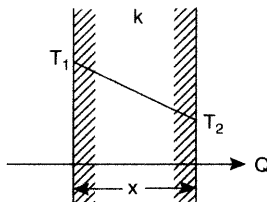


Figure 9.5. Conduction of heat through a plane wall

The rate of heat flow  $Q$  over the area  $A$  and a small distance  $dx$  may be written as:

$$Q = -kA \left( \frac{dT}{dx} \right) \quad (9.11)$$

which is often known as *Fourier's equation*, where the negative sign indicates that the temperature gradient is in the opposite direction to the flow of heat and  $k$  is the thermal conductivity of the material. Integrating for a wall of thickness  $x$  with boundary temperatures  $T_1$  and  $T_2$ , as shown in Figure 9.5:

$$Q = \frac{kA(T_1 - T_2)}{x} \quad (9.12)$$

Thermal conductivity is a function of temperature and experimental data may often be expressed by a linear relationship of the form:

$$k = k_0(1 + k'T) \quad (9.13)$$

where  $k$  is the thermal conductivity at the temperature  $T$  and  $k_0$  and  $k'$  are constants. Combining equations 9.11 and 9.13:

$$-k dT = -k_0(1 + k'T)dT = \frac{Q dx}{A}$$

Integrating between the temperature limits  $T_1$  and  $T_2$ ,

$$- \int_{T_1}^{T_2} k dT = (T_1 - T_2)k_0 \left\{ 1 + k' \left( \frac{T_1 + T_2}{2} \right) \right\} = Q \int_{x_1}^{x_2} \frac{dx}{A} \quad (9.14)$$

Where  $k$  is a linear function of  $T$ , the following equation may therefore be used:

$$k_a(T_1 - T_2) = Q \int_{x_1}^{x_2} \frac{dx}{A} \quad (9.15)$$

where  $k_a$  is the arithmetic mean of  $k_1$  and  $k_2$  at  $T_1$  and  $T_2$  respectively or the thermal conductivity at the arithmetic mean of  $T_1$  and  $T_2$ .

Where  $k$  is a non-linear function of  $T$ , some mean value,  $k_m$  will apply, where:

$$k_m = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} k dT \quad (9.16)$$

From Table 9.1 it will be seen that metals have very high thermal conductivities, non-metallic solids lower values, non-metallic liquids low values, and gases very low values. It is important to note that amongst metals, stainless steel has a low value, that water has a very high value for liquids (due to partial ionisation), and that hydrogen has a high value for gases (due to the high mobility of the molecules). With gases,  $k$  decreases with increase in molecular mass and increases with the temperature. In addition, for gases the dimensionless *Prandtl group*  $C_p\mu/k$ , which is approximately constant (where  $C_p$  is the specific heat at constant pressure and  $\mu$  is the viscosity), can be used to evaluate  $k$  at high temperatures where it is difficult to determine a value experimentally because of the formation of convection currents.  $k$  does not vary significantly with pressure, except where this is reduced to a value so low that the mean free path of the molecules becomes



Table 9.1. Thermal conductivities of selected materials

	Temp (K)	$k$ (Btu/h ft <sup>2</sup> °F/ft)	$k$ (W/mK)		Temp (K)	$k$ (Btu/h ft <sup>2</sup> °F/ft)	$k$ (W/mK)
<i>Solids—Metals</i>				<i>Liquids</i>			
Aluminium	573	133	230	Acetic acid 50%	293	0.20	0.35
Cadmium	291	54	94	Acetone	303	0.10	0.17
Copper	373	218	377	Aniline	273–293	0.1	0.17
Iron (wrought)	291	35	61	Benzene	303	0.09	0.16
Iron (cast)	326	27.6	48	Calcium chloride brine 30%	303	0.32	0.55
Lead	373	19	33	Ethyl alcohol 80%	293	0.137	0.24
Nickel	373	33	57	Glycerol 60%	293	0.22	0.38
Silver	373	238	412	Glycerol 40%	293	0.26	0.45
Steel 1% C	291	26	45	n-Heptane	303	0.08	0.14
Tantalum	291	32	55	Mercury	301	4.83	8.36
Admiralty metal	303	65	113	Sulphuric acid 90%	303	0.21	0.36
Bronze	—	109	189	Sulphuric acid 60%	303	0.25	0.43
Stainless Steel	293	9.2	16	Water	303	0.356	0.62
				Water	333	0.381	0.66
<i>Solids—Non-metals</i>				<i>Gases</i>			
Asbestos sheet	323	0.096	0.17	Hydrogen	273	0.10	0.17
Asbestos	273	0.09	0.16	Carbon dioxide	273	0.0085	0.015
Asbestos	373	0.11	0.19	Air	273	0.014	0.024
Asbestos	473	0.12	0.21	Air	373	0.018	0.031
Bricks (alumina)	703	1.8	3.1	Methane	273	0.017	0.029
Bricks (building)	293	0.4	0.69	Water vapour	373	0.0145	0.025
Magnesite	473	2.2	3.8	Nitrogen	273	0.0138	0.024
Cotton wool	303	0.029	0.050	Ethylene	273	0.0097	0.017
Glass	303	0.63	1.09	Oxygen	273	0.0141	0.024
Mica	323	0.25	0.43	Ethane	273	0.0106	0.018
Rubber (hard)	273	0.087	0.15				
Sawdust	293	0.03	0.052				
Cork	303	0.025	0.043				
Glass wool	—	0.024	0.041				
85% Magnesia	—	0.04	0.070				
Graphite	273	87	151				

comparable with the dimensions of the vessel; further reduction of pressure then causes  $k$  to decrease.

Typical values for Prandtl numbers are as follows:

Air	0.71	<i>n</i> -Butanol	50
Oxygen	0.63	Light oil	600
Ammonia (gas)	1.38	Glycerol	1000
Water	5–10	Polymer melts	10,000
		Mercury	0.02

The low conductivity of heat insulating materials, such as cork, glass wool, and so on, is largely accounted for by their high proportion of air space. The flow of heat through such materials is governed mainly by the resistance of the air spaces, which should be sufficiently small for convection currents to be suppressed.

It is convenient to rearrange equation 9.12 to give:

$$Q = \frac{(T_1 - T_2)A}{(x/k)} \quad (9.17)$$

where  $x/k$  is known as the *thermal resistance* and  $k/x$  is the *transfer coefficient*.

### Example 9.2.

Estimate the heat loss per square metre of surface through a brick wall 0.5 m thick when the inner surface is at 400 K and the outside surface is at 300 K. The thermal conductivity of the brick may be taken as 0.7 W/mK.

### Solution

From equation 9.12:

$$\begin{aligned} Q &= \frac{0.7 \times 1 \times (400 - 300)}{0.5} \\ &= \underline{\underline{140 \text{ W/m}^2}} \end{aligned}$$

### 9.3.2. Thermal resistances in series

It has been noted earlier that thermal resistances may be added together for the case of heat transfer through a complete section formed from different media in series.

Figure 9.6 shows a composite wall made up of three materials with thermal conductivities  $k_1$ ,  $k_2$ , and  $k_3$ , with thicknesses as shown and with the temperatures  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$  at the faces. Applying equation 9.12 to each section in turn, and noting that the same quantity of heat  $Q$  must pass through each area  $A$ :

$$T_1 - T_2 = \frac{x_1}{k_1 A} Q, \quad T_2 - T_3 = \frac{x_2}{k_2 A} Q \quad \text{and} \quad T_3 - T_4 = \frac{x_3}{k_3 A} Q$$

On addition: 
$$(T_1 - T_4) = \left( \frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} + \frac{x_3}{k_3 A} \right) Q \quad (9.18)$$

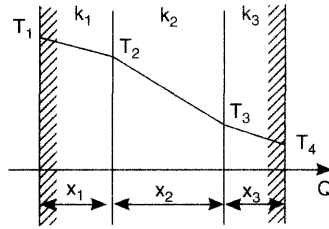


Figure 9.6. Conduction of heat through a composite wall

or:

$$Q = \frac{T_1 - T_4}{\Sigma(x_i/k_i A)}$$

$$= \frac{\text{Total driving force}}{\text{Total (thermal resistance/area)}} \quad (9.19)$$

**Example 9.3**

A furnace is constructed with 0.20 m of firebrick, 0.10 m of insulating brick, and 0.20 m of building brick. The inside temperature is 1200 K and the outside temperature is 330 K. If the thermal conductivities are as shown in Figure 9.7, estimate the heat loss per unit area and the temperature at the junction of the firebrick and the insulating brick.

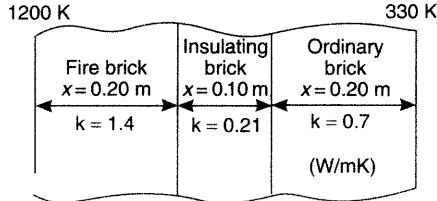


Figure 9.7. Data for Example 9.3

**Solution**

From equation 9.19:

$$Q = (1200 - 330) / \left[ \left( \frac{0.20}{1.4 \times 1} \right) + \left( \frac{0.10}{0.21 \times 1} \right) + \left( \frac{0.20}{0.7 \times 1} \right) \right]$$

$$= \frac{870}{(0.143 + 0.476 + 0.286)} = \frac{870}{0.905}$$

$$= \underline{\underline{961 \text{ W/m}^2}}$$

The ratio (Temperature drop over firebrick)/(Total temperature drop) = (0.143/0.905)

$$\therefore \text{Temperature drop over firebrick} = \left( \frac{870 \times 0.143}{0.905} \right) = 137 \text{ deg K}$$

Hence the temperature at the firebrick-insulating brick interface = (1200 - 137) = 1063 K

### 9.3.3. Conduction through a thick-walled tube

The conditions for heat flow through a thick-walled tube when the temperatures on the inside and outside are held constant are shown in Figure 9.8. Here the area for heat flow is proportional to the radius and hence the temperature gradient is inversely proportional to the radius.

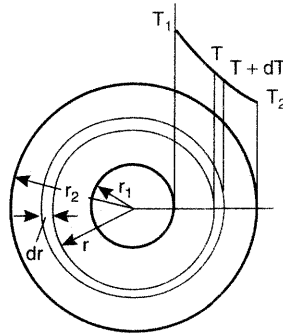


Figure 9.8. Conduction through thick-walled tube or spherical shell

The heat flow at any radius  $r$  is given by:

$$Q = -k2\pi rl \frac{dT}{dr} \quad (9.20)$$

where  $l$  is the length of tube.

Integrating between the limits  $r_1$  and  $r_2$ :

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi lk \int_{T_1}^{T_2} dT$$

or:

$$Q = \frac{2\pi lk(T_1 - T_2)}{\ln(r_2/r_1)} \quad (9.21)$$

This equation may be put into the form of equation 9.12 to give:

$$Q = \frac{k(2\pi r_m l)(T_1 - T_2)}{r_2 - r_1} \quad (9.22)$$

where  $r_m = (r_2 - r_1)/\ln(r_2/r_1)$ , is known as the *logarithmic mean radius*. For thin-walled tubes the arithmetic mean radius  $r_a$  may be used, giving:

$$Q = \frac{k(2\pi r_a l)(T_1 - T_2)}{r_2 - r_1} \quad (9.23)$$

### 9.3.4. Conduction through a spherical shell and to a particle

For heat conduction through a spherical shell, the heat flow at radius  $r$  is given by:

$$Q = -k4\pi r^2 \frac{dT}{dr} \quad (9.24)$$

Integrating between the limits  $r_1$  and  $r_2$ :

$$Q \int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi k \int_{T_1}^{T_2} dT$$

$$Q = \frac{4\pi k(T_1 - T_2)}{(1/r_1) - (1/r_2)} \quad (9.25)$$

An important application of heat transfer to a sphere is that of conduction through a stationary fluid surrounding a spherical particle or droplet of radius  $r$  as encountered for example in fluidised beds, rotary kilns, spray dryers and plasma devices. If the temperature difference  $T_1 - T_2$  is spread over a very large distance so that  $r_2 = \infty$  and  $T_1$  is the temperature of the surface of the drop, then:

$$\frac{Qr}{(4\pi r^2)(T_1 - T_2)k} = 1$$

or:

$$\frac{hd}{k} = Nu' = 2 \quad (9.26)$$

where  $Q/4\pi r^2(T_1 - T_2) = h$  is the heat transfer coefficient,  $d$  is the diameter of the particle or droplet and  $hd/k$  is a dimensionless group known as the *Nusselt number* ( $Nu'$ ) for the particle. The more general use of the Nusselt number, with particular reference to heat transfer by convection, is discussed in Section 9.4. This value of 2 for the Nusselt number is the theoretical minimum for heat transfer through a *continuous* medium. It is greater if the temperature difference is applied over a finite distance, when equation 9.25 must be used. When there is relative motion between the particle and the fluid the heat transfer rate will be further increased, as discussed in Section 9.4.6.

In this approach, heat transfer to a spherical particle by conduction through the surrounding fluid has been the prime consideration. In many practical situations the flow of heat from the surface to the internal parts of the particle is of importance. For example, if the particle is a poor conductor then the rate at which the particulate material reaches some desired average temperature may be limited by conduction inside the particle rather than by conduction to the outside surface of the particle. This problem involves unsteady state transfer of heat which is considered in Section 9.3.5.

Equations may be developed to predict the rate of change of diameter  $d$  of evaporating droplets. If the latent heat of vaporisation is provided by heat conducted through a hotter stagnant gas to the droplet surface, and heat transfer is the rate controlling step, it is shown by SPALDING<sup>(2)</sup> that  $d^2$  decreases linearly with time. A closely related and important practical problem is the prediction of the residence time required in a combustion chamber to ensure virtually complete burning of the oil droplets. Complete combustion is desirable to obtain maximum utilisation of energy and to minimise pollution of the atmosphere by partially burned oil droplets. Here a droplet is surrounded by a flame and heat conducted back from the flame to the droplet surface provides the heat to vaporise the oil and sustain the surrounding flame. Again  $d^2$  decreases approximately linearly with time though the derivation of the equation is more complex due to mass transfer effects, steep temperature gradients<sup>(3)</sup> and circulation in the drop<sup>(4)</sup>.

### 9.3.5. Unsteady state conduction

#### Basic considerations

In the problems which have been considered so far, it has been assumed that the conditions at any point in the system remain constant with respect to time. The case of heat transfer by conduction in a medium in which the temperature is changing with time is now considered. This problem is of importance in the calculation of the temperature distribution in a body which is being heated or cooled. If, in an element of dimensions  $dx$  by  $dy$  by  $dz$  (Figure 9.9), the temperature at the point  $(x, y, z)$  is  $\theta$  and at the point  $(x + dx, y + dy, z + dz)$  is  $(\theta + d\theta)$ , then assuming that the thermal conductivity  $k$  is constant and that no heat is generated in the medium, the rate of conduction of heat through the element is:

$$= -k \, dy \, dz \left( \frac{\partial \theta}{\partial x} \right)_{yz} \quad \text{in the } x\text{-direction}$$

$$= -k \, dz \, dx \left( \frac{\partial \theta}{\partial y} \right)_{zx} \quad \text{in the } y\text{-direction}$$

$$= -k \, dx \, dy \left( \frac{\partial \theta}{\partial z} \right)_{xy} \quad \text{in the } z\text{-direction}$$

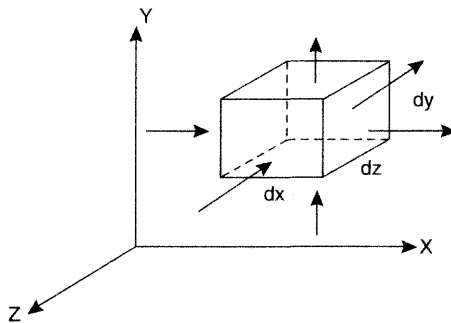


Figure 9.9. Element for heat conduction

The rate of change of heat content of the element is equal to *minus* the rate of increase of heat flow from  $(x, y, z)$  to  $(x + dx, y + dy, z + dz)$ . Thus the rate of change of the heat content of the element is:

$$\begin{aligned} &= k \, dy \, dz \left( \frac{\partial^2 \theta}{\partial x^2} \right)_{yz} \, dx + k \, dz \, dx \left( \frac{\partial^2 \theta}{\partial y^2} \right)_{zx} \, dy + k \, dx \, dy \left( \frac{\partial^2 \theta}{\partial z^2} \right)_{xy} \, dz \\ &= k \, dx \, dy \, dz \left[ \left( \frac{\partial^2 \theta}{\partial x^2} \right)_{yz} + \left( \frac{\partial^2 \theta}{\partial y^2} \right)_{zx} + \left( \frac{\partial^2 \theta}{\partial z^2} \right)_{zy} \right] \end{aligned} \quad (9.27)$$

The rate of increase of heat content is also equal, however, to the product of the heat capacity of the element and the rate of rise of temperature.

$$\text{Thus: } k \, dx \, dy \, dz \left[ \left( \frac{\partial^2 \theta}{\partial x^2} \right)_{yz} + \left( \frac{\partial^2 \theta}{\partial y^2} \right)_{zx} + \left( \frac{\partial^2 \theta}{\partial z^2} \right)_{xy} \right] = C_p \rho \, dx \, dy \, dz \frac{\partial \theta}{\partial t}$$

$$\begin{aligned} \text{or: } \frac{\partial \theta}{\partial t} &= \frac{k}{C_p \rho} \left[ \left( \frac{\partial^2 \theta}{\partial x^2} \right)_{yz} + \left( \frac{\partial^2 \theta}{\partial y^2} \right)_{zx} + \left( \frac{\partial^2 \theta}{\partial z^2} \right)_{xy} \right] \\ &= D_H \left[ \left( \frac{\partial^2 \theta}{\partial x^2} \right)_{yz} + \left( \frac{\partial^2 \theta}{\partial y^2} \right)_{zx} + \left( \frac{\partial^2 \theta}{\partial z^2} \right)_{xy} \right] \end{aligned} \quad (9.28)$$

where  $D_H = k/C_p \rho$  is known as the *thermal diffusivity*.

This partial differential equation is most conveniently solved by the use of the Laplace transform of temperature with respect to time. As an illustration of the method of solution, the problem of the unidirectional flow of heat in a continuous medium will be considered. The basic differential equation for the  $X$ -direction is:

$$\frac{\partial \theta}{\partial t} = D_H \frac{\partial^2 \theta}{\partial x^2} \quad (9.29)$$

This equation cannot be integrated directly since the temperature  $\theta$  is expressed as a function of two independent variables, distance  $x$  and time  $t$ . The method of solution involves transforming the equation so that the Laplace transform of  $\theta$  with respect to time is used in place of  $\theta$ . The equation then involves only the Laplace transform  $\bar{\theta}$  and the distance  $x$ . The Laplace transform of  $\theta$  is defined by the relation:

$$\bar{\theta} = \int_0^{\infty} \theta e^{-pt} \, dt \quad (9.30)$$

where  $p$  is a parameter.

Thus  $\bar{\theta}$  is obtained by operating on  $\theta$  with respect to  $t$  with  $x$  constant.

$$\text{Then: } \frac{\partial^2 \bar{\theta}}{\partial x^2} = \frac{\partial^2 \theta}{\partial x^2} \quad (9.31)$$

$$\begin{aligned} \text{and: } \frac{\partial \bar{\theta}}{\partial t} &= \int_0^{\infty} \frac{\partial \theta}{\partial t} e^{-pt} \, dt \\ &= \left[ \theta e^{-pt} \right]_0^{\infty} + p \int_0^{\infty} \theta e^{-pt} \, dt \\ &= -\theta_{t=0} + p \bar{\theta} \end{aligned} \quad (9.32)$$

Then, taking the Laplace transforms of each side of equation 9.29:

$$\frac{\partial \bar{\theta}}{\partial t} = D_H \frac{\partial^2 \bar{\theta}}{\partial x^2}$$

$$\text{or: } p \bar{\theta} - \theta_{t=0} = D_H \frac{\partial^2 \bar{\theta}}{\partial x^2} \quad (\text{from equations 9.31 and 9.32})$$

$$\text{and: } \frac{\partial^2 \bar{\theta}}{\partial x^2} - \frac{p}{D_H} \bar{\theta} = -\frac{\theta_{t=0}}{D_H}$$

If the temperature everywhere is constant initially,  $\theta_{t=0}$  is a constant and the equation may be integrated as a normal second-order differential equation since  $p$  is not a function of  $x$ .

$$\text{Thus:} \quad \bar{\theta} = B_1 e^{\sqrt{(p/D_H)x}} + B_2 e^{-\sqrt{(p/D_H)x}} + \theta_{t=0} p^{-1} \quad (9.33)$$

$$\text{and therefore:} \quad \frac{\partial \bar{\theta}}{\partial x} = B_1 \sqrt{\frac{p}{D_H}} e^{\sqrt{(p/D_H)x}} - B_2 \sqrt{\frac{p}{D_H}} e^{-\sqrt{(p/D_H)x}} \quad (9.34)$$

The temperature  $\theta$ , corresponding to the transform  $\bar{\theta}$ , may now be found by reference to tables of the Laplace transform. It is first necessary, however, to evaluate the constants  $B_1$  and  $B_2$  using the boundary conditions for the particular problem since these constants will in general involve the parameter  $p$  which was introduced in the transformation.

Considering the particular problem of the unidirectional flow of heat through a body with plane parallel faces a distance  $l$  apart, the heat flow is normal to these faces and the temperature of the body is initially constant throughout. The temperature scale will be so chosen that this uniform initial temperature is zero. At time,  $t = 0$ , one face (at  $x = 0$ ) will be brought into contact with a source at a constant temperature  $\theta'$  and the other face (at  $x = l$ ) will be assumed to be perfectly insulated thermally.

The boundary conditions are therefore:

$$\begin{aligned} t = 0, \quad \theta &= 0 \\ t > 0, \quad \theta &= \theta' \quad \text{when } x = 0 \\ t > 0, \quad \frac{\partial \theta}{\partial x} &= 0 \quad \text{when } x = l \end{aligned}$$

$$\text{Thus:} \quad \bar{\theta}_{x=0} = \int_0^\infty \theta' e^{-pt} dt = \frac{\theta'}{p}$$

$$\text{and:} \quad \left( \frac{\partial \bar{\theta}}{\partial x} \right)_{x=l} = 0$$

Substitution of these boundary conditions in equations 9.33 and 9.34 gives:

$$B_1 + B_2 = \frac{\theta'}{p}$$

$$\text{and:} \quad B_1 e^{\sqrt{(p/D_H)l}} - B_2 e^{-\sqrt{(p/D_H)l}} = 0 \quad (9.35)$$

Hence:

$$B_1 = \frac{(\theta'/p)e^{-\sqrt{(p/D_H)l}}}{e^{\sqrt{(p/D_H)l}} + e^{-\sqrt{(p/D_H)l}}}$$

and:

$$B_2 = \frac{(\theta'/p)e^{\sqrt{(p/D_H)l}}}{e^{\sqrt{(p/D_H)l}} + e^{-\sqrt{(p/D_H)l}}}$$



Then:

$$\begin{aligned}
 \bar{\theta} &= \frac{e^{(l-x)\sqrt{(p/D_H)}} + e^{-(l-x)\sqrt{(p/D_H)}}}{e^{\sqrt{(p/D_H)}l} + e^{-\sqrt{(p/D_H)}l}} \frac{\theta'}{p} \\
 &= \frac{\theta'}{p} (e^{(l-x)\sqrt{(p/D_H)}} + e^{-(l-x)\sqrt{(p/D_H)}}) (1 + e^{-2\sqrt{(p/D_H)}l})^{-1} (e^{-\sqrt{(p/D_H)}l}) \\
 &= \frac{\theta'}{p} (e^{-x\sqrt{(p/D_H)}} + e^{-(2l-x)\sqrt{(p/D_H)}}) (1 - e^{-2l\sqrt{(p/D_H)}} + \dots \\
 &\quad + (-1)^N e^{-2Nl\sqrt{(p/D_H)}} + \dots) \\
 &= \sum_{N=0}^{N=\infty} \frac{\theta'}{p} (-1)^N (e^{-(2lN+x)\sqrt{(p/D_H)}} + e^{-\{2(N+1)l-x\}\sqrt{(p/D_H)}}) \quad (9.36)
 \end{aligned}$$

The temperature  $\theta$  is then obtained from the tables of inverse Laplace transforms in the Appendix (Table 12, No 83) and is given by:

$$\theta = \sum_{N=0}^{N=\infty} (-1)^N \theta' \left( \operatorname{erfc} \frac{2lN+x}{2\sqrt{D_H t}} + \operatorname{erfc} \frac{2(N+1)l-x}{2\sqrt{D_H t}} \right) \quad (9.37)$$

where:

$$\operatorname{erfc} x = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\xi^2} d\xi$$

Values of  $\operatorname{erfc} x$  ( $= 1 - \operatorname{erf} x$ ) are given in the Appendix (Table 13) and in specialist sources.<sup>(5)</sup>

Equation 9.37 may be written in the form:

$$\frac{\theta}{\theta'} = \sum_{N=0}^{N=\infty} (-1)^N \left\{ \operatorname{erfc} \left[ F_{o_l}^{-1/2} \left( N + \frac{1}{2} \frac{x}{l} \right) \right] + \operatorname{erfc} \left[ F_{o_l}^{-1/2} \left( (N+1) - \frac{1}{2} \frac{x}{l} \right) \right] \right\} \quad (9.38)$$

where  $F_{o_l} = (D_H t / l^2)$  and is known as the *Fourier number*.

Thus:

$$\frac{\theta}{\theta'} = f \left( F_{o_l}, \frac{x}{l} \right) \quad (9.39)$$

The numerical solution to this problem is then obtained by inserting the appropriate values for the physical properties of the system and using as many terms in the series as are necessary for the degree of accuracy required. In most cases, the above series converge quite rapidly.

This method of solution of problems of unsteady flow is particularly useful because it is applicable when there are discontinuities in the physical properties of the material.<sup>(6)</sup> The boundary conditions, however, become a little more complicated, but the problem is intrinsically no more difficult.

A general method of estimating the temperature distribution in a body of any shape consists of replacing the heat flow problem by the analogous electrical situation and measuring the electrical potentials at various points. The heat capacity per unit volume  $C_p \rho$  is represented by an electrical capacitance, and the thermal conductivity  $k$  by an

electrical conductivity. This method can be used to take account of variations in the thermal properties over the body.

#### Example 9.4

Calculate the time taken for the distant face of a brick wall, of thermal diffusivity  $D_H = 0.0043 \text{ cm}^2/\text{s}$  and thickness  $l = 0.45 \text{ m}$ , to rise from 295 to 375 K, if the whole wall is initially at a constant temperature of 295 K and the near face is suddenly raised to 900 K and maintained at this temperature. Assume that all the flow of heat is perpendicular to the faces of the wall and that the distant face is perfectly insulated.

#### Solution

The temperature at any distance  $x$  from the near face at time  $t$  is given by:

$$\theta = \sum_{N=0}^{N=\infty} (-1)^N \theta' \left\{ \operatorname{erfc} \left[ \frac{2lN+x}{2\sqrt{D_H t}} \right] + \operatorname{erfc} \left[ \frac{2(N+1)l-x}{2\sqrt{D_H t}} \right] \right\} \quad (\text{equation 9.37})$$

The temperature at the distant face is therefore given by:

$$\theta = \sum_{N=0}^{N=\infty} (-1)^N \theta' 2 \operatorname{erfc} \left[ \frac{(2N+1)l}{2\sqrt{D_H t}} \right]$$

Choosing the temperature scale so that the initial temperature is everywhere zero, then:

$$\frac{\theta}{2\theta'} = \frac{375 - 295}{2(900 - 295)} = 0.066$$

$$D_H = 4.2 \times 10^{-7} \text{ m}^2/\text{s} \quad \therefore \quad \sqrt{D_H} = 6.5 \times 10^{-4}$$

Thus:

$$\begin{aligned} 0.066 &= \sum_{N=0}^{N=\infty} (-1)^N \operatorname{erfc} \left[ \frac{l(2N+1)}{2 \times 6.5 \times 10^{-4} t^{0.5}} \right] \\ &= \sum_{N=0}^{N=\infty} (-1)^N \operatorname{erfc} \left[ \frac{346(2N+1)}{t^{0.5}} \right] \\ &= \operatorname{erfc}(346t^{-0.5}) - \operatorname{erfc}(1038t^{-0.5}) + \operatorname{erfc}(1730t^{-0.5}) - \dots \end{aligned}$$

An approximate solution is obtained by taking the first term only, to give:

$$346t^{-0.5} = 1.30$$

from which

$$\begin{aligned} t &= 70\,840 \text{ s} \\ &= \underline{\underline{70.8 \text{ ks}}} \text{ or } \underline{\underline{19.7 \text{ h}}} \end{aligned}$$

#### Schmidt's method

Numerical methods have been developed by replacing the differential equation by a finite difference equation. Thus in a problem of unidirectional flow of heat:

$$\frac{\partial \theta}{\partial t} \approx \frac{\theta_{x(t+\Delta t)} - \theta_{x(t-\Delta t)}}{2\Delta t} \approx \frac{\theta_{x(t+\Delta t)} - \theta_{xt}}{\Delta t}$$

$$\begin{aligned} \frac{\partial^2 \theta}{\partial x^2} &\approx \frac{\left( \frac{\theta_{(x+\Delta x)t} - \theta_{xt}}{\Delta x} - \frac{\theta_{xt} - \theta_{(x-\Delta x)t}}{\Delta x} \right)}{\Delta x} \\ &= \frac{\theta_{(x+\Delta x)t} + \theta_{(x-\Delta x)t} - 2\theta_{xt}}{(\Delta x)^2} \end{aligned}$$

where  $\theta_{xt}$  is the value of  $\theta$  at time  $t$  and distance  $x$  from the surface, and the other values of  $\theta$  are at intervals  $\Delta x$  and  $\Delta t$  as shown in Figure 9.10.

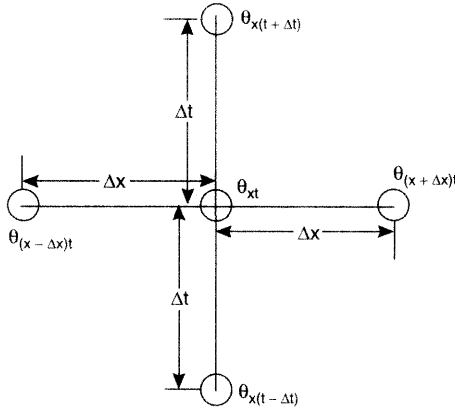


Figure 9.10. Variation of temperature with time and distance

Substituting these values in equation 9.29:

$$\theta_{x(t+\Delta t)} - \theta_{x(t-\Delta t)} = D_H \frac{2\Delta t}{(\Delta x)^2} (\theta_{(x+\Delta x)t} + \theta_{(x-\Delta x)t} - 2\theta_{xt}) \tag{9.40}$$

and:

$$\theta_{x(t+\Delta t)} - \theta_{xt} = D_H \frac{\Delta t}{(\Delta x)^2} (\theta_{(x+\Delta x)t} + \theta_{(x-\Delta x)t} - 2\theta_{xt}) \tag{9.41}$$

Thus, if the temperature distribution at time  $t$ , is known, the corresponding distribution at time  $t + \Delta t$  can be calculated by the application of equation 9.41 over the whole extent of the body in question. The intervals  $\Delta x$  and  $\Delta t$  are so chosen that the required degree of accuracy is obtained.

A graphical method of procedure has been proposed by SCHMIDT<sup>(7)</sup>. If the temperature distribution at time  $t$  is represented by the curve shown in Figure 9.11 and the points representing the temperatures at  $x - \Delta x$  and  $x + \Delta x$  are joined by a straight line, then the distance  $\theta_a$  is given by:

$$\begin{aligned} \theta_a &= \frac{\theta_{(x+\Delta x)t} + \theta_{(x-\Delta x)t}}{2} - \theta_{xt} \\ &= \frac{(\Delta x)^2}{2D_H \Delta t} (\theta_{x(t+\Delta t)} - \theta_{xt}) \quad (\text{from equation 9.41}) \end{aligned} \tag{9.42}$$

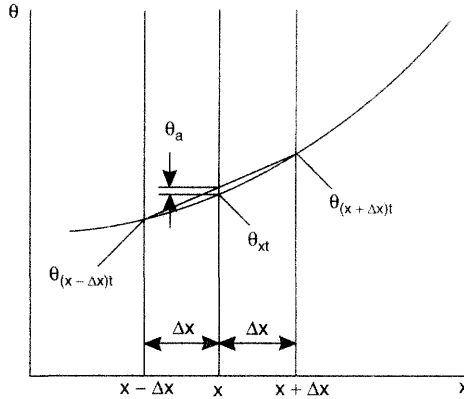


Figure 9.11. Schmidt's method

Thus,  $\theta_a$  represents the change in  $\theta_{x,t}$  after a time interval  $\Delta t$ , such that:

$$\Delta t = \frac{(\Delta x)^2}{2D_H} \tag{9.43}$$

If this simple construction is carried out over the whole of the body, the temperature distribution after time  $\Delta t$  is obtained. The temperature distribution after an interval  $2\Delta t$  is then obtained by repeating this procedure.

The most general method of tackling the problem is the use of the *finite-element* technique<sup>(8)</sup> to determine the temperature distribution at any time by using the finite difference equation in the form of equation 9.40.

**Example 9.5**

Solve Example 9.4 using Schmidt's method.

**Solution**

The development of the temperature profile is shown in Figure 9.12. At time  $t = 0$  the temperature is constant at 295 K throughout and the temperature of the hot face is raised to 900 K. The problem will be solved by taking relatively large intervals for  $\Delta x$ .

Choosing  $\Delta x = 50$  mm, the construction shown in Figure 9.12 is carried out starting at the hot face.

Points corresponding to temperature after a time interval  $\Delta t$  are marked 1, after a time interval  $2\Delta t$  by 2, and so on. Because the second face is perfectly insulated, the temperature gradient must be zero at this point. Thus, in obtaining temperatures at  $x = 450$  mm it is assumed that the temperature at  $x = 500$  mm will be the same as at  $x = 400$  mm, that is, horizontal lines are drawn on the diagram. It is seen that the temperature is less than 375 K after time  $23\Delta t$  and greater than 375 K after time  $25\Delta t$ .

Thus:  $t \approx 24\Delta t$

From equation 9.43:  $\Delta t = 5.0^2 / (2 \times 0.0042) = 2976$  s

Thus time required  $= 24 \times 2976 = 71400$  s

or:  $\underline{\underline{71.4 \text{ ks}}} = \underline{\underline{19.8 \text{ h}}}$

This value is quite close to that obtained by calculation, even using the coarse increments in  $\Delta x$ .

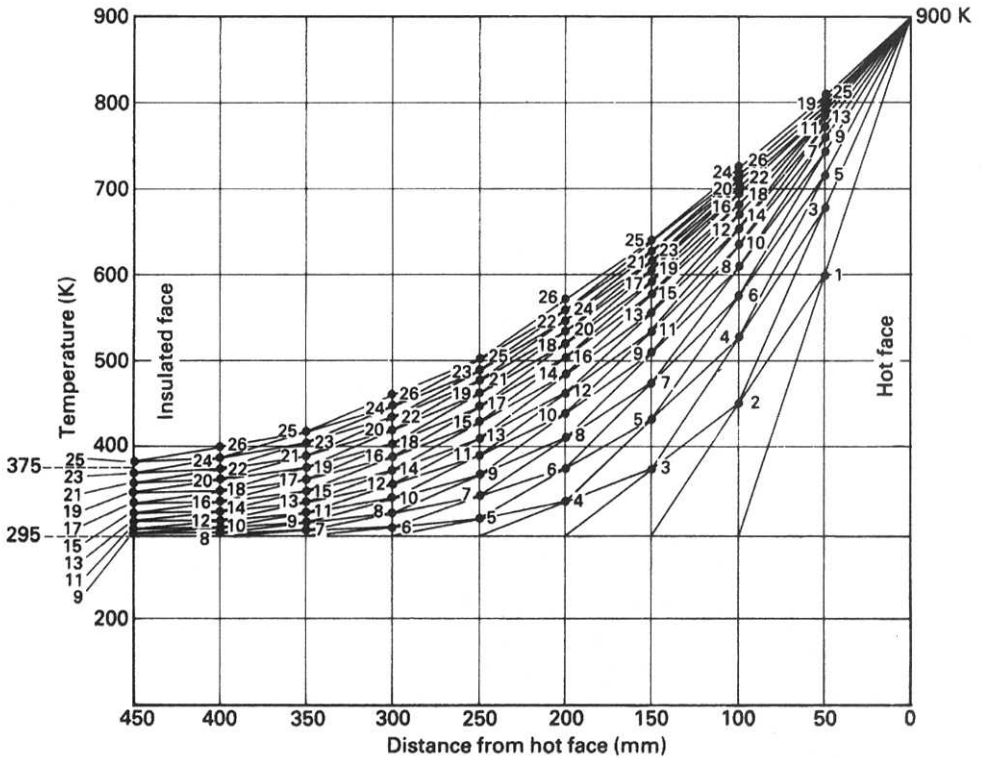


Figure 9.12. Development of temperature profile

### Heating and cooling of solids and particles

The exact mathematical solution of problems involving unsteady thermal conduction may be very difficult, and sometimes impossible, especially where bodies of irregular shapes are concerned, and other methods are therefore required.

When a body of characteristic linear dimension  $L$ , initially at a uniform temperature  $\theta_0$ , is exposed suddenly to surroundings at a temperature  $\theta'$ , the temperature distribution at any time  $t$  is found from dimensional analysis to be:

$$\frac{\theta' - \theta}{\theta' - \theta_0} = f\left(\frac{hL}{k}, D_H \frac{t}{L^2}, \frac{x}{L}\right) \quad (9.44)$$

where  $D_H$  is the thermal diffusivity ( $k_p/C_p\rho$ ) of the solid,  $x$  is distance within the solid body and  $h$  is the heat transfer coefficient in the fluid at the surface of the body.

Analytical solutions of equation 9.44 in the form of infinite series are available for some simple regular shapes of particles, such as rectangular slabs, long cylinders and spheres, for conditions where there is heat transfer by conduction or convection to or from the surrounding fluid. These solutions tend to be quite complex, even for simple shapes. The heat transfer process may be characterised by the value of the *Biot number*  $Bi$  where:

$$Bi = \frac{hL}{k_p} = \frac{L/k_p}{1/h} \quad (9.45)$$

where  $h$  is the external heat transfer coefficient,

$L$  is a characteristic dimension, such as radius in the case of a sphere or long cylinder, or half the thickness in the case of a slab, and

$k_p$  is the thermal conductivity of the particle.

The Biot number is essentially the ratio of the resistance to heat transfer within the particle to that within the external fluid. At first sight, it appears to be similar in form to the Nusselt Number  $Nu'$  where:

$$Nu' = \frac{hd}{k} = \frac{2hr_o}{k} \quad (9.46)$$

However, the Nusselt number refers to a single fluid phase, whereas the Biot number is related to the properties of both the fluid and the solid phases.

Three cases are now considered:

- (1) Very large Biot numbers,  $Bi \rightarrow \infty$
- (2) Very low Biot numbers,  $Bi \rightarrow 0$
- (3) Intermediate values of the Biot number.

(1) *Bi very large.* The resistance to heat transfer in the fluid is then low compared with that in the solid with the temperature of the surface of the particle being approximately equal to the bulk temperature of the fluid, and the heat transfer rate is independent of the Biot number. Equation 9.44 then simplifies to:

$$\frac{\theta' - \theta}{\theta' - \theta_0} = f\left(D_H \frac{t}{L^2}, \frac{x}{L}\right) = f\left(F_{OL}, \frac{x}{L}\right) \quad (9.47)$$

where  $F_{OL} \left(= D_H \frac{t}{L^2}\right)$  is known as the *Fourier* number, using  $L$  in this case to denote the characteristic length, and  $x$  is distance from the centre of the particle. Curves connecting these groups have been plotted by a number of workers for bodies of various shapes, although the method is limited to those shapes which have been studied experimentally.

In Figure 9.13, taken from CARSLAW and JAEGER<sup>(5)</sup>, the value of  $(\theta' - \theta_c)/(\theta' - \theta_0)$  is plotted to give the temperature  $\theta_c$  at the centre of bodies of various shapes, initially at a uniform temperature  $\theta_0$ , at a time  $t$  after the surfaces have been suddenly altered to and maintained at a constant temperature  $\theta'$ .

In this case  $(x/L)$  is constant at 0 and the results are shown as a function of the particular value of the Fourier number  $F_{OL} (D_H t/L^2)$ .

(2) *Bi very small.* (say,  $<0.1$ ). Here the main resistance to heat transfer lies within the fluid; this occurs when the thermal conductivity of the particle is very high and/or when the particle is very small. Under these conditions, the temperature within the particle is uniform and a "lumped capacity" analysis may be performed. Thus, if a solid body of volume  $V$  and initial temperature  $\theta_0$  is suddenly immersed in a volume of fluid large enough for its temperature  $\theta$  to remain effectively constant, the rate of heat transfer from the body may be expressed as:

$$-\rho C_p V \frac{d\theta}{dt} = hA_e(\theta - \theta') \quad (9.48)$$

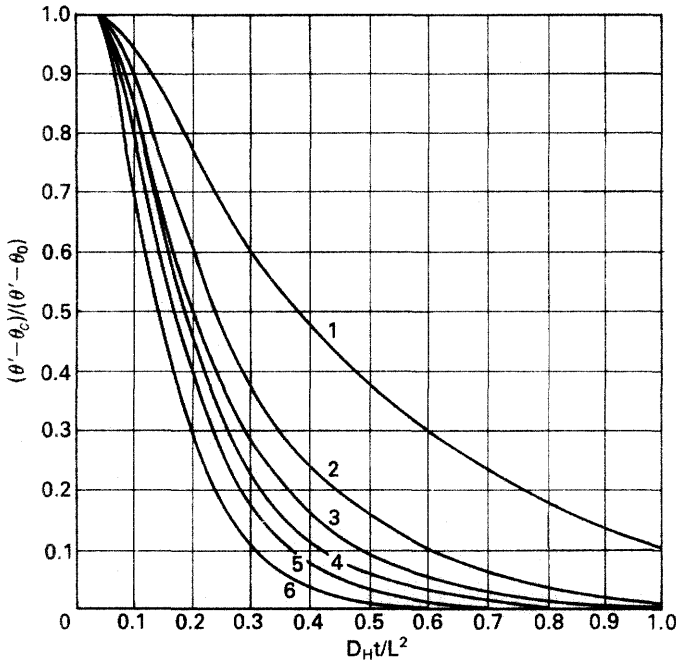


Figure 9.13. Cooling curve for bodies of various shapes: 1, slab ( $2L$  = thickness); 2, square bar ( $2L$  = side); 3, long cylinder ( $L$  = radius); 4, cube ( $2L$  = length of side); 5, cylinder ( $L$  = radius, length =  $2L$ ); 6, sphere ( $L$  = radius)

where  $A_e$  is the external surface area of the solid body.

$$\text{Then:} \quad \int_{\theta_0}^{\theta} \frac{d\theta}{\theta - \theta'} = - \int_0^t \frac{hA_e}{\rho C_v V} dt$$

$$\text{i.e.:} \quad \frac{\theta - \theta'}{\theta_0 - \theta'} = e^{-t/\tau} \quad (9.49)$$

where  $\tau = \frac{\rho C_p V}{hA_e}$  is known as the *response time constant*.

It will be noted that the relevant characteristic dimension in the Biot number is defined as the ratio of the volume to the external surface area of the particle ( $V/A_e$ ), and the higher the value of  $V/A_e$ , then the slower will be the response time. With the characteristic dimension defined in this way, this analysis is valid for particles of any shape at values of the Biot number less than 0.1

### Example 9.6

A 25 mm diameter copper sphere and a 25 mm copper cube are both heated in a furnace to  $650^\circ\text{C}$  (923 K). They are then annealed in air at  $95^\circ\text{C}$  (368 K). If the external heat transfer coefficient  $h$  is  $75 \text{ W/m}^2\text{K}$  in both cases, what is temperature of the sphere and of the cube at the end of 5 minutes?

The physical properties at the mean temperature for copper are:

$$\rho = 8950 \text{ kg/m}^3 \quad C_p = 0.38 \text{ kJ/kg K} \quad k_p = 385 \text{ W/mK}$$

### Solution

$$V/A_e \text{ for the sphere} = \frac{\frac{\pi}{6}d^3}{\pi d^2} = \frac{d}{6} = \frac{25 \times 10^{-3}}{6} = 4.17 \times 10^{-3} \text{ m}$$

$$V/A_e \text{ for the cube} = \frac{l^3}{6l^2} = \frac{l}{6} = \frac{25 \times 10^{-3}}{6} = 4.17 \times 10^{-3} \text{ m}$$

$$\therefore Bi = \frac{h(V/A_e)}{k} = \frac{75 \times 25 \times 10^{-3}}{385 \times 6} = 8.1 \times 10^{-4} \ll 0.1$$

The use of a lumped capacity method is therefore justified.

$$\tau = \frac{\rho C_p V}{h A_e} = \frac{8950 \times 380}{75} \times \frac{25 \times 10^{-3}}{6} = 189 \text{ s}$$

Then using equation 9.49:

$$\frac{\theta - 368}{923 - 368} = \exp\left(-\frac{5 \times 60}{189}\right)$$

and:

$$\theta = 368 + 0.2045(923 - 368) = 481 \text{ K} = 208^\circ\text{C}$$

Since the sphere and the cube have the same value of  $V/A_e$ , after 5 minutes they will both attain a temperature of 208°C.

(3) *Intermediate values of Bi*. In this case the resistances to heat transfer within the solid body and the fluid are of comparable magnitude. Neither will the temperature within the solid be uniform (case 1), nor will the surface temperature be equal to that in the bulk of the fluid (case 2).

Analytical solutions in the form of infinite series can be obtained for some regular shapes (thin plates, spheres and long cylinders (length  $\gg$  radius)), and numerical solutions using *finite element methods*<sup>(8)</sup> have been obtained for bodies of other shapes, both regular and irregular. Some of the results have been presented by HEISLER<sup>(9)</sup> in the form of charts, examples of which are shown in Figures 9.14–9.16 for thin slabs, long cylinders and spheres, respectively. It may be noted that in this case the characteristic length  $L$  is the half-thickness of the slab and the external radius  $r_o$  of the cylinder and sphere.

Figures 9.14–9.16 enable the temperature  $\theta_c$  at the centre of the solid (centre-plane, centre-line or centre-point) to be obtained as a function of the Fourier number, and hence of time, with the reciprocal of the Biot number ( $Bi^{-1}$ ) as parameter.

Temperatures at off-centre locations within the solid body can then be obtained from a further series of charts given by Heisler (Figures 9.17–9.19) which link the desired temperature to the centre-temperature as a function of Biot number, with location within the particle as parameter (that is the distance  $x$  from the centre plane in the slab or radius in the cylinder or sphere). Additional charts are given by Heisler for the quantity of heat transferred from the particle in a given time in terms of the initial heat content of the particle.

Figures 9.17–9.19 clearly show that, as the Biot number approaches zero, the temperature becomes uniform within the solid, and the lumped capacity method may be used for calculating the unsteady-state heating of the particles, as discussed in section (2). The charts are applicable for Fourier numbers greater than about 0.2.



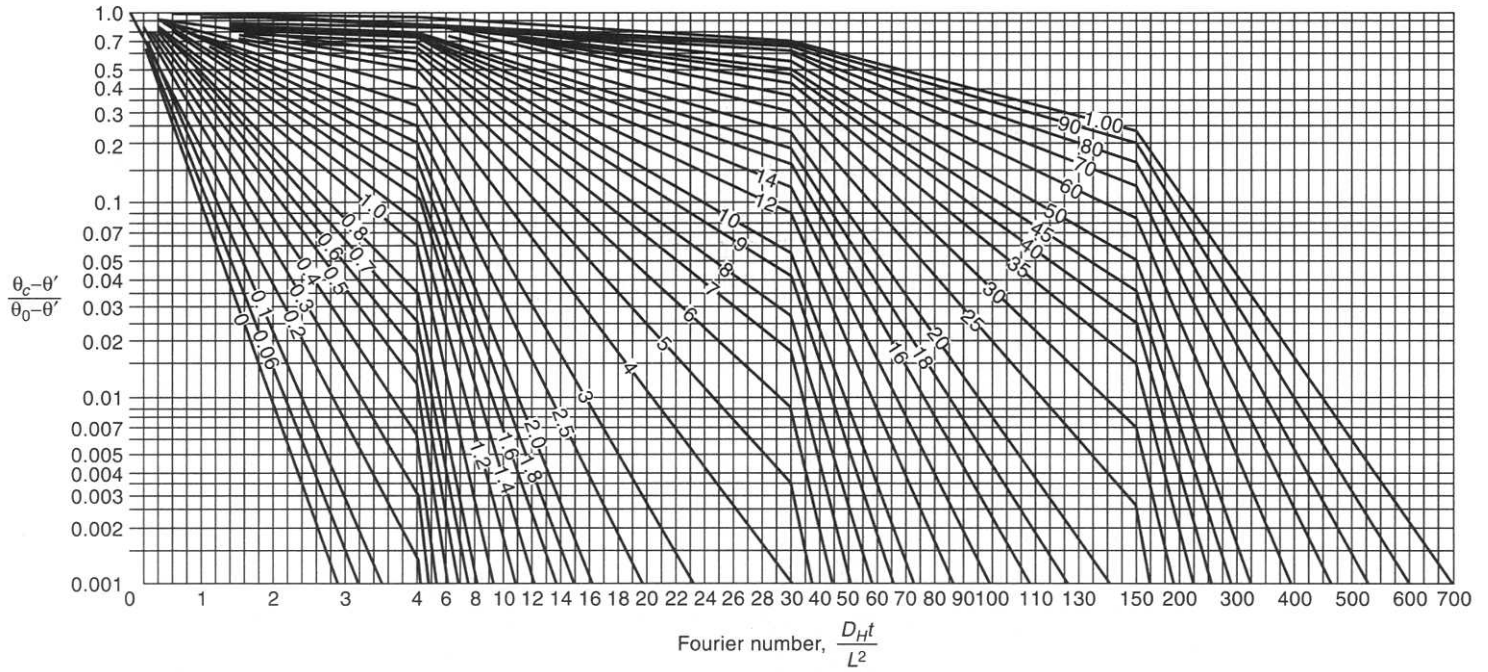


Figure 9.14. Mid-plane temperature for an infinite plate of thickness  $2L$ , for various values of parameters  $k_p/hL (= Bi^{-1})$

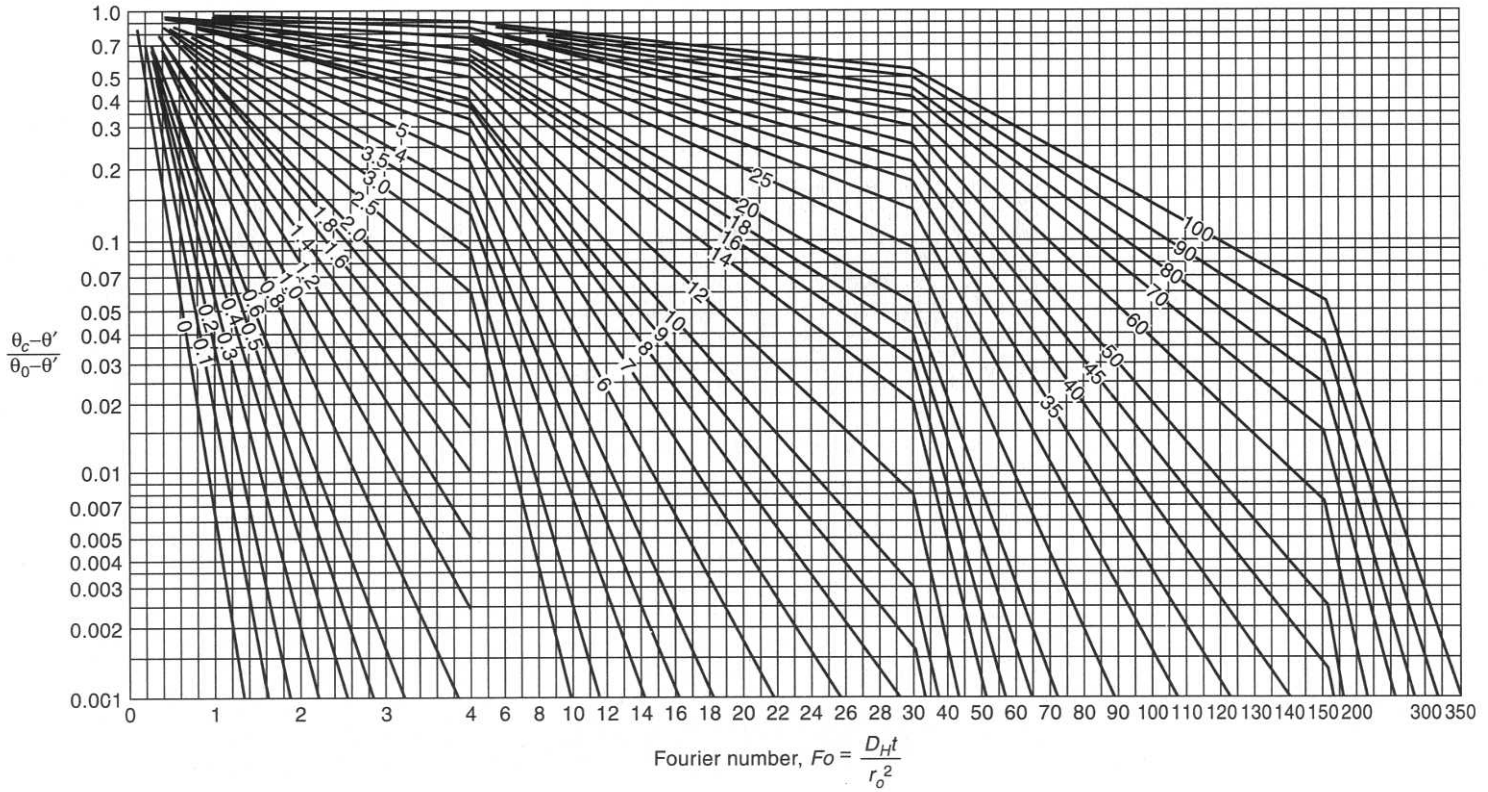


Figure 9.15. Axis temperature for an infinite cylinder of radius  $r_o$ , for various of parameters  $k_p / h r_o (= Bi^{-1})$

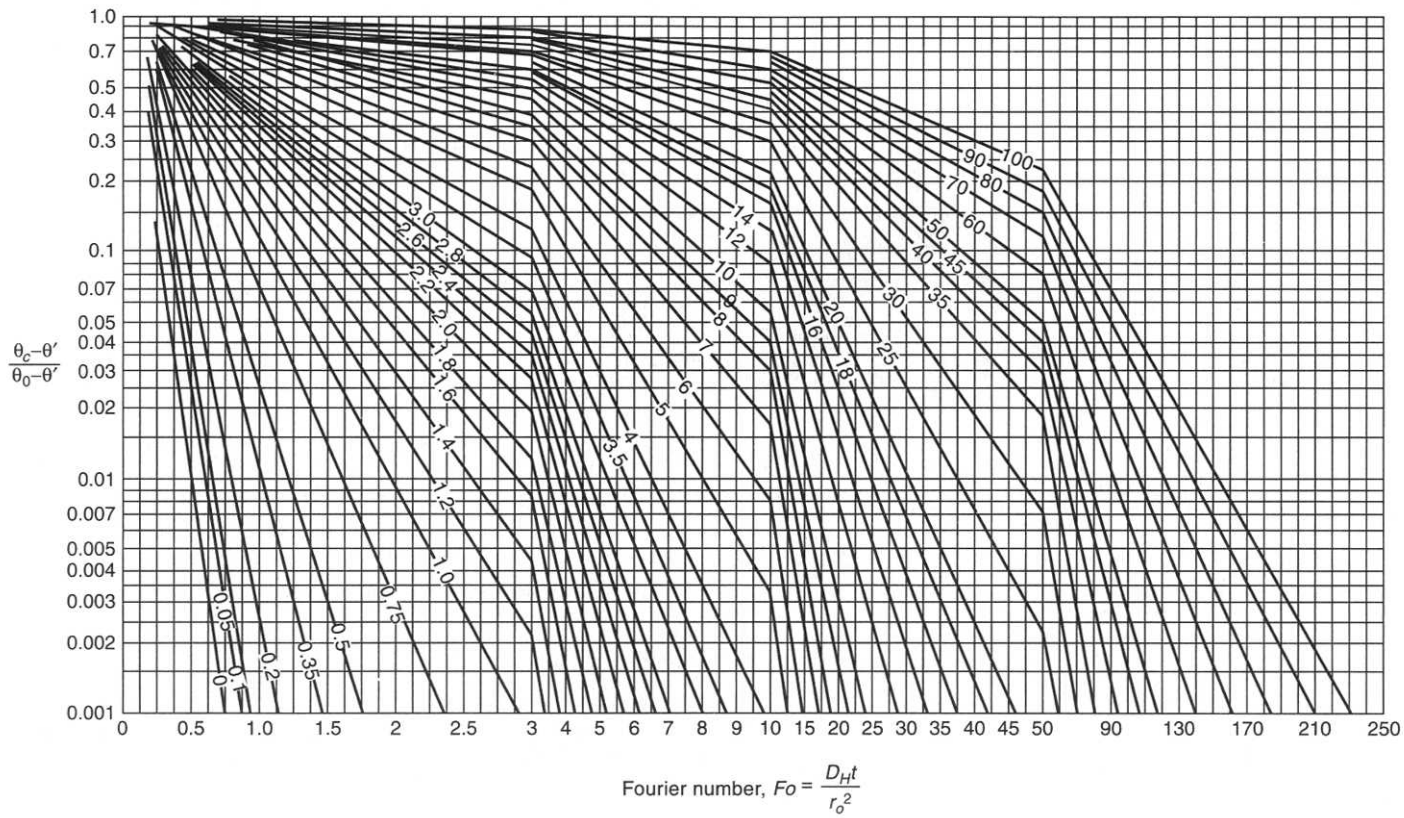


Figure 9.16. Centre-temperature for a sphere of radius  $r_o$ , for various values of parameters  $\frac{k_p}{hr_o} (= Bi^{-1})$

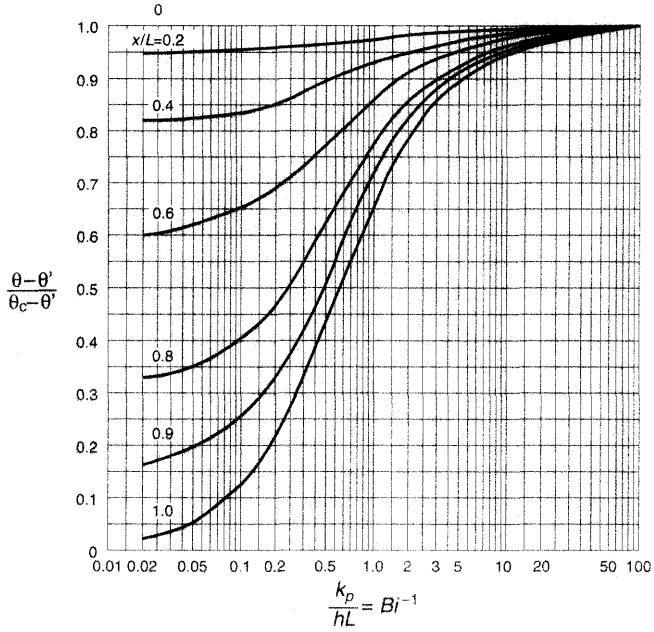


Figure 9.17. Temperature as a function of mid-plane temperature in an infinite plate of thickness  $2L$ .

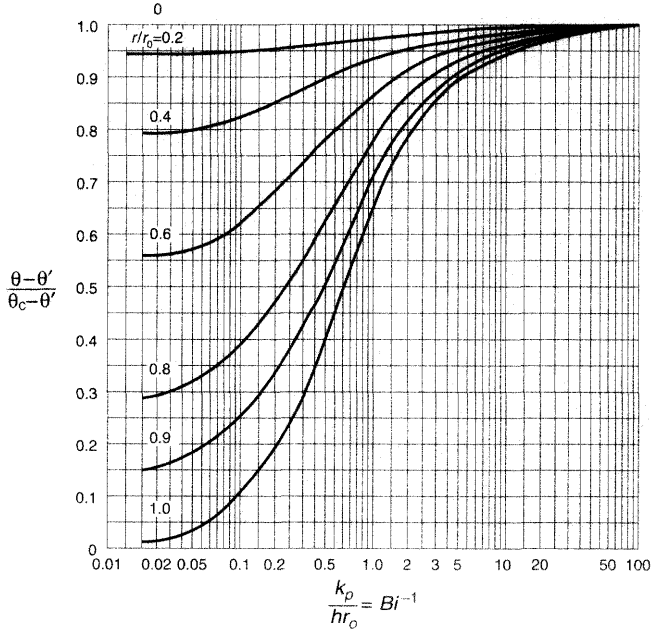


Figure 9.18. Temperature as a function of axis temperature in an infinite cylinder of radius  $r_o$ .

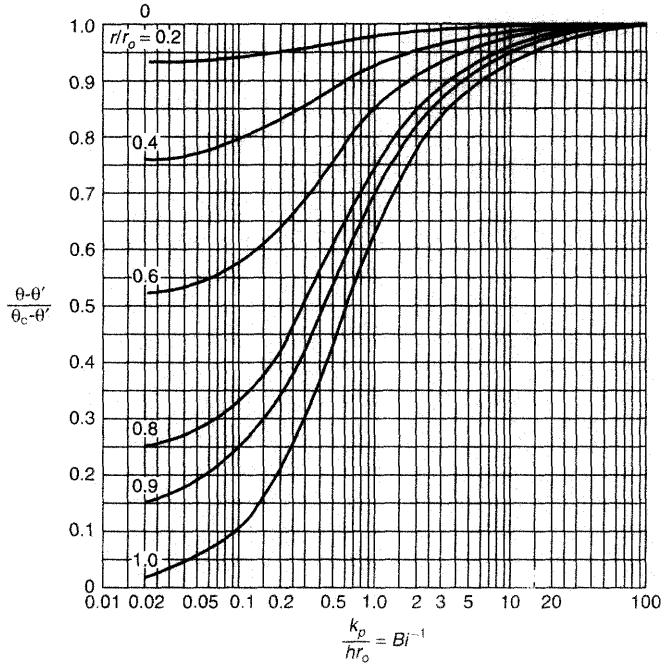


Figure 9.19. Temperature as a function of centre-temperature for a sphere of radius  $r_o$ .

### Example 9.7

A large thermoplastic sheet, 10 mm thick, at an initial temperature of  $20^\circ\text{C}$  (293 K), is to be heated in an oven in order to enable it to be moulded. The oven temperature is maintained at  $100^\circ\text{C}$  (373 K), the maximum temperature to which the plastic may be subjected, and it is necessary to ensure that the temperature throughout the whole of the sheet reaches a minimum of  $80^\circ\text{C}$  (353 K). Calculate the minimum length of time for which the sheet must be heated.

Thermal conductivity  $k_p$  of the plastic  $\quad\quad\quad = 2.5 \text{ W/mK}$

Thermal diffusivity of the surrounding fluid  $D_H = 2 \times 10^{-7} \text{ m}^2/\text{s}$

External heat transfer coefficient  $h \quad\quad\quad = 100 \text{ W/m}^2\text{K}$

### Solution

Throughout the heating process, the temperature within the sheet will be a minimum at the centre-plane ( $x = 0$ ) and therefore the required time is that for the centre to reach  $80^\circ\text{C}$  (353 K).

$$\text{For this process, the Biot number } Bi = \frac{hL}{k_p} = \frac{100 \times 5 \times 10^{-3}}{2.5} = 0.2 \text{ and } Bi^{-1} = 5$$

(since  $L$ , the half-thickness of the plate is 5 mm)

$$\text{The limiting value of } \frac{\theta' - \theta_c}{\theta' - \theta_0} = \frac{373 - 353}{373 - 293} = 0.25$$

From Figure 9.17, the Fourier number  $\frac{D_H t}{L^2} \approx 7.7$

$$\text{Thus: } t = \frac{7.7 \times (5 \times 10^{-3})^2}{2 \times 10^{-7}} = 960 \text{ s or } \underline{\underline{16 \text{ minutes}}}$$

### **Heating and melting of fine particles**

There are many situations in which particles are heated or cooled by a surrounding gas and these may be classified according to the degree of movement of the particle as follows:

#### *i) Static beds*

Although most beds of particles involve relatively large particle diameters, such as in pebble bed units used for the transfer of heat from flue gases to the incoming air for example, smaller particles, such as sand, are used in beds and, again, these are mainly used for heat recovery. One such application is the heating and cooling of buildings in hotter climes where the cool nocturnal air is used to cool a bed of particles which is then used to cool the incoming air during the heat of the day as it enters a building. In this way, an almost constant temperature may be achieved in a given enclosed environment in spite of the widely fluctuating ambient condition. A similar system has been used in less tropical areas where it is necessary to maintain a constant temperature in an environment in which heat is generated, such as a telephone exchange, for example. Such systems have the merit of very low capital and modest operating costs and, in most cases, the resistance to heat transfer by conduction within the solids is not dissimilar to the resistance in the gas film surrounding the particles.

#### *ii) Partial movement of particles*

The most obvious example of a process in which particles undergo only limited movement is the fluidised bed which is discussed in some detail in Volume 2. Applications here involve, not only heating and cooling, but also drying as in the case of grain dryers for example, and on occasions, chemical reaction as, for example, with fluidised-bed combustion. In such cases, conditions in the bed may, to all intents and purposes, be regarded as steady-state, with unsteady-state conduction taking place only in the entering 'process stream' which, by and large, is only a small proportion of the total bed mass in the bed.

#### *iii) Falling particles*

Particles fall by gravity through either static or moving gas streams in rotary dryers, for example, but they also fall through heating or cooling gases in specially designed columns. Examples here include the cooling of sand after it has been dried — again recovering heat in the process — salt cooling and also the spray drying of materials such as detergents which are sprayed as a concentrated solution of the material at the top of the tower and emerge as a dry powder. A similar situation occurs in fertiliser production where solid particles or granules are obtained from a spray of the molten material by counter-flow against a cooling gas stream. Convection to such materials is discussed in Section 9.4.6

One important problem involving unsteady state conduction of heat to particles is in the melting of powders in plasma spraying<sup>(10)</sup> where Biot numbers can range from 0.005 to 5. In this case, there is initially a very high relative velocity between the fluid and the powder. The plasmas referred to here are partially ionised gases with temperatures of around 10,000 K formed by electric discharges such as arcs. There is an increasing industrial use

of the technique of plasma spraying in which powders are injected into a high-velocity plasma jet so that they are both melted and projected at velocities of several hundred metres per second onto a surface. The molten particles with diameters typically of the order 10–100  $\mu\text{m}$  impinge to form an integral layer on the surface. Applications include the building up of worn shafts of pumps, for example, and the deposition of erosion-resistant ceramic layers on centrifugal pump impellers and other equipment prone to erosion damage. When a powder particle first enters the plasma jet, the relative velocity may be hundreds of metres per second and heat transfer to the particle is enhanced by convection, as discussed in Section 9.4.6. Often, and more particularly for smaller particles, the particle is quickly accelerated to essentially the same velocity as the plasma jet<sup>(2)</sup> and conduction becomes the main mechanism of heat transfer from plasma to particle. From a design point of view, neglecting the convective contribution will ease calculations and give a more conservative and safer estimate of the size of the largest particle which can be melted before it strikes the surface. In the absence of complications due to non-continuum conditions discussed later, the value of  $Nu' = hd/k$  is therefore often taken as 2, as in equation 9.26.

One complication which arises in the application of this equation to powder heating in high temperature plasmas lies in the dependence of  $k$ , the thermal conductivity of the gas or plasma surrounding the particle, on temperature. For example, the temperature of the particle surface may be 1000 K, whilst that of the plasma away from the particle may be about 10,000 K or even higher. The thermal conductivity of argon increases by a factor of about 20 over this range of temperature and that of nitrogen gas passes through a pronounced peak at about 7100 K due to dissociation–recombination effects. Thus, the temperature at which the thermal conductivity  $k$  is evaluated will have a pronounced effect on the value of the external heat transfer coefficient. A mean value of  $k$  would seem appropriate where:

$$(k)_{\text{mean}} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} k \, dT \quad (\text{equation 9.16})$$

Some workers have correlated experimental data in terms of  $k$  at the arithmetic mean temperature, and some at the temperature of the bulk plasma. Experimental validation of the true effective thermal conductivity is difficult because of the high temperatures, small particle sizes and variations in velocity and temperature in plasma jets.

In view of the high temperatures involved in plasma devices and the dependence of radiation heat transfer on  $T^4$ , as discussed in Section 9.5, it is surprising at first sight that conduction is more significant than radiation in heating particles in plasma spraying. The explanation lies in the small values of  $d$  and relatively high values of  $k$  for the gas, both of which contribute to high values of  $h$  for any given value of  $Nu'$ . Also the emissivities of most gases are, as seen later in Section 9.5, rather low.

In situations where the surrounding fluid behaves as a non-continuum fluid, for example at very high temperatures and/or at low pressures, it is possible for  $Nu'$  to be less than 2. A gas begins to exhibit non-continuum behaviour when the mean free path between collisions of gas molecules or atoms with each other is greater than about 1/100 of the characteristic size of the surface considered. The molecules or atoms are then sufficiently far apart on average for the gas to begin to lose the character of a homogeneous or continuum fluid which is normally assumed in the majority of heat transfer or fluid

dynamics problems. For example, with a particle of diameter  $25 \mu\text{m}$  as encountered in, for example, oil-burner sprays, pulverised coal flames, and in plasma spraying in air at room temperature and atmospheric pressure, the mean free path of gas molecules is about  $0.06 \mu\text{m}$  and the air then behaves as a continuum fluid. If, however, the temperature were say  $1800 \text{ K}$ , as in a flame, then the mean free path would be about  $0.33 \mu\text{m}$ , which is greater than  $1/100$  of the particle diameter. Non-continuum effects, leading to values of  $Nu'$  lower than 2 would then be likely according to theory<sup>(11,12)</sup>. The exact value of  $Nu'$  depends on the surface accommodation coefficient. This is a difficult parameter to measure for the examples considered here, and hence experimental confirmation of the theory is difficult. At the still higher temperatures that exist in thermal plasma devices, non-continuum effects should be more pronounced and there is limited evidence that values of  $Nu'$  below 1 are obtained<sup>(10)</sup>. In general, non-continuum effects, leading in particular to values of  $Nu'$  less than 2, would be more likely at high temperatures, low pressures, and small particle sizes. Thus, there is an interest in these effects in the aerospace industry when considering, for example, the behaviour of small particles present in rocket engine exhausts.

### 9.3.6. Conduction with internal heat source

If an electric current flows through a wire, the heat generated internally will result in a temperature distribution between the central axis and the surface of the wire. This type of problem will also arise in chemical or nuclear reactors where heat is generated internally. It is necessary to determine the temperature distribution in such a system and the maximum temperature which will occur.

If the temperature at the surface of the wire is  $T_o$  and the rate of heat generation per unit volume is  $Q_G$ , then considering unit length of a cylindrical element of radius  $r$ , the heat generated must be transmitted in an outward direction by conduction so that:

$$-k2\pi r \frac{dT}{dr} = \pi r^2 Q_G$$

Hence: 
$$\frac{dT}{dr} = -\frac{Q_G r}{2k} \quad (9.50)$$

Integrating: 
$$T = -\frac{Q_G r^2}{4k} + C$$

$T = T_o$  when  $r = r_o$  the radius of wire and hence:

$$T = T_o + Q_G \frac{r_o^2 - r^2}{4k}$$

or: 
$$T - T_o = \frac{Q_G r_o^2}{4k} \left(1 - \frac{r^2}{r_o^2}\right) \quad (9.51)$$

This gives a parabolic distribution of temperature and the maximum temperature will occur at the axis of the wire where  $(T - T_o) = Q_G r_o^2 / 4k$ . The arithmetic mean temperature difference,  $(T - T_o)_{av} = Q_G r_o^2 / 8k$ .



Since  $Q_G \pi r_o^2$  is the rate of heat release per unit length of the wire then, putting  $T_1$  as the temperature at the centre:

$$T_1 - T_o = \frac{\text{rate of heat release per unit length}}{4\pi k} \quad (9.52)$$

### Example 9.8

A fuel channel in a natural uranium reactor is 5 m long and has a heat release of 0.25 MW. If the thermal conductivity of the uranium is 33 W/mK, what is the temperature difference between the surface and the centre of the uranium element, assuming that the heat release is uniform along the rod?

### Solution

$$\begin{aligned} \text{Heat release rate} &= 0.25 \times 10^6 \text{ W} \\ &= \frac{0.25 \times 10^6}{5} = 5 \times 10^4 \text{ W/m} \end{aligned}$$

Thus, from equation 9.52:

$$\begin{aligned} T_1 - T_o &= \frac{5 \times 10^4}{4\pi \times 33} \\ &= \underline{\underline{121 \text{ deg K}}} \end{aligned}$$

It should be noted that the temperature difference is independent of the diameter of the fuel rod for a cylindrical geometry, and that the heat released per unit volume has been considered as being uniform.

In practice the assumption of the uniform heat release per unit length of the rod is not valid since the neutron flux, and hence the heat generation rate varies along its length. In the simplest case where the neutron flux may be taken as zero at the ends of the fuel element, the heat flux may be represented by a sinusoidal function, and the conditions become as shown in Figure 9.20.

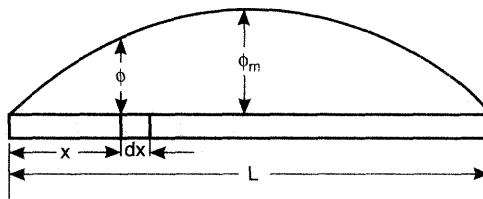


Figure 9.20. Variation of neutron flux along a length of fuel rod

Since the heat generated is proportional to the neutron flux, the heat  $dQ$  developed per unit time in a differential element of the fuel rod of length  $dx$  may be written as:

$$dQ = C \sin\left(\frac{\pi x}{L}\right) dx$$

The total heat generated by the rod  $Q$  is then given by:

$$Q = C \int_0^L \sin\left(\frac{\pi x}{L}\right) dx = \frac{2CL}{\pi}$$

Thus,  $C = \pi Q/2L$ . The heat release per unit length at any point is then given by:

$$\frac{dQ}{dx} = \frac{\pi Q}{2L} \sin\left(\frac{\pi x}{L}\right)$$

Substituting into equation 9.52 gives:

$$T_1 - T_o = \frac{\left(\frac{\pi Q}{2L}\right) \sin\left(\frac{\pi x}{L}\right)}{4\pi k} \quad (9.53)$$

It may be noted that when  $x = 0$  or  $x = L$ , then  $T_1 - T_o$  is zero as would be expected since the neutron flux was taken as zero at these positions.

## 9.4. HEAT TRANSFER BY CONVECTION

### 9.4.1. Natural and forced convection

Heat transfer by convection occurs as a result of the movement of fluid on a macroscopic scale in the form of eddies or circulating currents. If the currents arise from the heat transfer process itself, *natural convection* occurs, such as in the heating of a vessel containing liquid by means of a heat source situated beneath it. The liquid at the bottom of the vessel becomes heated and expands and rises because its density has become less than that of the remaining liquid. Cold liquid of higher density takes its place and a circulating current is thus set up.

In *forced convection*, circulating currents are produced by an external agency such as an agitator in a reaction vessel or as a result of turbulent flow in a pipe. In general, the magnitude of the circulation in forced convection is greater, and higher rates of heat transfer are obtained than in natural convection.

In most cases where convective heat transfer is taking place from a surface to a fluid, the circulating currents die out in the immediate vicinity of the surface and a film of fluid, free of turbulence, covers the surface. In this film, heat transfer is by thermal conduction and, as the thermal conductivity of most fluids is low, the main resistance to transfer lies there. Thus an increase in the velocity of the fluid over the surface gives rise to improved heat transfer mainly because the thickness of the film is reduced. As a guide, the film coefficient increases as (fluid velocity)<sup>*n*</sup>, where  $0.6 < n < 0.8$ , depending upon the geometry.

If the resistance to transfer is regarded as lying within the film covering the surface, the rate of heat transfer  $Q$  is given by equation 9.11 as:

$$Q = kA \frac{(T_1 - T_2)}{x}$$

The effective thickness  $x$  is not generally known and therefore the equation is usually rewritten in the form:

$$Q = hA(T_1 - T_2) \quad (9.54)$$

where  $h$  is the heat transfer coefficient for the film and  $(1/h)$  is the thermal resistance.

### 9.4.2. Application of dimensional analysis to convection

So many factors influence the value of  $h$  that it is almost impossible to determine their individual effects by direct experimental methods. By arranging the variables in a series of dimensionless groups, however, the problem is made more manageable in that the number of groups is significantly less than the number of parameters. It is found that the heat transfer rate per unit area  $q$  is dependent on those physical properties which affect flow pattern (viscosity  $\mu$  and density  $\rho$ ), the thermal properties of the fluid (the specific heat capacity  $C_p$  and the thermal conductivity  $k$ ) a linear dimension of the surface  $l$ , the velocity of flow  $u$  of the fluid over the surface, the temperature difference  $\Delta T$  and a factor determining the natural circulation effect caused by the expansion of the fluid on heating (the product of the coefficient of cubical expansion  $\beta$  and the acceleration due to gravity  $g$ ). Writing this as a functional relationship:

$$q = \phi[u, l, \rho, \mu, C_p, \Delta T, \beta g, k] \quad (9.55)$$

Noting the dimensions of the variables in terms of length **L**, mass **M**, time **T**, temperature  $\theta$ , heat **H**:

$q$	Heat transferred/unit area and unit time	$\mathbf{HL}^{-2}\mathbf{T}^{-1}$
$u$	Velocity	$\mathbf{LT}^{-1}$
$l$	Linear dimension	$\mathbf{L}$
$\mu$	Viscosity	$\mathbf{ML}^{-1}\mathbf{T}^{-1}$
$\rho$	Density	$\mathbf{ML}^{-3}$
$k$	Thermal conductivity	$\mathbf{HT}^{-1}\mathbf{L}^{-1}\theta^{-1}$
$C_p$	Specific heat capacity at constant pressure	$\mathbf{HM}^{-1}\theta^{-1}$
$\Delta T$	Temperature difference	$\theta$
$(\beta g)$	The product of the coefficient of thermal expansion and the acceleration due to gravity	$\mathbf{LT}^{-2}\theta^{-1}$

It may be noted that both temperature and heat are taken as fundamental units as heat is not expressed here in terms of **M**, **L**, **T**.

With nine parameters and five dimensions, equation 9.55 may be rearranged in four dimensionless groups.

Using the  $\Pi$ -theorem for solution of the equation, and taking as the recurring set:  $l, \rho, \mu, \Delta T, k$

The non-recurring variables are:  $q, u, (\beta g), C_p$

Then:

$$\begin{aligned} l &\equiv \mathbf{L} & \mathbf{L} &= l \\ \rho &\equiv \mathbf{ML}^{-3} & \mathbf{M} &= \rho l^3 = \rho l^3 \\ \mu &\equiv \mathbf{ML}^{-1}\mathbf{T}^{-1} & \mathbf{T} &= \mathbf{ML}^{-1}\mu^{-1} = \rho l^3 l^{-1} \mu^{-1} = \rho l^2 \mu^{-1} \\ \Delta T &\equiv \theta & \theta &= \Delta T \\ k &\equiv \mathbf{HL}^{-1}\mathbf{T}^{-1}\theta^{-1} & \mathbf{H} &= k \mathbf{L} \mathbf{T} \theta = k l \rho l^2 \mu^{-1} \Delta T = k l^3 \rho \mu^{-1} \Delta T \end{aligned}$$

The  $\Pi$  groups are then:

$$\Pi_1 = q\mathbf{H}^{-1}\mathbf{L}^2\mathbf{T} = qk^{-1}l^{-3}\rho^{-1}\mu\Delta T^{-1}l^2\rho l^2\mu^{-1} = qk^{-1}l\Delta T^{-1}$$

$$\Pi_2 = u\mathbf{L}^{-1}\mathbf{T} = ul^{-1}\rho l^2\mu^{-1} = u\rho l\mu^{-1}$$

$$\Pi_3 = C_p\mathbf{H}^{-1}\mathbf{M}\theta = C_p k^{-1}l^{-3}\rho^{-1}\mu\Delta T^{-1}\rho l^3\Delta T = C_p k^{-1}\mu$$

$$\Pi_4 = \beta g\mathbf{L}^{-1}\mathbf{T}^2\theta = \beta gl^{-1}\rho^2 l^4\mu^{-2}\Delta T = \beta g\Delta T\rho^2\mu^{-2}l^3$$

The relation in equation 9.55 becomes:

$$\frac{ql}{k\Delta T} = \frac{hl}{k} = \phi \left[ \left( \frac{l\rho}{\mu} \right) \left( \frac{C_p\mu}{k} \right) \left( \frac{\beta g\Delta T l^3 \rho^2}{\mu^2} \right) \right] \quad (9.56)$$

or:

$$Nu = \phi[Re, Pr, Gr]$$

This general equation involves the use of four dimensionless groups, although it may frequently be simplified for design purposes. In equation 9.56:

$hl/k$  is known as the *Nusselt* group  $Nu$  (already referred to in equation 9.46),

$l\rho/\mu$  the *Reynolds* group  $Re$ ,

$C_p\mu/k$  the *Prandtl* group  $Pr$ , and

$\beta g\Delta T l^3 \rho^2/\mu^2$  the *Grashof* group  $Gr$

It is convenient to define other dimensionless groups which are also used in the analysis of heat transfer. These are:

$l\rho C_p/k$  the *Peclet* group,  $Pe = RePr$ ,

$GC_p/kl$  the *Graetz* group  $Gz$ , and

$h/C_p\rho u$  the *Stanton* group,  $St = Nu/(RePr)$

It may be noted that many of these dimensionless groups are ratios. For example, the Nusselt group  $h/(k/l)$  is the ratio of the actual heat transfer to that by conduction over a thickness  $l$ , whilst the Prandtl group,  $(\mu/\rho)/(k/C_p\rho)$  is the ratio of the kinematic viscosity to the thermal diffusivity.

For conditions in which only natural convection occurs, the velocity is dependent on the buoyancy effects alone, represented by the Grashof number, and the Reynolds group may be omitted. Again, when forced convection occurs the effects of natural convection are usually negligible and the Grashof number may be omitted. Thus:

$$\text{for natural convection:} \quad Nu = f(Gr, Pr) \quad (9.57)$$

$$\text{and for forced convection:} \quad Nu = f(Re, Pr) \quad (9.58)$$

For most gases over a wide range of temperature and pressure,  $C_p\mu/k$  is constant and the Prandtl group may often be omitted, simplifying the design equations for the calculation of film coefficients with gases.

### 9.4.3. Forced convection in tubes

#### *Turbulent flow*

The results of a number of workers who have used a variety of gases such as air, carbon dioxide, and steam and of others who have used liquids such as water, acetone, kerosene, and benzene have been correlated by DITTUS and BOELTER<sup>(13)</sup> who used mixed units for their variables. On converting their relations using consistent (SI, for example) units, they become:

for heating of fluids:

$$Nu = 0.0241Re^{0.8}Pr^{0.4} \quad (9.59)$$

and for cooling of fluids:

$$Nu = 0.0264Re^{0.8}Pr^{0.3} \quad (9.60)$$

In these equations all of the physical properties are taken at the mean bulk temperature of the fluid  $(T_i + T_o)/2$ , where  $T_i$  and  $T_o$  are the inlet and outlet temperatures. The difference in the value of the index for heating and cooling occurs because in the former case the film temperature will be greater than the bulk temperature and in the latter case less. Conditions in the film, particularly the viscosity of the fluid, exert an important effect on the heat transfer process.

Subsequently MCADAMS<sup>(14)</sup> has re-examined the available experimental data and has concluded that an exponent of 0.4 for the Prandtl number is the most appropriate one for both heating and cooling. He also has slightly modified the coefficient to 0.023 (corresponding to Colburn's value, given below in equation 9.64) and gives the following equation, which applies for  $Re > 2100$  and for fluids of viscosities not exceeding  $2 \text{ mN s/m}^2$ :

$$Nu = 0.023Re^{0.8}Pr^{0.4} \quad (9.61)$$

WINTERTON<sup>(15)</sup> has looked into the origins of the "Dittus and Boelter" equation and has found that there is considerable confusion in the literature concerning the origin of equation 9.61 which is generally referred to as the Dittus-Boelter equation in the literature on heat transfer.

An alternative equation which is in many ways more convenient has been proposed by COLBURN<sup>(16)</sup> and includes the Stanton number ( $St = h/C_p\rho u$ ) instead of the Nusselt number. This equation takes the form:

$$j_H = StPr^{0.67} = 0.023Re^{-0.2} \quad (9.62)$$

where  $j_H$  is known as the *j-factor for heat transfer*.

It may be noted that:

$$\frac{h}{C_p\rho u} = \left(\frac{hd}{k}\right) \left(\frac{\mu}{ud\rho}\right) \left(\frac{k}{C_p\mu}\right)$$

or:

$$St = NuRe^{-1}Pr^{-1} \quad (9.63)$$

Thus, multiplying equation 9.62 by  $RePr^{0.33}$ :

$$Nu = 0.023Re^{0.8}Pr^{0.33} \quad (9.64)$$

which is a form of equations 9.59 and 9.60.

Again, the physical properties are taken at the bulk temperature, except for the viscosity in the Reynolds group which is evaluated at the mean film temperature taken as  $(T_{\text{surface}} + T_{\text{bulk fluid}})/2$ .

Writing a heat balance for the flow through a tube of diameter  $d$  and length  $l$  with a rise in temperature for the fluid from  $T_i$  to  $T_o$ :

$$h\pi dl \Delta T = \frac{\pi d^2}{4} C_p \rho u (T_o - T_i)$$

$$\text{or: } St = \frac{h}{C_p \rho u} = \frac{d(T_o - T_i)}{4l \Delta T} \quad (9.65)$$

where  $\Delta T$  is the mean temperature difference between the bulk fluid and the walls.

With *very viscous liquids* there is a marked difference at any position between the viscosity of the fluid adjacent to the surface and the value at the axis or at the bulk temperature of the fluid. SIEDER and TATE<sup>(17)</sup> examined the experimental data available and suggested that a term  $\left(\frac{\mu}{\mu_s}\right)^{0.14}$  be included to account for the viscosity variation and the fact that this will have opposite effects in heating and cooling. ( $\mu$  is the viscosity at the bulk temperature and  $\mu_s$  the viscosity at the wall or surface). They give a logarithmic plot, but do not propose a correlating equation. However, MCADAMS<sup>(14)</sup> gives the following equation, based on Sieder and Tate's work:

$$Nu = 0.027 Re^{0.8} Pr^{0.33} \left(\frac{\mu}{\mu_s}\right)^{0.14} \quad (9.66)$$

This equation may also be written in the form of the Colburn equation (9.62).

When these equations are applied to *heating or cooling of gases* for which the Prandtl group usually has a value of about 0.74, substitution of  $Pr = 0.74$  in equation 9.64 gives:

$$Nu = 0.020 Re^{0.8} \quad (9.67)$$

*Water* is very frequently used as the cooling medium and the effect of the variation of physical properties with temperature may be included in equation 9.64 to give a simplified equation which is useful for design purposes (Section 9.9.4).

There is a very big difference in the values of  $h$  for water and air for the same linear velocity. This is shown in Figures 9.21–9.23 and Table 9.2, all of which are based on the work of FISHENDEN and SAUNDERS<sup>(18)</sup>.

The effect of length to diameter ratio ( $l/d$ ) on the value of the heat transfer coefficient may be seen in Figure 9.24. It is important at low Reynolds numbers but ceases to be significant at a Reynolds number of about  $10^4$ .

It is also important to note that the film coefficient varies with the distance from the entrance to the tube. This is especially important at low ( $l/d$ ) ratios and an average value is given approximately by:

$$\frac{h_{\text{average}}}{h_{\infty}} = 1 + \left(\frac{d}{l}\right)^{0.7} \quad (9.68)$$

where  $h_{\infty}$  is the limiting value for a very long tube.

The roughness of the surface of the inside of the pipe can have an important bearing on rates of heat transfer to the fluid, although COPE<sup>(19)</sup>, using degrees of artificial roughness

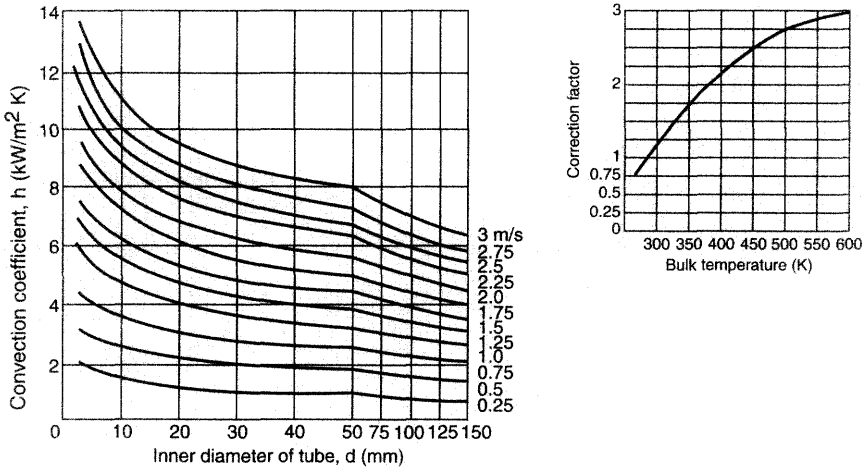


Figure 9.21. Film coefficients of convection for flow of water through a tube at 289 K

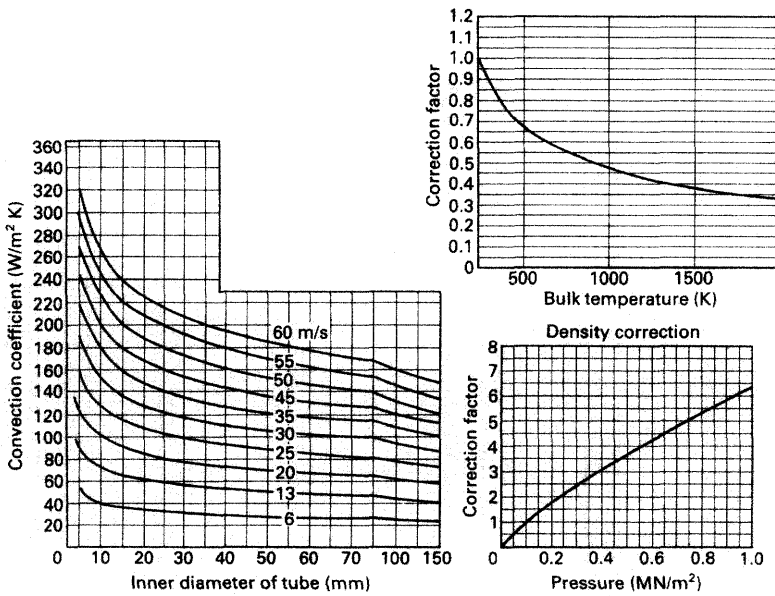


Figure 9.22. Film coefficients of convection for flow of air through a tube at various velocities (289 K, 101.3  $\text{kN/m}^2$ )

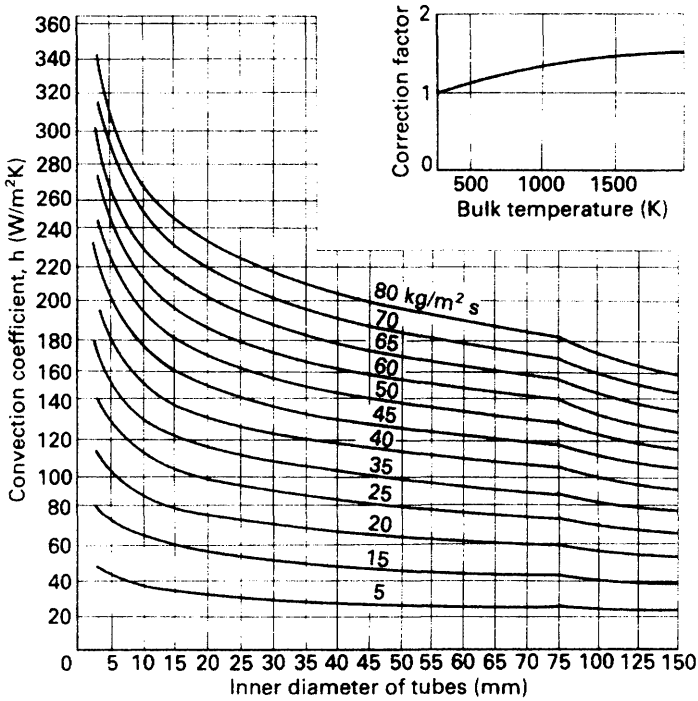


Figure 9.23. Film coefficients of convection for flow of air through a tube for various mass velocities (289 K, 101.3 kN/m<sup>2</sup>)

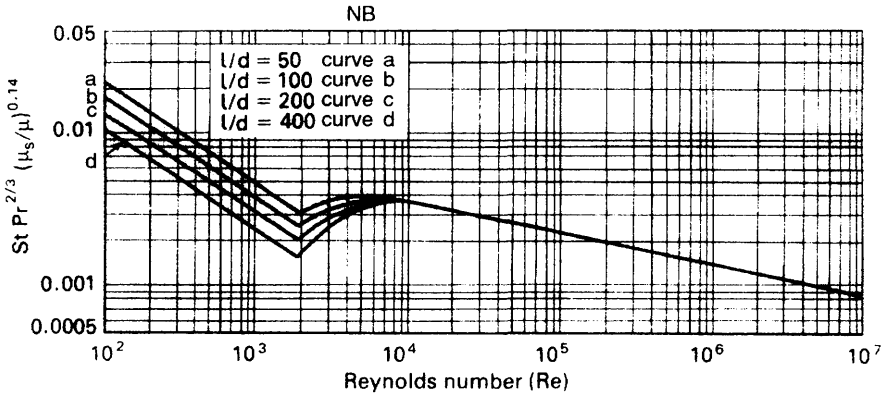


Figure 9.24. Effect of length:diameter ratio on heat transfer coefficient

ranging from 0.022 to 0.14 of the pipe diameter, found that, although the friction loss was some six times greater than for smooth tubes, the heat transfer was only 100–120 per cent higher. It was concluded that, for the same pressure drop, greater heat transfer was obtained from a smooth rather than a rough tube. The effect of a given scale deposit is usually less serious for gases than water because of the higher thermal resistance of



Table 9.2. Film coefficients for air and water (289 K and 101.3 kN/m<sup>2</sup>)

Inside diameter of tube		Velocity		Mass velocity		Film coefficient of heat transfer $h$	
(mm)	(in)	(m/s)	(ft/s)	(kg/m <sup>2</sup> s)	(lb/ft <sup>2</sup> h)	(W/m <sup>2</sup> K) [Ref.18]	(Btu/h ft <sup>2</sup> °F) [Ref.18]
<i>Air</i>							
25	1.0	5	16.4	6.11	4530	31.2	5.5
		10	32.8	12.2	9050	50.0	8.8
		20	65.6	24.5	18,100	84.0	14.8
		40	131	48.9	36,200	146	25.7
		60	197	73.4	54,300	211	37.2
50	2.0	5	16.4	6.11	4530	23.8	4.2
		10	32.8	12.2	9050	44.9	7.9
		20	65.6	24.5	18,100	77.8	13.7
		40	131	48.9	36,200	127	22.4
		60	197	73.4	54,300	181	31.9
75	3.0	5	16.4	6.11	4530	21.6	3.8
		10	32.8	12.2	9050	39.7	7.0
		20	65.6	24.5	18,100	71.0	12.5
		40	131	48.9	36,200	119	21.0
		60	197	73.4	54,300	169	29.8
<i>Water</i>							
25	1.0	0.5	1.64	488	361,000	2160	380
		1.0	3.28	975	722,000	3750	660
		1.5	4.92	1460	1,080,000	5250	925
		2.0	6.55	1950	1,440,000	6520	1150
		2.5	8.18	2440	1,810,000	7780	1370
50	2.0	0.5	1.64	488	361,000	1870	330
		1.0	3.28	975	722,000	3270	575
		1.5	4.92	1460	1,080,000	4540	800
		2.0	6.55	1950	1,440,000	5590	985
		2.5	8.18	2440	1,810,000	6700	1180
75	3.0	0.5	1.64	488	361,000	1760	310
		1.0	3.28	975	722,000	3070	540
		1.5	4.92	1460	1,080,000	4200	740
		2.0	6.55	1950	1,440,000	5220	920
		2.5	8.18	2440	1,810,000	6220	1100

the gas film, although layers of dust or of materials which sublime may seriously reduce heat transfer between gas and solid by as much as 40 per cent.

### Streamline flow

Although heat transfer to a fluid in streamline flow takes place solely by conduction, it is convenient to consider it here so that the results may be compared with those for turbulent flow.

In Chapter 3 it has been seen that, for streamline flow through a tube, the velocity distribution across a diameter is parabolic, as shown in Figure 9.25. If a liquid enters a section heated on the outside, the fluid near the wall will be at a higher temperature than that in the centre and its viscosity will be lower. The velocity of the fluid near the wall will therefore be greater in the heated section, and correspondingly less at the centre. The velocity distribution will therefore be altered, as shown. If the fluid enters a section where it is cooled, the same reasoning will show that the distribution in velocity will be altered

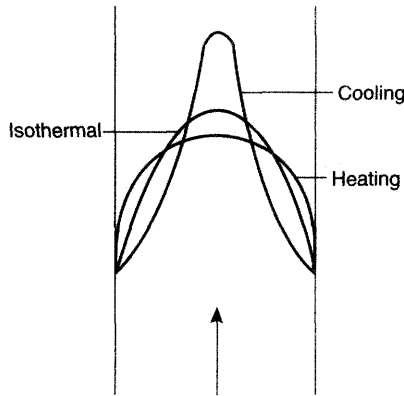


Figure 9.25. Effect of heat transfer on the velocity distribution for a liquid

to that shown. With a gas the conditions are reversed, because of the increase of viscosity with temperature. The heat transfer problem is therefore complex.

For the common problem of heat transfer between a fluid and a tube wall, the boundary layers are limited in thickness to the radius of the pipe and, furthermore, the effective area for heat flow decreases with distance from the surface. The problem can conveniently be divided into two parts. Firstly, heat transfer in the entry length in which the boundary layers are developing, and, secondly, heat transfer under conditions of fully developed flow. Boundary layer flow is discussed in Chapter 11.

For the region of fully developed flow in a pipe of length  $L$ , diameter  $d$  and radius  $r$ , the rate of flow of heat  $Q$  through a cylindrical surface in the fluid at a distance  $y$  from the wall is given by:

$$Q = -k2\pi L(r - y) \frac{d\theta}{dy} \tag{9.69}$$

Close to the wall, the fluid velocity is low and a negligible amount of heat is carried along the pipe by the flowing fluid in this region and  $Q$  is independent of  $y$ .

Thus:

$$\frac{d\theta}{dy} = -\frac{Q}{k2\pi L}(r - y)^{-1} \quad \text{and} \quad \left(\frac{d\theta}{dy}\right)_{y=0} = -\frac{Q}{2\pi kLr}$$

$$\frac{d^2\theta}{dy^2} = -\frac{Q}{k2\pi L}(r - y)^{-2} \quad \text{and} \quad \left(\frac{d^2\theta}{dy^2}\right)_{y=0} = -\frac{Q}{2\pi kLr^2}$$

Thus:

$$\left(\frac{d^2\theta}{dy^2}\right)_{y=0} = r^{-1} \left(\frac{d\theta}{dy}\right)_{y=0} \tag{9.70}$$

Assuming that the temperature of the walls remains constant at the datum temperature and that the temperature at any distance  $y$  from the walls is given by a polynomial, then:

$$\theta = a_0y + b_0y^2 + c_0y^3 \tag{9.71}$$

Thus:

$$\frac{d\theta}{dy} = a_0 + 2b_0y + 3c_0y^2 \quad \text{and} \quad \left(\frac{d\theta}{dy}\right)_{y=0} = a_0$$

$$\frac{d^2\theta}{dy^2} = 2b_0 + 6c_0y \quad \text{and} \quad \left(\frac{d^2\theta}{dy^2}\right)_{y=0} = 2b_0$$

Thus:  $2b_0 = \frac{a_0}{r}$  (from equation 9.65)

and:  $b_0 = \frac{a_0}{2r}$

If the temperature of the fluid at the axis of the pipe is  $\theta_s$  and the temperature gradient at the axis, from symmetry, is zero, then:

$$0 = a_0 + 2r \left(\frac{a_0}{2r}\right) + 3c_0r^2$$

giving:  $c_0 = -\frac{2a_0}{3r^2}$

and:  $\theta_s = a_0r + r^2 \left(\frac{a_0}{2r}\right) + r^3 \left(\frac{-2a_0}{3r^2}\right)$

$$= \frac{5}{6}a_0r$$

$$a_0 = \frac{6}{5} \frac{\theta_s}{r}$$

$$b_0 = \frac{3}{5} \frac{\theta_s}{r^2}$$

and:  $c_0 = -\frac{4}{5} \frac{\theta_s}{r^3}$

Thus:  $\frac{\theta}{\theta_s} = \frac{6}{5} \frac{y}{r} + \frac{3}{5} \left(\frac{y}{r}\right)^2 - \frac{4}{5} \left(\frac{y}{r}\right)^3$  (9.72)

Thus the rate of heat transfer per unit area at the wall:

$$q = -k \left(\frac{d\theta}{dy}\right)_{y=0}$$

$$= -\frac{6}{5} \frac{k\theta_s}{r}$$
 (9.73)

In general, the temperature  $\theta_s$  at the axis is not known, and the heat transfer coefficient is related to the temperature difference between the walls and the bulk fluid. The bulk temperature of the fluid is defined as the ratio of the heat content to the heat capacity of the fluid flowing at any section. Thus the bulk temperature  $\theta_B$  is given by:

$$\theta_B = \frac{\int_0^r C_p \rho \theta u_x 2\pi(r-y) dy}{\int_0^r C_p \rho u_x 2\pi(r-y) dy}$$

$$= \frac{\int_0^r \theta u_x(r-y) dy}{\int_0^r u_x(r-y) dy} \quad (9.74)$$

From Poiseuille's law (equation 3.30):

$$u_x = \frac{-\Delta P}{4\mu L} [r^2 - (r-y)^2] = \frac{-\Delta P}{4\mu L} (2ry - y^2)$$

Hence: 
$$u_s = \frac{-\Delta P}{4\mu L} r^2 \quad (9.75)$$

where  $u_s$  is the velocity at the pipe axis,

and: 
$$\frac{u_x}{u_s} = \frac{2y}{r} - \left(\frac{y}{r}\right)^2 \quad (9.76)$$

Thus: 
$$\begin{aligned} \int_0^r u_x(r-y) dy &= r^2 u_s \int_0^1 \left[ 2\frac{y}{r} - \left(\frac{y}{r}\right)^2 \right] \left(1 - \frac{y}{r}\right) d\left(\frac{y}{r}\right) \\ &= r^2 u_s \int_0^1 \left[ 2\left(\frac{y}{r}\right) - 3\left(\frac{y}{r}\right)^2 + \left(\frac{y}{r}\right)^3 \right] d\left(\frac{y}{r}\right) \\ &= \frac{1}{4} r^2 u_s \end{aligned} \quad (9.77)$$

Since: 
$$\frac{\theta}{\theta_s} = \frac{6y}{5r} + \frac{3}{5} \left(\frac{y}{r}\right)^2 - \frac{4}{5} \left(\frac{y}{r}\right)^3 \quad (\text{equation 9.72})$$

$$\begin{aligned} \int_0^r \theta u_x(r-y) dy &= r^2 u_s \theta_s \int_0^1 \left[ \frac{6y}{5r} + \frac{3}{5} \left(\frac{y}{r}\right)^2 - \frac{4}{5} \left(\frac{y}{r}\right)^3 \right] \left[ 2\left(\frac{y}{r}\right) - 3\left(\frac{y}{r}\right)^2 + \left(\frac{y}{r}\right)^3 \right] d\left(\frac{y}{r}\right) \\ &= r^2 u_s \theta_s \int_0^1 \left[ \frac{12}{5} \left(\frac{y}{r}\right)^2 - \frac{12}{5} \left(\frac{y}{r}\right)^3 - \frac{11}{5} \left(\frac{y}{r}\right)^4 + 3\left(\frac{y}{r}\right)^5 - \frac{4}{5} \left(\frac{y}{r}\right)^6 \right] d\left(\frac{y}{r}\right) \\ &= r^2 u_s \theta_s \left( \frac{4}{5} - \frac{3}{5} - \frac{11}{25} + \frac{1}{2} - \frac{4}{35} \right) \\ &= \frac{51}{350} r^2 u_s \theta_s \end{aligned} \quad (9.78)$$

Substituting from equations 9.77 and 9.78 in equation 9.74:

$$\begin{aligned} \theta_B &= \frac{\frac{51}{350} r^2 u_s \theta_s}{\frac{1}{4} r^2 u_s} \\ &= \frac{102}{175} \theta_s = 0.583 \theta_s \end{aligned} \quad (9.79)$$

The heat transfer coefficient  $h$  is then given by:

$$h = -\frac{q}{\theta_B}$$

where  $q$  is the rate of heat transfer per unit area of tube.

Thus, from equations 9.73 and 9.79:

$$h = \frac{6k\theta_s/5r}{0.583\theta_s} = \frac{2.06k}{r} = 4.1 \frac{k}{d}$$

and:

$$Nu = \frac{hd}{k} = 4.1 \quad (9.80)$$

This expression is applicable only to the region of fully developed flow. The heat transfer coefficient for the inlet length can be calculated approximately, using the expressions given in Chapter 11 for the development of the boundary layers for the flow over a plane surface. It should be borne in mind that it has been assumed throughout that the physical properties of the fluid are not appreciably dependent on temperature and therefore the expressions will not be expected to hold accurately if the temperature differences are large and if the properties vary widely with temperature.

For values of  $(RePr d/l)$  greater than 12, the following empirical equation is applicable:

$$Nu = 1.62 \left( RePr \frac{d}{l} \right)^{1/3} = 1.75 \left( \frac{GC_p}{kl} \right)^{1/3} \quad (9.81)$$

where  $G = (\pi d^2/4)\rho u$ , i.e. the mass rate of flow.

The product  $RePr$  is termed the Peclet number  $Pe$ .

Thus:

$$Pe = \frac{ud\rho C_p\mu}{\mu k} = \frac{C_p\rho ud}{k} \quad (9.82)$$

Equation 9.81 may then be written:

$$Nu = 1.62 \left( Pe \frac{d}{l} \right)^{1/3} \quad (9.83)$$

In this equation the temperature difference is taken as the arithmetic mean of the terminal values, that is:

$$\frac{(T_w - T_1) + (T_w - T_2)}{2}$$

where  $T_w$  is the temperature of the tube wall which is taken as constant.

If the liquid is heated almost to the wall temperature  $T_w$  (that is when  $GC_p/kl$  is very small) then, on equating the heat gained by the liquid to that transferred from the pipe:

$$GC_p(T_2 - T_1) = \pi dlh \frac{T_2 - T_1}{2}$$

or:

$$h = \frac{2 GC_p}{\pi dl} \quad (9.84)$$

For values of  $(RePr d/l)$  less than about 17, the Nusselt group becomes approximately constant at 4.1; the value given in equation 9.80.

Experimental values of  $h$  for *viscous oils* are greater than those given by equation 9.81 for heating and less for cooling. This is due to the large variation of viscosity with temperature and the correction introduced for turbulent flow may also be used here, giving:

$$Nu \left( \frac{\mu_s}{\mu} \right)^{0.14} = 1.86 \left( RePr \frac{d}{l} \right)^{1/3} = 2.01 \left( \frac{GC_p}{kl} \right)^{1/3} \quad (9.85)$$

$$\text{or: } Nu \left( \frac{\mu_s}{\mu} \right)^{0.14} = 1.86 \left( Pe \frac{d}{l} \right)^{1/3} \quad (9.86)$$

When  $(GC_p/kl) < 10$ , the outlet temperature closely approaches that of the wall and equation 9.84 applies. These equations have been obtained with tubes about 10 mm to 40 mm in diameter, and the length of unheated tube preceding the heated section is important. The equations are not entirely consistent since for very small values of  $\Delta T$  the constants in equations 9.81 and 9.85 would be expected to be the same. It is important to note, when using these equations for design purposes, that the error may be as much as  $\pm 25$  per cent for turbulent flow and greater for streamline conditions.

With laminar flow there is a marked influence of tube length and the curves shown in Figure 9.24 show the parameter  $l/d$  from 50 to 400.

Whenever possible, streamline conditions of flow are avoided in heat exchangers because of the very low heat transfer coefficients which are obtained. With very viscous liquids, however, turbulent conditions can be produced only if a very high pressure drop across the plant is permissible. In the processing industries, streamline flow in heat exchangers is most commonly experienced with heavy oils and brines at low temperatures. Since the viscosity of these materials is critically dependent on temperature, the equations would not be expected to apply with a high degree of accuracy.

#### 9.4.4. Forced convection outside tubes

##### *Flow across single cylinders*

If a fluid passes at right angles across a single tube, the distribution of velocity around the tube will not be uniform. In the same way the rate of heat flow around a hot pipe across which air is passed is not uniform but is a maximum at the front and rear, and a minimum at the sides, where the rate is only some 40 per cent of the maximum. The general picture is shown in Figure 9.26 but for design purposes reference is made to the average value.

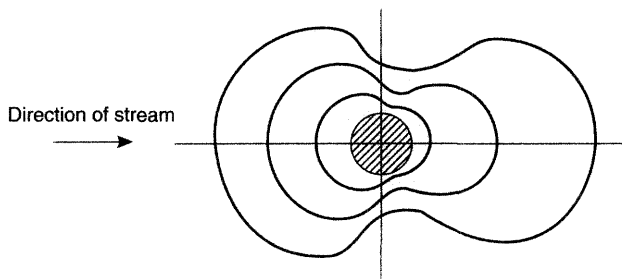


Figure 9.26. Distribution of the film heat transfer coefficient round a cylinder with flow normal to the axis for three different values of  $Re$

A number of workers, including, REIHER<sup>(20)</sup>, HILPERT<sup>(21)</sup>, GRIFFITHS and AWBERY<sup>(22)</sup>, have studied the flow of a hot gas past a single cylinder, varying from a thin wire to a

tube of 150 mm diameter. Temperatures up to 1073 K and air velocities up to 30 m/s have been used with Reynolds numbers ( $d_o u \rho / \mu$ ) from 1000 to 100,000 (where  $d_o$  is the cylinder diameter, or the outside tube diameter). The data obtained may be expressed by:

$$Nu = 0.26 Re^{0.6} Pr^{0.3} \quad (9.87)$$

Taking  $Pr$  as 0.74 for gases, this reduces to

$$Nu = 0.24 Re^{0.6} \quad (9.88)$$

DAVIS<sup>(23)</sup> has also worked with water, paraffin, and light oils and obtained similar results. For very low values of  $Re$  (from 0.2 to 200) with liquids the data are better represented by the equation:

$$Nu = 0.86 Re^{0.43} Pr^{0.3} \quad (9.89)$$

In each case the physical properties of the fluid are measured at the mean film temperature  $T_f$ , taken as the average of the surface temperature  $T_w$  and the mean fluid temperature  $T_m$ ; where  $T_m = (T_1 + T_2)/2$ .

### **Flow at right angles to tube bundles**

One of the great difficulties with this geometry is that the area for flow is continually changing. Moreover the degree of turbulence is considerably less for banks of tubes in line, as at (a), than for staggered tubes, as at (b) in Figure 9.27. With the small bundles which are common in the processing industries, the selection of the true mean area for flow is further complicated by the change in number of tubes in the rows.

The results of a number of workers for heat transfer to and from gases flowing across tube banks may be expressed by the equation:

$$Nu = 0.33 C_h Re_{\max}^{0.6} Pr^{0.3} \quad (9.90)$$

where  $C_h$  depends on the geometrical arrangement of the tubes, as shown in Table 9.3. GRIMISON<sup>(24)</sup> proposed this form of expression to correlate the data of HUGE<sup>(25)</sup> and PIERSON<sup>(26)</sup> who worked with small electrically heated tubes in rows of ten deep. Other workers have used similar equations. Some correction factors have been given by PIERSON<sup>(26)</sup> for bundles with less than ten rows although there are insufficient reported data from commercial exchangers to fix these values with accuracy. Thus for five rows a factor of 0.92 and for eight rows 0.97 is suggested.

These equations are based on the maximum velocity through the bundle. Thus for an in-line arrangement as is shown in Figure 9.27a,  $G'_{\max} = G'Y/(Y - d_o)$ , where  $Y$  is the pitch of the pipes at right-angles to direction of flow; it is more convenient here to use the mass flowrate per unit area  $G'$  in place of velocity. For staggered arrangements the maximum velocity may be based on the distance between the tubes in a horizontal line or on the diagonal of the tube bundle, whichever is the less.

It has been suggested that, for in-line arrangements, the constant in equation 9.90 should be reduced to 0.26, but there is insufficient evidence from commercial exchangers to confirm this.

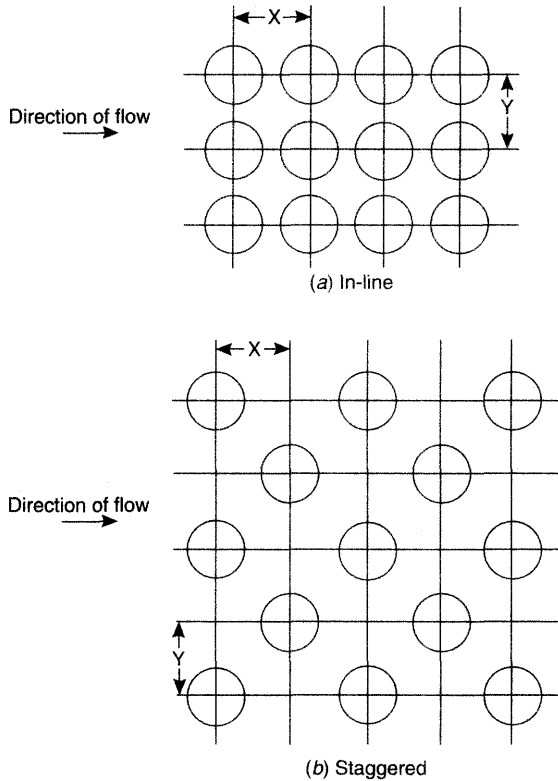


Figure 9.27. Arrangements of tubes in heat exchangers

Table 9.3.<sup>(18)</sup> Values of  $C_h$  and  $C_f$

$Re_{max}$	$X = 1.25d_o$				$X = 1.5d_o$			
	In-line		Staggered		In-line		Staggered	
	$C_h$	$C_f$	$C_h$	$C_f$	$C_h$	$C_f$	$C_h$	$C_f$
$Y = 1.25d_o$								
2000	1.06	1.68	1.21	2.52	1.06	1.74	1.16	2.58
20,000	1.00	1.44	1.06	1.56	1.00	1.56	1.05	1.74
40,000	1.00	1.20	1.03	1.26	1.00	1.32	1.02	1.50
$Y = 1.5d_o$								
2000	0.95	0.79	1.17	1.80	0.95	0.97	1.15	1.80
20,000	0.96	0.84	1.04	1.10	0.96	0.96	1.02	1.16
40,000	0.96	0.74	0.99	0.88	0.96	0.85	0.98	0.96

With liquids the same equation may be used, although for  $Re$  less than 2000, there is insufficient published work to justify an equation. MCADAMS,<sup>(27)</sup> however, has given a curve for  $h$  for a bundle with staggered tubes ten rows deep.

An alternative approach has been suggested by KERN<sup>(28)</sup> who worked in terms of the hydraulic mean diameter  $d_e$  for flow parallel to the tubes:



i.e.:

$$d_e = 4 \times \frac{\text{Free area for flow}}{\text{Wetted perimeter}}$$

$$= 4 \left( \frac{Y^2 - (\pi d_o^2/4)}{\pi d_o} \right)$$

for a square pitch as shown in Figure 9.28. The maximum cross-flow area  $A_s$  is then given by:

$$A_s = \frac{d_s l_B C'}{Y}$$

where  $C'$  is the clearance,  $l_B$  the baffle spacing, and  $d_s$  the internal diameter of the shell.

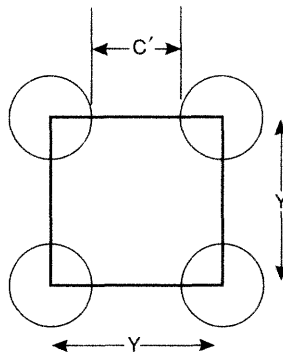


Figure 9.28. Clearance and pitch for tube layouts

The mass rate of flow per unit area  $G'_s$  is then given as rate of flow divided by  $A_s$ , and the film coefficient is obtained from a Nusselt type expression of the form:

$$\frac{h_o d_e}{k} = 0.36 \left( \frac{d_e G'_s}{\mu} \right)^{0.55} \left( \frac{C_p \mu}{k} \right)^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14} \quad (9.91)$$

There are insufficient published data to assess the relative merits of equations 9.90 and 9.91.

For 19 mm tubes on 25 mm square pitch:

$$d_e = 4 \frac{[25^2 - (\pi/4)19^2]}{\pi \times 19}$$

$$= 22.8 \text{ mm or } 0.023 \text{ m}$$

### Example 9.9

14.4 tonne/h (4.0 kg/s) of nitrobenzene is to be cooled from 400 to 315 K by heating a stream of benzene from 305 to 345 K.

Two tubular heat exchangers are available each with a 0.44 m i.d. shell fitted with 166 tubes, 19.0 mm o.d. and 15.0 mm i.d., each 5.0 m long. The tubes are arranged in two passes on 25 mm square pitch with a baffle spacing of 150 mm. There are two passes on the shell side and operation is to be countercurrent. With benzene passing through the tubes, the anticipated film coefficient on the tube side is 1000 W/m<sup>2</sup>K.

Assuming true cross-flow prevails in the shell, what value of scale resistance could be allowed if these units were used?

For nitrobenzene:  $C_p = 2380 \text{ J/kg K}$ ,  $k = 0.15 \text{ W/m K}$ ,  $\mu = 0.70 \text{ mN s/m}^2$

### Solution

(i) Tube side coefficient.

$h_i = 1000 \text{ W/m}^2 \text{ K}$  based on inside area

or: 
$$\frac{1000 \times 15.0}{19.0} = 790 \text{ W/m}^2 \text{ K based on outside area}$$

(ii) Shell side coefficient.

Area for flow = shell diameter  $\times$  baffle spacing  $\times$  clearance/pitch

$$= \frac{0.44 \times 0.150 \times 0.006}{0.025} = 0.0158 \text{ m}^2$$

Hence: 
$$G'_s = \frac{4.0}{0.0158} = 253.2 \text{ kg/m}^2\text{s}$$

Taking  $\mu/\mu_s = 1$  in equation 9.91:

$$h_o = 0.36 \frac{k}{d_e} \left( \frac{d_e G'_s}{\mu} \right)^{0.55} \left( \frac{C_p \mu}{k} \right)^{0.33}$$

The hydraulic mean diameter,

$$d_e = 4 \left[ \left( 25^2 - \frac{\pi \times 19.0^2}{4} \right) / (\pi \times 19.0) \right] = 22.8 \text{ mm or } 0.023 \text{ m}$$

and here: 
$$h_o = \left( \frac{0.15}{0.023} \right) 0.36 \left( \frac{0.023 \times 253.2}{0.70 \times 10^{-3}} \right)^{0.55} \left( \frac{2380 \times 0.70 \times 10^{-3}}{0.15} \right)^{0.33}$$

$$= 2.35 \times 143 \times 2.23 = 750 \text{ W/m}^2 \text{ K}$$

(iii) Overall coefficient.

The logarithmic mean temperature difference is given by:

$$\Delta T_m = \frac{(400 - 345) - (315 - 305)}{\ln(400 - 345)/(315 - 305)}$$

$$= 26.4 \text{ deg K}$$

The corrected mean temperature difference is then  $\Delta T_m \times F = 26.4 \times 0.8 = 21.1 \text{ deg K}$   
(Details of the correction factor for  $\Delta T_m$  are given in Section 9.9.3)

Heat load:  $Q = 4.0 \times 2380(400 - 315) = 8.09 \times 10^5 \text{ W}$

The surface area of each tube =  $0.0598 \text{ m}^2/\text{m}$

Thus: 
$$U_o = \frac{Q}{A_o \Delta T_m F} = \frac{8.09 \times 10^5}{2 \times 166 \times 5.0 \times 0.0598 \times 21.1}$$

$$= 386.2 \text{ W/m}^2 \text{ K}$$

(iv) Scale resistance.

If scale resistance is  $R_d$ , then:

$$R_d = \frac{1}{386.2} - \frac{1}{750} - \frac{1}{1000} = \underline{\underline{0.00026 \text{ m}^2 \text{ K/W}}}$$

This is a rather low value, though the heat exchangers would probably be used for this duty.

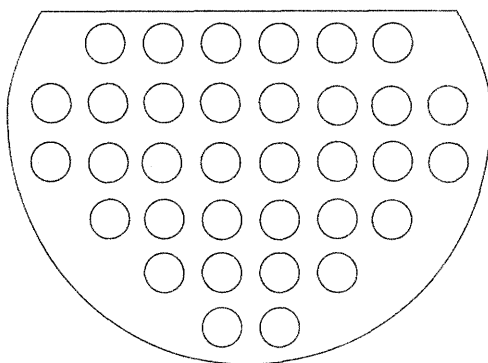


Figure 9.29. Baffle for heat exchanger

As discussed in Section 9.9 it is common practice to fit *baffles* across the tube bundle in order to increase the velocity over the tubes. The commonest form of baffle is shown in Figure 9.29 where it is seen that the cut-away section is about 25 per cent of the total area. With such an arrangement, the flow pattern becomes more complex and the extent of leakage between the tubes and the baffle, and between the baffle and the inside of the shell of the exchanger, complicates the problem, as discussed further in Section 9.9.6. Reference may also be made in Volume 6 and to the work of SHORT<sup>(29)</sup>, DONOHUE<sup>(30)</sup>, and TINKER<sup>(31)</sup>. The various methods are all concerned with developing a method of calculating the true area of flow and of assessing the probable influence of leaks. When using baffles, the value of  $h_o$ , as found from equation 9.89, is commonly multiplied by 0.6 to allow for leakage although more accurate approaches have been developed as discussed in Section 9.9.6.

The *drop in pressure* for the flow of a fluid across a tube bundle may be important because of the small pressure head available and because by good design it is possible to get a better heat transfer for the same drop in pressure.  $-\Delta P_f$  depends on the velocity  $u_t$  through the minimum area of flow and in Chapter 3 an equation proposed by GRIMISON<sup>(24)</sup> is given as:

$$-\Delta P_f = \frac{C_f j \rho u_t^2}{6} \quad (\text{equation 3.83})$$

Table 9.4.<sup>(18)</sup> Ratio of heat transfer to friction for tube bundles ( $Re_{\max} = 20,000$ )

	$X = 1.25d_o$			$X = 1.5d_o$		
	$C_h$	$C_f$	$C_h/C_f$	$C_h$	$C_f$	$C_h/C_f$
In-line						
$Y = 1.25d_o$	1	1.44	0.69	1	1.56	0.64
$Y = 1.5d_o$	0.96	0.84	1.14	0.96	0.96	1.0
Staggered						
$Y = 1.25d_o$	1.06	1.56	0.68	1.05	1.74	0.60
$Y = 1.5d_o$	1.04	1.10	0.95	1.02	1.16	0.88

where  $C_f$  depends on the geometry of the tube layout and  $j$  is the number of rows of tubes. It is found that the ratio of  $C_h$ , the heat transfer factor in equation 9.90, to  $C_f$

is greater for the in-line arrangement but that the actual heat transfer is greater for the staggered arrangement, as shown in Table 9.4.

The drop in pressure  $-\Delta P_f$  over the tube bundles of a heat exchanger is also given by:

$$-\Delta P_f = \frac{f' G_s'^2 (n+1) d_v}{2 \rho d_e} \quad (9.92)$$

where  $f'$  is the friction factor given in Figure 9.30,  $G_s'$  the mass velocity through bundle,  $n$  the number of baffles in the unit,  $d_v$  the inside shell diameter,  $\rho$  the density of fluid,  $d_e$  the equivalent diameter, and  $-\Delta P_f$  the drop in pressure.

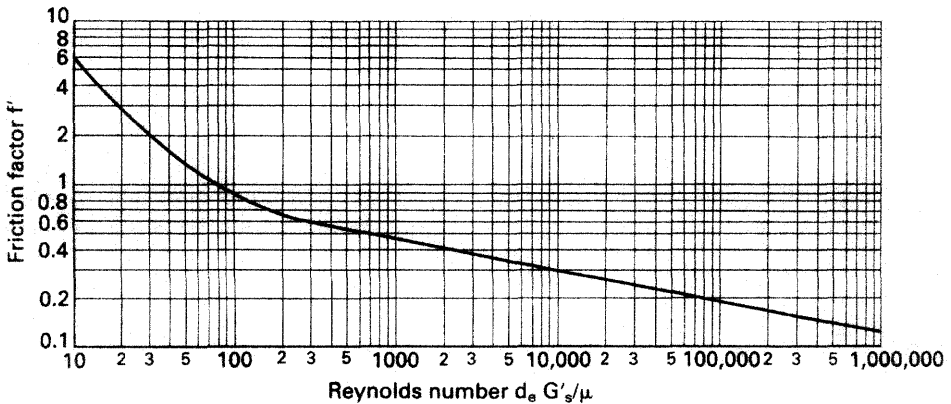


Figure 9.30. Friction factor for flow over tube bundles

### Example 9.10

54 tonne/h (15 kg/s) of benzene is cooled by passing the stream through the shell side of a tubular heat exchanger, 1 m i.d., fitted with 5 m tubes, 19 mm o.d. arranged on a 25 mm square pitch with 6 mm clearance. If the baffle spacing is 0.25 m (19 baffles), what will be the pressure drop over the tube bundle? ( $\mu = 0.5 \text{ mN s/m}^2$ ).

### Solution

Cross-flow area: 
$$A_s = \frac{1.0 \times 0.25 \times 0.006}{0.025} = 0.06 \text{ m}^2$$

Mass flow: 
$$G_s' = \frac{15}{0.06} = 250 \text{ kg/m}^2 \text{ s}$$

Equivalent diameter: 
$$d_e = \frac{4[0.025^2 - (\pi/4)0.019^2]}{\pi \times 0.019} = 0.0229 \text{ m}$$

Reynolds number through the tube bundle = 
$$\frac{250 \times 0.0229}{0.5 \times 10^{-3}} = 11450$$

From Figure 9.29: 
$$f' = 0.280$$

Density of benzene 
$$= 881 \text{ kg/m}^3$$

From equation 9.92:

$$-\Delta P_f = \frac{0.280 \times 250^2 \times 20 \times 1.0}{2 \times 881 \times 0.0229} = \underline{\underline{8674 \text{ N/m}^2}}$$

or:

$$\frac{8674}{881 \times 9.81} = \underline{\underline{1.00 \text{ m of benzene}}}$$

## 9.4.5. Flow in non-circular sections

### **Rectangular ducts**

For the heat transfer for fluids flowing in non-circular ducts, such as rectangular ventilating ducts, the equations developed for turbulent flow inside a circular pipe may be used if an equivalent diameter, such as the hydraulic mean diameter  $d_e$  discussed previously, is used in place of  $d$ .

The data for heating and cooling water in turbulent flow in rectangular ducts are reasonably well correlated by the use of equation 9.59 in the form:

$$\frac{hd_e}{k} = 0.023 \left( \frac{d_e G'}{\mu} \right)^{0.8} \left( \frac{C_p \mu}{k} \right)^{0.4} \quad (9.93)$$

Whilst the experimental data of COPE and BAILEY<sup>(32)</sup> are somewhat low, the data of WASHINGTON and MARKS<sup>(33)</sup> for heating air in ducts are well represented by this equation.

### **Annular sections between concentric tubes**

Concentric tube heat exchangers are widely used because of their simplicity of construction and the ease with which additions may be made to increase the area. They also give turbulent conditions at low volumetric flowrates.

In presenting equations for the film coefficient in the annulus, one of the difficulties is in selecting the best equivalent diameter to use. When considering the film on the outside of the inner tube, DAVIS<sup>(34)</sup> has proposed the equation:

$$\frac{hd_1}{k} = 0.031 \left( \frac{d_1 G'}{\mu} \right)^{0.8} \left( \frac{C_p \mu}{k} \right)^{0.33} \left( \frac{\mu}{\mu_s} \right)^{0.14} \left( \frac{d_2}{d_1} \right)^{0.15} \quad (9.94)$$

where  $d_1$  and  $d_2$  are the outer diameter of the inner tube, and the inner diameter of the outer tube, respectively.

CARPENTER *et al.*<sup>(35)</sup> suggest using the hydraulic mean diameter  $d_e = (d_2 - d_1)$  in the Sieder and Tate equation (9.66) and recommend the equation:

$$\frac{hd_e}{k} \left( \frac{\mu_s}{\mu} \right)^{0.14} = 0.027 \left( \frac{d_e G'}{\mu} \right)^{0.8} \left( \frac{C_p \mu}{k} \right)^{0.33} \quad (9.95)$$

Their data, which were obtained using a small annulus, are somewhat below those given by equation 9.95 for values of  $d_e G'/\mu$  less than 10,000, although this may be because the flow was not fully turbulent: with an index on the Reynolds group of 0.9, the equation fitted the data much better. There is little to choose between these two equations, but they both give rather high values for  $h$ .

For the viscous region, Carpenter's results are reasonably well correlated by the equation:

$$\frac{hd_e}{k} \left( \frac{\mu_s}{\mu} \right)^{0.14} = 2.01 \left( \frac{GC_p}{kl} \right)^{0.33} \quad (9.96)$$

$$= 1.86 \left[ \left( \frac{d_e G'}{\mu} \right) \left( \frac{C_p \mu}{k} \right) \left( \frac{d_1 + d_2}{l} \right) \right]^{1/3} \quad (9.97)$$

Equations 9.96 and 9.97 are the same as equations 9.85 and 9.86, with  $d_e$  replacing  $d$ .

These results have all been obtained with small units and mainly with water as the fluid in the annulus.

### Flow over flat plates

For the turbulent flow of a fluid over a flat plate the Colburn type of equation may be used with a different constant:

$$j_h = 0.037 Re_x^{-0.2} \quad (9.98)$$

where the physical properties are taken as for equation 9.64 and the characteristic dimension in the Reynolds group is the actual distance  $x$  along the plate. This equation therefore gives a point value for  $j_h$ .

### 9.4.6. Convection to spherical particles

In Section 9.3.4, consideration is given to the problem of heat transfer by conduction through a surrounding fluid to spherical particles or droplets. Relative motion between the fluid and particle or droplet causes an increase in heat transfer, much of which may be due to convection. Many investigators have correlated their data in the form:

$$Nu' = 2 + \beta'' Re'^n Pr^m \quad (9.99)$$

where values of  $\beta''$ , a numerical constant, and exponents  $n$  and  $m$  are found by experiment. In this equation,  $Nu' = hd/k$  and  $Re' = du\rho/\mu$ , the Reynolds number for the particle,  $u$  is the relative velocity between particle and fluid, and  $d$  is the particle diameter. As the relative velocity approaches zero,  $Re'$  tends to zero and the equation reduces to  $Nu' = 2$  for pure conduction.

ROWE *et al.*<sup>(36)</sup>, having analysed a large number of previous studies in this area and provided further experimental data, have concluded that for particle Reynolds numbers in the range 20–2000, equation 9.99 may be written as:

$$Nu' = 2.0 + \beta'' Re'^{0.5} Pr^{0.33} \quad (9.100)$$

where  $\beta''$  lies between 0.4 and 0.8 and has a value of 0.69 for air and 0.79 for water. In some practical situations the relative velocity between particle and fluid may change due to particle acceleration or deceleration, and the value of  $Nu'$  can then be time-dependent.

For mass transfer, which is considered in more detail in Chapter 10, an analogous relation (equation 10.233) applies, with the Sherwood number replacing the Nusselt number and the Schmidt number replacing the Prandtl number.

### 9.4.7. Natural convection

If a beaker containing water rests on a hot plate, the water at the bottom of the beaker becomes hotter than that at the top. Since the density of the hot water is lower than that of the cold, the water in the bottom rises and heat is transferred by natural convection. In the same way air in contact with a hot plate will be heated by natural convection currents, the air near the surface being hotter and of lower density than that some distance away. In both of these cases there is no external agency providing forced convection currents, and the transfer of heat occurs at a correspondingly lower rate since the natural convection currents move rather slowly.

For these processes which depend on buoyancy effects, the rate of heat transfer might be expected to follow a relation of the form:

$$Nu = f(Gr, Pr) \quad (\text{equation 9.57})$$

Measurements by SCHMIDT<sup>(37)</sup> of the upward air velocity near a 300 mm vertical plate show that the velocity rises rapidly to a maximum at a distance of about 2 mm from the plate and then falls rapidly. However, the temperature evens out at about 10 mm from the plate. Temperature measurements around horizontal cylinders have been made by RAY<sup>(38)</sup>.

Natural convection from horizontal surfaces to air, nitrogen, hydrogen, and carbon dioxide, and to liquids (including water, aniline, carbon tetrachloride, glycerol) has been studied by several workers, including DAVIS<sup>(39)</sup>, ACKERMANN<sup>(40)</sup>, FISHENDEN and SAUNDERS<sup>(18)</sup> and SAUNDERS<sup>(41)</sup>. Most of the results are for thin wires and tubes up to about 50 mm diameter; the temperature differences used are up to about 1100 deg K with gases and about 85 deg K with liquids. The general form of the results is shown in Figure 9.31, where  $\log Nu$  is plotted against  $\log (Pr Gr)$  for streamline conditions. The curve can be represented by a relation of the form:

$$Nu = C'(Gr Pr)^n \quad (9.101)$$

Numerical values of  $C'$  and  $n$ , determined experimentally for various geometries, are given in Table 9.5<sup>(42)</sup>. Values of coefficients may then be predicted using the equation:

$$\frac{hl}{k} = C' \left( \frac{\beta g \Delta T l^3 \rho^2 C_p \mu}{\mu^2 k} \right)^n \quad \text{or} \quad h = C' \left( \frac{\Delta T}{l} \right)^n k \left( \frac{\beta g \rho^2 C_p}{\mu k} \right)^n \quad (9.102)$$

Table 9.5. Values of  $C'$ ,  $C''$  and  $n$  for use in equations 9.102 and 9.105<sup>(42)</sup>

Geometry	$GrPr$	$C'$	$n$	$C''$ (SI units) (for air at 294 K)
<i>Vertical surfaces</i> ( $l$ = vertical dimension < 1 m)	$< 10^4$	1.36	0.20	
	$10^4 - 10^9$	0.59	0.25	1.37
	$> 10^9$	0.13	0.33	1.24
<i>Horizontal cylinders</i> ( $l$ = diameter < 0.2 m)	$10^{-5} - 10^{-3}$	0.71	0.04	
	$10^{-3} - 1.0$	1.09	0.10	
	$1.0 - 10^4$	1.09	0.20	
	$10^4 - 10^9$	0.53	0.25	1.32
	$> 10^9$	0.13	0.33	1.24
<i>Horizontal flat surfaces</i> (facing upwards)	$10^5 - 2 \times 10^7$	0.54	0.25	1.86
	$2 \times 10^7 - 3 \times 10^{10}$	0.14	0.33	
	(facing downwards)	$3 \times 10^5 - 3 \times 10^{10}$	0.27	0.25

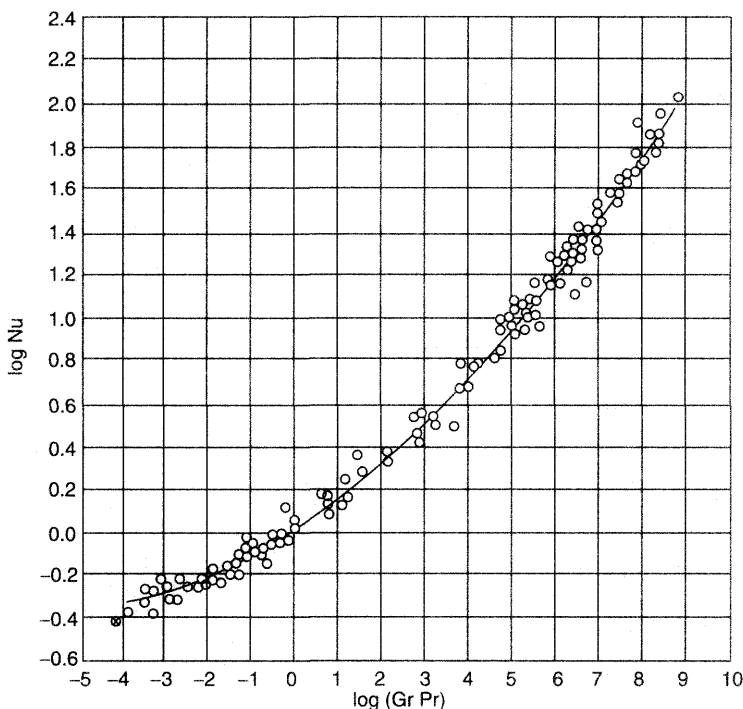


Figure 9.31. Natural convection from horizontal tubes

where the physical properties are at the mean of the surface and bulk temperatures and, for gases, the coefficient of cubical expansion  $\beta$  is taken as  $1/T$ , where  $T$  is the absolute temperature.

For vertical plates and cylinders, KATO *et al.*<sup>(43)</sup> have proposed the following equations for situations where  $1 < Pr < 40$ :

$$\text{For } Gr > 10^9: \quad Nu = 0.138Gr^{0.36}(Pr^{0.175} - 0.55) \quad (9.103)$$

$$\text{and for } Gr < 10^9: \quad Nu = 0.683Gr^{0.25}Pr^{0.25} \left( \frac{Pr}{0.861 + Pr} \right)^{0.25} \quad (9.104)$$

### Natural convection to air

Simplified dimensional equations have been derived for air, water and organic liquids by grouping the fluid properties into a single factor in a rearrangement of equation 9.102 to give:

$$h = C''(\Delta T)^n l^{3n-1} \quad (\text{W/m}^2\text{K}) \quad (9.105)$$

Values of  $C''$  (in SI units) are also given in Table 9.5 for air at 294 K. Typical values for water and organic liquids are 127 and 59 respectively.



**Example 9.11**

Estimate the heat transfer coefficient for natural convection from a horizontal pipe 0.15 m diameter, with a surface temperature of 400 K to air at 294 K

**Solution**

Over a wide range of temperature,  $k^4(\beta g \rho^2 C_p / \mu k) = 36.0$

For air at a mean temperature of  $0.5(400 + 294) = 347$  K,  $k = 0.0310$  W/m K (Table 6, Appendix A1)

$$\text{Thus: } \frac{\beta g \rho^2 C_p}{\mu k} = \frac{36.0}{0.0310^4} = 3.9 \times 10^7$$

From Equation 9.102:

$$\begin{aligned} GrPr &= 3.9 \times 10^7 (400 - 294) \times 0.15^3 \\ &= 1.39 \times 10^7 \end{aligned}$$

From Table 9.5:

$$n = 0.25 \quad \text{and} \quad C'' = 1.32$$

Thus, in Equation 9.104:

$$\begin{aligned} h &= 1.32(400 - 294)^{0.25} (0.15)^{(3 \times 0.25) - 1} \\ &= 1.32 \times 106^{0.25} \times 0.15^{-0.25} \\ &= \underline{\underline{6.81 \text{ W/m}^2 \text{ K}}} \end{aligned}$$

**Fluids between two surfaces**

For the transfer of heat from a hot surface across a thin layer of fluid to a parallel cold surface:

$$\frac{Q}{Q_k} = \frac{h\Delta T}{(k/x)\Delta T} = \frac{hx}{k} = Nu \quad (9.106)$$

where  $Q_k$  is the rate at which heat would be transferred by pure thermal conduction between the layers, a distance  $x$  apart, and  $Q$  is the actual rate.

For  $(Gr Pr) = 10^3$ , the heat transferred is approximately equal to that due to conduction alone, though for  $10^4 < Gr Pr < 10^6$ , the heat transferred is given by:

$$\frac{Q}{Q_k} = 0.15(Gr Pr)^{0.25} \quad (9.107)$$

which is noted in Figure 9.32. In this equation the characteristic dimension to be used for the Grashof group is  $x$ , the distance between the planes, and the heat transfer is independent of surface area, provided that the linear dimensions of the surfaces are large compared with  $x$ . For higher values of  $(Gr Pr)$ ,  $Q/Q_k$  is proportional to  $(Gr Pr)^{1/3}$ , showing that the heat transferred is not entirely by convection and is not influenced by the distance  $x$  between the surfaces.

A similar form of analysis has been given by KRAUSSOLD<sup>(44)</sup> for air between two concentric cylinders. It is important to note from this general analysis that a single layer of air will not be a good insulator because convection currents set in before it becomes 25 mm thick. The good insulating properties of porous materials are attributable to the

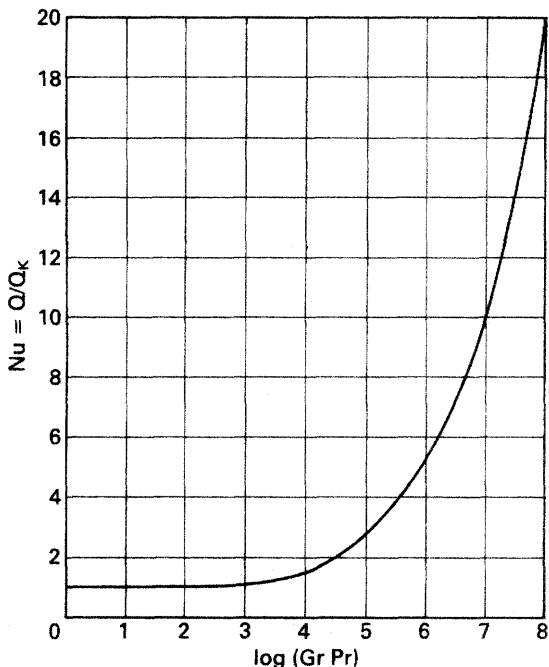


Figure 9.32. Natural convection between surfaces

fact that they offer a series of very thin layers of air in which convection currents are not present.

## 9.5. HEAT TRANSFER BY RADIATION

### 9.5.1. Introduction

It has been seen that heat transfer by conduction takes place through either a solid or a stationary fluid and heat transfer by convection takes place as a result of either forced or natural movement of a hot fluid. The third mechanism of heat transfer, radiation, can take place without either a solid or a fluid being present, that is through a vacuum, although many fluids are transparent to radiation, and it is generally assumed that the emission of thermal radiation is by "waves" of wavelengths in the range  $0.1-100 \mu\text{m}$  which travel in straight lines. This means that direct radiation transfer, which is the result of an interchange between various radiating bodies or surfaces, will take place only if a straight line can be drawn between the two surfaces; a situation which is often expressed in terms of one surface "seeing" another. Having said this, it should be noted that opaque surfaces sometimes cast shadows which inhibit radiation exchange and that indirect transfer by radiation can take place as a result of partial reflection from other surfaces. Although all bodies at temperatures in excess of absolute zero radiate energy in all directions, radiation is of especial importance from bodies at high temperatures such as those encountered in furnaces, boilers and high temperature reactors, where in addition to radiation from hot surfaces, radiation from reacting flame gases may also be a consideration.

**Solution**

If  $T$  K is the temperature of the liquid at time  $t$  s, then a heat balance on the vessel gives:

$$(1000 \times 4000) \frac{dT}{dt} = (600 \times 0.5)(393 - T) - (10 \times 6)(T - 293)$$

or: 
$$4,000,000 \frac{dT}{dt} = 135,480 - 360T$$

and: 
$$11,111 \frac{dT}{dt} = 376.3 - T.$$

The equilibrium temperature occurs when  $dT/dt = 0$ ,

that is when: 
$$\underline{\underline{T = 376.3 \text{ K.}}}$$

In heating from 293 to 353 K, the time taken is:

$$\begin{aligned} t &= 11,111 \int_{293}^{353} \frac{dT}{(376.3 - T)} \\ &= 11,111 \ln \left( \frac{83.3}{23.3} \right) \\ &= \underline{\underline{14,155 \text{ s (or 3.93 h)}}}. \end{aligned}$$

The steam is turned off for 7200 s and during this time a heat balance gives:

$$\begin{aligned} (1000 \times 4000) \frac{dT}{dt} &= -(10 \times 6)(T - 293) \\ 66,700 \frac{dT}{dt} &= 293 - T \end{aligned}$$

The change in temperature is then given by:

$$\begin{aligned} \int_{353}^T \frac{dT}{(293 - T)} &= \frac{1}{66,700} \int_0^{7200} dt \\ \ln \frac{-60}{293 - T} &= \frac{7200}{66,700} = 0.108 \end{aligned}$$

and: 
$$T = 346.9 \text{ K.}$$

The time taken to reheat the liquid to 353 K is then given by:

$$\begin{aligned} t &= 11,111 \int_{346.9}^{353} \frac{dT}{(376.3 - T)} \\ &= 11,111 \ln \left( \frac{29.4}{23.3} \right) \\ &= \underline{\underline{2584 \text{ s (0.72h)}}}. \end{aligned}$$

## 9.9. SHELL AND TUBE HEAT EXCHANGERS

### 9.9.1. General description

Since shell and tube heat exchangers can be constructed with a very large heat transfer surface in a relatively small volume, fabricated from alloy steels to resist corrosion and

be used for heating, cooling and for condensing a very wide range of fluids, they are the most widely used form of heat transfer equipment. Figures 9.62-9.64 show various forms of construction and a tube bundle is shown in Figure 9.65. The simplest type of unit, shown in Figure 9.62, has fixed tube plates at each end into which the tubes are expanded. The tubes are connected so that the internal fluid makes several passes up and

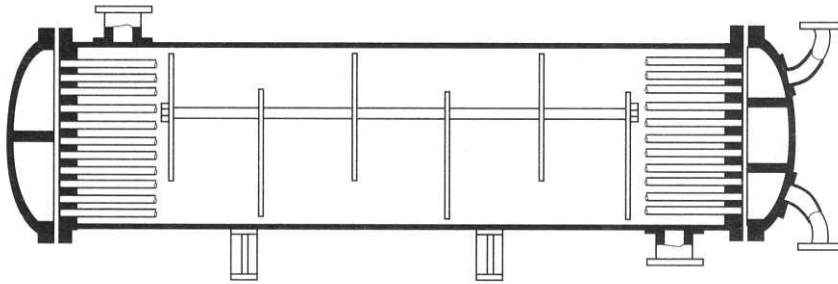


Figure 9.62. Heat exchanger with fixed tube plates (four tube, one shell-pass)

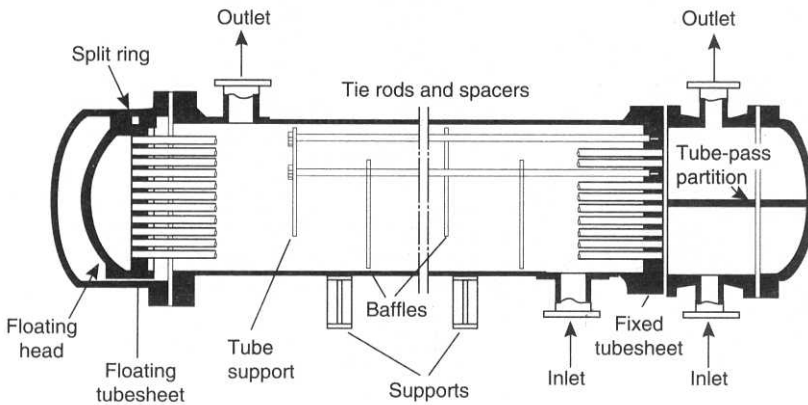


Figure 9.63. Heat exchanger with floating head (two tube-pass, one shell-pass)

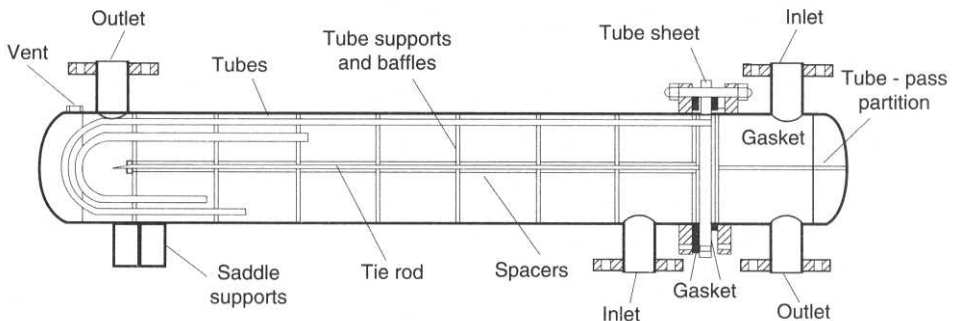


Figure 9.64. Heat exchanger with hairpin tubes

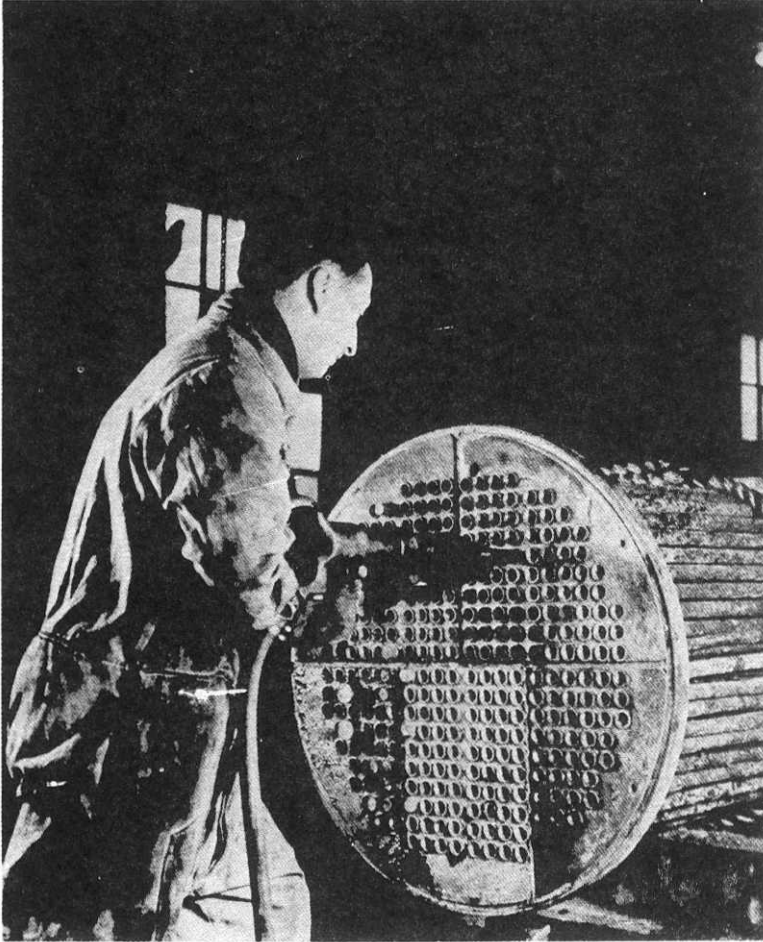


Figure 9.65. Expanding the ends of the tubes into the tube plate of a heat exchanger bundle

down the exchanger thus enabling a high velocity of flow to be obtained for a given heat transfer area and throughput of fluid. The fluid flowing in the shell is made to flow first in one sense and then in the opposite sense across the tube bundle by fitting a series of baffles along the length. These baffles are frequently of the segmental form with about 25 per cent cut away, as shown in Figure 9.29 to provide the free space to increase the velocity of flow across the tubes, thus giving higher rates of heat transfer. One problem with this type of construction is that the tube bundle cannot be removed for cleaning and no provision is made to allow for differential expansion between the tubes and the shell, although an expansion joint may be fitted to the shell.

In order to allow for the removal of the tube bundle and for considerable expansion of the tubes, a floating head exchanger is used, as shown in Figure 9.63. In this arrangement one tube plate is fixed as before, but the second is bolted to a floating head cover so that the tube bundle can move relative to the shell. This floating tube sheet is clamped

between the floating head and a split backing flange in such a way that it is relatively easy to break the flanges at both ends and to draw out the tube bundle. It may be noted that the shell cover at the floating head end is larger than that at the other end. This enables the tubes to be placed as near as possible to the edge of the fixed tube plate, leaving very little unused space between the outer ring of tubes and the shell.

Another arrangement which provides for expansion involves the use of hairpin tubes, as shown in Figure 9.64. This design is very commonly used for the reboilers on large fractionating columns where steam is condensed inside the tubes.

In these designs there is one pass for the fluid on the shell-side and a number of passes on the tube-side. It is often an advantage to have two or more shell-side passes, although this considerably increases the difficulty of construction and, very often therefore, several smaller exchangers are connected together to obtain the same effect.

The essential requirements in the design of a heat exchanger are, firstly, the provision of a unit which is reliable and has the desired capacity, and secondly, the need to provide an exchanger at minimum overall cost. In general, this involves using standard components and fittings and making the design as simple as possible. In most cases, it is necessary to balance the capital cost in terms of the depreciation against the operating cost. Thus in a condenser, for example, a high heat transfer coefficient is obtained and hence a small exchanger is required if a higher water velocity is used in the tubes. Against this, the cost of pumping increases rapidly with increase in velocity and an economic balance must be struck. A typical graph showing the operating costs, depreciation and the total cost plotted as a function of the water velocity in the tubes is shown in Figure 9.66.

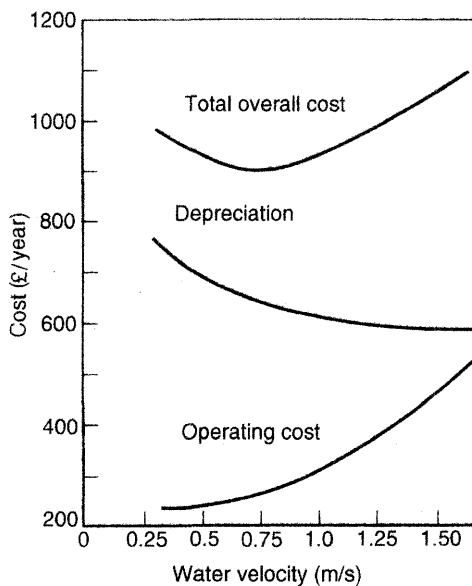


Figure 9.66. Effect of water velocity on annual operating cost of condenser

### 9.9.2. Basic components

The various components which make up a shell and tube heat exchanger are shown in Figures 9.63 and 9.64 and these are now considered. Many different mechanical arrangements are used and it is convenient to use a basis for classification. The standard published by the Tubular Exchanger Manufacturer's Association (TEMA<sup>(97)</sup>) is outlined here. It should be added that noting that SAUNDERS<sup>(98)</sup> has presented a detailed discussion of design codes and problems in fabrication.

Of the various *shell types* shown in Figure 9.67, the simplest, with entry and exit nozzles at opposite ends of a single pass exchanger, is the TEMA E-type on which most design methods are based, although these may be adapted for other shell types by allowing for the resulting velocity changes. The TEMA F-type has a longitudinal baffle giving two shell passes and this provides an alternative arrangement to the use of two shells required in order to cope with a close temperature approach or low shell-side flowrates. The pressure drop in two shells is some eight times greater than that encountered in the E-type design although any potential leakage between the longitudinal baffle and the shell in the F-type design may restrict the range of application. The so-called "split-flow" type of unit with a longitudinal baffle is classified as the TEMA G-type whose performance is superior although the pressure drop is similar to the E-type. This design is used mainly for reboilers and only occasionally for systems where there is no change of phase. The so-called "divided-flow" type, the TEMA J-type, has one inlet and two outlet nozzles and, with a pressure drop some one-eighth of the E-type, finds application in gas coolers and condensers operating at low pressures. The TEMA X-type shell has no cross baffles and hence the shell-side fluid is in pure counterflow giving extremely low pressure drops and again, this type of design is used for gas cooling and condensation at low pressures.

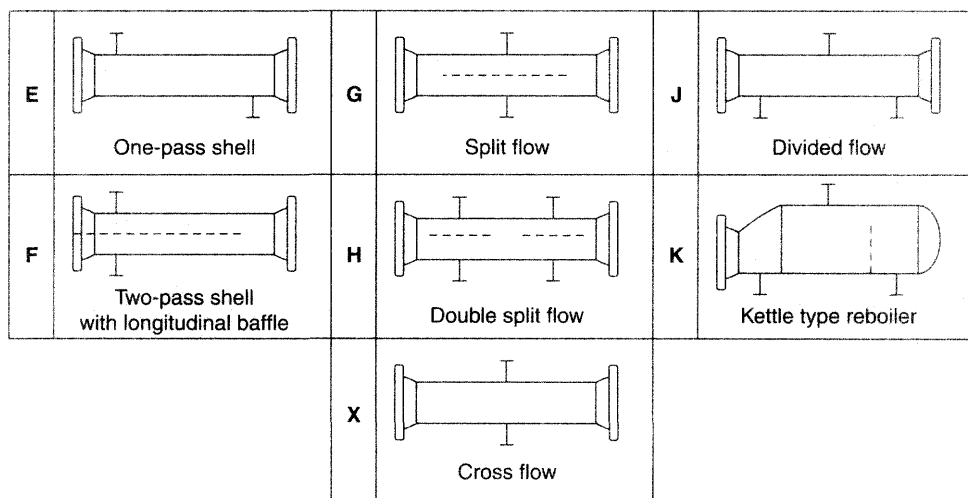


Figure 9.67. TEMA shell types

The *shell* of a heat exchanger is commonly made of carbon steel and standard pipes are used for the smaller sizes and rolled welded plate for the larger sizes (say 0.4–1.0 m).

The thickness of the shell may be calculated from the formula for thin-walled cylinders and a minimum thickness of 9.5 mm is used for shells over 0.33 m o.d. and 11.1 mm for shells over 0.9 m o.d. Unless the shell is designed to operate at very high pressures, the calculated wall thickness is usually less than these values although a corrosion allowance of 3.2 mm is commonly added to all carbon steel parts and thickness is determined more by rigidity requirements than simply internal pressure. The minimum shell thickness for various materials is given in BS3274<sup>(99)</sup>. A shell diameter should be such as to give as close a fit to the tube bundle as practical in order to reduce bypassing round the outside of the bundle. Typical values for the clearance between the outer tubes in the bundle and the inside diameter of the shell are given in Figure 9.68 for various types of exchanger.

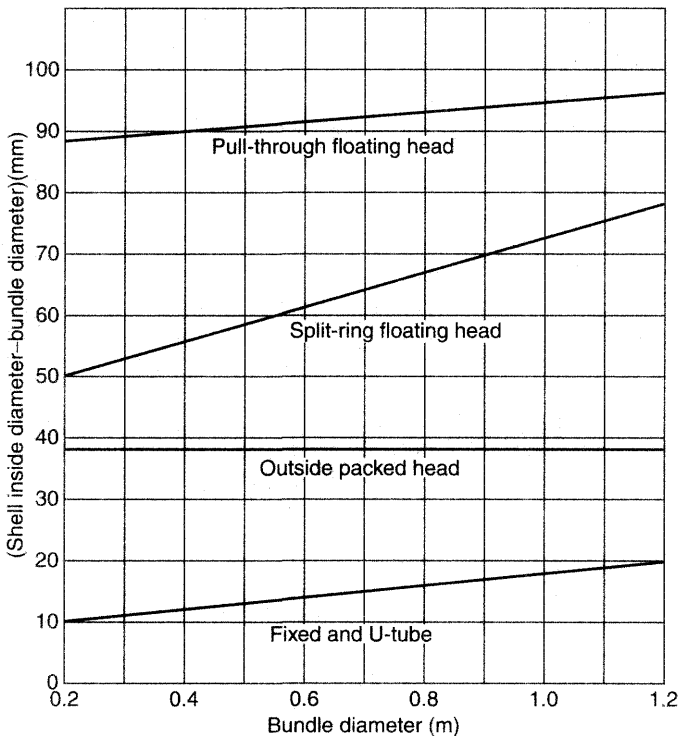


Figure 9.68. Shell-bundle clearance

The detailed design of the *tube bundle* must take into account both shell-side and tube-side pressures since these will both affect any potential leakage between the tube bundle and the shell which cannot be tolerated where high purity or uncontaminated materials are required. In general, tube bundles make use of a fixed tubesheet, a floating-head or U-tubes which are shown in Figures 9.62, 9.63 and 9.64 respectively. It may be noted here that the thickness of the fixed tubesheet may be obtained from a relationship of the form:

$$d_t = d_G \sqrt{0.25P/f} \quad (9.210)$$



where  $d_G$  is the diameter of the gasket (m),  $P$  the design pressure ( $\text{MN/m}^2$ ),  $f$  the allowable working stress ( $\text{MN/m}^2$ ) and  $d_t$  the thickness of the sheet measured at the bottom of the partition plate grooves. The thickness of the floating head tubesheet is very often calculated as  $\sqrt{2d_t}$ .

In selecting a *tube diameter*, it may be noted that smaller tubes give a larger heat transfer area for a given shell, although 19 mm o.d. tubes are normally the minimum size used in order to permit adequate cleaning. Although smaller diameters lead to shorter tubes, more holes have to be drilled in the tubesheet which adds to the cost of construction and increases the likelihood of tube vibration. Heat exchanger tubes are usually in the range 16 mm ( $\frac{5}{8}$  in) to 50 mm (2 in) O.D.; the smaller diameter usually being preferred as these give more compact and therefore cheaper units. Against this, larger tubes are easier to clean especially by mechanical methods and are therefore widely used for heavily fouling fluids. The tube thickness or gauge must be such as to withstand the internal pressure and also to provide an adequate corrosion allowance. Details of steel tubes used in heat exchangers are given in BS3606<sup>(100)</sup> and summarised in Table 9.12, and standards for other materials are given in BS3274<sup>(99)</sup>.

Table 9.12. Standard dimensions of steel tubes

Outside diameter $d_o$		Wall thickness		Cross sectional area for flow		Surface area per unit length	
(mm)	(in)	(mm)	(in)	(m <sup>2</sup> )	(ft <sup>2</sup> )	(m <sup>2</sup> /m)	(ft <sup>2</sup> /ft)
16	0.630	1.2	0.047	0.000145	0.00156	0.0503	0.165
		1.6	0.063	0.000129	0.00139		
		2.0	0.079	0.000113	0.00122		
20	0.787	1.6	0.063	0.000222	0.00239	0.0628	0.206
		2.0	0.079	0.000201	0.00216		
		2.6	0.102	0.000172	0.00185		
25	0.984	1.6	0.063	0.000373	0.00402	0.0785	0.258
		2.0	0.079	0.000346	0.00373		
		2.6	0.102	0.000308	0.00331		
		3.2	0.126	0.000272	0.00293		
30	1.181	1.6	0.063	0.000564	0.00607	0.0942	0.309
		2.0	0.079	0.000531	0.00572		
		2.6	0.102	0.000483	0.00512		
		3.2	0.126	0.000437	0.00470		
38	1.496	2.0	0.079	0.000908	0.00977	0.1194	0.392
		2.6	0.102	0.000845	0.00910		
		3.2	0.126	0.000784	0.00844		
50	1.969	2.0	0.079	0.001662	0.01789	0.1571	0.515
		2.6	0.102	0.001576	0.01697		
		3.2	0.126	0.001493	0.01607		

In general, the larger the *tube length*, the lower is the cost of an exchanger for a given surface area due to the smaller shell diameter, the thinner tube sheets and flanges and the smaller number of holes to be drilled, and the reduced complexity. Preferred tube lengths are 1.83 m (6 ft), 2.44 m (8 ft), 3.88 m (12 ft) and 4.88 m (16 ft); larger sizes are used where the total tube-side flow is low and fewer, longer tubes are required in order to obtain a required velocity. With the number of tubes per tube-side pass fixed in order to obtain a required velocity, the total length of tubes per tube-side pass is determined by the heat transfer surface required. It is then necessary to fit the tubes into a suitable shell

to give the desired shell-side velocity. It may be noted that with long tube lengths and relatively few tubes in a shell, it may be difficult to arrange sufficient baffles for adequate support of the tubes. For good all-round performance, the ratio of tube length to shell diameter is usually in the range 5–10.

*Tube layout and pitch*, considered in Section 9.4.4 and shown in Figure 9.69, make use of equilateral triangular, square and staggered square arrays. The triangular layout gives a robust tube sheet although, because the vertical and horizontal distances between adjacent tubes is generally greater in a square layout compared with the equivalent triangular pitch design, the square array simplifies maintenance and particularly cleaning on the shell-side. Good practice requires a minimum pitch of 1.25 times the tube diameter and/or a minimum web thickness between tubes of about 3.2 mm to ensure adequate strength for tube rolling. In general, the smallest pitch in triangular 30° layout is used for clean fluids in both laminar and turbulent flow and a 90° or 45° layout with a 6.4 mm clearance where mechanical cleaning is required. The bundle diameter,  $d_b$ , may be estimated from the following empirical equation which is based on standard tube layouts:

$$\text{Number of tubes, } N_t = a(d_b/d_o)^b \quad (9.211)$$

where the values of the constants  $a$  and  $b$  are given in Table 9.13. Tables giving the number of tubes that can be accommodated in standard shells using various tube sizes, pitches and numbers of passes for different exchanger types are given, for example, in KERN<sup>(28)</sup> and LUDWIG<sup>(101)</sup>.

Table 9.13. Constants for use with equation 9.211.

Number of passes		1	2	4	6	8
Triangular pitch*	$a$	0.319	0.249	0.175	0.0743	0.0365
	$b$	2.142	2.207	2.285	2.499	2.675
Square pitch*	$a$	0.215	0.156	0.158	0.0402	0.0331
	$b$	2.207	2.291	2.263	1.617	2.643

\*Pitch =  $1.25d_o$

Various *baffle designs* are shown in Figure 9.70. The cross-baffle is designed to direct the flow of the shell-side fluid across the tube bundle and to support the tubes against sagging and possible vibration, and the most common type is the segmental baffle which provides a baffle window. The ratio, baffle spacing/baffle cut, is very important in maximising the ratio of heat transfer rate to pressure drop. Where very low pressure drops are required, double segmental or “disc and doughnut” baffles are used to reduce the pressure drop by some 60 per cent. Triple segmental baffles and designs in which all the tubes are supported by all the baffles provide for low pressure drops and minimum tube vibration.

With regard to *baffle spacing*, TEMA<sup>(97)</sup> recommends that segmental baffles should not be spaced closer than 20 per cent of the shell inside diameter or 50 mm whichever is the greater and that the maximum spacing should be such that the unsupported tube lengths, given in Table 9.14, are not exceeded. It may be noted that the majority of failures due to vibration occur when the unsupported tube length is in excess of 80 per cent of the TEMA maximum; the best solution is to avoid having tubes in the baffle window.

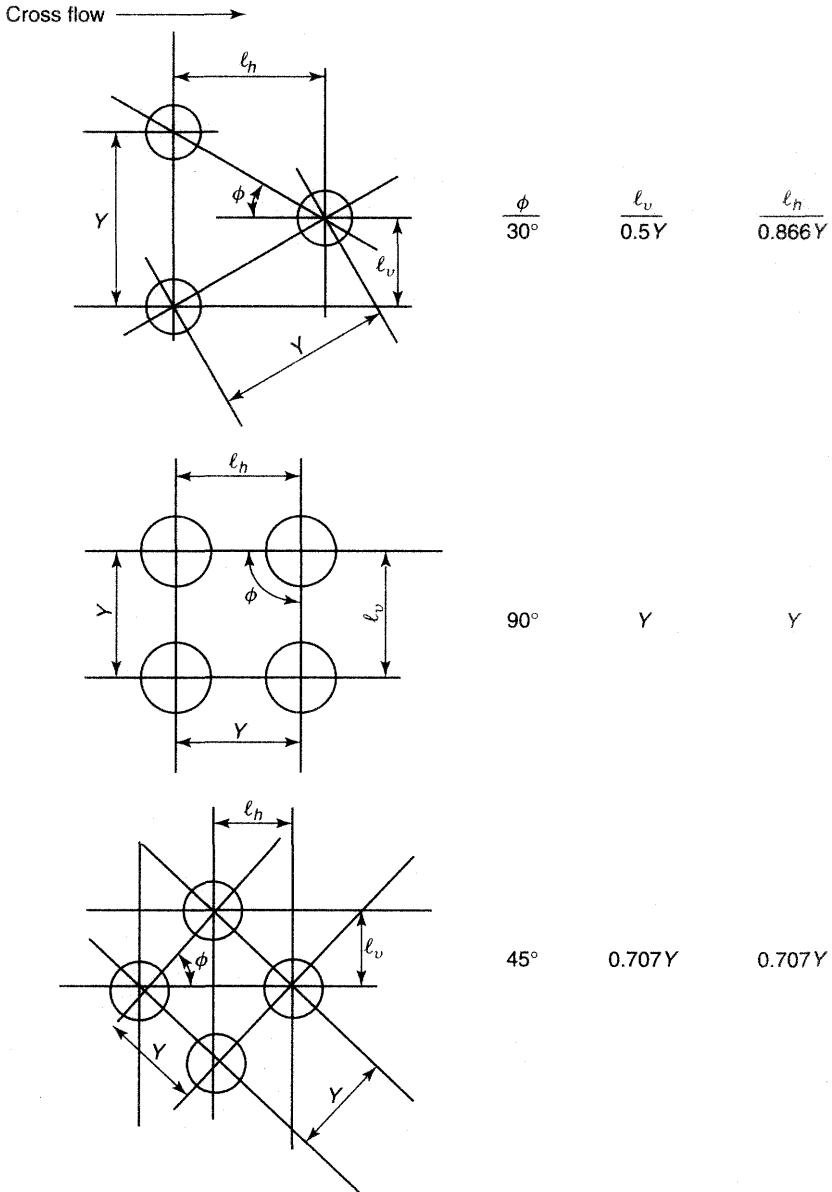


Figure 9.69. Examples of tube arrays<sup>(97)</sup>

**9.9.3. Mean temperature difference in multipass exchangers**

In an exchanger with one shell pass and several tube-side passes, the fluids in the tubes and shell will flow co-currently in some of the passes and countercurrently in the others. For given inlet and outlet temperatures, the mean temperature difference for countercurrent flow is greater than that for co-current or parallel flow, and there is no easy way of

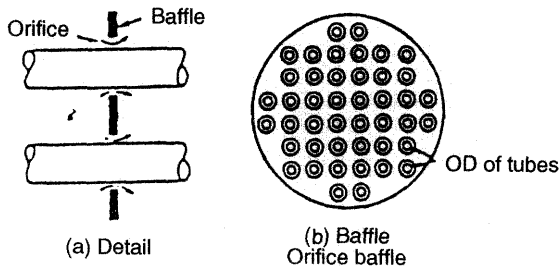
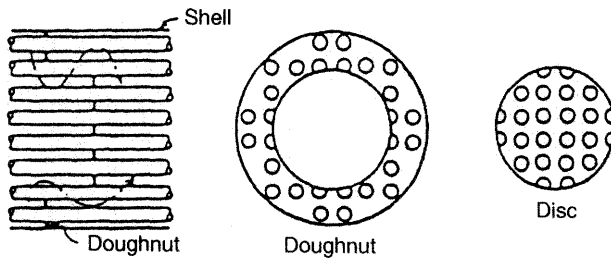
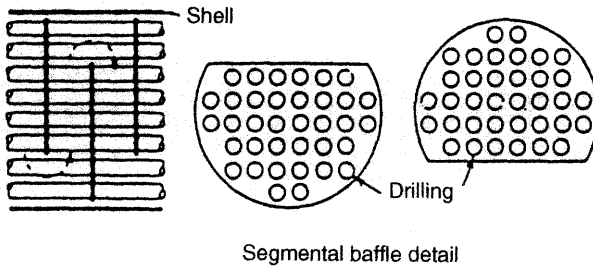
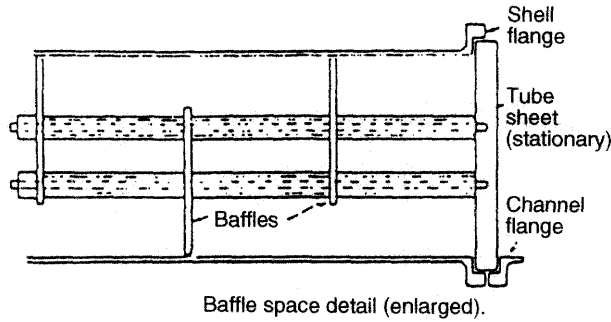


Figure 9.70. Baffle designs

finding the true temperature difference for the unit. The problem has been investigated by UNDERWOOD<sup>(102)</sup> and by BOWMAN *et al.*<sup>(103)</sup> who have presented graphical methods for calculating the true mean temperature difference in terms of the value of  $\theta_m$  which would be obtained for countercurrent flow, and a correction factor  $F$ . Provided the following conditions are maintained or assumed,  $F$  can be found from the curves shown in Figures 9.71–9.74.

Table 9.14. Maximum unsupported spans for tubes

Approximate tube OD. (mm)	Maximum unsupported span (mm)	
	Materials group A	Materials group B
19	1520	1321
25	1880	1626
32	2240	1930
38	2540	2210
50	3175	2794

Materials

Group A: Carbon and high alloy steel, low alloy steel, nickel-copper, nickel, nickel-chromium-iron.

Group B: Aluminium and aluminium alloys, copper and copper alloys, titanium and zirconium.

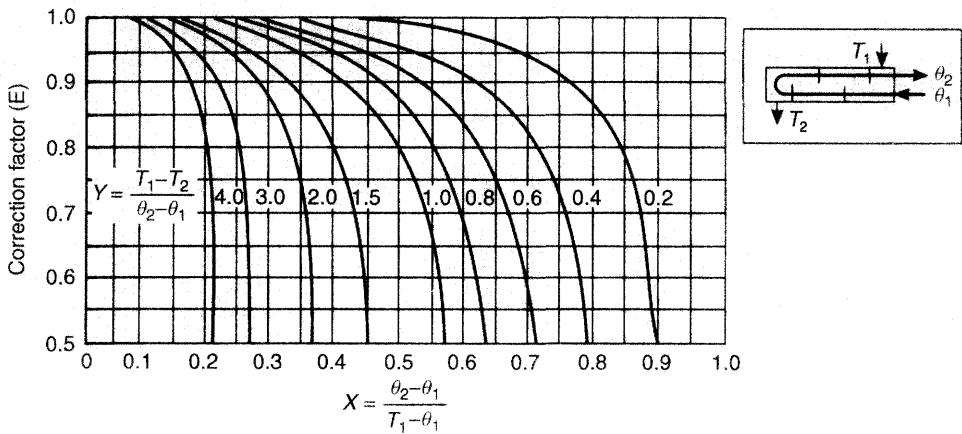


Figure 9.71. Correction for logarithmic mean temperature difference for single shell pass exchanger

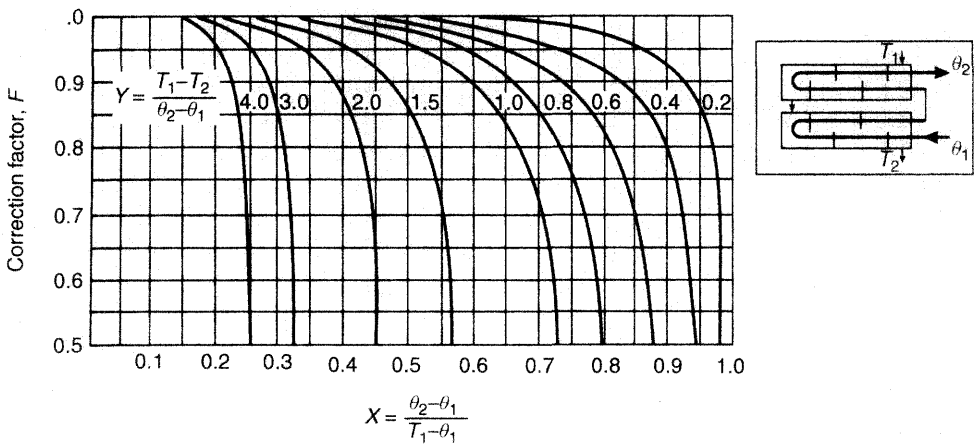


Figure 9.72. Correction for logarithmic mean temperature difference for double shell pass exchanger

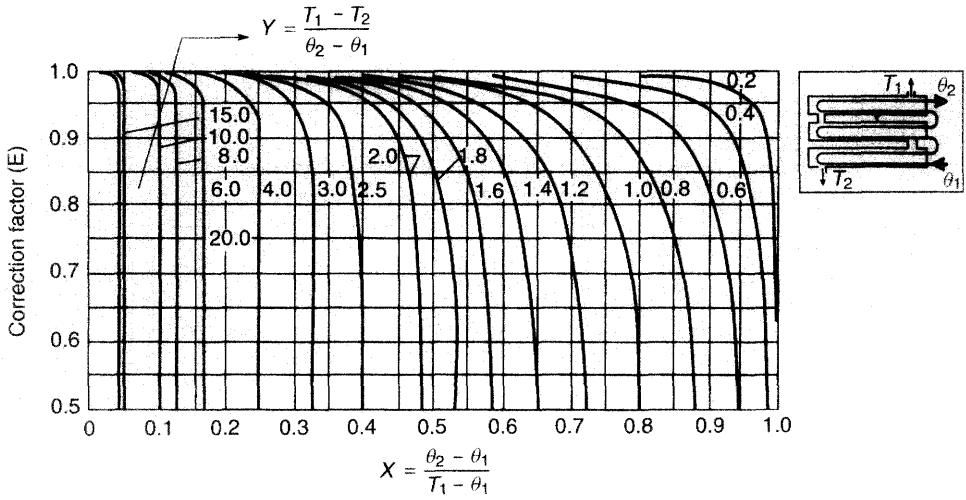


Figure 9.73. Correction for logarithmic mean temperature difference for three shell pass exchanger

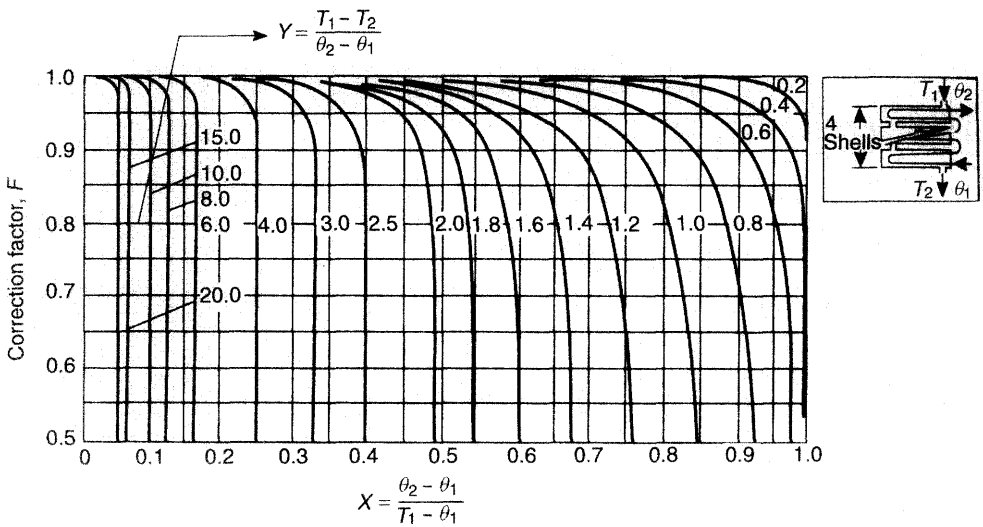


Figure 9.74. Correction for logarithmic mean temperature difference for four shell pass exchanger

- (a) The shell fluid temperature is uniform over the cross-section considered as constituting a pass.
- (b) There is equal heat transfer surface in each pass.
- (c) The overall heat transfer coefficient  $U$  is constant throughout the exchanger.
- (d) The heat capacities of the two fluids are constant over the temperature range.
- (e) There is no change in phase of either fluid.
- (f) Heat losses from the unit are negligible.

Then:

$$Q = UAF\theta_m \tag{9.212}$$

$F$  is expressed as a function of two parameters:

$$X = \frac{\theta_2 - \theta_1}{T_1 - \theta_1} \quad \text{and} \quad Y = \frac{T_1 - T_2}{\theta_2 - \theta_1} \tag{9.213}$$

If a one shell-side system is used Figure 9.71 applies, for two shell-side passes Figure 9.72, for three shell-side passes Figure 9.73, and for four shell-side passes Figure 9.74. For the case of a single shell-side pass and two tube-side passes illustrated in Figures 9.75a and 9.75b the temperature profile is as shown. Because one of the passes constitutes a parallel flow arrangement, the exit temperature of the cold fluid  $\theta_2$  cannot closely approach the hot fluid temperature  $T_1$ . This is true for the conditions shown in Figures 9.75a and 9.75b and UNDERWOOD<sup>(102)</sup> has shown that  $F$  is the same in both cases.

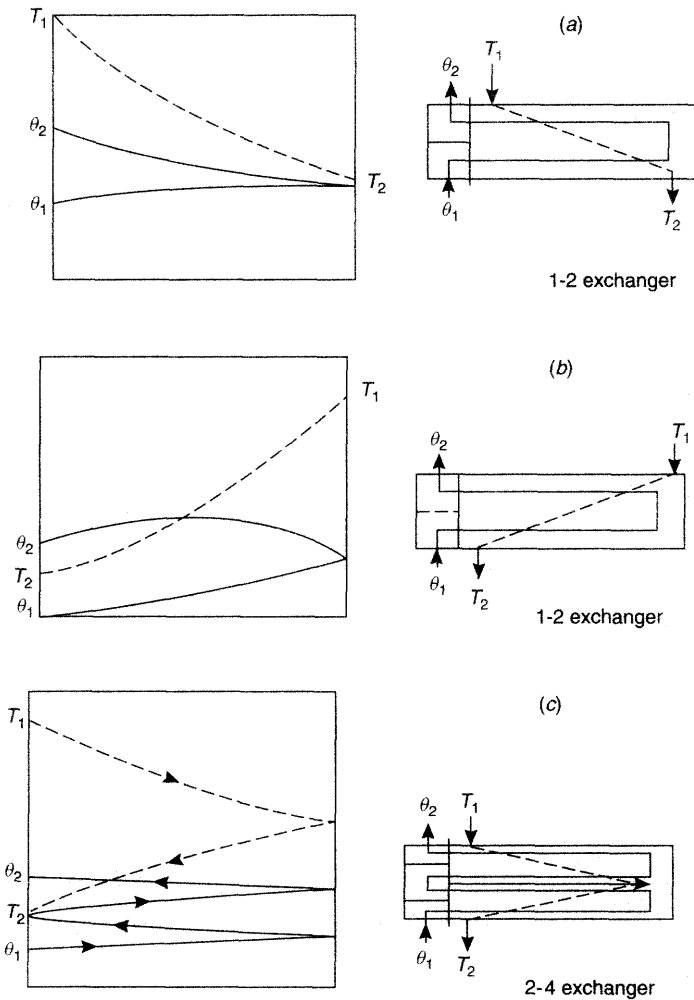


Figure 9.75. Temperature profiles in single and double shell pass exchangers

If, for example, an exchanger is required to operate over the following temperatures:

$$T_1 = 455 \text{ K}, T_2 = 372 \text{ K}$$

$$\theta_1 = 283 \text{ K}, \theta_2 = 388 \text{ K}$$

Then:

$$X = \frac{\theta_2 - \theta_1}{T_1 - \theta_1} = \frac{388 - 283}{455 - 283} = 0.6$$

and:

$$Y = \frac{T_1 - T_2}{\theta_2 - \theta_1} = \frac{455 - 372}{388 - 283} = 0.8$$

For a single shell pass arrangement, from Figure 9.71  $F$  is 0.65 and, for a double shell pass arrangement, from Figure 9.72  $F$  is 0.95. On this basis, a two shell-pass design would be used.

In order to obtain maximum heat recovery from the hot fluid,  $\theta_2$  should be as high as possible. The difference ( $T_2 - \theta_2$ ) is known as the *approach temperature* and, if  $\theta_2 > T_2$ , then a *temperature cross* is said to occur; a situation where the value of  $F$  decreases very rapidly when there is but a single pass on the shell-side. This implies that, in parts of the heat exchanger, heat is actually being transferred in the wrong direction. This may be illustrated by taking as an example the following data where equal ranges of temperature are considered:

Case	$T_1$	$T_2$	$\theta_1$	$\theta_2$	Approach ( $T_2 - \theta_2$ )	$X$	$Y$	$F$
1	613	513	363	463	50	0.4	1	0.92
2	573	473	373	473	0	0.5	1	0.80
3	543	443	363	463	cross of 20	0.55	1	0.66

If a temperature cross occurs with a single pass on the shell-side, a unit with two shell passes should be used. It is seen from Figure 9.75b that there may be some point where the temperature of the cold fluid is greater than  $\theta_2$  so that beyond this point the stream will be cooled rather than heated. This situation may be avoided by increasing the number of shell passes. The general form of the temperature profile for a two shell-side unit is as shown in Figure 9.75c. Longitudinal shell-side baffles are rather difficult to fit and there is a serious chance of leakage. For this reason, the use of two exchangers arranged in series, one below the other, is to be preferred. It is even more important to employ separate exchangers when three passes on the shell-side are required. On the very largest installations it may be necessary to link up a number of exchangers in parallel arranged as, say, sets of three in series as shown in Figure 9.76. This arrangement is preferable for any very large unit which would be unwieldy as a single system. When the total surface area is much greater than 250 m<sup>2</sup>, consideration should be given to using multiple smaller units even though the initial cost may be higher.

In many processing operations there may be a large number of process streams, some of which need to be heated and others cooled. An overall heat balance will indicate whether, in total, there is a net surplus or deficit of heat available. It is of great economic importance to achieve the most effective match of the hot and cold streams in the heat exchanger network so as to reduce to a minimum both the heating and cooling duties placed on the works utilities, such as supplies of steam and cooling water. This necessitates making the best use of the temperature driving forces. In considering the overall requirements there will be some point where the temperature difference between the hot and cold streams is a



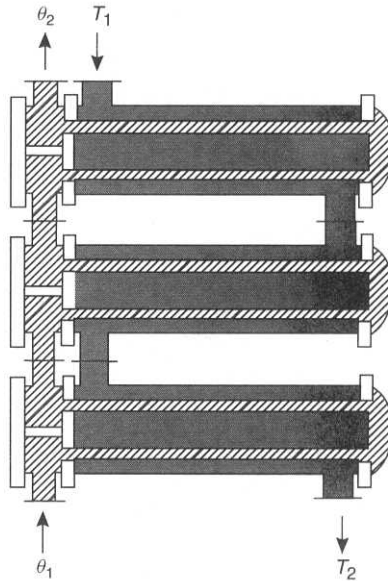


Figure 9.76. Set of three heat exchangers in series

minimum and this is referred to as the *pinch*. The lower the temperature difference at the pinch point, the lower will be the demand on the utilities, although it must be remembered that a greater area, (and hence cost) will be involved and an economic balance must therefore be struck. Heat exchanger networks are discussed in Volume 6 and, in detail, in the User Guide published by the Institution of Chemical Engineers<sup>(104)</sup>. Subsequently, LINNHOFF<sup>(105)</sup> has given an overview of the industrial application of pinch analysis to the design of networks in order to reduce both capital costs and energy requirements.

### Example 9.26

Using the data of Example 9.1, calculate the surface area required to effect the given duty using a multipass heat exchanger in which the cold water makes two passes through the tubes and the hot water makes a single pass through the shell.

### Solution

As in Example 9.1, the heat load = 1672 kW

With reference to Figure 9.71:  $T_1 = 360$  K,  $T_2 = 340$  K

and hence:

$$X = \frac{\theta_2 - \theta_1}{T_1 - \theta_1} = \frac{316 - 300}{360 - 300} = 0.267$$

and:

$$Y = \frac{T_1 - T_2}{\theta_2 - \theta_1} = \frac{360 - 340}{316 - 300} = 1.25$$

from Figure 9.58:

$$F = 0.97$$

and hence:

$$F\theta_m = (41.9 \times 0.97) = 40.6 \text{ K}$$

The heat transfer area is then:

$$A = \frac{1672}{2 \times 40.6} = \underline{\underline{20.6 \text{ m}^2}}$$

## 9.9.4. Film coefficients

### *Practical values*

In any item of heat transfer equipment, the required area of heat transfer surface for a given load is determined by the overall temperature difference  $\theta_m$ , the overall heat transfer coefficient  $U$  and the correction factor  $F$  in equation 9.212. The determination of the individual film coefficients which determine the value of  $U$  has proved difficult even for simple cases, and it is quite common for equipment to be designed on the basis of practical values of  $U$  rather than from a series of film coefficients. For the important case of the transfer of heat from one fluid to another across a metal surface, two methods have been developed for measuring film coefficients. The first requires a knowledge of the temperature difference across each film and therefore involves measuring the temperatures of both fluids and the surface of separation. With a concentric tube system, it is very difficult to insert a thermocouple into the thin tube and to prevent the thermocouple connections from interfering with the flow of the fluid. Nevertheless, this method is commonly adopted, particularly when electrical heating is used. It must be noted that when the heat flux is very high, as with boiling liquids, there will be an appreciable temperature drop across the tube wall and the position of the thermocouple is then important. For this reason, working with stainless steel, which has a relatively low thermal conductivity, is difficult.

The second method uses a technique proposed by WILSON<sup>(106)</sup>. If steam is condensing on the outside of a horizontal tube through which water is passed at various velocities, then the overall and film transfer coefficients are related by:

$$\frac{1}{U} = \frac{1}{h_o} + \frac{x_w}{k_w} + R_i + \frac{1}{h_i} \quad (\text{from equation 9.201})$$

provided that the transfer area on each side of the tube is approximately the same.

For conditions of turbulent flow the transfer coefficient for the water side,  $h_i = \epsilon u^{0.8}$ ,  $R_i$  the scale resistance is constant, and  $h_o$  the coefficient for the condensate film is almost independent of the water velocity. Thus, equation 9.201 reduces to:

$$\frac{1}{U} = (\text{constant}) + \frac{1}{\epsilon u^{0.8}}$$

If  $1/U$  is plotted against  $1/u^{0.8}$  a straight line, known as a Wilson plot, is obtained with a slope of  $1/\epsilon$  and an intercept equal to the value of the constant. For a clean tube  $R_i$  should be nil, and hence  $h_o$  can be found from the value of the intercept, as  $x_w/k_w$  will generally be small for a metal tube.  $h_i$  may also be obtained at a given velocity from the difference between  $1/U$  at that velocity and the intercept.

This technique has been applied by RHODES and YOUNGER<sup>(107)</sup> to obtain the values of  $h_o$  for condensation of a number of organic vapours, by PRATT<sup>(92)</sup> to obtain the inside coefficient for coiled tubes, and by COULSON and MEHTA<sup>(108)</sup> to obtain the coefficient for

Table 9.15. Thermal resistance of heat exchanger tubes

Gauge (BWG)	Thickness (mm)	Copper	Values of $x_w/k_w$ ( $m^2K/kW$ )			Aluminium
			Steel	Stainless steel	Admiralty metal	
18	1.24	0.0031	0.019	0.083	0.011	0.0054
16	1.65	0.0042	0.025	0.109	0.015	0.0074
14	2.10	0.0055	0.032	0.141	0.019	0.0093
12	2.77	0.0072	0.042	0.176	0.046	0.0123
Values of $x_w/k_w$ ( $ft^2h^\circ F/Btu$ )						
18	0.049	0.000018	0.00011	0.00047	0.000065	0.000031
16	0.065	0.000024	0.00014	0.00062	0.000086	0.000042
14	0.083	0.000031	0.00018	0.0008	0.00011	0.000053
12	0.109	0.000041	0.00024	0.001	0.00026	0.000070

Table 9.16. Thermal resistances of scale deposits from various fluids

	$m^2K/kW$	$ft^2h^\circ F/Btu$		$m^2K/kW$	$ft^2h^\circ F/Btu$
<i>Water*</i>			<i>Steam</i>		
distilled	0.09	0.0005	good quality, oil-free	0.052	0.0003
sea	0.09	0.0005			
clear river	0.21	0.0012	poor quality, oil-free	0.09	0.0005
untreated cooling tower	0.58	0.0033	exhaust from reciprocating engines	0.18	0.001
treated cooling tower	0.26	0.0015			
treated boiler feed	0.26	0.0015			
hard well	0.58	0.0033	<i>Liquids</i>		
			treated brine	0.27	0.0015
<i>Gases</i>			organics	0.18	0.001
air	0.25–0.50	0.0015–0.003	fuel oils	1.0	0.006
solvent vapours	0.14	0.0008	tars	2.0	0.01

\*For a velocity of 1 m/s ( $\approx 3$  ft/s) and temperatures of less than 320 K ( $122^\circ F$ )

an annulus. If the results are repeated over a period of time, the increase in the value of  $R_i$  can also be obtained by this method.

Typical values of thermal resistances and individual and overall heat transfer coefficients are given in Tables 9.15–9.18.

### Correlated data

Heat transfer data for turbulent flow *inside* conduits of uniform cross-section are usually correlated by a form of equation 9.66:

$$Nu = CRe^{0.8}Pr^{0.33}(\mu/\mu_s)^{0.14} \quad (9.214)$$

where, based on the work of SIEDER and TATE<sup>(17)</sup>, the index for the viscosity correction term is usually 0.14 although higher values have been reported. Using values of  $C$  of 0.021 for gases, 0.023 for non-viscous liquids and 0.027 for viscous liquids, equation 9.214 is sufficiently accurate for design purposes, and any errors are far outweighed by uncertainties in predicting shell-side coefficients. Rather more accurate tube-side data

Table 9.17. Approximate overall heat transfer coefficients  $U$  for shell and tube equipment

Hot side	Cold side	Overall $U$	
		W/m <sup>2</sup> K	Btu/h ft <sup>2</sup> °F
<i>Condensers</i>			
Steam (pressure)	Water	2000–4000	350–750
Steam (vacuum)	Water	1700–3400	300–600
Saturated organic solvents (atmospheric)	Water	600–1200	100–200
Saturated organic solvents (vacuum some non-condensable)	Water–brine	300–700	50–120
Organic solvents (atmospheric and high non-condensable)	Water–brine	100–500	20–80
Organic solvents (vacuum and high non-condensable)	Water–brine	60–300	10–50
Low boiling hydrocarbons (atmospheric)	Water	400–1200	80–200
High boiling hydrocarbons (vacuum)	Water	60–200	10–30
<i>Heaters</i>			
Steam	Water	1500–4000	250–750
Steam	Light oils	300–900	50–150
Steam	Heavy oils	60–400	10–80
Steam	Organic solvents	600–1200	100–200
Steam	Gases	30–300	5–50
Dowtherm	Gases	20–200	4–40
Dowtherm	Heavy oils	50–400	8–60
<i>Evaporators</i>			
Steam	Water	2000–4000	350–750
Steam	Organic solvents	600–1200	100–200
Steam	Light oils	400–1000	80–180
Steam	Heavy oils (vacuum)	150–400	25–75
Water	Refrigerants	400–900	75–150
Organic solvents	Refrigerants	200–600	30–100
<i>Heat exchangers (no change of state)</i>			
Water	Water	900–1700	150–300
Organic solvents	Water	300–900	50–150
Gases	Water	20–300	3–50
Light oils	Water	400–900	60–160
Heavy oils	Water	60–300	10–50
Organic solvents	Light oil	100–400	20–70
Water	Brine	600–1200	100–200
Organic solvents	Brine	200–500	30–90
Gases	Brine	20–300	3–50
Organic solvents	Organic solvents	100–400	20–60
Heavy oils	Heavy oils	50–300	8–50

may be obtained by using correlations given by the Engineering Sciences Data Unit and, based on this work, BUTTERWORTH<sup>(109)</sup> offers the equation:

$$St = E Re^{-0.205} Pr^{-0.505} \quad (9.215)$$

where:

$$\text{the Stanton Number } St = NuRe^{-1} Pr^{-1}$$

and

$$E = 0.22 \exp[-0.0225(\ln Pr)^2]$$

Equation 9.215 is valid for Reynolds Numbers in excess of 10,000. Where the Reynolds Number is less than 2000, the flow will be laminar and, provided natural convection effects

Table 9.18. Approximate film coefficients for heat transfer

	$h_i$ or $h_o$	
	W/m <sup>2</sup> K	Btu/ft <sup>2</sup> h °F
<i>No change of state</i>		
water	1700–11,000	300–2000
gases	20–300	3–50
organic solvents	350–3000	60–500
oils	60–700	10–120
<i>Condensation</i>		
steam	6000–17,000	1000–3000
organic solvents	900–2800	150–500
light oils	1200–2300	200–400
heavy oils (vacuum)	120–300	20–50
ammonia	3000–6000	500–1000
<i>Evaporation</i>		
water	2000–12,000	30–200
organic solvents	600–2000	100–300
ammonia	1100–2300	200–400
light oils	800–1700	150–300
heavy oils	60–300	10–50

are negligible, film coefficients may be estimated from a form of equation 9.85 modified to take account of the variation of viscosity over the cross-section:

$$Nu = 1.86(RePr)^{0.33}(d/l)^{0.33}(\mu/\mu_s)^{0.14} \quad (9.216)$$

The minimum value of the Nusselt Number for which equation 9.216 applies is 3.5. Reynolds Numbers in the range 2000–10,000 should be avoided in designing heat exchangers as the flow is then unstable and coefficients cannot be predicted with any degree of accuracy. If this cannot be avoided, the lesser of the values predicted by Equations 9.214 and 9.216 should be used.

As discussed in Section 9.4.3, heat transfer data are conveniently correlated in terms of a heat transfer factor  $j_h$ , again modified by the viscosity correction factor:

$$j_h = StPr^{0.67}(\mu/\mu_s)^{-0.14} \quad (9.217)$$

which enables data for laminar and turbulent flow to be included on the same plot, as shown in Figure 9.77. Data from Figure 9.77 may be used together with equation 9.217 to estimate coefficients with heat exchanger tubes and commercial pipes although, due to a higher roughness, the values for commercial pipes will be conservative. Equation 9.217 is rather more conveniently expressed as:

$$Nu = (hd/k) = j_h RePr^{0.33}(\mu/\mu_s)^{-0.14} \quad (9.218)$$

It may be noted that whilst Figure 9.77 is similar to Figure 9.24, the values of  $j_h$  differ due to the fact that KERN<sup>(28)</sup> and other workers define the heat transfer factor as:

$$j_H = NuPr^{-0.33}(\mu/\mu_s)^{-0.14} \quad (9.219)$$

Thus the relationship between  $j_h$  and  $j_H$  is:

$$j_h = j_H/Re \quad (9.220)$$

As discussed in Section 9.4.3, by incorporating physical properties into equations 9.214 and 9.216, correlations have been developed specifically for water and equation 9.221, based on data from EAGLE and FERGUSON<sup>(110)</sup> may be used:

$$h = 4280(0.00488T - 1)u^{0.8}/d^{0.2} \quad (9.221)$$

which is in SI units, with  $h$  (film coefficient) in  $\text{W/m}^2\text{K}$ ,  $T$  in  $\text{K}$ ,  $u$  in  $\text{m/s}$  and  $d$  in  $\text{m}$ .

### Example 9.27

Estimate the heat transfer area required for the system considered in Examples 9.1 and 9.26, assuming that no data on the overall coefficient of heat transfer are available.

### Solution

As in the previous examples,

$$\text{heat load} = 1672 \text{ kW}$$

and:

$$\text{corrected mean temperature difference, } F\theta_m = 40.6 \text{ deg K}$$

In the tubes;

$$\text{mean water temperature, } T = 0.5(360 + 340) = 350 \text{ K}$$

Assuming a tube diameter,  $d = 19 \text{ mm}$  or  $0.019 \text{ m}$  and a water velocity,  $u = 1 \text{ m/s}$ , then, in equation 9.221:

$$h_i = 4280((0.00488 \times 350) - 1)1.0^{0.8}/0.019^{0.2} = 10610 \text{ W/m}^2\text{K} \text{ or } 10.6 \text{ kW/m}^2\text{K}$$

From Table 9.18, an estimate of the shell-side film coefficient is:

$$h_o = 0.5(1700 + 11000) = 6353 \text{ W/m}^2\text{K} \text{ or } 6.35 \text{ kW/m}^2\text{K}$$

For steel tubes of a wall thickness of  $1.6 \text{ mm}$ , the thermal resistance of the wall, from Table 9.15 is:

$$x_w/k_w = 0.025 \text{ m}^2\text{K/kW}$$

and the thermal resistance for treated water, from Table 9.16, is  $0.26 \text{ m}^2\text{K/kW}$  for both layers of scale. Thus, in Equation 9.201:

$$\begin{aligned} (1/U) &= (1/h_o) + (x_w/k_w) + R_i + R_o + (1/h_i) \\ &= (1/6.35) + 0.025 + 0.52 + (1/10.6) = 0.797 \text{ m}^2\text{K/kW} \end{aligned}$$

and:

$$U = 1.25 \text{ kW/m}^2\text{K}$$

The heat transfer area required is then:

$$A = Q/F\theta_m U = 1672/(40.6 \times 1.25) = \underline{\underline{32.9 \text{ m}^2}}$$

As discussed in Section 9.4.4, the complex flow pattern on the *shell-side* and the great number of variables involved make the prediction of coefficients and pressure drop very difficult, especially if leakage and bypass streams are taken into account. Until about 1960, empirical methods were used to account for the difference in the performance

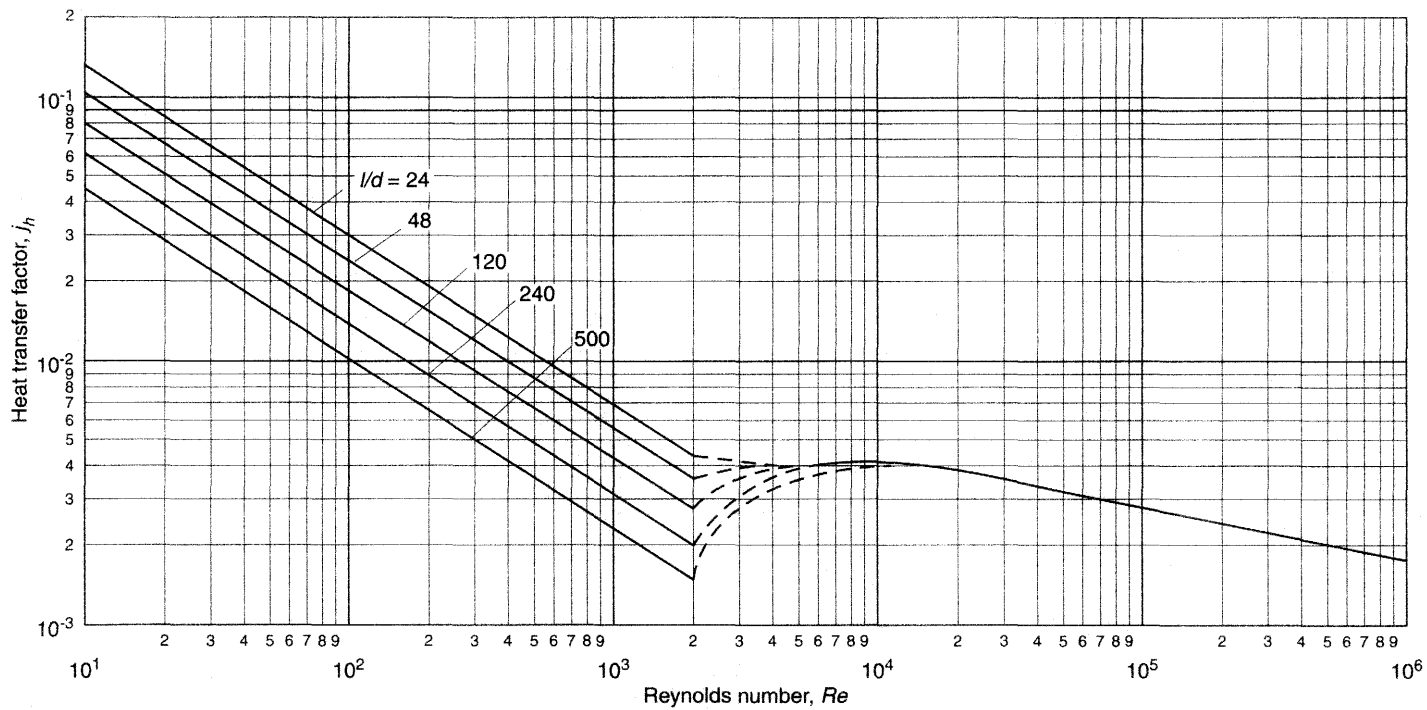


Figure 9.77. Heat transfer factor for flow inside tubes

of real exchangers as compared with that for cross-flow over ideal tube banks. The methods of KERN<sup>(28)</sup> and DONOHUE<sup>(111)</sup> are typical of these "bulk flow" methods and their approach, together with more recent methods involving an analysis of the contribution to heat transfer by individual streams in the shell, are discussed in Section 9.9.6.

Special correlations have also been developed for liquid metals, used in recent years in the nuclear industry with the aim of reducing the volume of fluid in the heat transfer circuits. Such fluids have high thermal conductivities, though in terms of heat capacity per unit volume, liquid sodium, for example, which finds relatively widespread application, has a value of  $C_p\rho$  of only 1275 kJ/m<sup>3</sup> K.

Although water has a much greater value, it is unsuitable because of its high vapour pressure at the desired temperatures and the corresponding need to use high-pressure piping. Because of their high thermal conductivities, liquid metals have particularly low values of the Prandtl number (about 0.01) and they behave rather differently from normal fluids under conditions of forced convection. Some values for typical liquid metals are given in Table 9.19.

Table 9.19. Prandtl numbers of liquid metals

Metal	Temperature (K)	Prandtl number $Pr$
Potassium	975	0.003
Sodium	975	0.004
Na/K alloy (56:44)	975	0.06
Mercury	575	0.008
Lithium	475	0.065

The results of work on sodium, lithium, and mercury for forced convection in a pipe have been correlated by the expression:

$$Nu = 0.625(RePr)^{0.4} \quad (9.222)$$

although the accuracy of the correlation is not very good. With values of Reynolds number of about 18,000 it is quite possible to obtain a value of  $h$  of about 11 kW/m<sup>2</sup> K for flow in a pipe.

## 9.9.5. Pressure drop in heat exchangers

### **Tube-side**

Pressure drop on the tube-side of a shell and tube exchanger is made up of the friction loss in the tubes and losses due to sudden contractions and expansions and flow reversals experienced by the tube-side fluid. The friction loss may be estimated by the methods outlined in Section 3.4.3 from which the basic equation for isothermal flow is given by equation 3.18 which can be written as:

$$-\Delta P_t = 4j_f(l/d_i)(\rho u^2) \quad (9.223)$$

where  $j_f$  is the dimensionless friction factor. Clearly the flow is not isothermal and it is usual to incorporate an empirical correction factor to allow for the change in physical



properties, particularly viscosity, with temperature to give:

$$-\Delta P_t = 4j_f(l/d_i)(\rho u^2)(\mu/\mu_s)^m \quad (9.224)$$

where  $m = -0.25$  for laminar flow ( $Re < 2100$ ) and  $-0.14$  for turbulent flow ( $Re > 2100$ ). Values of  $j_f$  for heat exchanger tubes are given in Figure 9.78 which is based on Figure 3.7.

There is no entirely satisfactory method for estimating losses due to contraction at the tube inlets, expansion at the exits and flow reversals, although KERN<sup>(28)</sup> suggests adding four velocity heads per pass, FRANK<sup>(112)</sup> recommends 2.5 velocity heads and BUTTERWORTH<sup>(113)</sup> 1.8. LORD *et al.*<sup>(114)</sup> suggests that the loss per pass is equivalent to a tube length of 300 diameters for straight tubes and 200 for U-tubes, whilst EVANS<sup>(115)</sup> recommends the addition of 67 tube diameters per pass. Another approach is to estimate the number of velocity heads by using factors for pipe-fittings as discussed in Section 3.4.4 and given in Table 3.2. With four tube passes, for example, there will be four contractions equivalent to a loss of  $(4 \times 0.5) = 2$  velocity heads, four expansions equivalent to a loss of  $(4 \times 1.0) = 4$  velocity heads and three  $180^\circ$  bends equivalent to a loss of  $(3 \times 1.5) = 4.5$  velocity heads. In this way, the total loss is 10.5 velocity heads, or 2.6 per pass, giving support to Frank's proposal of 2.5. Using this approach, equation 9.224 becomes:

$$-\Delta P_{\text{total}} = N_P[4j_f(l/d_i)(\mu/\mu_s)^m + 1.25](\rho u^2) \quad (9.225)$$

where  $N_P$  is the number of tube-side passes. Additionally, there will be expansion and contraction losses at the inlet and outlet nozzles respectively, and these losses may be estimated by adding one velocity head for the inlet, and 0.5 of a velocity head for the outlet, based on the nozzle velocities. Losses in the nozzles are only significant for gases at pressures below atmospheric.

### Shell-side

As discussed in Section 9.4.4, the prediction of pressure drop, and indeed heat transfer coefficients, in the shell is very difficult due to the complex nature of the flow pattern in the segmentally baffled unit. Whilst the baffles are intended to direct fluid across the tubes, the actual flow is a combination of cross-flow between the baffles and axial or parallel flow in the baffle windows as shown in Figure 9.79, although even this does not represent the actual flow pattern because of leakage through the clearances necessary for the fabrication and assembly of the unit. This more realistic flow pattern is shown in Figure 9.80 which is based on the work of TINKER<sup>(116)</sup> who identifies the various streams in the shell as follows:

- A—fluid flowing through the clearance between the tube and the hole in the baffle.
- B—the actual cross-flow stream.
- C—fluid flowing through the clearance between the outer tubes and the shell.
- E—fluid flowing through the clearance between the baffle and the shell.
- F—fluid flowing through the gap between the tubes because of any pass-partition plates. This is especially significant with a vertical gap.

Because stream A does not bypass the tubes, it is the pressure drop rather than the heat transfer which is affected. Streams C, E and F bypass the tubes, thus reducing the effective

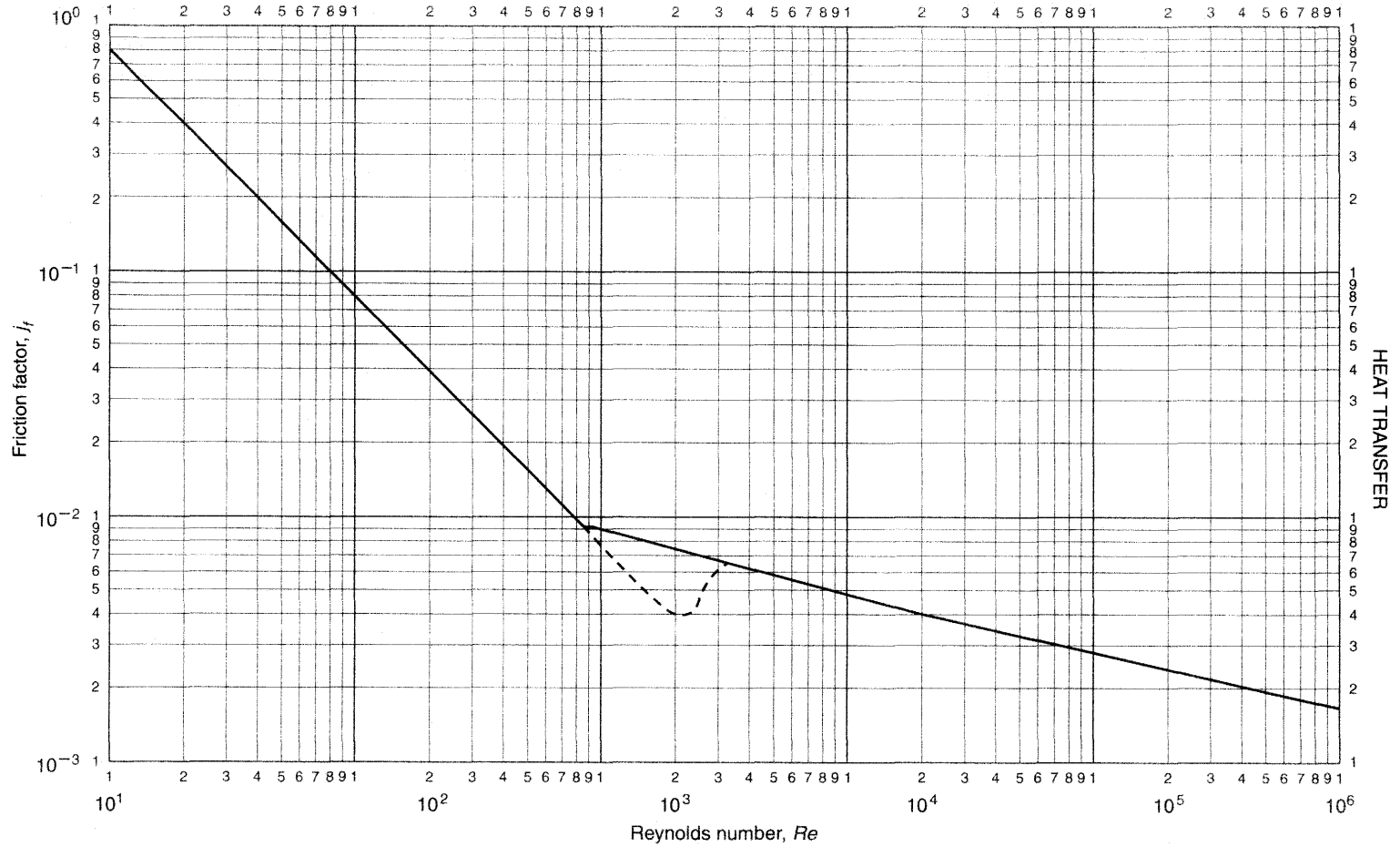


Figure 9.78. Tube-side friction factors<sup>(28)</sup>.

Note: The friction factor  $j_f$  is the same as the friction factor for pipes  $\phi (= R/\rho u^2)$ , defined in Chapter 3

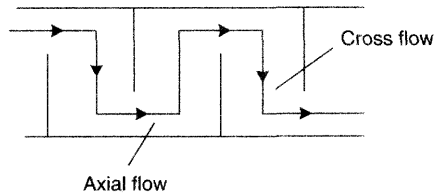
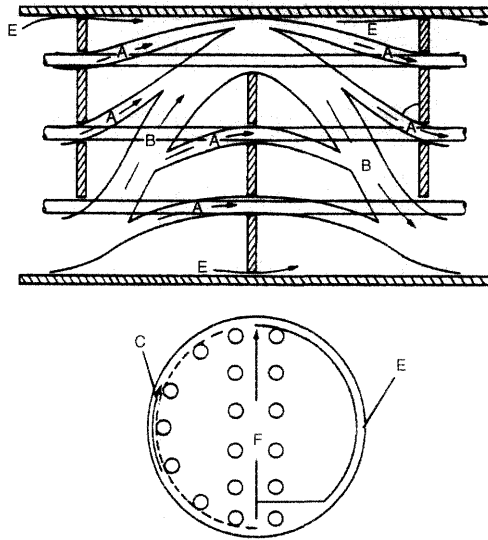


Figure 9.79. Idealised main stream flow

Figure 9.80. Shell-side leakage and by-pass paths<sup>(116)</sup>

heat transfer area. Stream C, the main bypass stream, is most significant in pull-through bundle units where there is of necessity a large clearance between the bundle and the shell, although this can be reduced by using horizontal sealing strips. In a similar way, the flow of stream F may be reduced by fitting dummy tubes. As an exchanger becomes fouled, clearances tend to plug and this increases the pressure drop. The whole question of shell-side pressure drop estimation in relation to design procedures is now discussed.

### 9.9.6. Heat exchanger design

#### **Process conditions**

A first-stage consideration in the design process is the allocation of fluids to either shell or tubes and, by and large, the more corrosive fluid is passed through the tubes to reduce the costs of expensive alloys and clad components. Similarly, the fluid with the greatest fouling tendency is also usually passed through the tubes where cleaning is easier. Furthermore, velocities through the tubes are generally higher and more readily controllable and can be adjusted to reduce fouling. Where special alloys are in contact with hot fluids, the