CHAPTER 5

Flow of Multiphase Mixtures

5.1. INTRODUCTION

The flow problems considered in previous chapters are concerned with homogeneous fluids, either single phases or suspensions of fine particles whose settling velocities are sufficiently low for the solids to be completely suspended in the fluid. Consideration is now given to the far more complex problem of the flow of multiphase systems in which the composition of the mixture may vary over the cross-section of the pipe or channel; furthermore, the components may be moving at different velocities to give rise to the phenomenon of "slip" between the phases.

Multiphase flow is important in many areas of chemical and process engineering and the behaviour of the material will depend on the properties of the components, the flowrates and the geometry of the system. In general, the complexity of the flow is so great that design methods depend very much on an analysis of the behaviour of such systems in practice and, only to a limited extent, on theoretical predictions. Some of the more important systems to be considered are:

Mixtures of liquids with gas or vapour.
Liquids mixed with solid particles ("hydraulic transport").
Gases carrying solid particles wholly or partly in suspension ("pneumatic transport").
Multiphase systems containing solids, liquids and gases.

Mixed materials may be transported horizontally, vertically, or at an inclination to the horizontal in pipes and, in the case of liquid-solid mixtures, in open channels. Although there is some degree of common behaviour between the various systems, the range of physical properties is so great that each different type of system must be considered separately. Liquids may have densities up to three orders of magnitude greater than gases but they do not exhibit any significant compressibility. Liquids themselves can range from simple Newtonian liquids such as water, to non-Newtonian fluids with very high apparent viscosities. These very large variations in density and viscosity are responsible for the large differences in behaviour of solid-gas and solid-liquid mixtures which must, in practice, be considered separately. For, all multiphase flow systems, however, it is important to understand the nature of the interactions between the phases and how these influence the *flow patterns*—the ways in which the phases are distributed over the cross-section of the pipe or duct. In design it is necessary to be able to predict *pressure drop* which, usually, depends not only on the flow pattern, but also on the relative velocity of the phases; this *slip velocity* will influence the *hold-up*, the fraction of the pipe volume which is occupied by a particular phase. It is important to note that, in the flow of a two-component mixture, the hold-up (or *in situ* concentration) of a component will differ from that in the mixture discharged at the end of the pipe because, as a result of *slip* of the phases relative to one another, their residence times in the pipeline will not be the same. Special attention is therefore focused on three aspects of the flow of these complex mixtures.

(1) The flow patterns.
(2) The hold-up of the individual phases and their relative velocities.
(3) The relationship between pressure gradient in a pipe and the flowrates and physical properties of the phases.

The difference in density between the phases is important in determining flow pattern. In gas-solid and gas-liquid mixtures, the gas will always be the lighter phase, and in liquid-solid systems it will be usual for the liquid to be less dense than the solid. In vertical upward flow, therefore, there will be a tendency for the lighter phase to rise more quickly than the denser phase giving rise to a slip velocity. For a liquid-solid or gas-solid system this *slip* velocity will be close to the terminal falling velocity of the particles. In a liquid-gas system, the slip velocity will depend on the flow pattern in a complex way. In all cases, there will be a net upwards force resulting in a transference of energy from the faster to the slower moving phase, and a vertically downwards gravitational force will be balanced by a vertically upwards drag force. There will be axial symmetry of flow.

In horizontal flow, the flow pattern will inevitably be more complex because the gravitational force will act perpendicular to the pipe axis, the direction of flow, and will cause the denser component to flow preferentially nearer the bottom of the pipe. Energy transfer between the phases will again occur as a result of the difference in velocity, but the net force will be horizontal and the suspension mechanism of the particles, or the dispersion of the fluid will be a more complex process. In this case, the flow will not be symmetrical about the pipe axis.

In practice, many other considerations will affect the design of an installation. For example, wherever solid particles are present, there is the possibility of *blockage* of the pipe and it is therefore important to operate under conditions where the probability of this occurring is minimised. Solids may be *abrasive* and cause undue wear if the velocities are too high or changes in direction of flow are too sudden. Choice of suitable materials of construction and operating conditions is therefore important. In pneumatic transport, *electrostatic charging* may take place and cause considerable increase in pressure gradient.

5.2. TWO-PHASE GAS (VAPOUR)–LIQUID FLOW

5.2.1. Introduction

Some of the important features of the flow of two-phase mixtures composed of a liquid together with a gas or vapour are discussed in this section. There are many applications in the chemical and process industries, ranging from the flow of mixtures of oil and gas from well heads to flow of vapour-liquid mixtures in boilers and evaporators.

Because of the presence of the two phases, there are considerable complications in describing and quantifying the nature of the flow compared with conditions with a single phase. The lack of knowledge of the velocities at a point in the individual phases makes it impossible to give any real picture of the velocity distribution. In most cases the gas phase, which may be flowing with a much greater velocity than the liquid, continuously accelerates the liquid thus involving a transfer of energy. Either phase may be in *streamline* or in *turbulent* flow, though the most important case is that in which both phases are turbulent. The criterion for streamline or turbulent flow of a phase is whether the Reynolds number for its flow at the same rate on its own is less or greater than 1000–2000. This distinction is to some extent arbitrary in that injection of a gas into a liquid initially in streamline flow may result in turbulence developing.

If there is no heat transfer to the flowing mixture, the mass rate of flow of each phase will remain substantially constant, though the volumetric flowrates (and velocities) will increase progressively as the gas expands with falling pressure. In a boiler or evaporator, there will be a progressive vaporisation of the liquid leading to a decreased mass flowrate of liquid and corresponding increase for the vapour, with the total mass rate of flow remaining constant. The volumetric flowrate will increase very rapidly as a result of the combined effects of falling pressure and increasing vapour/liquid ratio.

A gas-liquid mixture will have a lower density than the liquid alone. Therefore, if in a U-tube one limb contains liquid and the other a liquid-gas mixture, the equilibrium height in the second limb will be higher than in the first. If two-phase mixture is discharged at a height less than the equilibrium height, a continuous flow of liquid will take place from the first to the second limb, provided that a continuous feed of liquid and gas is maintained. This principle is used in the design of the air lift pump described in Chapter 8.

Consideration will now be given to the various flow regimes which may exist and how they may be represented on a "Flow Pattern Map"; to the calculation and prediction of hold-up of the two phases during flow; and to the calculation of pressure gradients for gas-liquid flow in pipes. In addition, when gas-liquid mixtures flow at high velocities serious erosion problems can arise and it is necessary for the designer to restrict flow velocities to avoid serious damage to equipment.

A more detailed treatment of the subject is given by GOVIER and AZIZ[1], by CHISHOLM[2] and by HEWITT[3].

5.2.2. Flow regimes and flow patterns

Horizontal flow

The flow pattern is complex and is influenced by the diameter of the pipe, the physical properties of the fluids and their flowrates. In general, as the velocities are increased and as the gas-liquid ratio increases, changes will take place from "bubble flow" through to "mist flow" as shown in Figure 5.1[1–7]; the principal characteristics are described in Table 5.1. At high liquid-gas ratios, the liquid forms the continuous phase and at low values it forms the disperse phase. In the intervening region, there is generally some instability; and sometimes several flow regimes are lumped together. In plug flow and slug flow, the gas is flowing faster than the liquid and liquid from a slug tends to become detached, to move as a relatively slow moving film along the surface of the pipe and then to be reaccelerated when the next liquid slug catches it up. This process can account for a significant proportion of the total energy losses. Particularly in short pipelines, the flow develops an oscillating pattern arising largely from discontinuities associated with the expulsion of successive liquid slugs.

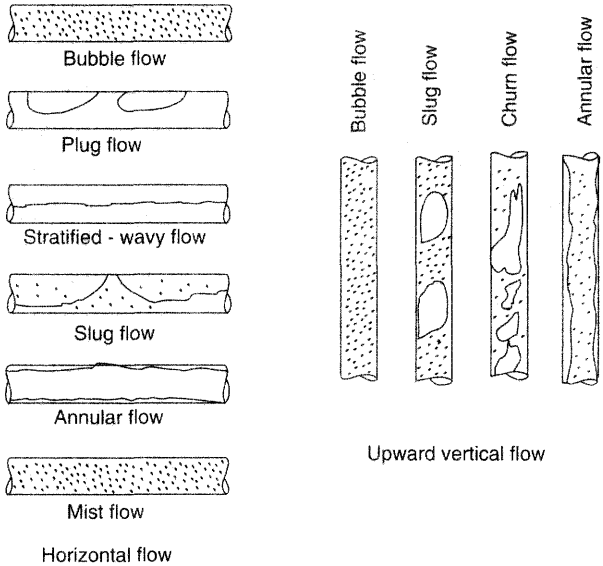

Figure 5.1. Flow patterns in two-phase flow

Table 5.1. Flow regimes in horizontal two-phase flow

Regime	Description	Typical velocities (m/s) Liquid	Vapour
1. Bubble flow[a]	Bubbles of gas dispersed throughout the liquid	1.5–5	0.3–3
2. Plug flow[a]	Plugs of gas in liquid phase	0.6	<1.0
3. Stratified flow	Layer of liquid with a layer of gas above	<0.15	0.6–3
4. Wavy flow	As stratified but with a wavy interface due to higher velocities	<0.3	>5
5. Slug flow[a]	Slug of gas in liquid phase	Occurs over a wide range of velocities	
6. Annular flow[b]	Liquid film on inside walls with gas in centre		>6
7. Mist flow[b]	Liquid droplets dispersed in gas		>60

[a] Frequently grouped together as *intermittent* flow
[b] Sometimes grouped as *annular/mist* flow

The regions over which the different types of flow can occur are conveniently shown on a "Flow Pattern Map" in which a function of the gas flowrate is plotted against a function of the liquid flowrate and boundary lines are drawn to delineate the various regions. It should be borne in mind that the distinction between any two flow patterns is not clear-cut and that these divisions are only approximate as each flow regime tends to merge in with its neighbours; in any case, the whole classification is based on highly subjective observations. Several workers have produced their own maps[4–8].

Most of the data used for compiling such maps have been obtained for the flow of water and air at near atmospheric temperature and pressure, and scaling factors have been introduced to extend their applicability to other systems. However, bearing in mind the diffuse nature of the boundaries between the regimes and the relatively minor effect of changes in physical properties, such a refinement does not appear to be justified. The flow pattern map for horizontal flow illustrated in Figure 5.2 which has been prepared by CHHABRA and RICHARDSON[9] is based on those previously presented by MANDHANE *et al.*[8] and WEISMAN *et al.*[7] The axes of this diagram are superficial liquid velocity u_L and superficial gas velocity u_G (in each case the volumetric flowrate of the phase divided by the total cross-sectional area of the pipe).

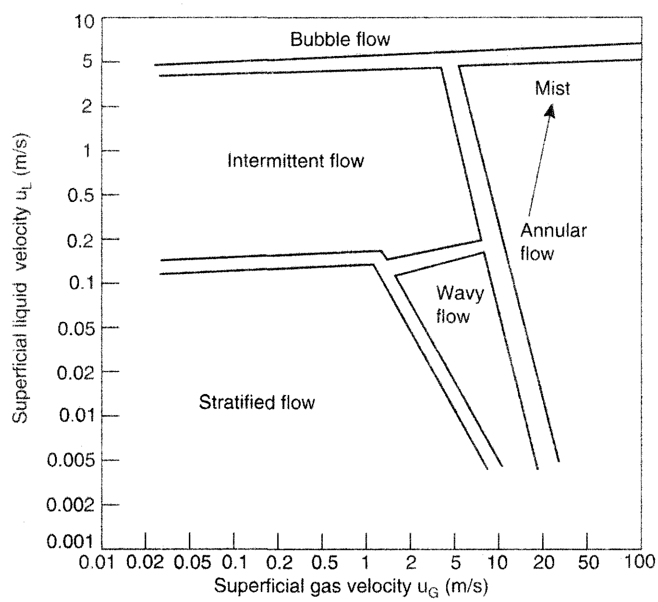

Figure 5.2. Flow pattern map

Slug flow should be avoided when it is necessary to obviate unsteady conditions, and it is desirable to design so that annular flow still persists at loadings down to 50 per cent of the normal flow rates. Even though in many applications both phases should be turbulent, excessive gas velocity will lead to a high pressure drop, particularly in small pipes.

Although most of the data relate to flow in pipes of small diameters (<42 mm), results of experiments carried out in a 205 mm pipe fit well on the diagram. The flow pattern map, shown in Figure 5.2, also gives a good representation of results obtained for the flow of mixtures of gas and shear-thinning non-Newtonian liquids, including very highly shear-thinning suspensions (power law index $n \approx 0.1$) and viscoelastic polymer solutions.

Vertical flow

In vertical flow, axial symmetry exists and flow patterns tend to be somewhat more stable. However, with slug flow in particular, oscillations in the flow can occur as a result of sudden changes in pressure as liquid slugs are discharged from the end of the pipe.

The principal flow patterns are shown in Figure 5.1. In general, the flow pattern map (Figure 5.2) is also applicable to vertical flow. Further reference to flow of gas–liquid mixtures in vertical pipes is made in Section 8.4.1 with reference to the operation of the air-lift pump.

5.2.3. Hold-up

Because the gas always flows at a velocity greater than that of the liquid, the *in situ* volumetric fraction of liquid at any point in a pipeline will be greater than the input volume fraction of liquid; furthermore it will progressively change along the length of the pipe as a result of expansion of the gas.

There have been several experimental studies of two-phase flow in which the hold-up has been measured, either directly or indirectly. The direct method of measurement involves suddenly isolating a section of the pipe by means of quick-acting valves and then determining the quantity of liquid trapped.[(10,11)] Such methods are cumbersome and are subject to errors arising from the fact that the valves cannot operate instantaneously. Typical of the indirect methods is that in which the pipe cross-section is scanned by γ-rays and the hold-up is determined from the extent of their attenuation.[(12,13,14)]

LOCKHART and MARTINELLI[(15)] expressed hold-up in terms of a parameter X, characteristic of the relative flowrates of liquid and gas, defined as:

$$X = \sqrt{\frac{-\Delta P_L}{-\Delta P_G}} \tag{5.1}$$

where $-\Delta P_L$ and $-\Delta P_G$ are the frictional pressure drops which would arise from the flow of the respective phases on their own at the same rates. Their correlation is reproduced in Figure 5.3. As a result of more recent work it is now generally accepted that the correlation overpredicts values of liquid hold-up. Thus FAROOQI and RICHARDSON[(16)], the results of whose work are also shown in Figure 5.3, have given the following expression for liquid hold-up ϵ_L for co-current flow of air and Newtonian liquids in horizontal pipes:

$$\left.\begin{aligned} \epsilon_L &= 0.186 + 0.0191X & 1 < X < 5 \\ \epsilon_L &= 0.143X^{0.42} & 5 < X < 50 \\ \epsilon_L &= \frac{1}{0.97 + 19/X} & 50 < X < 500 \end{aligned}\right\} \tag{5.2}$$

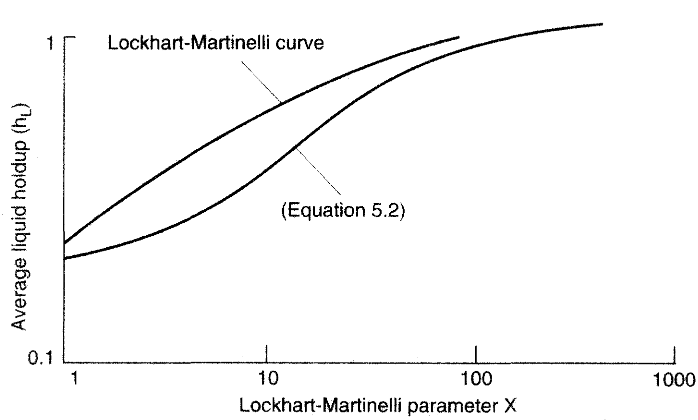

Figure 5.3. Correlation for average liquid hold-up, ϵ_L

It should be noted that, for turbulent flow of each phase, pressure drop is approximately proportional to the square of velocity and X is then equal to the ratio of the superficial velocities of the liquid and gas.

Equation 5.2 is found to hold well for non-Newtonian shear-thinning suspensions as well, provided that the liquid flow is turbulent. However, for laminar flow of the liquid, equation 5.2 considerably overpredicts the liquid hold-up ϵ_L. The extent of overprediction increases as the degree of shear-thinning increases and as the liquid Reynolds number becomes progressively less. A modified parameter X' has therefore been defined[16,17] for a power-law fluid (Chapter 3) in such a way that it reduces to X both at the superficial velocity u_L equal to the transitional velocity $(u_L)_c$ from streamline to turbulent flow and when the liquid exhibits Newtonian properties. The parameter X' is defined by the relation

$$X' = X\left(\frac{u_L}{(u_L)_c}\right)^{1-n} \tag{5.3}$$

where n is the *power-law* index. It will be seen that the correction factor becomes progressively more important as n deviates from unity and as the velocity deviates from the critical velocity. Equation 5.2 may then be applied provided X' is used in place of X in the equation.

Thus, in summary, liquid hold-up can be calculated using equation 5.2 for:

Newtonian fluids in laminar or turbulent flow.
Non-Newtonian fluids in *turbulent flow only*.

Equation 5.2, with the modified parameter X' used in place of X, may be used for laminar flow of shear-thinning fluids whose behaviour can be described by the *power-law* model.

A knowledge of hold-up is particularly important for vertical flow since the hydrostatic pressure gradient, which is frequently the major component of the total pressure gradient, is directly proportional to liquid hold-up. However, in slug flow, the situation is complicated by the fact that any liquid which is in the form of an annular film surrounding the gas slug does not contribute to the hydrostatic pressure[14].

5.2.4. Pressure, momentum, and energy relations

Methods for determining the drop in pressure start with a physical model of the two-phase system, and the analysis is developed as an extension of that used for single-phase flow. In the *separated flow* model the phases are first considered to flow separately; and their combined effect is then examined.

The total pressure gradient in a horizontal pipe, $(-\mathrm{d}P_{TPF}/\mathrm{d}l)$, consists of two components which represent the frictional and the acceleration pressure gradients respectively, or:

$$\frac{-\mathrm{d}P_{TPF}}{\mathrm{d}l} = \frac{-\mathrm{d}P_f}{\mathrm{d}l} + \frac{-\mathrm{d}P_a}{\mathrm{d}l} \tag{5.4}$$

A momentum balance for the flow of a two-phase fluid through a horizontal pipe and an energy balance may be written in an expanded form of that applicable to single-phase fluid flow. These equations for two-phase flow cannot be used in practice since the individual phase velocities and local densities are not known. Some simplification is possible if it is assumed that the two phases flow separately in the channel occupying fixed fractions of the total area, but even with this assumption of separated flow regimes, progress is difficult. It is important to note that, as in the case of single-phase flow of a compressible fluid, it is no longer possible to relate the shear stress to the pressure drop in a simple form since the pressure drop now covers both frictional and acceleration losses. The shear at the wall is proportional to the total rate of momentum transfer, arising from friction and acceleration, so that the total drop in pressure $-\Delta P_{TPF}$ is given by:

$$-\Delta P_{TPF} = (-\Delta P_f) + (-\Delta P_a) \tag{5.5}$$

The pressure drop due to acceleration is important in two-phase flow because the gas is normally flowing much faster than the liquid, and therefore as it expands the liquid phase will accelerate with consequent transfer of energy. For flow in a vertical direction, an additional term $-\Delta P_{\text{gravity}}$ must be added to the right hand side of equation 5.5 to account for the hydrostatic pressure attributable to the liquid in the pipe, and this may be calculated approximately provided that the liquid hold-up is known.

Analytical solutions for the equations of motion are not possible because of the difficulty of specifying the flow pattern and of defining the precise nature of the interaction between the phases. Rapid fluctuations in flow frequently occur and these cannot readily be taken into account. For these reasons, it is necessary for design purposes to use correlations which have been obtained using experimental data. Great care should be taken, however, if these are used outside the limits used in the experimental work.

Practical methods for evaluating pressure drop

Probably the most widely used method for estimating the drop in pressure due to friction is that proposed by LOCKHART and MARTINELLI[(15)] and later modified by CHISHOLM[(18)]. This is based on the physical model of separated flow in which each phase is considered separately and then a combined effect formulated. The two-phase pressure drop due to friction $-\Delta P_{TPF}$ is taken as the pressure drop $-\Delta P_L$ or $-\Delta P_G$ that would arise for either phase flowing alone in the pipe at the stated rate, multiplied by some factor Φ_L^2 or Φ_G^2. This factor is presented as a function of the ratio of the individual single-phase pressure drops and:

$$\frac{-\Delta P_{TPF}}{-\Delta P_G} = \Phi_G^2 \tag{5.6}$$

$$\frac{-\Delta P_{TPF}}{-\Delta P_L} = \Phi_L^2 \tag{5.7}$$

The relation between Φ_G and Φ_L and X (defined by equation 5.1) is shown in Figure 5.4, where it is seen that separate curves are given according to the nature of the flow of the two phases. This relation was developed from studies on the flow in small tubes of up to 25 mm diameter with water, oils, and hydrocarbons using air at a pressure of up to 400 kN/m^2. For mass flowrates per unit area of L' and G' for the liquid and gas, respectively, Reynolds numbers $Re_L(L'd/\mu_L)$ and $Re_G(G'd/\mu_G)$ may be used as criteria for defining the flow regime; values less than 1000 to 2000, however, do not necessarily imply that the fluid is in truly laminar flow. Later experimental work showed that the total pressure has an influence and data presented by GRIFFITH[(19)] may be consulted where

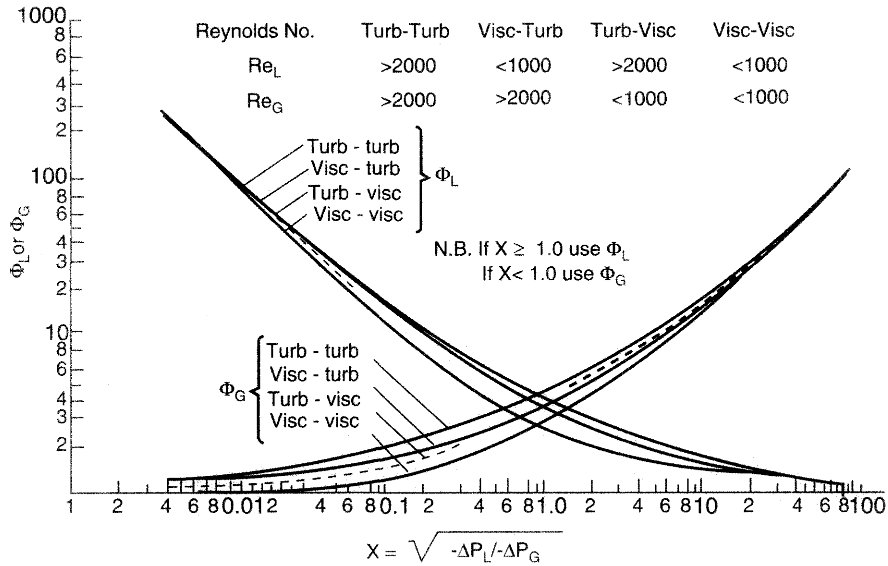

Figure 5.4. Relation between Φ and X for two-phase flow

pressures are in excess of 3 MN/m^2. CHISHOLM[18] has developed a relation between Φ_L and X which he puts in the form:

$$\Phi_L^2 = 1 + \frac{c}{X} + \frac{1}{X^2} \tag{5.8}$$

where c has a value of 20 for turbulent/turbulent flow, 10 for turbulent liquid/streamline gas, 12 for streamline liquid/turbulent gas, and 5 for streamline/streamline flow. If the densities of the fluids are significantly different from those of water and air at atmospheric temperature and pressure, the values of c are somewhat modified.

CHENOWETH and MARTIN[20,21] have presented an alternative method for calculating the drop in pressure, which is empirical and based on experiments with pipes of 75 mm and pressures up to 0.7 MN/m^2. They have plotted the volume fraction of the inlet stream that is liquid as abscissa against the ratio of the two-phase pressure drop to that for liquid flowing at the same volumetric rate as the mixture. An alternative technique has been described by BAROCZY[22]. If heat transfer gives rise to evaporation then reference should be made to work by DUKLER *et al*[23].

An illustration of the method of calculation of two-phase pressure drop is included here as Example 5.1.

Critical flow

For the flow of a compressible fluid, conditions of sonic velocity may be reached, thus limiting the maximum flowrate for a given upstream pressure. This situation can also occur with two-phase flow, and such critical velocities may sometimes be reached with a drop in pressure of only 30 per cent of the inlet pressure.