

Solution to Homework N° 5: mass transfer

a.

1. Let us assume that the concentration of gas at the interface with the liquid remains equal to C_{int} along the whole column.

The mass balance written for the gas that diffuses in the volume of liquid present in a segment dz of column is given by:

$$\frac{dVC}{dt} = Q(z)C(z) - Q(z+dz)C(z+dz) + \quad (1)$$

$$+kA_{int}(C_{int} - C(z)) \quad (2)$$

where the term on the left hand side represents the accumulation of mass over time in the control volume, terms on the right represent respectively convective flows entering in/exiting from the control volume (liquid film moving downward) and mass transfer through the gas/liquid interface. In particular, we have:

- (a) $dVC/dt = 0$ at steady state
- (b) $Q(z) = Q(z+dz)$ for mass conservation of the liquid phase
- (c) $A_{int} = 2\pi(R-\delta)dz \simeq 2\pi R dz$ for the interface (cylindrical surface on which the liquid film flows)

where $C(z)$ is the concentration of gas in the liquid at the generic height z along the column. Using the derivative to rewrite the convective flow (or using Taylor to express the concentration at $C(z+dz)$) the mass balance becomes:

$$0 = -Q \frac{dC(z)}{dz} dz + k2\pi R dz (C_{int} - C(z)) \quad (3)$$

$$\frac{dC(z)}{dz} = \frac{2\pi k R}{Q} (C_{int} - C(z)) \quad (4)$$

which can be integrated by separation of variables:

$$\frac{dC(z)}{C_{int} - C(z)} = \frac{2\pi R D}{Q} 0.69 \left(\frac{zv_0}{D}\right)^{0.5} dz \quad (5)$$

$$\frac{dC(z)}{C_{int} - C(z)} = 0.69 \frac{2\pi R}{Q} \left(\frac{Dv_0}{z}\right)^{0.5} dz \quad (6)$$

between $C(0) = 0$ and $C(L)$ and between 0 and L . The concentration $C(0) = 0$ because there is no H_2S in the washing liquid at column inlet. To determine v_0 we know that $Q = 2\pi R \delta v_0$ and $v_0 = Q/2\pi R \delta$.

2. Equation integration gives:

$$\ln \frac{C_{int} - C(z)}{C_{int} - C(0)} = -\frac{0.69}{v_0 \delta} (Dv_0)^{0.5} 2z^{0.5} \quad (7)$$

$$\frac{C_{int} - C(z)}{C_{int} - C(0)} = \exp\left(-\frac{0.69}{v_0 \delta} (Dv_0)^{0.5} 2z^{0.5}\right) \quad (8)$$

For the concentration profile we get:

$$C_{int} - C(z) = \quad (9)$$

$$= (C_{int} - C(0)) \exp\left(-\frac{0.69}{v_0 \delta} (Dv_0)^{0.5} 2z^{0.5}\right) \quad (10)$$

$$C(z) = C_{int} \left(1 - \exp\left(-\frac{0.69}{v_0 \delta} (Dv_0)^{0.5} 2z^{0.5}\right)\right) \quad (11)$$

and imposing $C(L) = 0.1C_{int}$ we can calculate the length of the column L for which we reach the target absorption.

b.

1. The river is a convective flow system that exchanges oxygen through the interface with the outside air. Indicated with x the coordinate along the flow direction, the mass balance is given by:

$$\frac{VC(x)}{dx} = Q(x)C(x) - Q(x+dx)C(x+dx) + \quad (12)$$

$$+kA_{int}(C_{sat} - C(x)) \quad (13)$$

where $C(x)$ is the concentration of dissolved oxygen in water at the generic position x along the river and C_{sat} is the concentration of oxygen at the air/liquid interface. The first term is the mass variation of oxygen in the control volume, which is null in stationary conditions. The budget can be rewritten as:

$$0 = -Q \frac{dC(x)}{dx} dx + k_l A_{int} (C_{sat} - C(x)) \quad (14)$$

where $Q = Q(x) = Q(x+dx)$ and $A_{int} = w dx$ with w river width. This equation must be integrated between $C(x=0) = 2 \text{ mg/l}$ and $C(L)$.

2. Integrating the equation by separation of variables we have:

$$\frac{dC(x)}{dx} = \frac{k_l w}{Q} (C_{sat} - C(x)) \quad (15)$$

$$\frac{dC(x)}{C_{sat} - C(x)} = \frac{k_l w}{Q} dx \quad (16)$$

$$\ln \frac{C_{sat} - C(x)}{C_{sat} - C(0)} = -\frac{k_l w}{Q} x \quad (17)$$

$$\frac{C_{sat} - C(x)}{C_{sat} - C(0)} = \exp\left(-\frac{k_l w}{Q} x\right) \quad (18)$$

and the variation of concentration along the river is:

$$C_{sat} - C(x) = (C_{sat} - C(0)) \exp\left(-\frac{k_l w}{uw h} x\right) \quad (19)$$

$$C(x) = C_{sat} - (C_{sat} - C(0)) \exp\left(-\frac{k_l w}{uw h} x\right) \quad (20)$$

Imposing $C(x = L) = 6 \text{ mg/l}$, we can derive the value of L within which the level of oxygen goes back to acceptable values.

c.

1. The mass balance for the gas absorbed in the liquid phase is:

$$\frac{dVC(z)}{dz} = \frac{\Gamma}{\rho}\pi d(C(z) - C(z + dz)) + \quad (21)$$

$$+ kA_{int}(C_{sat} - C(z)) \quad (22)$$

where d is column diameter. The first term on the left is null if the column works in stationary conditions. The convective flow can be rewritten as:

$$\frac{\Gamma}{\rho}\pi d(C(z) - C(z + dz)) = -\frac{\Gamma}{\rho}\pi d\frac{dC(z)}{dz}dz \quad (23)$$

and the interface area is given by $A_{int} = \pi ddz$. The budget is simplified into:

$$\frac{dC(z)}{dz} = \frac{k\rho}{\Gamma}(C_{sat} - C(z)) \quad (24)$$

where k is the local mass transfer coefficient. The integration of the equation allows to derive the expression of the liquid concentration at a distance L and, if this is known, the value of k . Integrating between 0 and L we have:

$$\frac{dC(z)}{C_{sat} - C(z)} = \frac{k\rho}{\Gamma}dz \quad (25)$$

$$\ln \frac{C_{sat} - C(z)}{C_{sat} - C(0)} = -\frac{k\rho}{\Gamma}z \quad (26)$$

$$\frac{C_{sat} - C(z)}{C_{sat} - C(0)} = \exp\left(-\frac{k\rho}{\Gamma}z\right) \quad (27)$$

The concentration profile along the column is:

$$C_{sat} - C(z) = (C_{sat} - C(0))\exp\left(-\frac{k\rho}{\Gamma}z\right) \quad (28)$$

$$C(z) = C_{sat} - (C_{sat} - C(0))\exp\left(-\frac{k\rho}{\Gamma}z\right) \quad (29)$$

Assume that the concentration C , expressed in moles/m^3 , is zero when the liquid enters the column. At a distance L the concentration is:

$$C(L) = C_{sat} \left(1 - \exp\left(-\frac{k\rho}{\Gamma}L\right)\right) = 0.4C_{sat} \quad (30)$$

from which we can obtain the local mass transfer coefficient:

$$k = -\frac{\Gamma}{\rho L} \ln \frac{C_{sat} - C(L)}{C_{sat}} = -\frac{\Gamma}{\rho L} \ln 0.6 \quad (31)$$

The molar flow of CO_2 absorbed in the liquid phase within the generic distance z is calculated as the

variation of CO_2 in the liquid phase between section z and inlet section:

$$\dot{m}_{int}(z) = \frac{\Gamma\pi d}{\rho}(C(z) - C(0)) = C(z)\frac{\Gamma\pi d}{\rho} \quad (32)$$

At the outlet section, $z = L$, the transferred flow can be expressed using the coefficient of global transfer k_L as:

$$\dot{m}_{int}(L) = k_L A_{int,L}(C(L) - C(0)) \quad (33)$$

where $A_{int,L} = \pi dL$. To link the local mass transfer coefficient to the global (equipment) mass transfer coefficient one can proceed by equating the two formulas:

$$C(L)\frac{\Gamma}{\rho}\pi d = k_L\pi dL(C(L) - C(0)) \quad (34)$$

and expressing Γ as a function of k using equation 30:

$$\Gamma = -\frac{\rho k L}{\ln \frac{C_{sat} - C(L)}{C_{sat} - C(0)}} = \frac{\rho k L}{\ln \frac{C_{sat} - C(0)}{C_{sat} - C(L)}} \quad (35)$$

from which we get:

$$\frac{k\pi dL}{\ln \frac{C_{sat} - C(0)}{C_{sat} - C(L)}}C(L) = k_L\pi dL(C(L) - C(0)) \quad (36)$$

$$\frac{k\pi dL}{\ln \frac{C_{sat} - C(0)}{C_{sat} - C(L)}} \cdot \quad (37)$$

$$\cdot [C(L) - C_{sat} - (C(0) - C_{sat})] = \quad (38)$$

$$= kA_{int,L} \frac{[C(L) - C_{sat} - (C(0) - C_{sat})]}{\ln \frac{C_{sat} - C(0)}{C_{sat} - C(L)}} = \quad (39)$$

$$= kA_{int,L}\Delta C_{ml} = k_L A_{int,L}(C(L) - C(0)) \quad (40)$$

$$(41)$$

Finally we can write:

$$k_L = k / \ln \frac{C_{sat} - C(0)}{C_{sat} - C(L)} = \frac{\Gamma}{\rho L} \quad (42)$$

The global (equipment) mass transfer coefficient therefore depends on the film flow rate and on the length of the column.

d.

1. In this configuration, liquid drops enter from the top of the scrubber with a null VOC concentration, $C(0) = 0$. As they move down by gravity, they get rich in VOC concentration by effect of mass transfer through the interface, $A_{int} = \pi D_p^2/4$, with D_p diameter of the drop. $J(z)$ is the mass transferred per unit area of the drop. The mass balance of VOC written for the control volume associated to

a single drop that moves along the column is given by:

$$\frac{dVC(t)}{dt} = A_{int}J(z) \quad (43)$$

In this case, the mass transfer does not occur in stationary conditions for the single drop ($VdC/dt \neq 0$), while remaining stationary at the equipment level. If the gas is still, the drop would move down at the terminal speed

$$v_p = \tau_p g = \frac{\rho_p D_p^2}{18\mu} g \quad (44)$$

This speed must be compared with that of the gas rising in the column ($v_{gas} = Q/A$, with A section of the column). The velocity of the downward drops is given by $v'_p = v_p - v_{gas}$. In these conditions, the time coordinate (time from the entry of the drop in the column) and the spatial coordinate (space along the column) are linked to each other as:

$$z = v'_p t \rightarrow t = z/v'_p$$

and the balance equation can be rewritten as:

$$\frac{VdC(t)}{dt} = \frac{Vv'_p dC(z)}{dz} = A_{int}J(z) \quad (45)$$

$$\frac{dC(z)}{dz} = \frac{A_{int}}{Vv'_p} J(z) = \quad (46)$$

$$\frac{dC(z)}{dz} = \frac{\pi D_p^2 6}{\pi D_p^3 v'_p} D \frac{C_{sat} - C(z)}{D_p/2} \quad (47)$$

$$\frac{dC(z)}{dz} = \frac{12D}{D_p^2 v'_p} (C_{sat} - C(z)) \quad (48)$$

2. The equation in this form can be integrated in z .

$$\frac{dC(z)}{C_{sat} - C(z)} = \frac{12D}{D_p^2 v'_p} dz \quad (49)$$

$$\ln \frac{C_{sat} - C(z)}{C_{sat} - C(0)} = -\frac{12D}{D_p^2 v'_p} z \quad (50)$$

$$\frac{C_{sat} - C(z)}{C_{sat} - C(0)} = \exp\left(-\frac{12D}{D_p^2 v'_p} z\right) \quad (51)$$

We get:

$$C_{sat} - C(z) = (C_{sat} - C(0)) \exp\left(\frac{-12D}{D_p^2 v'_p} z\right) \quad (52)$$

from which it is possible to derive the relationship between VOC concentration in the drop at the base of the column and the size of the drop. If we write the terminal velocity v'_p as a function of D_p we can calculate the value of D_p which allows to achieve the mass transfer in the available length.

e.

1. The mass balance on a slice of liquid of height dz inside the bubble oxygenator provides:

$$0 = QC(z) - QC(z + dz) + KA_{int}(C_{sat} - C(z)) \quad (53)$$

which in differential form becomes

$$Q \frac{dC}{dz} dz = KA_{int}(C_{sat} - C(z)) \quad (54)$$

As for a packed column, the interface area can be calculated as:

$$A_{int} = adV = a \cdot Sdz \quad (55)$$

where a is the interface area for unit volume and S is the column section. a depends on the amount of gas present in the unit volume and on the size of gas bubbles. By rearranging the balance equation we obtain:

$$\frac{dC}{dz} = \frac{Ka}{v}(C_{sat} - C(z)) \quad (56)$$

where $v = Q/S$ is the surface velocity of the fluid in the column. This equation can be integrated by separation of variables taking into account the boundary condition $C(0) = 0$ at the base of the oxygenator and provides:

$$C(z) = C_{sat} \left(1 - \exp\left[-\frac{Kaz}{v}\right]\right) \quad (57)$$

where z is the position along the column.

2. The degree of oxygenation achieved is:

$$C(L) = C_{sat} \left(1 - \exp\left[-\frac{KaL}{v}\right]\right) = 0.08C_{sat} \quad (58)$$

3. Considering the mass of injected oxygen fixed, to maximize the transfer in the liquid phase we should generate the maximum interfacial area possible. It is preferable to generate bubbles of very small diameter which, with the same volume of inflated gas,

$$V_{O_2} = n_b \cdot \pi d_b^3 / 6 \quad (59)$$

allow to maximize the interfacial surface

$$A_{O_2} = n_b \cdot \pi d_b^2 \quad (60)$$

and the specific interfacial area value (per unit of volume of inflated gas):

$$a_{O_2} = \frac{n_b \cdot \pi d_b^2}{n_b \cdot \pi d_b^3 / 6} = 6/d_b \quad (61)$$

The specific area of the bubble is larger for smaller bubbles. However, there is a limit to the minimum bubble size below which the column will not work that is given by the impossibility for the bubbles to go up against the flow of liquid. To calculate the critical size of bubbles, we need to calculate the rising velocity:

$$v_{O_2} = \tau_p g = \frac{(\rho - \rho_g) d_b^2 g}{18\mu} \quad (62)$$

The critical diameter is the one for which $v_{O_2} < v$.

f.

1. The volumetric flow rate of gas to be processed is $Q = \dot{m}/WL = 2 \text{ m}^3/\text{s}$. Indicating as C , [kg/m^3], the concentration of H_2S in the gas and fixing the origin of the reference system at the gas inlet section, the mass balance written for the gas for a control volume of height dz of the biofilter gives:

$$0 = QC(z) - QC(z + dz) - KA_{int}(C(z) - C_{liq}) \quad (63)$$

which in differential form becomes

$$Q \frac{dC}{dz} dz = -KA_{int}(C(z) - C_{liq}) = -KA_{int}C(z) \quad (64)$$

As for a packed column, the interfacial area in the biofilter control volume control can be calculated as:

$$A_{int} = adV = a \cdot Sdz \quad (65)$$

where a is the interfacial area per unit volume and S is the section of the column. By rearranging the balance equation we obtain:

$$\frac{dC}{dz} = -\frac{Ka}{v}C(z) \quad (66)$$

where $v = Q/S$ is the surface velocity of gas in the column. This equation can be integrated by separation of variables taking into account the boundary condition $C(0) = C_{in}$ at the biofilter inlet and gives:

$$C(z) = C_{in} \exp\left(-\frac{Kaz}{v}\right) \quad (67)$$

2. Biofilter efficiency is defined as

$$\eta = \frac{\dot{m}_{\text{H}_2\text{S},in} - \dot{m}_{\text{H}_2\text{S},out}}{\dot{m}_{\text{H}_2\text{S},in}} = \quad (68)$$

$$\frac{C_{\text{H}_2\text{S},in} - C_{\text{H}_2\text{S},out}}{C_{\text{H}_2\text{S},in}} = 1 - \frac{C_{\text{H}_2\text{S},out}}{C_{\text{H}_2\text{S},in}} \quad (69)$$

If the abatement efficiency of the biofilter should be equal to $\eta = 0.9$, the concentration in the gas phase at biofilter outlet must be $C(z = H) = (1 - \eta)C_{in}$. From equation 67 we get:

$$\ln \frac{C_{in}}{C(z)} = \frac{Kaz}{v} \quad (70)$$

from which substituting the values we obtain $a = a_{min} = (v/KH) \ln(1./0.1) = 13815.5 \text{ m}^2/\text{m}^3$. Any packing having a specific area $a > a_{min}$ will be adequate to achieve mass transfer with the desired efficiency.

g.

1. The velocity of the washing liquid in the lifting tube is $v = 4Q/\pi D_{pipe}^2 = 1.52 \text{ m/s}$, which corresponds to turbulent flow ($Re = \rho_l v D_{pipe} / \mu_L = 152788$). Using the Blasius formula for the friction coefficient we get:

$$f = 0.079 \cdot Re^{-0.25} = 0.004 \quad (71)$$

Applying Bernoulli equation between the supply tank and the top of the column we have:

$$p_{atm} + \rho g H_{serb} + \Delta p_{pump} = \quad (72)$$

$$p_{atm} + \rho g H + 2f \frac{H}{D} \rho v^2 + \frac{1}{2} \rho v^2 \quad (73)$$

where Δp_{pump} is the pump head, in [Pa]. Given that the tank is at ground level ($H_{serb} = 0$) and that the contribution of the kinetic term is negligible compared to losses due to friction, we obtain:

$$\Delta p_{pump} = \rho g H + 2f \frac{H}{D} \rho v^2 = \quad (74)$$

$$1.962 \cdot 10^5 + 3696.64 = 1.9989 \cdot 10^5 \text{ Pa} \quad (75)$$

The first term (energy per unit volume needed for lifting the fluid) is much greater than the head loss term. The power of the pump that allows to lift the fluid is $P = \Delta p_{pump} \cdot Q = 2.398 \text{ kW}$.

2. To allow the sedimentation of limestone particles along the vertical stretch of the pipe, the deposition rate of the particles must be higher than the fluid entrainment to have net downward motion for the solid suspended phase. To calculate the sedimentation rate, we consider the balance of forces acting on the stationary particle, which are the weight force, the thrust of Archimedes (buoyancy) and the friction force. Taken a reference system oriented upward we have:

$$\pi \frac{D_p^3}{6} g(\rho - \rho_p) v_p + \frac{1}{2} C_D \pi \frac{D_p^2}{4} \rho (v - v_p) \|v - v_p\| = 0 \quad (76)$$

with $C_D = f(Re_p)$, from which

$$(v - v_p) = \sqrt{\frac{4(\rho_p - \rho)gD_p}{3C_D\rho}} \quad (77)$$

If particles move in the Stokes regime, $C_D = 24/Re_p$:

$$v - v_p = \frac{\rho_p D_p^2}{18\mu} \frac{\rho_p - \rho}{\rho_p} g = 1.079 \text{ m/s} \quad (78)$$

Calculating the particle Reynolds number, $Re_p = \rho D_p v_p / \mu = 2158$ we find that the velocity should be recalculated using the right formula for the drag coefficient C_D until we obtain convergence on the velocity value. In a pair of iterations we get $v - v_p = 0.148$ m/s. To have sedimentation it should be $v_p = v - 0.148 < 0$ (the particle velocity should be downward directed), therefore $v < 0.148$ m/s (the flow rate should be reduced 10.27 times).

3. The mass balance written in stationary conditions for a slice of liquid of height dx using a reference system centered at the liquid inlet section gives

$$0 = QC(x) - QC(x + dx) + KA_{int}(C_{sat} - C(x)) \quad (79)$$

where $A_{int} = \pi D_{col} dx$. Rearranging the terms we get:

$$Q \frac{dC(x)}{dx} dx = KA_{int}(C_{sat} - C(x)) \quad (80)$$

The boundary condition is $C(x = 0) = 0$. If K is constant, separating the variables and integrating we get:

$$\ln \frac{C_{sat} - C(x)}{C_{sat} - C(0)} = -\frac{K\pi D_{col} x}{Q} \quad (81)$$

and

$$C(x) = C_{sat} \left(1 - \exp \left(-\frac{K\pi D_{col} x}{Q} \right) \right) \quad (82)$$

4. If K depends on the thickness and varies with the position along the column, x , it is necessary to separate the variables appropriately before integrating the differential equation:

$$\frac{dC(x)}{C_{sat} - C(x)} = \frac{\pi D_{col}}{Q} K(x) dx = Ax^{-0.5} dx \quad (83)$$

where $A = \pi D_{col} / Q \cdot (6D\Gamma / \pi\rho\delta)^{0.5}$ is function of Q ($A \propto Q^{-1/3}$) and yet constant in x . Upon integration we get:

$$\ln \frac{C_{sat} - C(x)}{C_{sat} - C(0)} = -2Ax^{0.5} \quad (84)$$

from which

$$C(x) = C_{sat} \left(1 - \exp(-2Ax^{0.5}) \right) \quad (85)$$

Since $A \propto Q^{-1/3}$, a flow rate (and film thickness) reduction leads to a reduction of the column height required to achieved the target absorption.

h.

1. The mass balance written in stationary conditions for a slice of liquid of height dx using a reference system centered at the liquid inlet section gives:

$$0 = QC(x) - QC(x + dx) + kA_{int}(C_{sat} - C(x)) \quad (86)$$

where $A_{int} = aSdx$ and S is the column section. Rearranging the terms we get:

$$Q \frac{dC(x)}{dx} dx = kaS(C_{sat} - C(x)) \quad (87)$$

The boundary condition is $C(x = 0) = 0$. The correlation available to calculate the value of k indicates that k is constant along x , but depends on the packing:

$$k = 25 \frac{D}{d} \left(\frac{dv_o}{\nu} \right)^{0.45} \cdot \left(\frac{\nu}{D} \right)^{0.5} = A \cdot d^{-0.55} \quad (88)$$

By separating the variables and integrating we get:

$$\ln \frac{C_{sat} - C(x)}{C_{sat} - C(0)} = -\frac{kaSx}{Q} \quad (89)$$

and

$$C(x) = C_{sat} \left(1 - \exp \left(-\frac{kaSx}{Q} \right) \right) \quad (90)$$

2. To identify the most suitable type of packing to use we must evaluate which value of ka (terms that depend on the packing) you must have to achieve the desired absorption. From Equation 101 we have:

$$\ln \frac{C_{sat} - C(H)}{C_{sat}} = \ln 0.25 = -\frac{kaSH}{Q} \quad (91)$$

from which we get

$$ka = -\frac{Q}{SH} \ln 0.25 = 0.00346 \quad (92)$$

which is the minimum value of the product for which is possible to achieve the absorption target. Calculating the value of ka for each packing, those having a value larger than minimum are suitable packings.

From calculation we get: and only the third and fourth

d	k	a	ak
$5.0 \cdot 10^{-2}$	$1.28 \cdot 10^{-5}$	$1.05 \cdot 10^2$	$1.35 \cdot 10^{-3}$
$3.8 \cdot 10^{-2}$	$1.49 \cdot 10^{-5}$	$1.50 \cdot 10^2$	$2.23 \cdot 10^{-3}$
$2.5 \cdot 10^{-2}$	$1.88 \cdot 10^{-5}$	$2.50 \cdot 10^2$	$4.69 \cdot 10^{-3}$
$1.3 \cdot 10^{-2}$	$2.69 \cdot 10^{-5}$	$4.65 \cdot 10^2$	$1.25 \cdot 10^{-2}$

turn out to be adequate for the absorption task.

i.

1. To write the mass balance we consider the control volume associated to an air bubble rising inside the tank. From the mass conservation of NH_3 present in the gaseous phase, taken x as the vertical with $x = 0$ at the base of the tank, we have

$$\frac{dV_b C(x)}{dt} = kA_{int}(C_{int} - C(x)) \quad (93)$$

where $V_b = \pi d_b^3/6$ is the bubble volume and $A_{int} = \pi d_b^2$ is the interfacial area available for the mass transfer. Over time, the bubble rises up along the column. The relationship between the position of the bubble and the time since injection is given by $x = v_b \cdot t$ if the bubble rises at a constant speed (equal to its terminal velocity). The rising velocity depends on the size of the bubble and assuming that the bubble moves in Stokes regime:

$$v_b = \tau_p g \frac{\rho - \rho_b}{\rho_b} = \frac{(\rho - \rho_b)gd_b^2}{18\mu} \quad (94)$$

By replacing space with time in the balance equation we obtain:

$$\frac{\pi d_b^3}{6} \frac{(\rho - \rho_b)gd_b^2}{18\mu} \frac{dC(x)}{dx} = k\pi d_b^2(C_{int} - C(x)) \quad (95)$$

Simplifying, separating the variables and integrating along the liquid column we get:

$$\frac{dC(x)}{C_{int} - C(x)} = \frac{108\mu k}{g\Delta\rho d_b^3} dx \quad (96)$$

from which

$$\ln \frac{C_{int} - C(x)}{C_{int} - C(0)} = -\frac{108\mu k}{g\Delta\rho d_b^3} x \quad (97)$$

from which the concentration profile of NH_3 stripped along the column is obtained:

$$C(x) = C_{int} \left(1 - \exp\left(-\frac{108\mu k}{g\Delta\rho d_b^3} x\right) \right) \quad (98)$$

2. For the given value of k , Equation 98 indicates that, the smaller is the bubble, the larger will be the concentration of stripped ammonia.

j.

1. The mass balance written in stationary conditions for the gas phase contained in a slice of column of height dx (reference system with origin at the gas inlet section) provides:

$$0 = QC(x) - QC(x + dx) - kA_{int}(C(x) - C_{int}) \quad (99)$$

where $A_{int} = aSdx$, S is the column section and a is the specific surface section of the packing. Rearranging the terms we have:

$$Q \frac{dC(x)}{dx} dx = kaS(C_{int} - C(x)) \quad (100)$$

The boundary condition is $C(x = 0) = C_{in} = 10 \mu g/m^3$ and we assume $C_{liq} = 0$ as suggested by the text. By separating the variables and integrating we get:

$$\ln \frac{C(x)}{C_{in}} = -\frac{kaSx}{Q} \quad (101)$$

from which

$$C(x) = C_{in} \exp\left(-\frac{kaSx}{Q}\right) \quad (102)$$

2. To calculate the amount of VOC removed, we need to evaluate the variation of the VOC present in gas phase between inlet and outlet of the washing column. The current flow rate of the gas stream is calculated by converting the Nm^3/s in m^3/s . The gas density at normal condition is

$$\rho_{cn} = \frac{Mp}{RT} = 1,17 \text{ kg}/m^3 \quad (103)$$

the gas density at actual (working) condition is

$$\rho = \frac{Mp}{RT_c} = 1,08 \text{ kg}/m^3 \quad (104)$$

and the volumetric flow rate is:

$$Q = \frac{\rho_{cn}Q_{cn}}{\rho} = 3,8 \text{ m}^3/s. \quad (105)$$

From problem data we have $C(H) = 0.14C_{in} = 1.4 \mu g/m^3$ and the amount of VOC absorbed is $\dot{m} = Q(C_{in} - C(H)) = 32.61 \mu g/s$.

3. To evaluate if the solution proposed by the consultant is adequate, you can recalculate the concentration value at the outlet if $a = a'$. It results $C(H) = 0.0196C_{in} = 0.196 \mu g/m^3$ which is less than the limit, therefore the solution is adequate.

k.

1. In the release test, the solid material is present in a packed form it is wetted by a liquid solvent able to extract the species of interest from the solid, accumulating them in the liquid phase. To limit the dimensions of the equipment, the liquid is recirculated, that is, at each passage the solvent wetting the solid is more and more concentrated in solute. If the amount of solute extracted from the solid phase does not substantially change the characteristics of the solid (C_{int} is constant), the solvent

recirculating column configuration is equivalent to a column configuration without recirculation but much longer, where the length depends on the velocity of the flow and the number of times it is recirculated during the test.

The mass balance written in stationary conditions for the liquid phase contained in a column slice of height dx (reference system with origin at the liquid inlet section) provides:

$$0 = QC(x) - QC(x + dx) + kA_{int}(C_{int} - C(x)) \quad (106)$$

where $A_{int} = aSdx$ with S column section and a specific surface area of packing. Rearranging the terms we have:

$$Q \frac{dC(x)}{dx} dx = kaS(C_{int} - C(x)) \quad (107)$$

The boundary condition is $C(x = 0) = 0$ and we assume C_{int} constant as suggested by the text. The length of the equivalent column (treatment without recirculation) depends on the liquid superficial velocity, $v_{liq} = Q_{lavaggio}/A = 0.0127 \text{ m/s}$ with $A = \pi D^2/4$, and on the duration of the test:

$$L_{eq} = v_{liq} t_{test} = 1100 \text{ m} \quad (108)$$

By separating the variables and integrating we get:

$$\ln \frac{C_{int} - C(x)}{C_{int}} = -\frac{kaSx}{Q} \quad (109)$$

from which

$$C(x) = C_{int} \left(1 - \exp\left(-\frac{kaSx}{Q}\right) \right) \quad (110)$$

2. To evaluate the transfer rate, we need to replace the numerical values in equation ?? to derive the value of k :

$$k = \ln \frac{C_{int}}{C_{int} - C(H')} \frac{Q}{aS H'} \quad (111)$$

where $C(H')$ is species concentration in the eluate at the end of the test. From calculation we get $k = 1.158 \cdot 10^{-10} \text{ m/s}$.

l. _____

1. To write the mass balance we consider the control volume associated to a nitrogen bubble rising

into the tank. From the mass conservation of NH_3 present in the gaseous phase, taken x along the vertical with $x = 0$ at the base of the tank, we have:

$$\frac{dV_b C(x)}{dt} = kA_{int}(C_{NH_3,int} - C(x)) \quad (112)$$

where $V_b = \pi d_b^3/6$ is the bubble volume and $A_{int} = \pi d_b^2$ is the interfacial area available for mass transfer. Over time, the bubble rises up along the column. The relationship between the position of the bubble and the time since injection is given by $x = v_b \cdot t$ if the bubble rises at constant speed (equal to its terminal velocity). The rising velocity depends on bubble size and assuming that the bubble moves in Stokes regime:

$$v_b = \tau_p g \frac{\rho - \rho_b}{\rho_b} = \frac{(\rho - \rho_b) g d_b^2}{18\mu} \quad (113)$$

By replacing space with time in the balance equation we obtain:

$$\frac{\pi d_b^3}{6} \frac{(\rho - \rho_b) g d_b^2}{18\mu} \frac{dC(x)}{dx} = k \pi d_b^2 (C_{NH_3,int} - C(x)) \quad (114)$$

Simplifying, separating the variables and integrating along the stripping column we get:

$$\frac{dC(x)}{C_{NH_3,int} - C(x)} = \frac{108\mu k}{g\Delta\rho d_b^3} dx \quad (115)$$

and

$$\ln \frac{C_{NH_3,int} - C(x)}{C_{NH_3,int} - C(0)} = -\frac{108\mu k}{g\Delta\rho d_b^3} x \quad (116)$$

from which we can calculate the concentration profile of NH_3 stripped along the column:

$$C(x) = C_{NH_3,int} \left(1 - \exp\left(-\frac{108\mu k}{g\Delta\rho d_b^3} x\right) \right) \quad (117)$$

2. For the given value of k and the desired NH_3 concentration for outgoing bubbles, equation 116 allows to derive the size of the bubble needed to carry out the transfer: the smaller the bubble the larger the concentration of stripped ammonia. Replacing numerical values we get $d_b = 84.4 \mu\text{m}$. This is the maximum value of bubble diameter to inject to achieve ammonia concentration reduction in the liquid phase.