Solution of Homework N^o 3: multiphase flow transport systems

a.

1. To identify the flow regime we need to calculate gas and liquid superficial velocities which are given by:

$$U_{Oil,sup} = \frac{W_{oil}}{\rho_{oil} A} = 0.69 \; m/s \tag{1}$$

$$U_{Gas,sup} = \frac{W_{gas}}{\rho_{gas} A} = 0.35 \; m/s \tag{2}$$

From Mandane map, these superficial velocity values identify an intermittent flow regime (elongated bubble flow).

2. Reynolds numbers are:

$$Re_{oil} = \frac{U_{oil,sup}\rho_{oil} D}{\mu_{oil}} = 2.75 \cdot 10^4 \tag{3}$$

$$Re_{gas} = \frac{U_{gas,sup}\rho_{gas} D}{\mu_{gas}} = 3.099 \cdot 10^4 \tag{4}$$

indicating turbulent flow for both phases. Friction factor coefficients calculated using Blasius law are

$$f_{oil} = 0.079 \cdot Re_{oil}^{-0.25} = 0.00613 \tag{5}$$

$$f_{gas} = 0.079 \cdot Re_{gas}^{-0.25} = 0.00595 \tag{6}$$

Pressure losses for monophase oil and gas flows are:

$$\Delta p_{oil} = 2 f_{oil} \frac{L}{D} \rho_{oil} v_{oil}^2 = 55679.5 \; Pa \tag{7}$$

$$\Delta p_{gas} = 2 f_{gas} \frac{L}{D} \rho_{gas} v_{gas}^2 = 32.17 \; Pa \tag{8}$$

Martinelli parameter is $X = \sqrt{\Delta p_{oil}/\Delta p_{gas}} = 41.60$. The pressure drop correction factor is

$$\Phi_L(X)^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \tag{9}$$

and using $C = 20$ for turbulent gas and turbulent liquid flow, we get $\Phi_L^2 = 1.48$ and the two phase pressure drop is:

$$\Delta P_{TPF} = \Phi_L(X)^2 \Delta p_{oil} = 82480.5 \; Pa \tag{10}$$

b.

1. The expected flow regime depends on the values of gas and liquid superficial velocities:

$$U_{L,sup} = \frac{W_L}{\rho_L A} = 1.00 \; m/s \tag{11}$$

$$U_{G,sup} = \frac{W_G}{\rho_G A} = 10.00 \; m/s \tag{12}$$

From Mandane map the flow regime is intermittent (slug flow).

2. In intermittent flow regimes, especially in the slug flow, the flow transport system is prone to mechanical stresses and large pressure drops. To avoid these flow conditions, we can choose to conveying the multiphase mixture using a larger (or smaller) pipe diameter. If we choose a pipe diameter three times larger ($D = 0.6 \; m$) we move toward stratified flow conditions (moving to the bottom left corner in the map). In these conditions, superficial flow velocities are: $U_{L,sup} = 0.11 \; m/s$ and $U_{G,sup} = 1.11 \; m/s$, corresponding to stratified flow on the map.

3. Reynolds number are

$$Re_L = \frac{U_{L,sup}\rho_L D}{\mu_L} = 7.026 \cdot 10^4 \tag{13}$$

$$Re_G = \frac{U_{G,sup}\rho_G D}{\mu_G} = 6.224 \cdot 10^4 \tag{14}$$

indicating turbulent flow in the gas and in the liquid phase. Friction factors f calculated using Blasius law are:

$$f_L = 0.079 \cdot Re_L^{-0.25} = 0.00485 \tag{15}$$

$$f_G = 0.079 \cdot Re_G^{-0.25} = 0.00500 \tag{16}$$

and monophase pressure losses in the liquid and gas are:

$$\Delta p_L = 2 f_L \frac{L}{D} \rho_L v_{Ll}^2 = 9.45 \; Pa \tag{17}$$

$$\Delta p_G = 2 f_G \frac{L}{D} \rho_G v_G^2 = 1.44 \; Pa \tag{18}$$

from which we calculate the Martinelli parameter $X = \sqrt{\Delta p_L/\Delta p_G} = 2.56$. Using the analytical formula to calculate the pressure drop correction factor we get

$$\Phi_L(X)^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \tag{19}$$

and using $C = 20$ we get $\Phi_L^2 = 8.96$. The two phase flow pressure drop is:

$$\Delta P_{TPF} = \Phi_L(X)^2 \Delta p_L = 84.75 \; Pa \tag{20}$$

4. Additional pressure drops to lift the two phase mixture at height 2 m depends on liquid hold-up. Using the analytical formula to calculate the liquid hold-up we get

$$\lambda_L(X) = 0.186 + 0.0191 \cdot X \quad \text{for} \;\; 1 < X < 5 \tag{21}$$

and

$$\Delta P_{TPF,Vert} = \lambda_L(X) g \rho_l H = 4378.54 \; Pa \tag{22}$$

The additional pressure drop to lift the fluid is larger than viscous pressure losses.

c.

To calculate the pressure drop for the pneumatic transport of carbon particles we use the equations available from the slides. From problem data we can calculate:

1. Mass loading and saltation velocity:

$$Z = W_s/W_g = 19.3 = 1/10^\delta \cdot Fr^x \quad (23)$$
$$\delta = 1.44D_p + 1.96 = 1.96 \quad \text{con } D_p = [m] \quad (24)$$
$$x = 1.1 * (10^3 \cdot D_p) + 2.5 = 2.82 \quad \text{con } D_p = [m] \quad (25)$$

   from which we can calculate

$$Fr = (10 \cdot 10^\delta)^{1/x} = 14.18 \quad (26)$$
$$U_{G,salt} = Fr \cdot (gD)^{0.5} = 6.77\ m/s \quad (27)$$

   For the pneumatic transport system to work continuously, avoiding particle deposition at pipe walls, the gas velocity should be $U_{G,sup} \geq 2U_{G,salt}$.

2. Superficial velocity of gas and solid phases

$$U_{G,sup} = \frac{W_g}{\rho A} = 59.8\ m/s \quad (28)$$
$$U_{S,sup} = \frac{W_s}{\rho_p A} = 0.997\ m/s \quad (29)$$

   The condition $U_{G,sup} > 2U_{G,salt}$ is satisfied.

3. Volumetric fractions:

$$Q_G = \frac{W_g}{\rho} = 2.485 \cdot 10^{-2} m^3/s \quad (30)$$
$$Q_S = \frac{W_s}{\rho_p} = 4.143 \cdot 10^{-4} m^3/s \quad (31)$$
$$\epsilon = \frac{Q_G}{Q_G + Q_S} = 0.984 \quad (32)$$
$$\epsilon_p = 1 - \epsilon = 0.164 \quad (33)$$

4. Effective flow velocities:

$$U_{G,eff} = U_{G,sup}/\epsilon = 60.82\ m/s \quad (34)$$
$$U_{S,eff} = U_{S,sup}/\epsilon_p = 60.82\ m/s \quad (35)$$

5. Calculation of the different terms (inertial, frictional) contributing to he overall pressure drop for the gas-solid system:
   pressure losses due to fluid/particles acceleration

$$\Delta P_{acc,gas} = 0.5 \cdot \rho \cdot \epsilon U^2_{G,eff} = 2.196 \cdot 10^3 Pa \quad (36)$$
$$\Delta P_{acc,part} = 0.5 \cdot \rho_p \cdot \epsilon_p U^2_{S,eff} = 4.245 \cdot 10^4 Pa \quad (37)$$

   pressure losses due to fluid friction at wall

$$\Delta P_{att,gas} = 2f\frac{L}{D}\rho U^2_{G,eff} \quad (38)$$
$$(39)$$

   where f is given by Blasius law:

$$Re_G = \frac{U_{G,eff}\rho D}{\mu} = 9.38 \cdot 10^4 \quad (40)$$
$$f = 0.079 \cdot Re^{-0.25} = 4.51 \cdot 10^{-3} \quad (41)$$
$$\Delta P_{att,gas} = 2.628 \cdot 10^3 Pa \quad (42)$$

   pressure losses due to solid friction

$$\Delta P_{att,part} = f_s \cdot Z\frac{L}{2D}\rho U^2_{G,eff} \quad (43)$$

   where f_s is given as an empirical function of particle settling velocity

$$f_s = 0.082 \cdot Z^{-0.3} Fr^{-0.86} Fr_s^{0.25} (D/D_p)^{0.1} \quad (44)$$

   where

$$U_{S,term} = g\frac{\rho_p D_p^2}{18\mu} = 3.565\ m/s \quad (45)$$
$$Fr_S = U_{S,term}/(gD)^{0.5} = 7.505 \quad (46)$$
$$Fr = U_{G,eff}/(gD)^{0.5} = 128.04 \quad (47)$$
$$f_S = 1.33 \cdot 10^{-3} \quad (48)$$

   from which we get

$$\Delta P_{att,part} = 3.748 \cdot 10^3\ Pa \quad (49)$$

6. Overall pressure drops are given by:

$$\Delta P_{TOT} = \Delta P_{acc,gas} + \Delta P_{acc,part} + \quad (50)$$
$$+\Delta P_{att,gas} + \Delta P_{att,part} = 5.1026 \cdot 10^4 Pa \quad (51)$$

Comparing the contribution of the different terms in this case (short length of pipe) we find that the pressure loss due to solid acceleration is prevailing.

d.

1. To verify if the pneumatic transport is in the dilute regime we need to calculate the solid to gas mass flow rates ratio. The gas mass flow rate is:

$$W_g = \dot{m}_{aria} = \rho_{cn} \cdot Q \quad (52)$$

   where Q is the volumetric flow rate in Nm^3/s and ρ_{cn} is the air density evaluated at normal temperature and pressure ($p = 10^5\ Pa$, $T = 273\ K$). Density at normal condition is calculated using the ideal gas law:

$$\rho_{cn} = \frac{pM}{RT} = 1.27\ kg/m^3 \quad (53)$$

   from which we get $\dot{m}_{aria} = 37.976\ kg/s$. The mass flow rate of fibers is $W_s = \dot{m}_{sol} = 7.683\ kg/s$ and the mass loading ratio, $Z = \dot{m}_{sol}/\dot{m}_{aria}$ is 0.202 (¡1, dilute flow).

2. To calculate the pressure drop for the pneumatic transport of fibers we need the equations available from the slides. From problem data we calculate:

(a) Mass loading and saltation velocity:

$$Z = W_s/W_g = 0.202 = 1/10^\delta \cdot Fr^x \quad (54)$$
$$\delta = 1.44 D_p + 1.96 = 1.96 \quad (55)$$
$$x = 1.1 * (10^3 \cdot D_p) + 2.5 = 4.70 \quad (56)$$

from which we calculate

$$Fr = (0.202 \cdot 10^\delta)^{1/x} = 1.862 \quad (57)$$
$$U_{G,salt} = Fr \cdot (gD)^{0.5} = 6.683\ m/s \quad (58)$$

To work properly, avoiding particles deposition at pipe wall, the gas velocity should be $U_{G,sup} \geq 2U_{G,salt}$.

(b) Superficial velocity of gas and solid flows:

$$U_{G,sup} = \frac{W_g}{\rho A} = 27.325\ m/s \quad (59)$$
$$U_{S,sup} = \frac{W_s}{\rho_p A} = 1.65 \cdot 10^{-2}\ m/s \quad (60)$$

(c) Solid and gas volume fractions:

$$Q_G = \frac{W_g}{\rho} = 36.269\ m^3/s \quad (61)$$
$$Q_S = \frac{W_s}{\rho_p} = 2.20 \cdot 10^{-2} m^3/s \quad (62)$$
$$\epsilon = \frac{Q_G}{Q_G + Q_S} = 0.999 \quad (63)$$
$$\epsilon_p = 1 - \epsilon = 6.05 \cdot 10^{-4} \quad (64)$$

(d) Effective velocities of gas and solid:

$$U_{G,eff} = U_{G,sup}/\epsilon = 27.34\ m/s \quad (65)$$
$$U_{S,eff} = U_{S,sup}/\epsilon_p = 27.34\ m/s \quad (66)$$

(e) Calculation of the terms (inertial, viscous) contributing to overall pressure drop:
pressure drops due to particles/fluid acceleration

$$\Delta P_{acc,gas} = 0.5 \cdot \rho \cdot \epsilon U^2_{G,eff} = 3.91 \cdot 10^2 Pa \quad (67)$$
$$\Delta P_{acc,part} = 0.5 \cdot \rho_p \cdot \epsilon_p U^2_{S,eff} = 7.91 \cdot 10^1 Pa \quad (68)$$

pressure losses due to fluid/particles acceleration

$$\Delta P_{att,gas} = 2f\frac{L}{D}\rho U^2_{G,eff} \quad (69)$$
$$(70)$$

where f is given by Blasius law:

$$Re_G = \frac{U_{G,eff}\rho D}{\mu} = 2.067 \cdot 10^6 \quad (71)$$
$$f = 0.079 \cdot Re^{-0.25} = 2.083 \cdot 10^{-3} \quad (72)$$
$$\Delta P_{att,gas} = 4.265 \cdot 10^2 Pa \quad (73)$$

pressure loss due to solid friction

$$\Delta P_{att,part} = f_s \cdot Z\frac{L}{2D}\rho U^2_{G,eff} \quad (74)$$

where the solid friction factor is calculated from the empirical formula as a function of particle settling velocity

$$f_s = 0.082 \cdot Z^{-0.3} Fr^{-0.86} Fr_s^{0.25} (D/D_p)^{0.1} \quad (75)$$

where

$$U_{S,term} = g\frac{\rho_p D_p^2}{18\mu} = 42.389\ m/s \quad (76)$$
$$Fr_S = U_{S,term}/(gD)^{0.5} = 11.87 \quad (77)$$
$$Fr = U_{G,eff}/(gD)^{0.5} = 7.656 \quad (78)$$
$$f_S = 8.159 \cdot 10^{-2} \quad (79)$$

From calculations we get

$$\Delta P_{att,part} = 8.4486 \cdot 10^2\ Pa \quad (80)$$

(f) Overall pressure drop for the pneumatic transport of fibers is:

$$\Delta P_{TOT} = \Delta P_{acc,gas} + \Delta P_{acc,part} + \quad (81)$$
$$+\Delta P_{att,gas} + \Delta P_{att,part} = 1.7416 \cdot 10^3 Pa \quad (82)$$

(g) If there are bends at the end of the line, additional pressure drops will be proportional to the pressure losses calculated for the gas phase along an horizontal pipe whose equivalent length depends on the solid mass flow rate conveyed. From the graph available in the slides, the equivalent length for each bend (90^o, wide radius, $R = 5\ D$) is $L_{eq} = 16\ m$ for the given solid mass flow rate ($\simeq 7.7\ kg/s$). Additional pressure drops are given by

$$\Delta P_{bend} = 2f\frac{L_{eq}}{D}\rho U^2_{G,eff} \cdot n_{bend} = 160.56\ Pa \quad (83)$$

where $L_{eq} = 17\ m$ and $n_{bend} = 4$.

In this case the overall pressure drop becomes

$$\Delta P_{TOT} = \Delta P_{acc,gas} + \Delta P_{acc,part} + \quad (84)$$
$$+\Delta P_{att,gas} + \Delta P_{att,part} + \Delta P_{bend} = 1.902 \cdot 10^3 Pa \quad (85)$$

Comparing the different terms, the one prevailing is due to the solid friction.