Solution of Homework N^o 1: storage/transport of incompressible fluids

a.

1. Given the pipe diameter D and the volumetric flow rate Q to be pumped we can calculate the fluid velocity along the pipeline $v = Q/A = 4Q/\pi D^2 = 2.55\ m/s$. Bernoulli equation written between point (A) at the free surface of the tank and point (B) at the pipeline outlet we get:

$$p_A + \frac{1}{2}\rho v_A^2 + \rho g h_A + \Delta p_{pump} = p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B + l_v \quad (1)$$

where Δp_{pump} is the prevalence of the pump (i.e. the energy per unit volume added to the fluid). The equation can be simplified assuming $v_B = v$, $p_A = p_B = p_{atm}$ and $v_A = 0$ in the tank:

$$\Delta p_{pump} = \frac{1}{2}\rho v^2 + \rho g(h_B - h_A) + l_v \quad (2)$$

Viscous losses are given by:

$$l_v = 2f\frac{L}{D}\rho v^2 \quad (3)$$

with f given by Blasius law for a smooth pipe:

$$f = 0.079 Re^{-0.25} = 0.079\left(\frac{\rho v D}{\mu}\right)^{-0.25} = 0.0035 \quad (4)$$

The prevalence of the pump is $\Delta p_{pump} = 2.08 \cdot 10^5\ Pa$ and the pumping power is:

$$P = Q \cdot \Delta p_{pump} = 4.17\ kW \quad (5)$$

2. For a rough pipe with surface roughness $k = 1\ mm$, we need to solve Colebrook equation:

$$\frac{1}{\sqrt{f}} = -1.7\ln\left(\frac{k}{D} + \frac{4.67}{Re\sqrt{f}}\right) + 2.28 \quad (6)$$

which can be iteratively solved starting from the value of f calculated for a smooth pipe. After a few iterations we get $f = 0.00985$ and $P = 4.5\ kW$.

b.

1. The condensate extraction system can be sketched as a number of pipe segments (each $\Delta h = 4\ m$ long) of different diameter connected in series. At each connection, the flow rate to be drained increases by a quote represented by the flow rate drained by a single gutter. The surface on which the condensate drained by each gutter forms is:

$$S_c = \pi D \Delta h = 100.53\ m^2 \quad (7)$$

where Δh is the distance between gutters. The volumetric flow rate drained by each gutter is

$$Q_c = \frac{\dot{q} S_c}{\rho} = 1.01 \cdot 10^{-3}\ m^3/s \quad (8)$$

For each vertical segment of the draining pipe, the minimum pipe diameter is fixed by equating the pressure drop due to viscous losses to the variation of potential energy

$$\rho g \Delta h = 2f\frac{\Delta h}{D}\rho v^2 \quad (9)$$

Writing the fluid velocity as a function of flow rate and using Blasius equation for the friction factor we get

$$D = \left(k\rho^{-0.25}\mu^{0.25}Q^{1.75}\right)^{1/4.75} \quad (10)$$

where $k = 0.0245$ is a constant value. Equation 10 fixes, for each flow rate, the minimum diameter of the draining pipe. Since the flow rate increases stepwise at each gutter discharge point moving from the top to the bottom of the tower, we have $Q = 1.01 \cdot 10^{-3} m^3/s$ and $D_{I^a}^{min} = 0.017m$ for the first draining pipe (at the top) and $Q_{tot} = n_c \cdot Q_c = 2.02 \cdot 10^{-2} m^3/s$ and $D_{n_c}^{min} = 0.053m$ at the bottom of the tower.

c.

1. From the volume of the tank to be filled (B) and the time for re-filling, we calculate the flow rate as:

$$Q = \frac{V_B}{t} = 0.0044\ m^3/s \quad (11)$$

During the tank loading, the fluid velocity is:

$$v = \frac{4Q}{\pi D^2} = 0.56\ m/s \quad (12)$$

Bernoulli equation written between tank A and B gives:

$$p_A + \frac{1}{2}\rho v_A^2 + \rho g h_A + \Delta p_{pump} = p_B + \frac{1}{2}\rho v_B^2 + \rho g h + l_v \quad (13)$$

Since $p_A = p_B = p_{atm}$ and the fluid velocity is negligible ($\simeq 0$) inside the tanks the prevalence of the pump is given by

$$\Delta p_{pump} = \rho g(h - h_A) + l_v \quad (14)$$

The energy per unit volume given by the pump is needed to lift the fluid up to tank B winning the viscous losses along the pipeline. Viscous losses are given by:

$$l_v = 2f\frac{L}{D}\rho v^2 \tag{15}$$

where the friction factor given by Blasius is $f = 0.079Re^{-0.25} = 0.0051$ and $l_v = 4510\ Pa$. The energy per unit volume required for lifting is $\rho g(h - h_A) = 5.788 \cdot 10^5 Pa$ (larger than viscous losses) and we calculate $\Delta p_{pump} = 5.83300 \cdot 10^5\ Pa$ and $P = Q \cdot \Delta p_{pump} = 2.6\ kW$.

d.

1. Pipeline cost includes capital costs (for piping and power station equipments) and operating costs (energy to run the pumping station) given by:

$$C_I = \frac{k_t D L_{tot} + k_p P/10^3}{N_y} \text{ on annual basis} \tag{16}$$

$$C_E = k_e N_h P/10^3 \tag{17}$$

$$C_{tot} = C_I + C_E = \frac{k_t DL + k_p P/10^3}{N_y} + k_e N_h P/10^3 = \tag{18}$$

$$= \frac{k_t L}{N_y} D + \left(\frac{k_p}{N_y} + k_e N_h\right)\frac{P}{10^3} \tag{19}$$

where $P = \Delta pQ/\eta$ is the pumping power of the pump to be installed. Named 1 the node upstream the pipe loop and 2 the node downstream the pipe loop, the overall length of the pipe is $L_{tot} = L_{A1} + 2L_{12} + L_{2B} = 10520\ m$. If the branch of the loop without the pump is closed by a valve, the prevalence of the pump is given by Bernoulli equation written between tank A and B

$$\rho g h_A + \Delta p_{pump} = 2f\frac{L'}{D}\rho v^2 \tag{20}$$

where $L' = 6520\ m$ is the length of pipeline generating viscous losses, the pressure above tank A and B is the same and the velocity is negligible. We get

$$\Delta p_{pump} = 2f\frac{L'}{D}\rho v^2 - rhogh_A \geq 0 \tag{21}$$

If the pipeline diameter is large enough, we do not need a pump to push the fluid from A to B. Equation 21 equated to 0 can be solved to compute how large the pipe diameter should be. Using Blasius equation to calculate f and writing the velocity as a function of flow rate $v = 4Q/\pi D^2$ we get:

$$2 \cdot 0.079\left(\frac{4\rho}{\pi\mu}\right)^{-0.25}\left(\frac{4}{\pi}\right)^2 L'Q^{1.75}\frac{1}{D^{4.74}} = \tag{22}$$

$$= \frac{k}{D^{4.74}} = gh_A \tag{23}$$

where $k = 167.22$ from which we get $D = 0.967\ m$. If $D > 0.967\ m$ we do not need to install a pump. This option could be economically optimum (i.e. the cheapest) if the pipeline cost is lower than the costs necessary to install and operate the pump. To calculate the economically optimal diameter of the pipeline we need to calculate the pumping power and to write overall costs as a function of pipeline diameter. The pumping power to be installed is $P = \Delta pQ/\eta$, given by

$$P = \frac{\rho Q}{\eta}\left(\frac{k}{D^{4.74}} - gh_A\right) = k_1 D^{-4.75} - k_2 \tag{24}$$

which for $\eta = 1$ gives $k_1 = 3.3444 \cdot 10^5$ and $k_2 = 3.924 \cdot 10^5$. Overall costs are written as

$$C_{tot}(D) = k_a D + k_b P = k_a + k_b(k_1 D^{-4.75} + k_2) \tag{25}$$

where $k_a = 9.468 \cdot 10^5$ and $k_b = 0.925$. The optimum diameter minimizing the overall costs can be calculated from:

$$\frac{\mathrm{d}C_{tot}(D)}{\mathrm{d}D} = k_a - 4.75k_b \cdot k_1 D^{-5.75} = 0 \tag{26}$$

$$\frac{\mathrm{d}C_{tot}(D)}{\mathrm{d}D} = k_a - 4.75k_b \cdot k_1 D^{-5.75} = 0 \tag{27}$$

and is $D = 1.079\ m$. Using this value of D the pumping power is negative, meaning that the pump is not at all required (D is larger than the minimum D able to convey the flow under gravity). Therefore, the optimum pipe diameter is $D = 0.967\ m$ (or the larger commercial size) and no pump should be installed.

2. The pump is not required. The overall cost is $C_{tot} = 915.55\ kEuro$.

3. From the previous point we know that if the same flow rate of fluid flows along a single branch of the loop we do not need a pump. If the fluid can flow through both branches of the loop and they have the same D and L, the flow rate will divide half on the top and half on the bottom branch. Bernoulli equation written between tanks A and B gives:

$$\rho g h_A = 2f_{A1}\frac{L_{A1} + L2B}{D}\rho v^2 + 2f_{12}\frac{L_{12}}{D}\rho v_{12}^2 \tag{28}$$

where $v_{12} = v/2$. Using Blasius for f and writing v as a function of Q we get:

$$gh_A = 2 \cdot 0.079\left(\frac{4\rho}{\pi\mu}\right)^{-0.25}\left(\frac{4}{\pi}\right)^2 \cdot \tag{29}$$

$$\cdot Q^{1.75}(LA1 + 2B + L_{12}2^{-1.75})\frac{1}{D^{4.74}} = k_c\frac{1}{D^{4.74}} \tag{30}$$

with $k_c = 95.133$ from which we calculate $D = 0.858\ m$ (lower than before). Plant cost becomes $812.972\ kEuro$.

Without any prior knowledge about the plant, we can solve the problem writing one Bernoulli equation for each branch of the pipe and the continuity equation for one of the two nodes of the loop. These equations are: Bernoulli A-1:

$$p_{atm} + \rho g h_A = p_1 + 2f\frac{L_{A1}}{D}\rho\left(\frac{4Q}{\pi D^2}\right)^2 \quad (31)$$

Bernoulli 2-B:

$$p_2 = p_{atm} + 2f\frac{L_{2B}}{D}\rho\left(\frac{4Q}{\pi D^2}\right)^2 \quad (32)$$

Bernoulli 1-2 (branch without the pump)

$$p_1 = p_2 + 2f\frac{L_{12}}{D}\rho\left(\frac{4QK_Q}{\pi D^2}\right)^2 \quad (33)$$

Bernoulli 1-2 (branch with pump)

$$p_1 + \Delta p_{pump} = p_2 + 2f\frac{L_{12}}{D}\rho\left(\frac{4Q(1-K_Q)}{\pi D^2}\right)^2 \quad (34)$$

In Equations 33 and 34 the flow rate along each branch is unknown. Selecting the branch without the pump as a reference point, the flow rate along this branch can be indicated as a fraction K_Q of the flow rate flowing from A to B. From the continuity equation at node 1, in the branch with the pump the flow rate will be $(1-K_Q)Q$. Matching Bernoulli equations 31 and 32, the prevalence of the pump can be written as a function of pipe diameter as

$$p_1 - p_2 = \rho g h_A - 2f\frac{L_{A1}+L_{2B}}{D}\rho\left(\frac{4Q}{\pi D^2}\right)^2 = \quad (35)$$

$$= \rho g h_A - k(L_{1A}+L_{2B})D^{-4.75} \quad (36)$$

and from 34

$$\Delta p_{pump} = 2f\frac{L_{12}}{D}\rho\left(\frac{4Q(1-K_Q)}{\pi D^2}\right)^2 - (p_1 - p_2) = \quad (37)$$

$$= kL_{12}[(1-K_Q)^{1.75} - K_Q^{1.75}]D^{-4.75} = \quad (38)$$

$$= kL_{12}(1-2K_Q)^{1.75}D^{-4.75} \quad (39)$$

from which we can calculate the pumping power $P = kL_{12}(1-K_Q)Q(1-2K_Q)^{1.75}D^{-4.75}$ which is a function of pipe diameter and K_Q. Combining equations 36 and 33 we derive the relationship between K_Q and D:

$$\rho g h_A - k(L_{1A}+L_{2B})D^{-4.75} = k(K_Q^{1.75}L_{12})D^{-4.75} \quad (40)$$

and

$$L_{1A} + L_{2B} + K_Q^{1.75}L_{12} = \frac{\rho g h_A}{k}D^{4.75} = \quad (41)$$

from which $K_Q = f(D)$. This equation indicates that, depending on the pipe diameter and the prevalence of the pump, the flow rate can divide differently along the two branches. Overall cost, which is a function of D and K_Q, can be derived as a composite function:

$$\frac{\mathrm{d}C_{tot}(D,K_Q(D))}{\mathrm{d}D} = \frac{\mathrm{d}C_{tot}(D,K_Q(D))}{\mathrm{d}K_Q}\frac{\mathrm{d}K_Q(D)}{\mathrm{d}D} = 0 \quad (42)$$

equated to zero to determine the value of D_{ott}.

e. ____________________________________

1. The oil volumetric flow rate is $Q = W/\rho = 1.875\ m^3/s$ and the velocity in the pipe is $v = 4Q/\pi D^2 = 1.53\ m/s$. Reynolds number is:

$$Re = \frac{\rho v D}{\mu} = 127500 \quad (43)$$

from which we can calculate a trial value for the friction factor f using Blasius equation, $f = 0.079Re^{-0.25} = 0.004$ Given $\epsilon = k/D$, from Colebrook equation

$$\frac{1}{\sqrt{f}} = -1.7\ln\left(\epsilon + \frac{4.67}{Re\sqrt{f}}\right) + 2.28 \quad (44)$$

we get after a few iterations $f = 0.0056$. The friction factor is larger than the value calculated for a smooth pipe.

The points along the pipeline where the static pressure could be the lowest are those at the end of long lines which are at highest quote. For the pipeline under investigation, the point of minimum static pressure is most likely located at the top of the hill, C. Bernoulli equation written between point A and a generic point along the pipeline gives:

$$p_A + \Delta p_{pump} = p(x) + \rho g h(x) + 2f\frac{x}{D}\rho v^2 + \frac{1}{2}\rho v^2 \quad (45)$$

where x is the distance from point A and $h(x)$ is the geometrical height of point x. The kinetic term is negligible compared to viscous losses (long pipeline) and the variation of the static pressure along the pipeline is:

$$p(x) = p_A + \Delta p_{pump} - \rho g h(x) - 2f\frac{x}{D}\rho v^2 \quad (46)$$

The static pressure decreases as the height of the pipe and the pipe length increase. Given that the energy loss due to viscous friction is generally lower than the energy required to lift the fluid, the highest geometrical quote is the point where the static pressure will be the minimum. Fixing $p_C = p_v = 2.4\ kPa$ and writing Bernoulli equation between A and C we get:

$$p_A + \Delta p_{pump} = p_C + \rho g h_C + 2f\frac{L_{AC}}{D}\rho v^2 \qquad (47)$$

from which $\Delta p_{pump} = 3.78307 \cdot 10^5 Pa$. If $p_C = 2.4\ kPa$, we can use Bernoulli written between C and B to verify that the fluid has enough energy in C to flow to B. Bernoulli from C to B gives:

$$p_C + \rho g h_C = p_B + 2f\frac{L_{CB}}{D}\rho v^2 \qquad (48)$$

from which

$$p_B = p_C + \rho g h_C - 2f\frac{L_{CB}}{D}\rho v^2 \qquad (49)$$

Being $L_{BC} < L_{AB}$, we can calculate $p_B = 3.44 \cdot 10^5\ Pa > p_{atm}$ which means that there is enough energy for the transport of fluid. The static pressure at the end of the pipeline is larger than the static pressure at the top of the hill. The pumping power is $P = Q \cdot \Delta p_{pump} = 709\ kW$.

f.

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1. The volumetric flow rate and the velocity in the pipelines entering/exiting from the loop are $Q = W/\rho = 0.02\ m^3/s$ and $v = Q/A = 4Q/\pi D^2 = 1.13\ m/s$. The Reynolds number is $Re = \rho v D/\mu = 169500$ and the friction factor, according to Blasius law, is $f = 0.079Re^{-0.25} = 0.0039$. Bernoulli equation written between tank A and the point upstream the loop (1) is:

$$p_A + \rho g h_A = p_1 + \frac{1}{2}\rho v_{A1}^2 + 2f\frac{L_{A1}}{D}\rho v_{A1}^2 \qquad (50)$$

where we assume that the velocity in the tank is negligible and the kinetic term is negligible compared to the viscous term. Using the numerical values we get $p_1 = 2.63 \cdot 10^5\ Pa$. Bernoulli equation written between the point downstream the loop (2) and tank B, we get:

$$p_2 = p_B + 2f\frac{L_{2B}}{D}\rho v_{2B}^2 = 2.3279 \cdot 10^5\ Pa \qquad (51)$$

where we neglect the kinetic term and assume $v_{A1} = v_{2B}$. Being $p_1 > p_2$, we know the flow direction along the two branches of the loop (from 1 to 2). Bernoulli equation written for the branch without the pump allows to calculate the flow rate along this branch:

$$p_1 = p_2 + 2f\frac{L_{12}}{D}\rho v_{12}^2 \qquad (52)$$

where using Blasius we get

$$v_{12} = \left[\frac{(p_1 - p_2)D^{1.25}}{0.158\rho^{0.75}\mu^{0.25}L_{12}}\right]^{\frac{1}{1.75}} = 0.32\ m/s \qquad (53)$$

From continuity equation written for one of the nodes (1 or 2) we get:

$$v_{A1} = v_{1P2} + v_{12} \rightarrow v_{1P2} = 0.81\ m/s \qquad (54)$$

from which $Re = 121500$ and $f = 0.0042$. Bernoulli equation written for the branch with the pump gives

$$\Delta p_{pump} = p_2 - p_1 + 2f\frac{L_{1P2}}{D}\rho v_{1P2}^2 = 1.178 \cdot 10^5 Pa \qquad (55)$$

from which we can calculate the pumping power $P = 1.686\ kW$.

2. Lets assume that the flow rate leaking from the pipe is small compared to the flow rate still moving along the pipe. In this condition, the leakage should not change significantly the pressure at nodes 1 and 2 and the flow rates along the branches of the plant. The pressure at point C is given by Bernoulli equation written between points C and 2

$$p_R = p_2 + 2f\frac{L_{R2}}{D}\rho v_{12}^2 = 2.6979 \cdot 10^5\ Pa \qquad (56)$$

If we use the orifice equation to calculate the outflow velocity from the hole we get

$$v_e = 0.62\sqrt{\frac{2(p_r - p_{atm})/\rho}{1 - (A_R/A)^2}} \qquad (57)$$

where A_R is the leakage cross section and A is the pipe cross section. Being $A_R/A = 0.0028$ we get $v_e = 18.42\ m/s$ and $Q_e = 0.92\ 10^{-3} m^3/s$. The flow rate along the branch 12 is $Q_{12} = 14.3 \cdot 10^{-3} m^3/s$, which is about 15 times larger than that leaking from the hole, therefore the leakage will not change significantly the static pressure in the pipeline.

If the leaking flow rate is significant compared to that circulating in the pipe without leakage, the initial assumption is wrong and the problem should be solved re-writing Bernoulli equations for each branch in the plant (A1, 1P2, 1R, R2, 2B), the continuity equation for each (-1) node of the plant (1, R and 2) and the orifice equation in R. In this way we get 5+2+1 equations in 8 unknowns (p_1, p_2, p_R, v_{A1}, v_{1R}, v_{R2}, v_{1P2} and v_e) which should be solved to find the new working conditions for the pipeline.

g.

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1. If valve V is closed, the fluid velocity along the pipeline is $v = 4Q/\pi D^2 = 3.02\ m/s$, $Re = 59433.6$

and Blasius equation for smooth pipes gives $f = 0.079 Re^{-0.25} = 0.0051$. The prevalence of the pump is (Bernoulli between tank A and B)

$$p_A + \rho g h_A + \Delta p_{pump} = p_B + \rho g h_B + 2f\frac{L_{AB}}{D}\rho v^2 \quad (58)$$

where the assume a negligible velocity of fluid inside the tanks. Being $p_A = p_B$, isolating Δp_{pump} we get

$$\Delta p_{pump} = \rho g(h_B - h_A) + 2f\frac{L_{AB}}{D}\rho v^2 \quad (59)$$

where the first term is the energy per unit volume required to lift the fluid and the second term is the energy loss due to viscous effects. In this case, viscous losses are larger than energy for lift (the fluid velocity along the pipe is quite large) and we get

$$\Delta p_{pump} = 2.41326 cdot 10^5 + 11.25163 \cdot 10^5 = 13.665 \cdot 10^5 Pa \quad (60)$$

The pumping power is $P = Q\Delta p_{pump} = 46.60\ kW$.

2. Opening the valve V, the flow rate along each branch will change. To calculate the flow rates we need to write Bernoulli equation for each branch (A-1, 1-P-2, 2-B e 1-V-2) and the continuity equation for one of the two nodes (1) or (2). The unknowns are flow velocities in each branch (A1, 1V2 e 1P2) and the static pressure at nodes 1 and 2. Since we can not predict which is the flow direction in each branch, we start writing Bernoulli for the branches for which we have mode data. Bernoulli between 1 and 2 (with the pump) is:

$$p_1 + \Delta p_{pump} = p_2 \rightarrow p_2 - p_1 = \Delta p_{pump} > 0 \quad (61)$$

Being larger than 0, we conclude that the flow will be from 2 to 1 in the branch with the valve:

$$p_2 = p_1 + 2f\frac{L_{12}}{D}\rho v_{1V2}^2 = p_1 + kL_{12}v_{1V2}^{1.75} \quad (62)$$

with $k = 91.156$. If the pumping power is the same as before, we can write $\Delta p_{pump} = P/Q_P$ with Q_P the flowrate along the branch with the pump. Equating equations 61 and 62 we can write the flowrates moving in the branches of the loop as:

$$Q_P = \frac{P}{kL_{12}v_{1V2}^{1.75}} = \frac{0.5112}{v_{1V2}^{1.75}} \quad (L = 1000\ m) \quad (63)$$

and

$$Q_{1V2} = \frac{\pi D^2}{4}v_{1V2} = 0.0113 v_{1V2} \quad (64)$$

These flow rates are equal if $v_{1V2} = 4\ m/s$. If v_{1V2} is larger, $Q_{1V2} > Q_P$. Using this information to write the continuity equation at the node we get:

$$Q_{1V2} = Q_P + Q_{B2} \quad \text{if} \quad v_{1V2} > 4\ m/s \quad (65)$$

i.e. the flow rate goes from B to A, whereas

$$Q_P = Q_{1V2} + Q_{2B} \quad \text{if} \quad v_{1V2} < 4\ m/s \quad (66)$$

indicating that the flow rate is transferred from A to B. Supposing $v_{1V2} > 4\ m/s$, the continuity equation gives

$$Q_{B2} = Q_{1V2} - Q_P = \frac{\pi D^2}{4}v_{B2} \rightarrow v_{B2} = f(v_{12}) \quad (67)$$

and Bernoulli equation between B and A along the branch without pump gives

$$\rho g(h_B - h_A) = k(L_{B2} + L_{1A})\rho v_{B2}^{1.75} + kL_{1V2}\rho v_{1V2}^{1.75} \quad (68)$$

The right end side is a function of v_{1V2} only. If this equation can not be solved or calculation gives $v_{1V2} < 4$ we need to rewrite the continuity equation as

$$Q_{B2} = Q_P - Q_{1V2} = \frac{\pi D^2}{4}v_{2B} \rightarrow v_{2B} = f(v_{1V2}) \quad (69)$$

and the Bernoulli equation from A to B along the branch with the pump gives

$$\rho g(h_A - h_B) + \frac{P}{Q_P} = k(L_{B2} + L_{1A})\rho v_{2B}^{1.75} \quad (70)$$

where Q_P and v_{2B} are functions of v_{1V2}. Solving this equation we find the value of v_{1V2}. Solving numerically we get $v_{1V2} = 3.3741\ m/s$, $v_Q = 5.38\ m/s$ and $v_{A1} = 2.01\ m/s$.

When the valve is open enough to induce a lower pressure drop (short equivalent length), the threshold value of velocity v_{1V2} changes and the direction of flow transfer inverts. If $L = 100\ m$:

$$Q_P = \frac{P}{kL_{12}v_{1V2}^{1.75}} = \frac{5.112}{v_{1V2}^{1.75}} \quad (L = 100\ m) \quad (71)$$

and

$$Q_{1V2} = \frac{\pi D^2}{4}v_{1V2} = 0.0113 v_{1V2} \quad (72)$$

Flow rates become equal if $v_{1V2} = 9.2\ m/s$. If v_{1V2} is larger, $Q_{1V2} > Q_P$. From this point on, the procedure is the same as before. When the closure of the valve is reduced, for the same pressure drop between the nodes the flow rate along the branch with the valve increases and the transfer of fluid from B to A becomes possible. For $L_{1V2} = 100\ m$ we get $v_{1V2} = 8.8624\ m/s$, $v_P = 9.93\ m/s$ and $v_{B2} = 1.07\ m/s$.

h.

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1. Given the flowrate and the pipe diameter, the velocity in along branch (a) is known and equal to $v_a = 4Q/\pi D^2 = 2.23\ m/s$. Since $Q_B = Q_C$ and

$D = cost$, fluid velocity in branch (b) and (c) will be $v_b = v_c = v_a/2 = 1.115\ m/s$. Since the fluid is moving at the same velocity along branch (b) and (c) which are at the same height and have different length, the larger pressure loss will be along branch (c). In branch (b) we will use the valve (partially closed) to generate the additional pressure loss required to balance the shorter length of the pipe ($L_{eq} = 7\ km$).

Bernoulli equation written from points A and C gives:

$$p_A+\frac{1}{2}\rho v_A^2+\rho g h_A+\Delta p_{pump} = p_C+\frac{1}{2}\rho v_C^2+\rho g h_C+2f_{AN}\frac{L_{AN}}{D}\rho v_{AN}^2+2f_{NC}\frac{L_{NC}}{D}\rho v_{NC}^2 \quad (73)$$

which can be simplified considering that (i) the velocity is negligible and (ii) the pressure is the same in the tanks. Using Blasius equation to calculate the friction factor we get $f_{AN} = 0.0032$, $f_{NC} = 0.0039$ and viscous losses equal to $l_{v,AN} = 18.67 \cdot 10^5 Pa$ and $l_{v,NC} = 7.11 \cdot 10^5 Pa$ from which the prevalence of the pump is calculated as $\Delta p_{pump} = 25.78 \cdot 10^5 Pa$ and the pumping power is $P = 146.477\ kW$.

2. Indicating D_1 as the optimum diameter for branch (a) D_2 as the optimum diameter in branches (b) and (c). the prevalence of the pump is calculated writing Bernoulli between A and C:

$$\Delta p_{pump} = 2f_{AN}\frac{L_{AN}}{D_1}\rho v_{AN}^2 + 2f_{NC}\frac{L_{NC}}{D_2}\rho v_{NC}^2 \quad (74)$$

which can be rewritten as

$$\Delta p_{pump} = \frac{K_A}{D_1^{4.75}} + \frac{K_C}{D_2^{4.75}} \quad (75)$$

where $K_A = 549.72$ and $K_C = 204.29$. The pumping power is $P = Q\Delta p_{pump}$. Annual costs are given by (1) investment costs for piping:

$$C_T = \frac{k_t(D_1 L_{AN} + D_2(L_{NB} + L_{NC}))}{N_y} \quad (76)$$

(2) investment and operative costs for the pumping station

$$C_P + C_E = \left(k_E N_h + \frac{k_p}{N_y}\right)\frac{Q}{10^3}\left(\frac{K_A}{D_1^{4.75}} + \frac{K_C}{D_2^{4.75}}\right) \quad (77)$$

The total costs is a function of both D_1 and D_2. The minimum of this function can be calculated deriving $C_T+C_P+C_E$ with respect to each variable and equating the derivatives to zero:

$$\frac{\mathrm{d}C_{TOT}}{\mathrm{d}D_1} = \frac{k_t L_{AN}}{N_y}+\left(k_E N_h + \frac{k_p}{N_y}\right)\frac{Q}{10^3}K_A(-4.75)D_1^{-5.75} = 0 \quad (78)$$

$$\frac{\mathrm{d}C_{TOT}}{\mathrm{d}D_2} = \frac{k_t(L_{NC} + L_{NB})}{N_y}+\left(k_E N_h + \frac{k_p}{N_y}\right)\frac{Q}{10^3}K_B(-4.75)D_2^{-5.75} = 0 \quad (79)$$

Calculations give $D_1 = 0.206\ m$ and $D_2 = 0.131\ m$.

i.

1. he the valve V is closed, the flow moves from the pressurized tank to tank B. The volumetric flow rate is $Q = w/\rho = 9.5 \cdot 10^{-3}\ m^3/s$ and the fluid velocity in the pipe is $v = 4Q/\pi D^2 = 1.21\ m/s$. Using Blasius equation to calculate the friction factor we get $f = 0.0042$. Bernoulli equation written between points A and B gives:

$$p_0 + \rho g h_A = p_{atm} + 2f\frac{L_1 + L_2}{D}\rho v^2 \quad (80)$$

from which we can calculate $p_0 = 1.147 \cdot 10^5\ Pa$.

2. From mass conservation at node N we get:

$$Q_{AN} = Q_{NC} + Q_{NB} = \frac{w_B + w_c}{\rho} = 0.0175\ m^3/s \quad (81)$$

and along branch N-B the volumetric flow rate and velocity are the same as before. Bernoulli between points N-B gives the value of pressure at node N

$$p_N = p_{atm} + 2f\frac{L_2}{D}\rho v_{NB}^2 = 1.1969 \cdot 10^5\ Pa \quad (82)$$

Bernoulli between N-C allows to calculate the pipe diameter

$$p_N = p_{atm} + \rho g h_c + 2f\frac{h_c}{D_{NC}}\rho v_{NC}^2 = \quad (83)$$

$$= p_{atm} + \rho g h_c + 20.079\left(\frac{4Q\rho}{\pi\mu}\right)^{-0.25}\left(\frac{4Q}{\pi}\right)^2 h_c \rho D_{NC}^{-4.75} \quad (84)$$

which is $D_{NC} = 0.122\ m$. Along branch A-N we know the volumetric flow rate $Q = 17.5 \cdot 10^{-3}\ m^3/s$ and the velocity in the pipe $v = 4Q/\pi D^2 = 2.228\ m/s$. Using Blasius equation to calculate the friction factor we get $f = 0.0036$. Bernoulli between A-N allows to calculate the pressure inside the tank:

$$p_0 + \rho g h_A = p_N + 2f\frac{L_1}{D}\rho v_{AN}^2 \quad (85)$$

which is $p_0 = 1.24 \cdot 10^5\ Pa$.

j.

1. Given the initial level of liquid inside the tank, h_0, and the height of the overflow discharging valve, $\bar{h}$, the volume to be filled is:

$$V = \frac{\pi D^2}{4}\left(\bar{h} - h_0\right) = 5969 m^3 \quad (86)$$

Until the overflow valve is inactive, the mass conservation for the liquid in the tank gives

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \rho S\frac{\mathrm{d}h}{\mathrm{d}t} = \rho Q_{in} \quad (87)$$

where S is the tank cross section. By integration we get:

$$h(t) = h(0) + \frac{Q_{in}}{S}t \tag{88}$$

and

$$t = \frac{S(h(t) - h(o))}{Q_{in}} = \simeq 3h\ 18\ min \tag{89}$$

2. When the overflow valve is active, the mass conservation becomes

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \rho S\frac{\mathrm{d}h}{\mathrm{d}t} = \rho Q_{in} - \rho Q_{out} \tag{90}$$

and dividing by the density of the fluid

$$S\frac{\mathrm{d}h}{\mathrm{d}t} = Q_0 - Q_{out} \tag{91}$$

The level is maximum when $\mathrm{d}h/\mathrm{d}t = 0$ i.e. $Q_0 = Q_{out}$. Q_{out} is the flow rate exiting from the overflow valve which depends on the liquid level above the overflow. Bernoulli written between the free surface of the tank and the flow outlet section gives:

$$p_{atm} + \rho g h' = p_{atm} + \frac{1}{2}\rho v_{out}^2 \tag{92}$$

and

$$v_{out} = \sqrt{2gh'} \tag{93}$$

which is the efflux velocity (Torricelli). The outgoing flowrate is

$$Q_{out} = f(h', d) = A_{sfioro}v_{out} = \frac{\pi d^2}{4}\sqrt{2gh'} \tag{94}$$

which can be equated to Q_0 to give

$$h' = 0.0207\frac{1}{d^4} \tag{95}$$

3. Given the basin diameter, the maximum distance at which the radial jet can move to is $r = (D_m - D)/2 = 10\ m$. Assuming that the air friction on the jet can be neglected, the radial distance reached by the jet depends on the fluid velocity at the outlet section and by the time of flight, which is the time spent before reaching the ground. The jet velocity is maximum when the liquid level above the overflow valve is maximum, i.e. at equilibrium when $h = h'$ and $v_{out} = 4Q_0/\pi d^2$. Jet moves at constant velocity in the radial direction:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = v_{out} \tag{96}$$

and accelerating at a constant rate in the vertical direction:

$$\frac{\mathrm{d}z}{\mathrm{d}t} = gt \tag{97}$$

From the vertical component we get the time of flight:

$$\overline{h} = \frac{1}{2}gt_{max}^2 \to t_{max} = \sqrt{2overlineh/g} \tag{98}$$

which depends only on the jet outlet height. Using this value in 96 we get:

$$r = v_{out}t_{max} = 4Q_0/\pi d^2\sqrt{2\overline{h}/g} \tag{99}$$

from which we can derive d

$$d = \sqrt{\frac{4Q_0}{\pi r}\sqrt{\frac{2\overline{h}}{g}}} = 0.358\ m \tag{100}$$

This is the (minimum) diameter producing a jet velocity sufficiently high to reach the wall of the basin. Choosing any larger diameter, the jet be contained by the basin. For this value, $h' = 1.25\ m$ and the equilibrium height in the tank is $\overline{h} + h' = 21.25\ m$. To calculate the time required to fill the volume between the overflow discharge valve and the alarm level $h* = 21\ m$ we use the mass conservation equation for the tank

$$\frac{\pi D^2}{4}\frac{dh}{dt} = Q_0 - Q_{out} \tag{101}$$

where:

$$Q_{out} = \frac{\pi d^2}{4}\sqrt{2g(h - \overline{h})} \tag{102}$$

and separating the variables

$$\frac{\mathrm{d}h}{\frac{4Q}{\pi D^2} - \frac{d^2}{D^2}\sqrt{2g(h - \overline{h})}} = \mathrm{d}t \tag{103}$$

which can be integrated between $\overline{h}$ and $h*$. Grouping and substituting the constant values we get

$$\frac{\mathrm{d}h}{a - b\sqrt{h - \overline{h}}} = \mathrm{d}t \tag{104}$$

which can be integrated by variable substitution

$$y = a - b\sqrt{h - \overline{h}} \tag{105}$$

$$\mathrm{d}y = -\frac{b\mathrm{d}h}{\sqrt{h - \overline{h}}} \tag{106}$$

to get

$$\mathrm{d}h = -\frac{2(a - y)}{b^2}\mathrm{d}y \tag{107}$$

Finally, integrating we get

$$-\frac{2(a - y)}{b^2}\frac{\mathrm{d}y}{y} = -\frac{2}{b^2}\left(\frac{a}{y} - 1\right)\mathrm{d}y = \mathrm{d}t \tag{108}$$

and

$$t = -\frac{2}{b^2}\left(a\ln\frac{y(t)}{y(0)} - (y(t) - y(0))\right) \tag{109}$$

from which $t = 1858\ s \simeq 31\ min$.

k.

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1. To solve the problem we need to write the mass conservation for each phase. For air:

$$\frac{\mathrm{d}m_G}{\mathrm{d}t} = 0 \quad \rightarrow m_G = nM = cost \tag{110}$$

because the gas is an isolated system (no mass exchange through the volume boundaries). From ideal gas law

$$pV_G = nRT \rightarrow n = \frac{pV_G}{RT} = 12.89\ moli \tag{111}$$

where V_G is the volume of the gas inside the tank, $V_G = \pi D^2 (H - H_L)/4$, The mass of gas is $m_G = nM = 373.99\ kg$. When the oil starts flowing, the volume of oil inside the tank decreases whereas the volume available for the as increases. From mass conservation

$$p(0)V_G(0) = p(t)V_G(t) \tag{112}$$

$$\rightarrow p(0)(H - H_L(0)) = p(t)(H - H_L(t)) \tag{113}$$

and

$$p(t) = p(0)\frac{(H - H_L(0))}{(H - H_L(t))} \tag{114}$$

Mass conservation for the oil gives

$$\frac{\mathrm{d}m_L}{\mathrm{d}t} = \rho_L \frac{\pi D^2}{4}\frac{\mathrm{d}H_L}{\mathrm{d}t} = -\rho_L \frac{\pi d^2}{4} v_{out} \tag{115}$$

where d is the pipe diameter and v_{out} is given by Bernoulli equation written between a point in the tank and the outflow section of the pipe:

$$p(t) + \rho_L g H_L(t) = p_{atm} + \frac{1}{2}\rho_L v_{out}^2 + 2f\frac{L}{d}\rho_L v_{out}^2 \tag{116}$$

When the oil stops flowing, its velocity is $v_{out} = 0$, and using Bernoulli

$$p(t) = p_{atm} - \rho_L g H_L(t) \tag{117}$$

which can be used to get

$$p(0)\frac{(H - H_L(0))}{(H - H_L(t))} = p_{atm} - \rho_L g H_L(t) \tag{118}$$

This is a quadratic equation in H_L which can be solved to calculate the value of the equilibrium height $H_L = 4.124\ m$.

2. To calculate the time required to reach the equilibrium we need to integrate the mass conservation equation, using for v_{out} the relationship obtained from Bernoulli equation. Assuming that the value of the friction factor f is known, we get:

$$v_{out} \left(\frac{p(t) + \rho_L g H_L(t) - p_{atm}}{\frac{1}{2}\rho_L + 2f\frac{L}{d}\rho_L} \right)^{0.5} \tag{119}$$

where using for $p(t)$ the value given by 114 we get $v_{out}(t) = f(H_L(t))$. From the mass conservation for the liquid, by variables separation and numerical integration we get the function describing the evolution of the liquid level in time. Since the stopping condition is an equilibrium state, this will be reached asymptotically in infinite time.