Homework N^o 2: storage/transport of compressible fluids

a.

In a spray painting plant, N_2 ($M = 28\ kg/kmol$, $\mu = 1.78 \cdot 10^{-5} Pa \cdot s$) is used as atomizing gas for the spray painting guns. The storage tank A (volume $V_A = 5\ m^3$) of the department is periodically filled withdrawing the gas from an different storage tank (B) of larger volume maintained at $P_B = 25\ atm$ (constant). The line connecting tanks B and A is $L = 50\ m$ long and pipe diameter is $D = 0.05\ m$.

1. Calculate the gas flow rate transferred along the line when the shut-off valve between the two tanks is opened (initial pressure in tank A equal to atmospheric pressure); hypothesize isothermal transformations for the gas ($T = 293\ K$) and friction factor equal to $f = 0.003$;
2. Calculate the mass transferred from B to A to load the tank up to a pressure $P_A = 15\ atm$;
3. If the valve between B and A is left open, the pressure in tank A can rise up to a maximum of 20 atm before a rupture disk ($d = 2.5\ cm$) breaks. Calculate whether, in the event of disk breakage, the outgoing flow of nitrogen is sonic.
4. Determine how long the outgoing flow remains sonic if nitrogen supply from reservoir B is interrupted.

b.

A gas tank (volume $V = 40\ m^3$) contains ethylene (C_2H_4, $M = 28\ kg/kmol$, viscosity $\mu = 1.1 \cdot 10^{-4} Pa \cdot s$) initially at pressure $p_0 = 30 \cdot 10^5 Pa$ and temperature $T = 200\ K$ and connected through a long pipe ($L = 1500\ m$, diameter $d = 0.2\ m$) to a reactor operating at atmospheric pressure.

1. calculate the flow rate fed to the reactor when the valve placed at the pipe inlet is opened (assume isothermal transformations and $f = 0.003$);
2. calculate the time needed to halve the pressure in the tank;
3. determine the mass of gas discharged from the tank up to that time.

c.

A gas storage tank (temperature $T = 20^oC$ and initial pressure $p_0 = 10\ atm$) contains methane ($MM = 16\ kg/kmol$). The tank volume is 10 m^3. The tank is equipped with a vent valve V_1 ($d = 2\ cm$) and is connected through a valve V_2 to a pipe ($L = 250\ m$, $D = 2.5\ cm$) with the second end open to the atmosphere. At starting time the valve V_1 is open and the valve V_2 is closed. Considering isothermal transformation for the gas:

1. check if the flow of methane exiting from valve V_1 is sonic;
2. after 40. s, valve V_1 is closed and valve V_2 is opened. Calculate the mass of gas discharged from the tank up to that moment;
3. calculate the mass flow rate of the gas moving along the pipeline (assume $f = 0.003$).

d.

A tank (volume $V = 10\ m^3$, $T = 293\ K$) is filled by a constant mass flow rate $w_{in} = 2.50\ kg/s$ of natural gas (molar mass $M = 16\ kg/kmol$, $\mu = 1.8 \cdot 10^{-5} Pa$, $\gamma = 1.3$) and delivers (isothermally) gas through a pipeline (diameter $D = 0.1\ m$, length $L = 800\ m$) to a burner operating at atmospheric pressure.

1. Determine the value of pressure in the tank at steady state working conditions;
2. During periodic maintenance operations of the burner, the valve connecting the tank to the burner is closed whereas the tank continues to be filled, increasing its storage pressure. A safety valve prevents pressure rising above 20 atm. Calculate the time elapsed before the opening of the safety valve;
3. Calculate the specific flow rate of gas exiting from the valve when it opens (assume adiabatic outflow from the valve).

e.

A natural gas pipeline ($M_{CH_4} = 16\ kg/kmol$, $R = 8314\ J/kmolK$) is composed of segments of pipe diameter ($D = 0.3\ m$, $L = 4\ km$) interconnected by recompressing stations. Assuming isothermal flow (293 K) and friction coefficient equal to $f = 0.003$),

1. calculate the pressure at which each recompressing station should compress the gas to guarantee a flow rate equal to $Q = 35\ m^3/s$ evaluated at room temperature 293 K and pressure $1. \cdot 10^5\ Pa$ if the pressure along the pipeline should never fall below the value of $1.5 \cdot 10^5\ Pa$;
2. calculate the power of the compressor (assume $\eta = 1$);
3. Due to an accident, the tube is cut immediately upstream the compressor inlet. Calculate the mass flow rate of gas discharged and the pressure at the broken section ($A = 3\ cm^2$).

f.

In a cylindrical tank (volume $V = 10\ m^3$) contains a chemical reagent ($M = 24\ kg/kmol$, $\gamma = 1.4$) at initial pressure $p_i = 2\ atm$ and temperature $T = 300\ K$. Due to a chain reaction, an increase in the number of moles is produced inside the tank according to the law:

$$\dot{n}(t) = \dot{n}_0 \exp[kt] \tag{1}$$

with $k = 0.1\ s^{-1}$ and $\dot{n}_0 = 0.1\ mol/s$. When the pressure in the tank rises up to 15 atm, a safety valve (section $A = 5\ cm^2$) opens discharging the gas into the atmosphere.

1. calculate the time after which the safety valve opens (consider isothermal transformations for the gas inside the tank);
2. calculate the variation of pressure inside the tank after the opening of the safety valve (hypothesize adiabatic outflow of gas from the tank).

g.

A storage tank (volume $V = 10\ m^3$, temperature $T = 20^oC$, initial pressure $p_0 = 10\ atm$) contains methane ($MM = 16\ kg/kmol$). For accidental reasons, a small hole of diameter $D = 2\ cm$, is produced in the tank wall through which the methane is free to escape. Assuming adiabatic transformations for the gas ($\gamma = 1.33$),

1. check if the outgoing flow of methane is sonic;
2. calculate the specific flow rate of the outgoing gas at starting time;
3. calculate the time t necessary in order to reduce the pressure inside the tank up to 3 atm;
4. calculate the total mass of gas discharged from tank in time t.

h.

A pipe (diameter $D = 5\ cm$ and length $L = 300\ m$) is used to deliver a mass flow rate of oxygen ($MM = 32\ kg/kmol$, $\mu = 1.8 \cdot 10^{-5} Pa \cdot s$) $w = 0.80\ kg/s$ from tank A to tank B.

1. If tank B is maintained at atmospheric pressure, the tube is smooth and all the gas transformations are isothermal ($T = 298\ K$), check is the flow discharged in tank B is sonic.
2. Calculate the value of pressure in tank A to supply the design flow rate to tank B.
3. Calculate the flow rate of gas exiting from tank A if a hole of $d = 2\ cm$ is produced for accidental reasons in the wall of the tank.

i.

Gas is transferred from tank A to tank B using a long pipe (length L) tilted upward of α degrees from the horizontal. The pressures p_A and p_B are both much greater than the atmospheric pressure, such that the density of the gas is high. Assuming that the transport from A to B is isothermal, calculate, if p_A and p_B are given, the flow rate of gas transferred between the two tanks

1. if frictional losses are negligible compared to gravitational losses ($\rho g h$);
2. if frictional losses are comparable to gravitational losses.

j.

A pipe (diameter $D = 0.1\ m$, $L = 800\ m$ long) is used to extract natural gas ($M = 16\ kg/kmol$, $\mu = 1.8 \cdot 10^{-5} Pa \cdot s$) from a well ($T = 293\ K$, $V = 100000\ m^3$, $p_o = 25 \cdot 10^5 Pa$).

1. Assuming isothermal transformations for the gas, calculate the mass flow rate of gas extracted from the well at starting time (environmental pressure is atmospheric).
2. To transport the gas to the storage tank it is enough to have a pressure equal to $1.5 \cdot 10^5 Pa$ at the end of of the pipe. The pipe is equipped with a laminarization valve to regulate the flow. Calculate the time to exhaustion of the well if the flow rate of gas extracted is maintained equal to $G = 180\ kg/m^2s$.

k.

A cylindrical tank (volume 10 m^3) containing chlorine gas ($R = 8314\ J/kgK$, $M = 34\ kg/kmol$, $\mu = 1.4 \cdot 10^{-4} Pa \cdot s$) at pressure $p = 1\ MPa$ is connected through a horizontal duct (diameter $d = 0.05\ m$, length $L = 50\ m$) to the atmosphere. Assuming isothermal transformation for the gas ($T = 293\ K$)

1. Calculate the time during which the outflow remains critical;
2. Calculate the time during which the flow would remain critical if the tank is discharging gas directly into the atmosphere (through a pipe of negligible length having a diameter $d = 0.05\ m$).