Pressure drop through particle beds/porous media

a.

Consider a pipe filled with packing material and a fluid flow (gas or liquid) through it. The objective is to derive design equations to calculate the pressure drop as a function of fluid flow rate and packing characteristics.

Figure 1 shows a sketch of Darcy experiment, in which he measured the pressure drop variation at small flow rates for different packings.

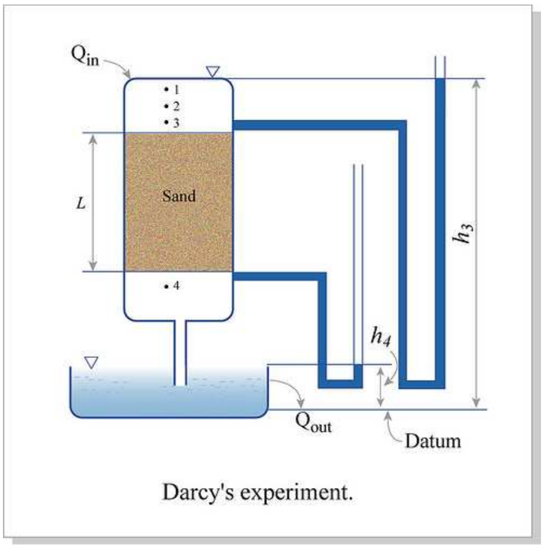

Figura 1. Sketch of Darcy experiment.

He derived a relationship between pressure drop, bed thickness and flow rate of the type:

$$Q = A\frac{\Delta p}{L}K \tag{1}$$

where K is costant of proportionality, expressed in $[m^2/Pa \cdot s]$. The linear relationship between flow rate and pressure drop suggests that viscous effects dominate and that it should be possible to separate the effect of fluid viscosity μ from packing characteristics which are jointly summarized by K. Moving into more general variables we can define the superficial velocity $v = Q/A$ where A is the cross section of the bed, and define a different proportionality constant B ($K = B/\mu$) which depends only on properties of the bed material:

$$v = \frac{\Delta p}{L}\frac{B}{\mu} \tag{2}$$

B represents the permeability of the medium and is expressed in $[m^2]$. The typical range of variation for B is $10^{-10} \div 10^{-8} m^2]$. A large value of permeability B indicates that is rather easy for the flow to pass through the medium (a large velocity/flow rate is produced at the expense of a small pressure drop).

B is a function of two main packing material/bed properties: the specific surface area, which depends on the size and shape of packing material, and the packing density. The pressure drop is due to friction which is produced at the surface of the packing. We can define the specific surface of the packing material S as the ratio between the surface, A_{sup}, and the volume, V, of the unit object of packing. For spherical packing with diameter d:

$$S = \frac{A_{sup}}{V} = \frac{\pi d^2}{\pi d^3/6} = \frac{6}{d} \tag{3}$$

and for cubical packing with size d

$$S = \frac{6d^2}{d^3} = \frac{6}{d} \tag{4}$$

Whichever the shape of the particle/object costituting the bed, S depends on geometry and can be used to identify a characteristic size (units of S are $m^2/m^3 = 1/m$), $d \propto 1/S$. For packing material of complex shape, we can define the specific surface area as:

$$S = \frac{6}{\phi d_{eq}} \tag{5}$$

where ϕ and d_{eq} are the sphericity and the characteristic size of the packing. The characteristic size is the diameter of the sphere having the same volume of the packing:

$$d_{eq} = \left(\frac{6 \cdot V}{\pi}\right)^{1/3} \tag{6}$$

whereas the sphericity is defined as the ratio between the surface area of packing, A_{sup} and the surface area of the sphere of size d_{eq}:

$$\phi = \frac{A_{sup}}{\pi d_{eq}^2} \tag{7}$$

From the definition of S we have:

$$S = \frac{A_{sup}}{V} = \frac{\phi \pi d_{eq}^2}{\pi d_{eq}^3/6} = \frac{6}{\phi d_{eq}} \tag{8}$$

We can define also the specific surface of the bed S_B as the ratio of the surface offered by the packing divided by the volume of the bed, V_{bed}:

$$S_B = \frac{\text{Surface of packing}}{\text{Volume of bed}} = \frac{S \cdot V_{bed}(1-\epsilon)}{V_{bed}} \tag{9}$$

where ϵ is the void fraction, i.e. the fraction of V_{bed} not filled by the packing material and the space left for fluid flow. The way the single particles/objects constituting the beds are packed determines the packing density which can be measured as the space left for the fluid in a reference volume of bed, V_{bed}:

$$\epsilon = \frac{V_{empty}}{V_{bed}} \quad . \tag{10}$$

For regular packing, ϵ can be calculated based on geometrical considerations: the packing of spherical objects of diameter d arranged in a cubic pattern is given by:

$$\epsilon_{cubic} = \frac{d^3 - \pi d^3/6}{d^3} = 1 - \pi/6 = 0.476 \tag{11}$$

whereas for the same objects arranged in a tetrahedric pattern is $0.26 = \epsilon_{tetrahedron} < \epsilon_{cubic}$. In most of packed beds/porous material, packing elements are are not of the same size and shape and the packing is random. Therefore, both S_B and ϵ are obtained from experimental measurements. For example, the void fraction can be obtained by infiltrometric tests measuring the volume of liquid necessary to fill the empty space of the solid bed; the specific surface can be measured by adsorption tests, evaluating the surface exposed to the flow by amount of substance which can be absorbed onto it. In general, $\epsilon = 0.3 \div 0.5$ for traditional packings and can rise up to $0.8 \div 0.9$ for new generations packing and foams. The larger is ϵ, the lower is the resistance of flow through the bed.

The pressure drop across a bed can be calculated similarly to the pressure drop along a pipe if we consider the flow channelling along the series of inter-connected voids encountered in the flow direction. Indicating by d' the diameter of the pipe and by l' the length of the pipe, we can express the pressure drop as:

$$\Delta p = 2f \frac{l'}{d'} \rho v'^2 \tag{12}$$

where v' is the fluid velocity inside the pore channels of the bed, d' is the characteristic size of these channels and $f = 16/Re$ is the friction factor ($Re = v'd'\rho/\mu$). This relationship gives

$$v' = \frac{d'^2}{32\mu} \frac{\Delta p}{l'} \tag{13}$$

We can define d' as the hydraulic diameter

$$d_H = \frac{4A'}{p'} = \frac{A' \cdot l'}{p' \cdot l'} = \frac{V\epsilon}{V \cdot S_B} = \frac{\epsilon}{(1-\epsilon)S} \tag{14}$$

were $A' = A \cdot \epsilon$ is the bed cross section area available for fluid flow and p' is the wetted perimeter which is directly related to the specific surface area of the bed, S_B. Based on Equation 14 we can rewrite the relationship between velocity and pressure drop as:

$$v' = \frac{v}{\epsilon} = \frac{\epsilon^2}{(1-\epsilon)^2 S^2} \frac{1}{32\mu} \frac{\Delta p}{l'} = \frac{\epsilon^2}{(1-\epsilon)^2 S^2} \frac{1}{32\mu} \frac{\Delta p}{L} \tag{15}$$

where v is the superficial flow velocity through the bed, i.e. the flowrate divided by the cross section area of the bed and $l' \simeq L$. The same equation can be recasted in a more general form:

$$v = \frac{\epsilon^3}{(1-\epsilon)^2 S^2} \frac{1}{K\mu} \frac{\Delta p}{L} \qquad \text{(Karman law)} \tag{16}$$

where K depends on the shape of the irregular channel along the bed ($K = 5$ is the Kozeny constant derived from experiments).

Karman law is valid for laminar flow through the bed, when viscous force prevails against inertia. We can define the Reynolds number as:

$$Re' = \frac{v'd'\rho}{\mu} = \frac{v}{\epsilon} \frac{\epsilon}{S(1-\epsilon)} \frac{\rho}{\mu} = \frac{v\rho}{S(1-\epsilon)\mu} \tag{17}$$

Following the same approach used to derive the functional expression for the friction factor in pipes, we can relate the friction force at the wall of the tortuous channel R' to the pressure drop writing a force balance equation:

$$\Delta p[\epsilon A] = R'[SL(1-\epsilon)A] \tag{18}$$

where the terms in brackets in the left and right side of the equation represent the fraction of the bed section area available for the flow and the overall surface of the walls of the tortous channel. From this equation:

$$R' = \frac{\Delta p}{L} \frac{\epsilon}{(1-\epsilon)} \frac{1}{S} \tag{19}$$

This quantity can be made dimensionless using the dynamic pressure term $\rho v'^2$:

$$f = \frac{R'}{\rho v'^2} = \frac{\Delta p}{L} \frac{\epsilon}{(1-\epsilon)} \frac{1}{S} \frac{\epsilon^2}{\rho v^2} \tag{20}$$

and based on the Buckingham theorem, it can be expressed as a function of a single dimensionless parameter,

$$Re' = \frac{\rho v' d'}{\mu} = \frac{v}{\epsilon} \frac{\epsilon}{(1-\epsilon)S} \frac{\rho}{\mu} \tag{21}$$

From the fitting of experimental data made dimensionless as described above we get:

$$f = 5Re'^{-1} + 0.4Re'^{-0.1} \tag{22}$$

The first term of the friction factor dominates at low Reynolds number (leading to the Carman-Kozeny equation); the second term of the friction factor dominates at larger Reynolds numers (leading to Burke-Plummer equation). The pressure drop across the bed can be calculated as:

$$\frac{\Delta p}{L} = f \frac{(1-\epsilon)}{\epsilon^3} S\rho v^2 \tag{23}$$

which using f from Equation 22 gives:

$$\frac{\Delta p}{L} = 5\mu \frac{(1-\epsilon)^2}{\epsilon^3} vS^2 + 0.4Re^{-0.1} S\rho v^2 \frac{(1-\epsilon)}{\epsilon^3} \tag{24}$$

and for spherical packings $S = 6/d$ becomes:

$$\frac{\Delta p}{L} = 180\mu \frac{(1-\epsilon)^2}{\epsilon^3 d^2} v + 2.4Re^{-0.1} \rho v^2 \frac{(1-\epsilon)}{\epsilon^3 d} \tag{25}$$

which is of the same form of the Ergun equation

$$\frac{\Delta p}{L} = 150 \frac{(1-\epsilon)^2}{\epsilon^3} \frac{\mu v}{d^2} + 1.75 \frac{(1-\epsilon)}{\epsilon^3} \frac{\rho v^2}{d} \quad \text{Ergun equation} \tag{26}$$